ELC N205: Electromagnetics 1 Tutorials

Department of Communications and Computer Engineering

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Agenda

- Infinite Transmission Line
- Finite Transmission Line
- Transmission Line (input) Impedance
- Power calculations
- Transmission Line measurement

Infinite Transmission Line

Only forward voltage and current waves exit over the line

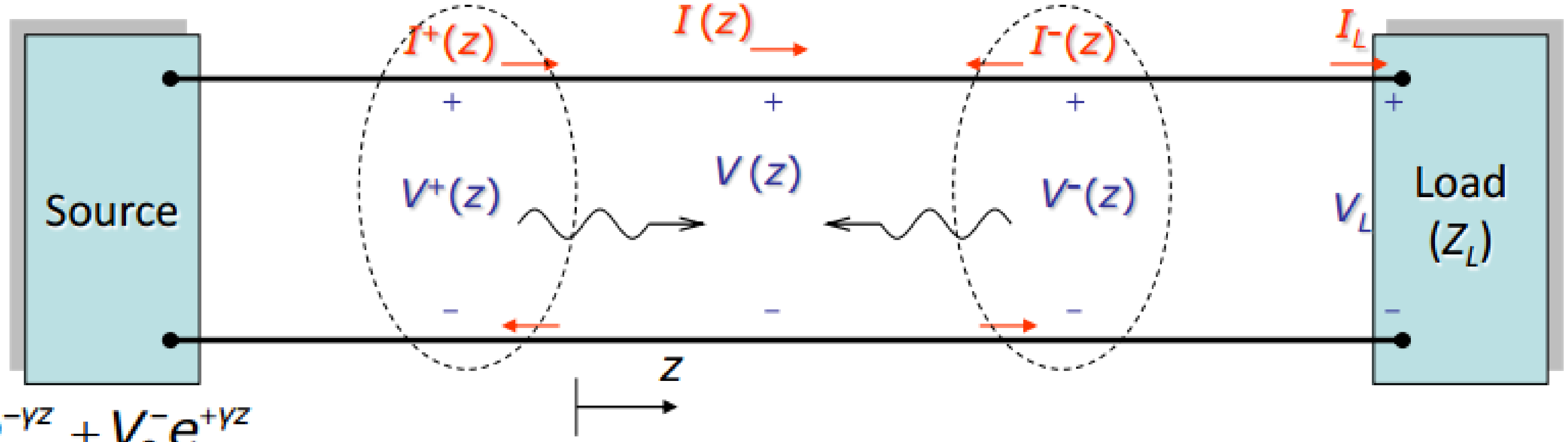
$$V(z) = V^+(z) = V_o^+ e^{-\gamma z}$$

$$I(z) = I^+(z) = I_o^+ e^{-\gamma z}$$

$$Z(z) = \frac{V(z)}{I(z)} = \frac{V_o^+}{I_o^+} = Z_o$$

Non-practical case

Finite Transmission Line



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{1}{Z_0}\left(V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}\right)$$

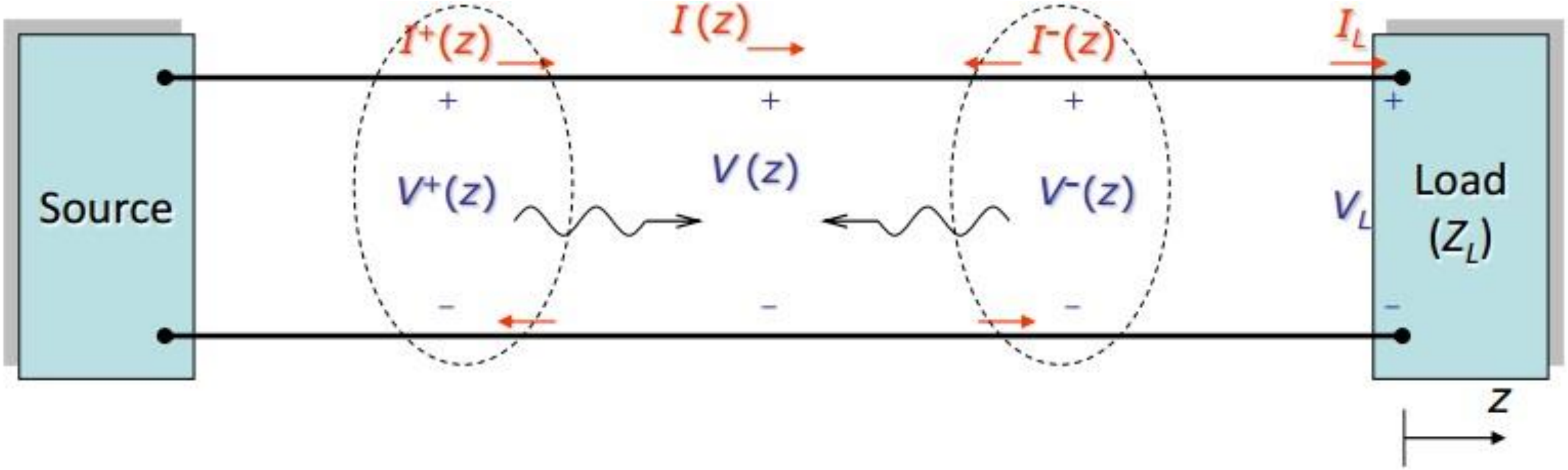
$$\frac{V_L}{I_L} = Z_L = \frac{V_L^+ + V_L^-}{\frac{1}{Z_0}\left(V_L^+ - V_L^-\right)}$$

$$\frac{V_L^-}{V_L^+} = \frac{V_0^- e^{\gamma z_L}}{V_0^+ e^{-\gamma z_L}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma_L = \rho e^{j\varphi}$$

$$V(z) = V_0^+\left(e^{-\gamma z} + \Gamma_L e^{-2\gamma z_L} e^{+\gamma z}\right)$$

$$I(z) = \frac{V_0^+}{Z_0}\left(e^{-\gamma z} - \Gamma_L e^{-2\gamma z_L} e^{+\gamma z}\right)$$

Finite Transmission Line



$$V(z) = V_0^+\left(e^{-\gamma z} + \Gamma_L e^{+\gamma z}\right)$$

$$I(z) = \frac{V_0^+}{Z_0}\left(e^{-\gamma z} - \Gamma_L e^{+\gamma z}\right)$$

Transmission Line (input) Impedance

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma d}{Z_0 + Z_L \tanh \gamma d}$$

Matched Load ($Z_L = Z_0$)

$$Z_{in} = Z_0$$

Half-Wave Lossless Section

$$Z_{in} = Z_L$$

Open Circuit ($Z_L \to \infty$)

$$Z_{in} = Z_0 \coth \gamma d = -jR_0 \cot \beta d$$

Short Circuit ($Z_L = 0$)

$$Z_{in} = Z_0 \tanh \gamma d = jR_0 \tan \beta d$$

Quarter-Wave Lossless Section

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

Power calculations

$$P(z) = \frac{1}{2} Re\{ V(z) \times I^*(z)\}$$

$$P_{in} = \frac{1}{2} Re\{V_{in} \times I_{in}^*\} = \frac{1}{2} \left|\frac{V_{in}}{Z_{in}}\right|^2 R_{in} = \frac{1}{2} |I_{in}|^2 R_{in},$$

P_{in} is the delivered power to the TL.

$$P_{load} = \frac{1}{2} Re\{V_l \times I_l^*\} = \frac{1}{2} \left|\frac{V_l}{Z_l}\right|^2 R_l = \frac{1}{2} |I_l|^2 R_l,$$ P_{load} is the load consumed power.

Transmission Line measurement

Determination of $\boldsymbol{\gamma}$ and Z_0

Open-circuit line: $Z_L \to \infty$ $\qquad Z_{io} = Z_o \coth(\gamma \ell)$

Short-circuit line: $Z_L = 0$ $\qquad Z_{is} = Z_o \tanh(\gamma \ell)$

$$\frac{Z_{is}}{Z_{io}} = \tanh^2(\gamma \ell)$$

$$Z_o = \sqrt{Z_{io} \cdot Z_{is}}$$

$$\gamma = \frac{1}{\ell} \tanh^{-1} \sqrt{\frac{Z_{is}}{Z_{io}}}$$

Exercise VI (part one)

1) **P.9–15** A generator with an open-circuit voltage $v_g(t) = 10 \sin 8000\pi t$ (V) and internal impedance $Z_g = 40 + j30$ (Ω) is connected to a 50 (Ω) distortionless line. The line has a resistance of 0.5 (Ω/m), and its lossy dielectric medium has a loss tangent of 0.18%. The line is 50 (m) long and is terminated in a matched load. Find (a) the instantaneous expressions for the voltage and current at an arbitrary location on the line, (b) the instantaneous expressions for the voltage and current at the load, and (c) the average power transmitted to the load.

2) **P.9–16** The input impedance of an open- or short-circuited *lossy* transmission line has both a resistive and a reactive component. Prove that the input impedance of a very short section ℓ of a slightly lossy line ($\alpha\ell \ll 1$ and $\beta\ell \ll 1$) is approximately

a) $Z_{in} = (R + j\omega L)\ell$ with a short-circuit termination.

b) $Z_{in} = (G - j\omega C)/[G^2 + (\omega C)^2]\ell$ with an open-circuit termination.

(Assignment)

Exercise VI (part one)

3) **P.9–18** A 2 (m) lossless air-spaced transmission line having a characteristic impedance 50 (Ω) is terminated with an impedance $40 + j30$ (Ω) at an operating frequency of 200 (MHz). Find the input impedance.

4) **P.9–19** The open-circuit and short-circuit impedances measured at the input terminals of an air-spaced transmission line 4 (m) long are $250\underline{/-50^\circ}$ (Ω) and $360\underline{/20^\circ}$ (Ω), respectively.

- **a)** Determine Z_0, α, and β of the line.
- **b)** Determine R, L, G, and C.

5) **P.9–20** Measurements on a 0.6 (m) lossless coaxial cable at 100 (kHz) show a capacitance of 54 (pF) when the cable is open-circuited and an inductance of 0.30 (μH) when it is short-circuited.

- **a)** Determine Z_0 and the dielectric constant of its insulating medium.
- **b)** Calculate the X_{io} and X_{is} at 10 (MHz). (Assignment)