ELC N205: Electromagnetics 1 Tutorials

Department of Communications and Computer Engineering

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Agenda

- Normal Incidence at a Plane Conducting Boundary
- Normal incidence at Plane Dielectric Boundary
- Wave Impedance of the Total Field
- Normal Incidence at Multiple Dielectric Layers

Normal Incidence at a Plane Conducting Boundary

Reflection Coefficient $\Gamma = \dfrac{E_{ro}}{E_{io}} = -1$

Transmission Coefficient $\mathrm{T} = \dfrac{E_{to}}{E_{io}} = 0$

"Standing Wave"

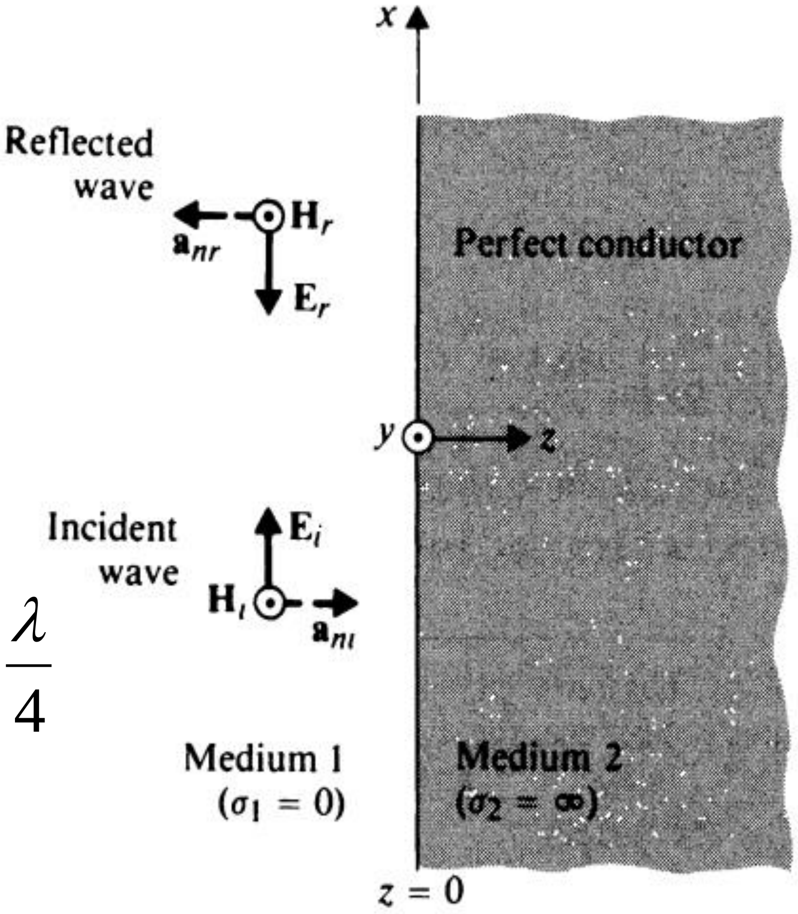
$$\underline{E}_1(z) = -2\, j\, E_{io} \sin(\beta_1 z)\ \underline{u}_x$$

$$\underline{H}_1(z) = 2\frac{E_{io}}{\eta_1} \cos(\beta_1 z) \underline{u}_y$$

$\underline{E}_1 = \min \quad @ \quad z_{\min} = -n\dfrac{\lambda}{2}$

$\underline{E}_1 = \max \quad @ \quad z_{\max} = -(2n+1)\dfrac{\lambda}{4}$

$$\underline{\mathscr{P}}_{1av} = \frac{1}{2}\mathrm{Re}\left\{\underline{E}_1 \times \underline{H}_1^*\right\} = 0$$



Normal Incidence at a Plane Dielectric Boundary

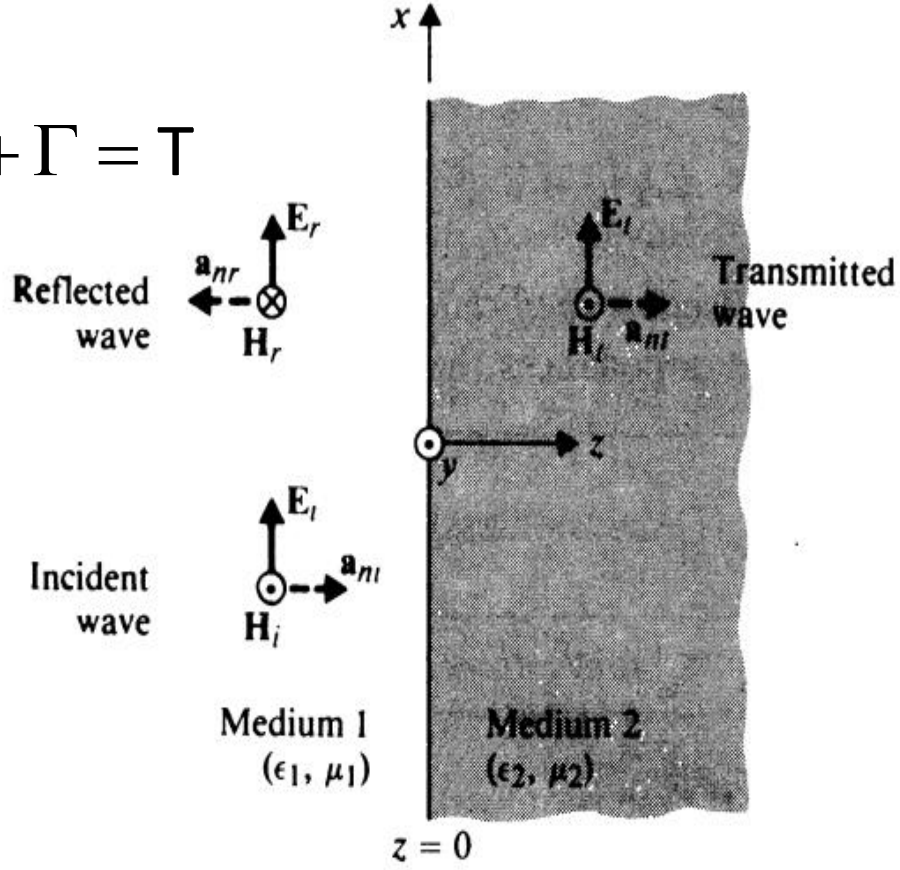
Reflection Coefficient

$$\Gamma \equiv \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Transmission Coefficient

$$\mathrm{T} \equiv \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$1 + \Gamma = \mathrm{T}$$



"Traveling & Standing Waves"

$$\underline{E}_1(z) = [E_{io}\,\mathrm{T}\, e^{-j\beta_1 z} + E_{io}\Gamma\,(j\,2\sin\beta_1 z)]\,\underline{u}_x$$

$$\underline{E}_1(z) = E_{io}\, e^{-j\beta_1 z}(1 + \Gamma\, e^{+j2\beta_1 z})\,\underline{u}_x$$

$$\underline{E}_t(z) = \mathrm{T}\, E_{io}\, e^{-j\beta_2 z}\,\underline{u}_x$$

Normal Incidence at a Plane Dielectric Boundary

$$\underline{H}_1(z) = \frac{E_{io}}{\eta_1} e^{-j\beta_1 z}\left(1-\Gamma e^{+j2\beta_1 z}\right)\underline{u}_y$$

$$\underline{H}_t(z) = \mathrm{T}\,\frac{E_{io}}{\eta_2} e^{-j\beta_2 z}\,\underline{u}_y$$

$\Gamma > 0\ (\eta_2 > \eta_1)$.

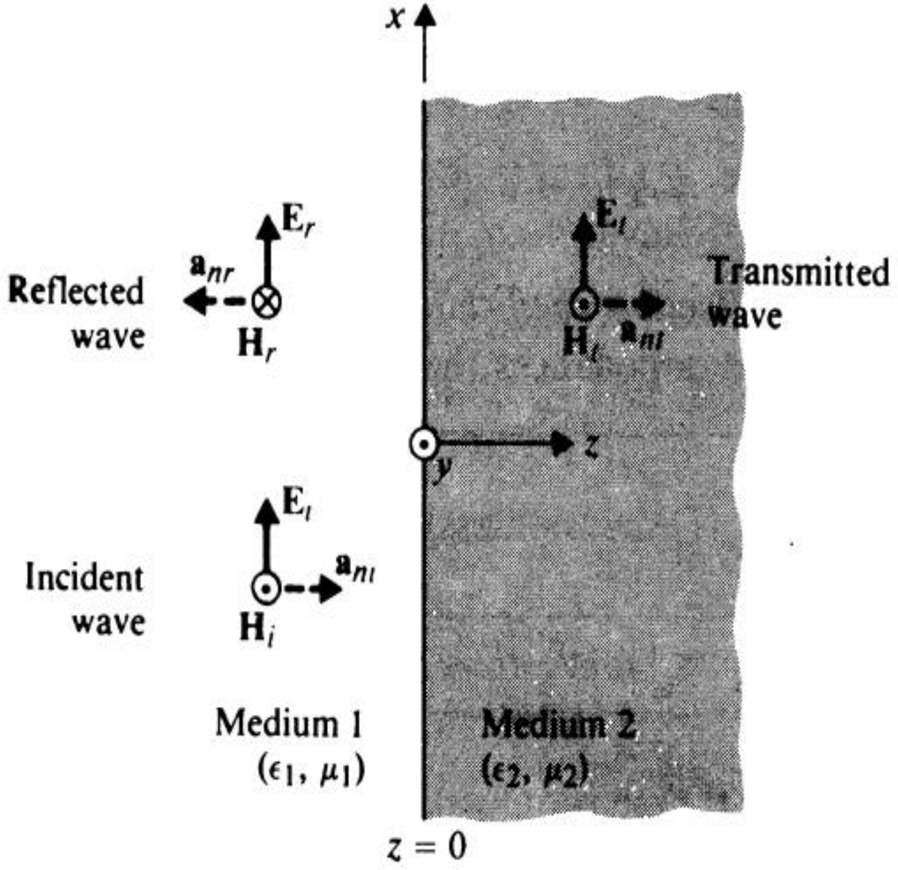
The maximum value of $|\mathbf{E}_1(z)|$ is $E_{i0}(1+\Gamma)$

$$z_{\max} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2}, \qquad n = 0, 1, 2, \ldots$$

The minimum value of $|\mathbf{E}_1(z)|$ is $E_{i0}(1-\Gamma)$

$$z_{\min} = -\frac{(2n+1)\pi}{2\beta_1} = -\frac{(2n+1)\lambda_1}{4}, \qquad n = 0, 1, 2, \ldots$$

$$S = \frac{|\underline{E}|_{\max}}{|\underline{E}|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$



Normal Incidence at a Plane Dielectric Boundary

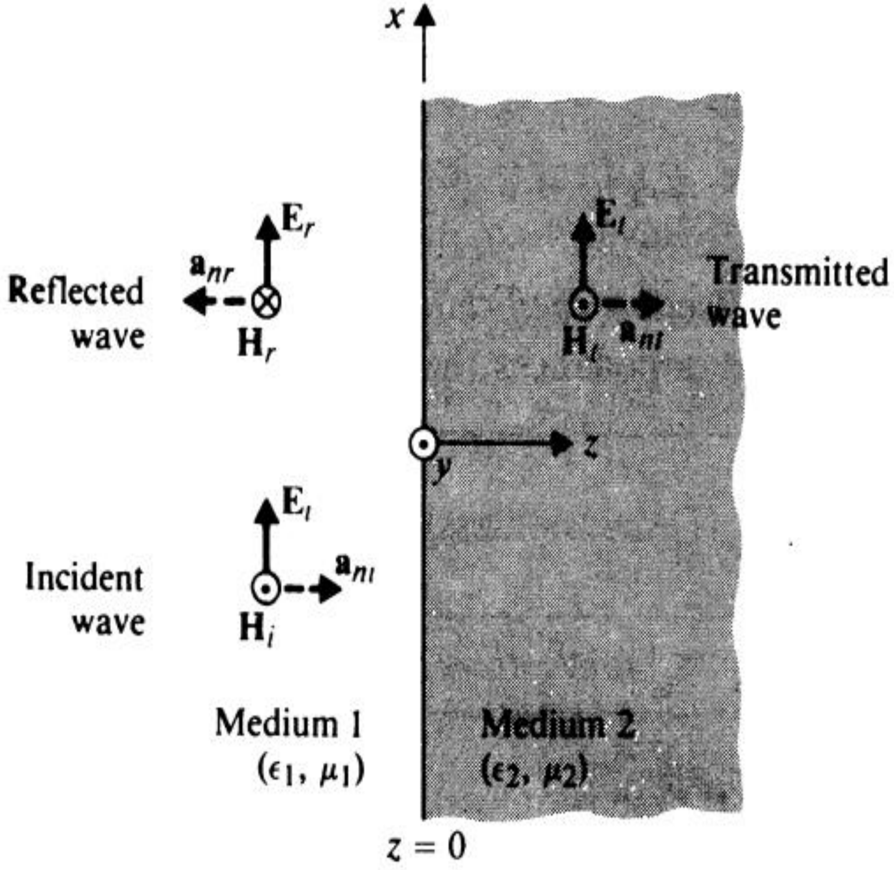
$$(\underline{\mathscr{P}}_{av})_1 = \frac{E_{io}^2}{2\eta_1}(1-\Gamma^2)\,\underline{u}_z$$

$$(\underline{\mathscr{P}}_{av})_2 = \frac{E_{io}^2}{2\eta_2}\mathrm{T}^2\;\underline{u}_z$$

For lossless Media:

$$(\underline{\mathscr{P}}_{av})_1 = (\underline{\mathscr{P}}_{av})_2$$

$$(1-\Gamma^2) = \frac{\eta_1}{\eta_2}\mathrm{T}^2$$



Exercise IV(part one)

1) **P.8–20** A uniform plane electromagnetic wave propagates in the $+z$- (downward) direction and impinges normally at $z = 0$ on an ocean surface. Let the magnetic field at $z = 0$ be $\mathbf{H}(0, t) = \mathbf{a}_y H_0 \cos 10^4 t$ (A/m).

- **a)** Determine the skin depth. (For the ocean: Conductivity $= \sigma$, permeability $= \mu_0$.)
- **b)** Find the expressions for $\mathbf{H}(z, t)$ and $\mathbf{E}(z, t)$.
- **c)** Find the power loss per unit area (in terms of H_0) into the ocean.

2) **P.8–21** A right-hand circularly polarized plane wave represented by the phasor

$$\mathbf{E}(z) = E_0(\mathbf{a}_x - j\mathbf{a}_y)e^{-j\beta z}$$

impinges normally on a perfectly conducting wall at $z = 0$.

- **a)** Determine the polarization of the reflected wave.
- **b)** Find the induced current on the conducting wall.
- **c)** Obtain the instantaneous expression of the total electric intensity based on a cosine time reference.

Exercise IV(part one)

3) **P.8–27** A uniform plane wave in air with $\mathbf{E}_i(z) = \mathbf{a}_x 10e^{-j6z}$ (V/m) is incident normally on an interface at $z = 0$ with a lossy medium having a dielectric constant 2.5 and a loss tangent 0.5. Find the following:

- **a)** The instantaneous expressions for $\mathbf{E}_r(z, t)$, $\mathbf{H}_r(z, t)$, $\mathbf{E}_t(z, t)$, and $\mathbf{H}_t(z, t)$, using a cosine reference.
- **b)** The expressions for time-average Poynting vectors in air and in the lossy medium.

(Assignment)

Wave Impedance of the Total Field

Definition

The wave impedance of the total field at any plane parallel to the plane boundary as the ratio of the total electric field intensity to the total magnetic field intensity.

With a z-dependent uniform plane wave, $Z(z) = \dfrac{\text{Total } E_x(z)}{\text{Total } H_y(z)} \quad (\Omega).$

For a single wave propagating in the + z-direction in an unbounded medium, the wave impedance equals the intrinsic impedance of the medium(η); for a single wave traveling in the — z-direction, it is — the intrinsic impedance($-\eta$) for all z.

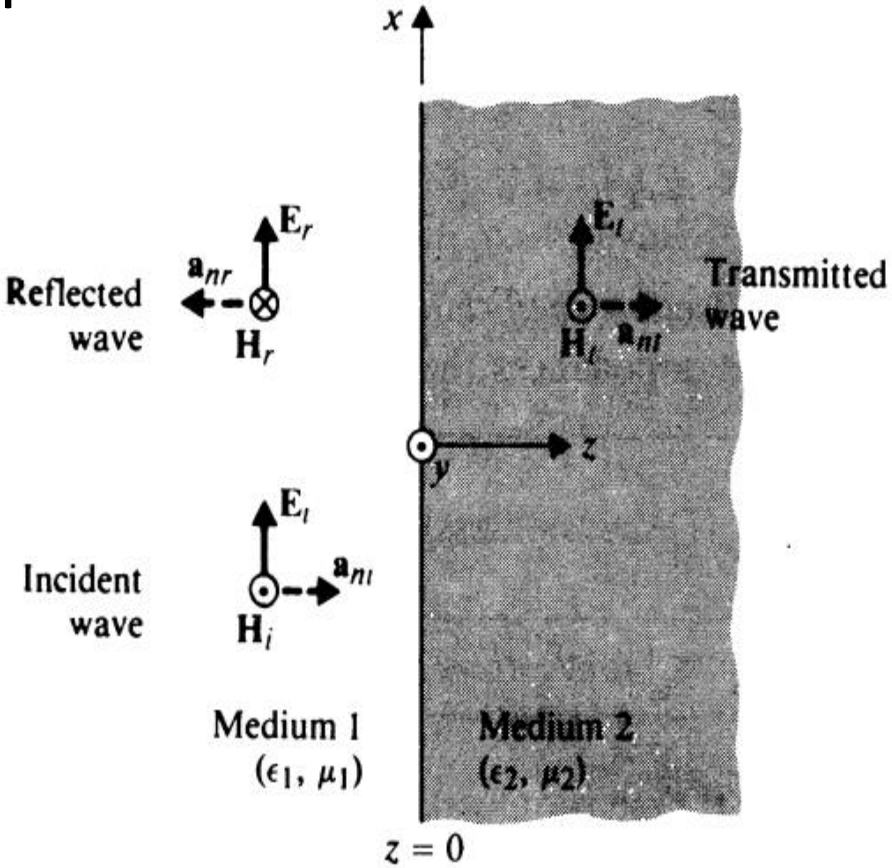
Wave Impedance of the Total Field

In the case of a uniform plane wave incident from medium 1 normally on a plane boundary with an infinite medium 2.

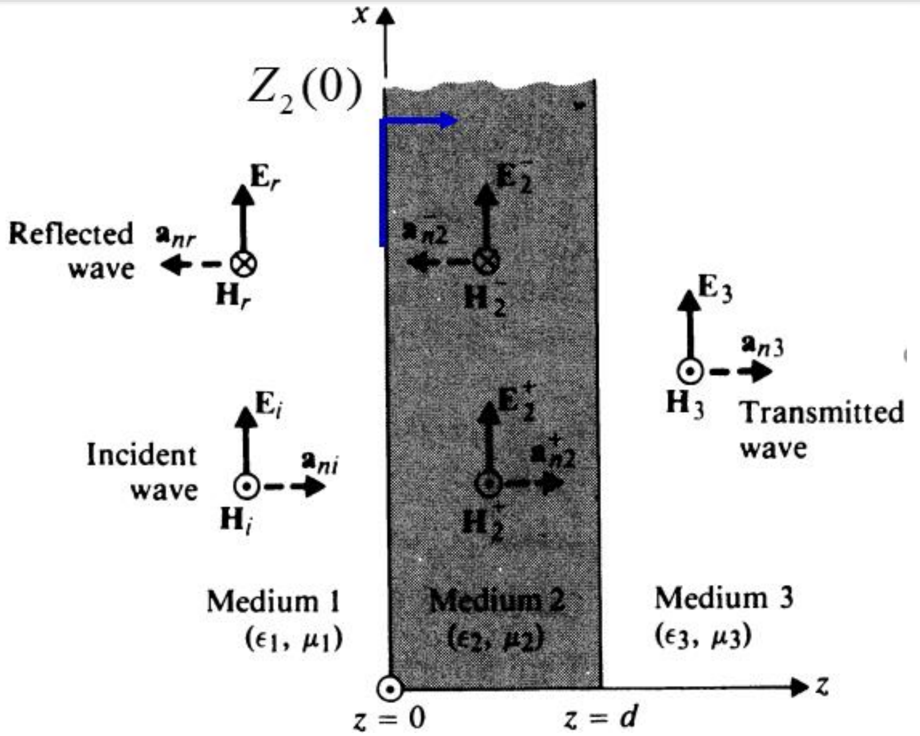
$$E_{1x}(z) = E_{i0}(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}),$$

$$H_{1y}(z) = \frac{E_{i0}}{\eta_1}(e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}).$$

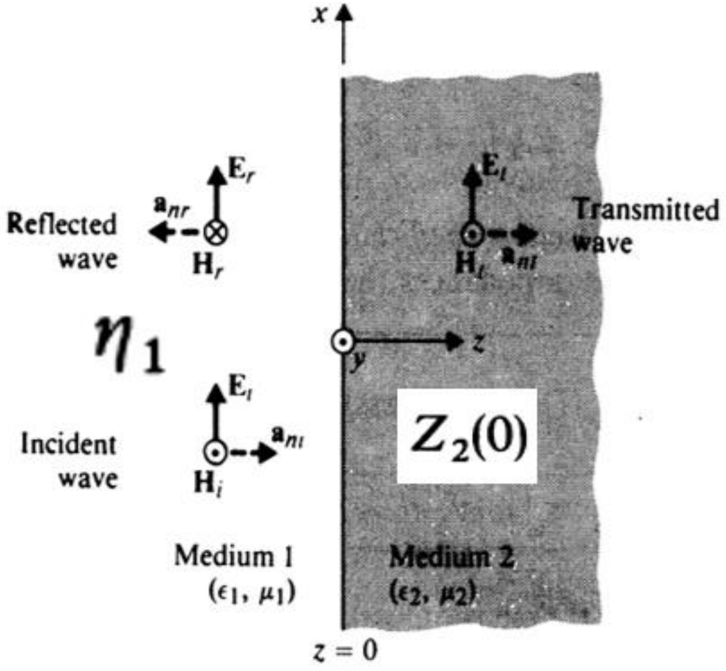
$$Z_1(-\ell) = \frac{E_{1x}(-\ell)}{H_{1y}(-\ell)} = \eta_1 \frac{e^{j\beta_1\ell} + \Gamma e^{-j\beta_1\ell}}{e^{j\beta_1\ell} - \Gamma e^{-j\beta_1\ell}} = \eta_1 \frac{\eta_2 \cos\beta_1\ell + j\eta_1 \sin\beta_1\ell}{\eta_1 \cos\beta_1\ell + j\eta_2 \sin\beta_1\ell}$$



Normal Incidence at Multiple Dielectric Layers



$\Rightarrow$



$$Z_2(0) = \eta_2 \frac{\eta_3 \cos \beta_2 d + j\eta_2 \sin \beta_2 d}{\eta_2 \cos \beta_2 d + j\eta_3 \sin \beta_2 d} \quad \Rightarrow \quad \Gamma_0 = \frac{E_{r0}}{E_{i0}} = -\frac{H_{r0}}{H_{i0}} = \frac{Z_2(0) - \eta_1}{Z_2(0) + \eta_1}.$$

Normal Incidence at a Plane Dielectric Boundary

Special Cases

Quarter-Wavelength Layer

$$d = (2n+1)\frac{\lambda_2}{4}, \qquad n = 0, 1, 2, \ldots$$

$$Z_2(0) = \frac{\eta_2^2}{\eta_3}$$

$$When\ \eta_2 = \sqrt{\eta_1 \eta_3}$$

Half-Wavelength Layer

$$d = n\frac{\lambda_2}{2}, \qquad n = 0, 1, 2, \ldots$$

$$Z_2(0) = \eta_3$$

$$When\ \eta_3 = \eta_1$$

Dielectric coating on a Conductor

$$\eta_3 = 0$$

$$Z_2(0) = j\eta_2 \tan(\beta_2 d)$$

No reflection occurs when a uniform plane wave in medium 1 impinges normally on the interface with medium 2.

Exercise IV(part two)

1) **P.8–30** A transparent dielectric coating is applied to glass ($\epsilon_r = 4$, $\mu_r = 1$) to eliminate the reflection of red light [$\lambda_0 = 0.75$ (μm)].

   a) Determine the required dielectric constant and thickness of the coating.

   b) If violet light [$\lambda_0 = 0.42$ (μm)] is shone normally on the coated glass, what percentage of the incident power will be reflected?

2) **P.8–32** A uniform plane wave with

$$\mathbf{E}_i(z, t) = \mathbf{a}_x E_{i0} \cos \omega \left(t - \frac{z}{u_p} \right)$$

in medium 1 (ϵ_1, μ_1) is incident normally onto a lossless dielectric slab (ϵ_2, μ_2) of a thickness d backed by a perfectly conducting plane, as shown in Fig. 8–22. Find

   a) $(\mathscr{P}_{av})_1$ **b)** $(\mathscr{P}_{av})_2$

   c) Determine the thickness d that makes $\mathbf{E}_1(z, t)$ the same as if the dielectric slab were absent.