ELC N205: Electromagnetics 1 Tutorials

Department of Communications and Computer Engineering

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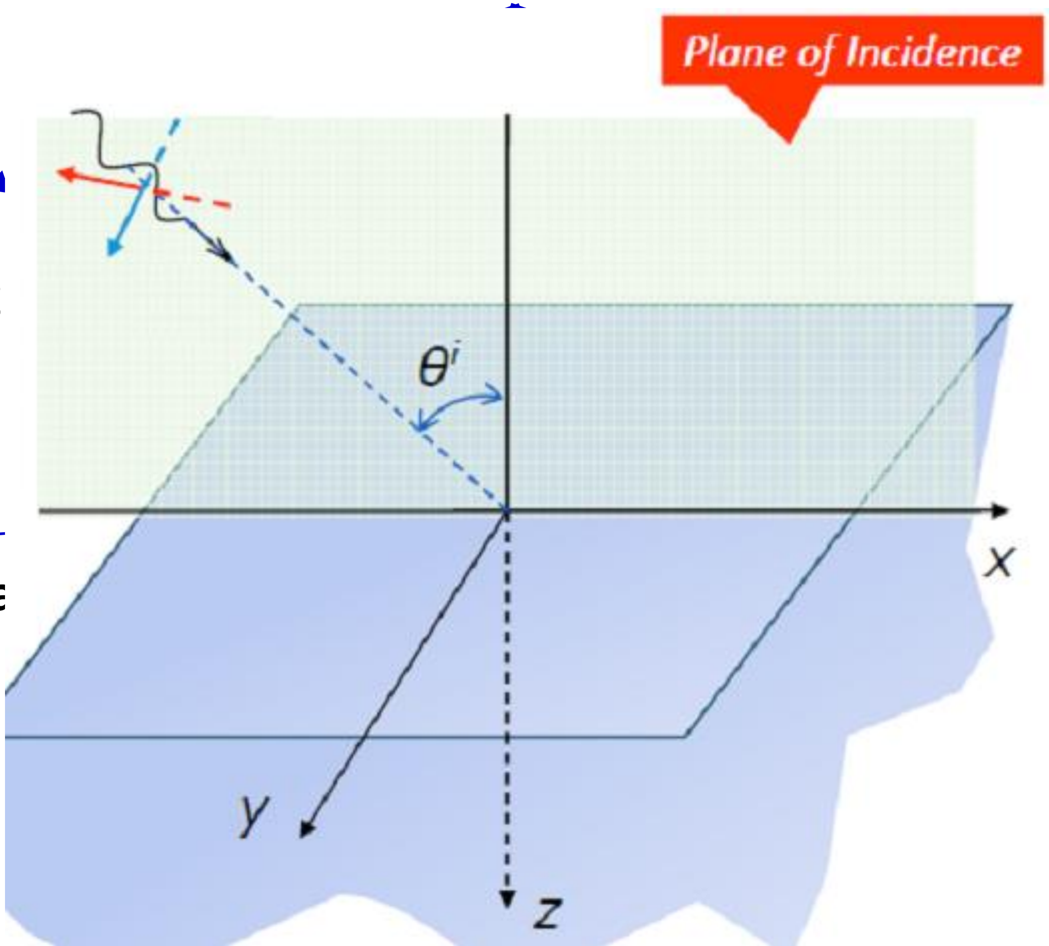
Agenda

- Oblique Incidence at a Plane Boundary
- Oblique Incidence at a Plane Conducting Boundary
- Oblique Incidence at a Plane Dielectric Boundary
- No reflection Conditions

Oblique Incidence at a Plane Boundary



Horizontal (perpendic
polarization

**Component perpendicula
to the plane of incidence**

Vertical (Parallel)
polarization

**Component parallel to the
plane of incidence**

Oblique Incidence at a Plane Conducting Boundary

Case I: Perpendicular Polarization $\underline{E}^i = E_o^i \, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \; \underline{u}_y$

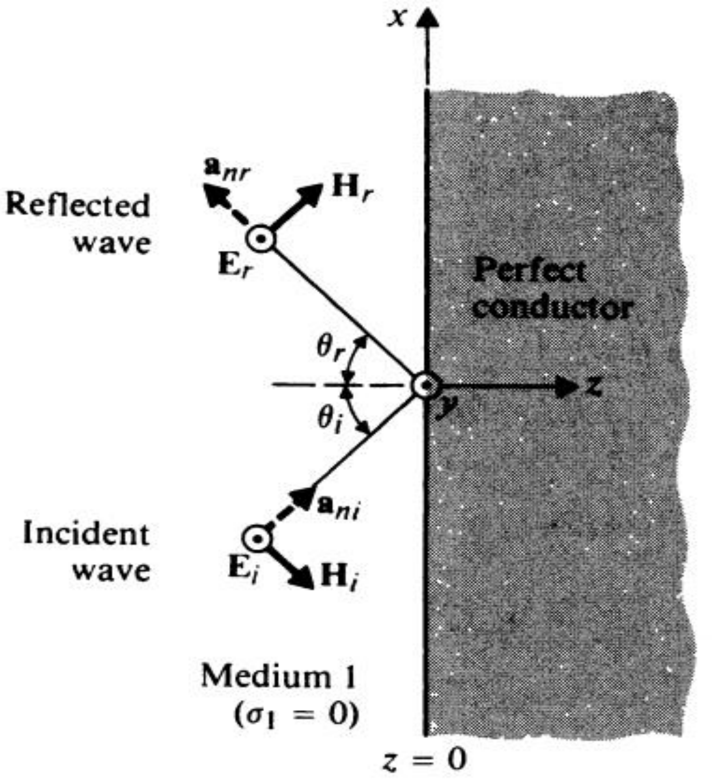
$$\underline{E}_1 = -2jE_o^i \sin(\beta_1 z\cos\theta_i)\, e^{-j\beta_1 x\sin\theta_i} \; \underline{u}_y$$

$$\underline{H}_1 = -2\frac{E_o^i}{\eta_1}\left[\begin{array}{l}\cos\theta_i \cos(\beta_1 z\cos\theta_i) e^{-j\beta_1 x\sin\theta_i} \; \underline{u}_x \\ + j\sin\theta_i \sin(\beta_1 z\cos\theta_i) e^{-j\beta_1 x\sin\theta_i} \; \underline{u}_z\end{array}\right]$$

E_{1y} and H_{1x} form a standing wave in the z-direction

E_{1y} and H_{1z} form a **non-uniform travelling wave** in the x-direction

$$\mathbf{P}_{1av} = 2\frac{|E_o^i|^2}{\eta_1}\sin\theta_i \sin^2(\beta_1 z\cos\theta_i) \quad \underline{u}_x$$



Snell's Law of Reflection

$$\theta_r = \theta_i$$

Oblique Incidence at a Plane Conducting Boundary

Case II : Parallel Polarization

$$\underline{H}^i = \frac{E_o^i}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \underline{u}_y$$

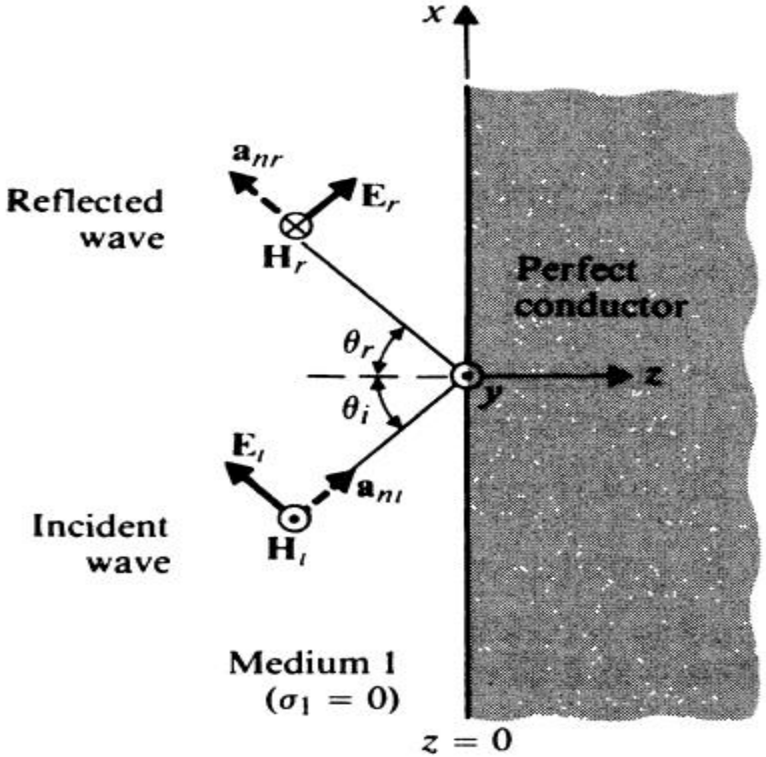
$$\underline{E}_1 = -2jE_o^i \cos\theta_i \sin(\beta_1 z\cos\theta_i) e^{-j\beta_1 x\sin\theta_i} \underline{u}_x + 2E_o^i \sin\theta_i \cos(\beta_1 z\cos\theta_i) e^{-j\beta_1 x\sin\theta_i} \underline{u}_z$$

$$\underline{H}_1 = 2\frac{E_o^i}{\eta_1} \cos(\beta_1 z\cos\theta_i) e^{-j\beta_1 x\sin\theta_i} \underline{u}_y$$

E_{1x} and H_{1y} form a standing wave in the z-direction

E_{1z} and H_{1y} form a **non-uniform travelling wave** in the x-direction

TE & TM modes can be supported in a parallel-plate waveguide.



Snell's Law of Reflection

$$\theta_r = \theta_i$$

Oblique Incidence at a Plane Dielectric Boundary

Case I: Perpendicular Polarization

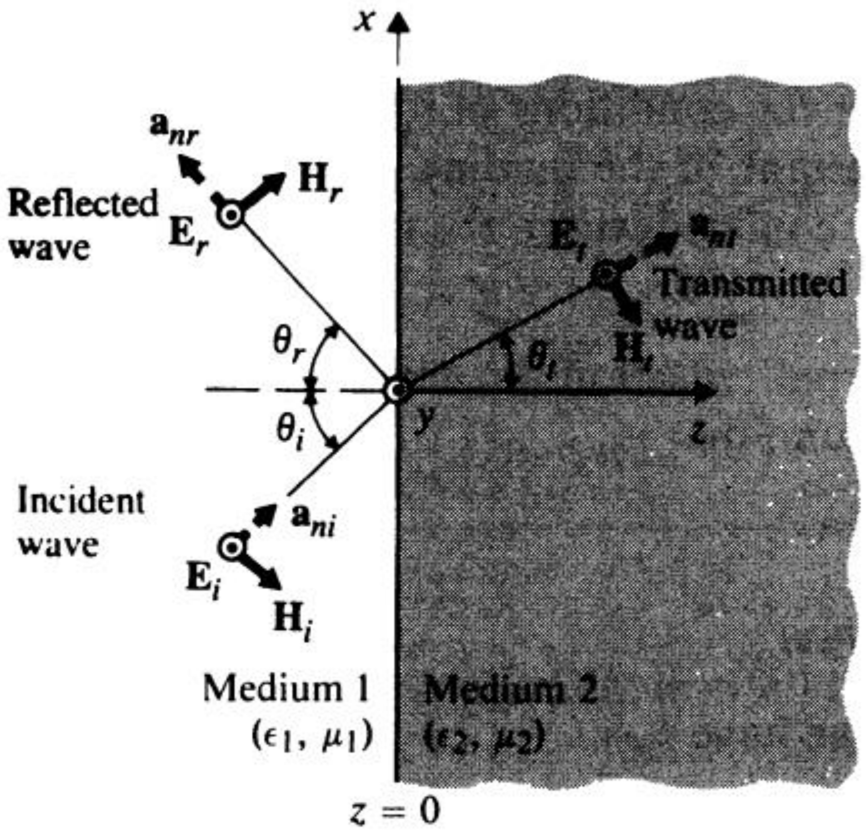
Snell's Law of reflection $\theta_r = \theta_i$

Snell's Law of refraction $$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} = \frac{\beta_1}{\beta_2} = \frac{n_1}{n_2}$$

$$\Gamma_\perp = \frac{E_o^r}{E_o^i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\mathrm{T}_\perp = \frac{E_o^t}{E_o^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$1 + \Gamma_\perp = \mathrm{T}_\perp$$



Oblique Incidence at a Plane Dielectric Boundary

Case II : Parallel Polarization

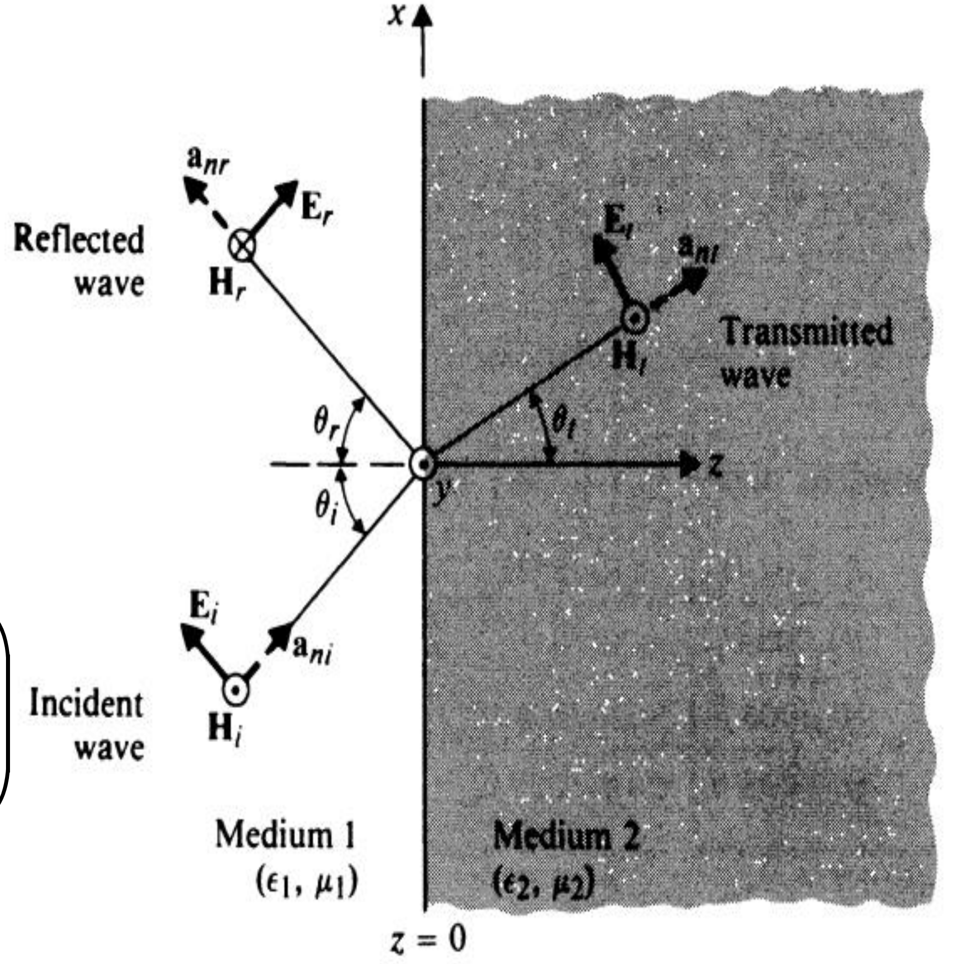
Snell's Law of reflection $\theta_r = \theta_i$

Snell's Law of refraction $\dfrac{\sin\theta_t}{\sin\theta_i} = \sqrt{\dfrac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} = \dfrac{\beta_1}{\beta_2} = \dfrac{n_1}{n_2}$

$$\Gamma_{//} = \frac{E_o^r}{E_o^i} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$1 + \Gamma_{//} = \mathrm{T}_{//}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

$$\mathrm{T}_{//} = \frac{E_o^t}{E_o^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$



No reflection Conditions (Brewster Angle)

Perpendicular Polarization

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\eta_2 \cos\theta_{B\perp} = \eta_1 \cos\theta_t$$

$$\sin^2 \theta_{B\perp} = \frac{1 - \mu_1\varepsilon_2 / \mu_2\varepsilon_1}{1 - (\mu_1/\mu_2)^2}$$

$\theta_{B\perp}$ *does not exist*

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$$

$$\mu_1 = \mu_2$$

Parallel Polarization

$$\Gamma_{//} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_{B//}$$

$$\sin^2 \theta_{B//} = \frac{1 - \mu_2\varepsilon_1 / \mu_1\varepsilon_2}{1 - (\varepsilon_1/\varepsilon_2)^2}$$

$$\sin\theta_{B//} = \frac{1}{\sqrt{1 + (\varepsilon_1/\varepsilon_2)}}$$

Exercise IV(part one)

1) **P.8–22** A uniform sinusoidal plane wave in air with the following phasor expression for electric intensity

$$\mathbf{E}_i(x, z) = \mathbf{a}_y 10 e^{-j(6x+8z)} \qquad \text{(V/m)}$$

is incident on a perfectly conducting plane at $z = 0$.

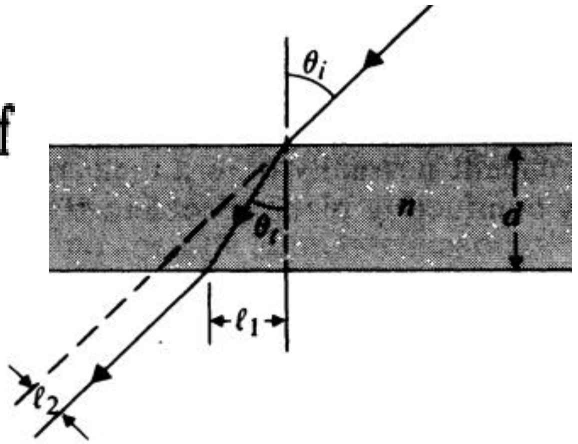
- **a)** Find the frequency and wavelength of the wave.
- **b)** Write the instantaneous expressions for $\mathbf{E}_i(x, z; t)$ and $\mathbf{H}_i(x, z; t)$, using a cosine reference.
- **c)** Determine the angle of incidence.
- **d)** Find $\mathbf{E}_r(x, z)$ and $\mathbf{H}_r(x, z)$ of the reflected wave.
- **e)** Find $\mathbf{E}_1(x, z)$ and $\mathbf{H}_1(x, z)$ of the total field.

2) **P.8–23** Repeat Problem P.8–22 for $\mathbf{E}_i(y, z) = 5(\mathbf{a}_y + \mathbf{a}_z\sqrt{3})e^{j6(\sqrt{3}y - z)}$ (V/m). (Assignment)

Exercise IV(part three)

3) **P.8–35** A 10 (kHz) parallelly polarized electromagnetic wave in air is incident obliquely on an ocean surface at a near-grazing angle $\theta_i = 88°$. Using $\epsilon_r = 81$, $\mu_r = 1$, and $\sigma = 4$ (S/m) for seawater, find (a) the angle of refraction θ_t, (b) the transmission coefficient $\tau_{||}$, (c) $(\mathscr{P}_{av})_t/(\mathscr{P}_{av})_i$, and (d) the distance below the ocean surface where the field intensity has been diminished by 30 (dB).

4) **P.8–36** A light ray is incident from air obliquely on a transparent sheet of thickness d with an index of refraction n, as shown in Fig. 8–24. The angle of incidence is θ_i. Find (a) θ_t, (b) the distance ℓ_1 at the point of exit, and (c) the amount of the lateral displacement ℓ_2 of the emerging ray.



Exercise IV(part three)

5) **P.8–37** A uniform plane wave with perpendicular polarization represented by Eqs. (8–196) and (8–197) is incident on a plane interface at $z = 0$, as shown in Fig. 8–16. Assuming $\epsilon_2 < \epsilon_1$ and $\theta_i > \theta_c$, (a) obtain the phasor expressions for the transmitted field $(\mathbf{E}_t, \mathbf{H}_t)$, and (b) verify that the average power transmitted into medium 2 vanishes.

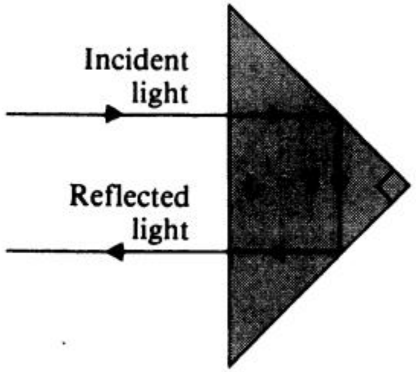
6) **P.8–38** A uniform plane wave of angular frequency ω in medium 1 having a refractive index n_1 is incident on a plane interface at $z = 0$ with medium 2 having a refractive index n_2 $(<n_1)$ at the critical angle. Let E_{i0} and E_{t0} denote the amplitudes of the incident and refracted electric field intensities, respectively.

- **a)** Find the ratio E_{t0}/E_{i0} for perpendicular polarization.
- **b)** Find the ratio E_{t0}/E_{i0} for parallel polarization.
- **c)** Write the instantaneous expressions of $\mathbf{E}_i(x, z; t)$ and $\mathbf{E}_t(x, z; t)$ for perpendicular polarization in terms of the parameters ω, n_1, n_2, θ_i, and E_{i0}.

Exercise IV(part three)

7) **P.8–39** An electromagnetic wave from an underwater source with perpendicular polarization is incident on a water–air interface at $\theta_i = 20°$. Using $\epsilon_r = 81$ and $\mu_r = 1$ for fresh water, find (a) critical angle θ_c, (b) reflection coefficient $\Gamma_\perp$, (c) transmission coefficient $\tau_\perp$, and (d) attenuation in dB for each wavelength into the air.

8) **P.8–40** Glass isosceles triangular prisms shown in Fig. 8–25 are used in optical instruments. Assuming $\epsilon_r = 4$ for glass, calculate the percentage of the incident light power reflected back by the prism.

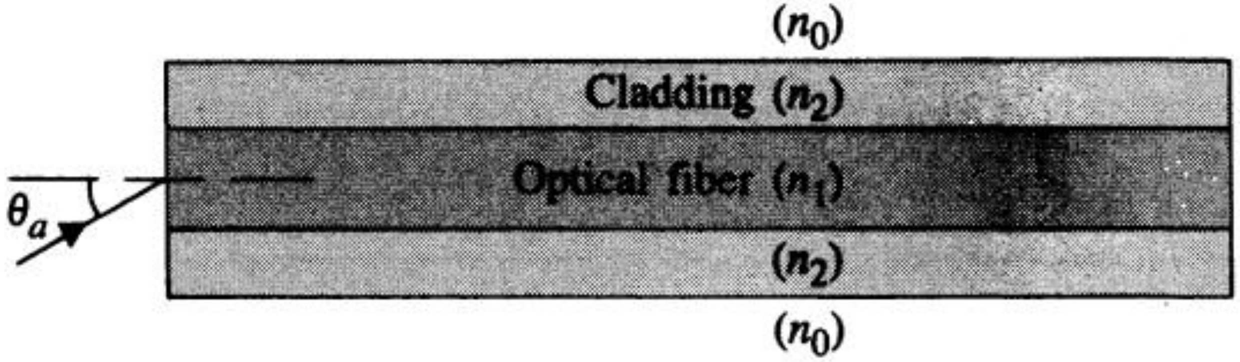


Exercise IV(part three)

9) **P.8–41** For preventing interference of waves in neighboring fibers and for mechanical protection, individual optical fibers are usually cladded by a material of a lower refractive index, as shown in Fig. 8–26, where $n_2 < n_1$.

a) Express the maximum angle of incidence θ_a in terms of n_0, n_1, and n_2 for meridional rays incident on the core's end face to be trapped inside the core by total internal reflection. (***Meridional rays*** are those that pass through the fiber axis. The angle θ_a is called the ***acceptance angle***, and $\sin\theta_a$ the ***numerical aperture*** (N.A.) of the fiber.)

b) Find θ_a and N.A. if $n_1 = 2$, $n_2 = 1.74$, and $n_0 = 1$.



Exercise IV(part three)

10) **P.8–46** A perpendicularly polarized uniform plane wave in air of frequency f is incident obliquely at an angle of incidence θ_i on a plane boundary with a lossy dielectric medium that is characterized by a complex permittivity $\epsilon_2 = \epsilon' - j\epsilon''$. Let the incident electric field be

$$\mathbf{E}_i(x, z) = \mathbf{a}_y E_{i0} e^{-jk_0(x \sin\theta_i - z\cos\theta_i)}.$$

a) Find the expressions of the transmitted electric and magnetic field intensity phasors in terms of the given parameters.

b) Show that the angle of refraction is complex and that $\mathbf{H}_t$ is elliptically polarized.