Second Year
Electric and Magnetic Fields ELC 205A
Fall 2018

Quiz 2

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Model Answer

Write your final answers in the boxes found at the end of each question.
Zero points assigned to empty boxes.
Only the answers inside the boxes will be considered,

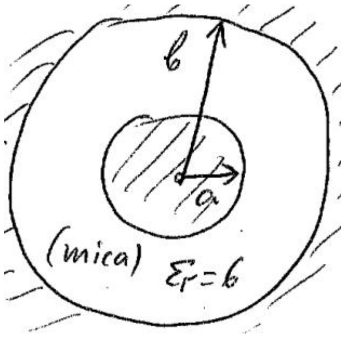
Total number of points in this Exam 10 points.

Question 1 (5 points)

A spherical capacitor is made of two concentric spherical shells of radii $a = 1$ cm and $b = 2$ cm.

The insulator is mica of relative permittivity $\varepsilon_r = 6$.

$a = 1$ cm
$b = 2$ cm



1) At what value of r (the distance from the center of the spherical shells) is the magnitude of the electric field $|\mathbf{E}(r)|$ maximum?

A) at r = a = 1 cm
B) at r = b = 2 cm
C) at r = 0
D) at r = 1.5 cm
E) at r = 2.5 cm

(1 point; 0.5 point for steps & 0.5 point for final answer)

From Gauss law, $\vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{r^2}\hat{a}_r$

Therefore, $|\vec{E}|$ is max when $r = a = 1$ cm

$$|\vec{E}|_{max} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{a^2}$$

2) What is the breakdown voltage of the capacitor if the dielectric strength of mica is $E_{ds} = 200$ MV/m?

A) 0.55 MV
B) 1.45 MV
C) 1 MV
D) -1.45 MV
E) -1 MV

(1 point; 0.5 point for steps & 0.5 point for final answer)

$$V_{ab} = \int_a^b E_r\,dr = \int_a^b \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{r^2}\,dr$$

$$V_{ab} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(-\frac{1}{r}\right)\Big|_a^b = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$|\vec{E}|_{max} = E_{ds} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{a^2}$$

$$\Rightarrow \frac{Q}{4\pi\varepsilon_0\varepsilon_r} = E_{ds}a^2 = 2\times10^8\times10^{-4} = 2\times10^4 \text{, V.m}$$

$$\Rightarrow V_{ab} = 2\times10^4\left(\frac{1}{0.01} - \frac{1}{0.02}\right) = 2\times10^4(100-50)$$

$$\Rightarrow V_{ab} = V_{max} = 2\times50\times10^4 = 10^6 \text{ V} = 1 \text{ MV}$$

3) If the capacitor is biased at 100 V, find the electric field vector **E**, the electric density vector **D** at the surface of the inner sphere (i.e., at r = a).

A) 0.6 $\underline{u}_r$ KV/m, 111.23 $\underline{u}_r$ nC/m^2 C) 20 $\underline{u}_r$ KV/m, 106.25 $\underline{u}_r$ μC/m^2

B) 2 $\underline{u}_r$ V/m, 0.10625 $\underline{u}_r$ nC/m^2 D) 20 $\underline{u}_r$ KV/m, 1.0625 $\underline{u}_r$ μC/m^2

E) None of the above

(1 point; 0.5 point for steps & 0.5 point for final answer)

$$V_{ab} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) = 100$$

$$\Rightarrow \frac{Q}{4\pi\varepsilon_0\varepsilon_r} = \frac{100}{\frac{1}{0.01} - \frac{1}{0.02}} = \frac{100}{100-50} = 2$$

$$\Rightarrow \vec{E} = \frac{2}{r^2}\hat{a}_r \text{ V/m}; \quad \vec{D} = \varepsilon_0\varepsilon_r\vec{E} = \varepsilon_0 6\times\frac{2}{r^2}\hat{a}_r$$

$$\Rightarrow \vec{D} \approx \frac{1.0625}{r^2}\times10^{-10}\hat{a}_r \text{ C/m}^2$$

$$\vec{E}(r=a) = \hat{a}_r\frac{2}{10^{-4}} = 20000\,\hat{a}_r \text{ V/m}$$

$$\vec{D}(r=a) \approx 1.0625\times10^{-6}\hat{a}_r \text{ C/m}^2$$

4) Find the free surface charge density (ρ_{sf}) at the surface of the inner sphere (i.e., at r = a).

A) 111.23 nC/m^2
B) 0.10625 nC/m^2
C) 1.0625 µC/m^2
D) - 0.10625 nC/m^2
E) -1.0625 µC/m^2
F) None of the above

(0.5 point for final answer)

$\rho_{sf} = D_N (r=a) \approx 1.0625 \times 10^{-6}\ C/m^2$

5) Find the capacitance C of the capacitor.

A) 1 pF
B) 10.92 pF
C) 15.423 pF
D) 3.33 pF
E) None of the above

(1.5 points; 0.5 point for steps, & 1 point for the choice and the correct final answer)

$Q = \rho_{sf} \times 4\pi a^2 = 1.0625 \times 10^{-6} \times 4\pi \times 10^{-4}$

$\Rightarrow Q \approx 13.35177 \times 10^{-10}\ C$

$C = \frac{Q}{V_{ab}} = \frac{13.35177 \times 10^{-10}}{100} \approx 13\ pF$

Question 2 (Each requirement worths 1 point; 0.5 point for steps & 0.5 point for final answer) (5 points)

The magnetic field is given as $\underline{H} = \underline{u}_{\varphi}\ 20\rho\ A/m$ in the volume of a resistor shaped as a cylindrical rod whose axis is aligned with the z axis. The resistive region has specific conductivity $\sigma = 5\ S/m$. The resistor's electrodes are circular of radius $a = 1\ mm$ and are located at $z = -3\ mm$ and $z = 3\ mm$, respectively.

1) Find the expression for the current density $\underline{J}$ inside the resistor.

2) Find the expression for the electric field $\underline{E}$ inside the resistor.

3) Find the total current I through the surface defined by $0 \le \rho \le a$, $0 \le \varphi \le 2\pi$, $z = 0$.

4) Find the voltage between the two electrodes.

5) Find the resistance R of the resistor.

$$\vec{J} = \nabla \times \vec{H} = \frac{1}{\rho} \frac{\partial (\rho H_\phi)}{\partial \rho} \hat{a}_z$$

$$\vec{J} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (20\rho^2) \hat{a}_z = \frac{1}{\rho} 40\rho \hat{a}_z$$

$$\vec{J} = \boxed{40 \hat{a}_z \ , \ A/m^2}$$

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{40 \hat{a}_z}{5} = \boxed{8 \hat{a}_z \ , \ V/m}$$

$$I = \iint \vec{J} \cdot d\vec{s} = J \times \pi a^2 = 40\pi \times 10^{-6}$$

$$I \approx \boxed{125.7 \times 10^{-6} \ , \ A}$$

$$V = E \times L = 8 \times 6 \times 10^{-3} = \boxed{48 \ mV}$$

$$R = \frac{V}{I} = \frac{48 \times 10^{-3}}{0.125664 \times 10^{-3}} \approx \boxed{382 \ \Omega}$$