ELC N205: Electromagnetics 1 Tutorials

Department of Communications and Computer Engineering

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Agenda

- Uniform Plane waves
- Solutions of wave equations in free space
- Uniform Plane waves characteristics
  - In free space
  - In lossless medium
- Polarization

Uniform Plane waves

Definition

Uniform ⇒ Field components vary in one direction & time.

Plane ⇒ Field has the same direction, same magnitude, and same phase in infinite planes perpendicular to the direction of propagation.

So, if we assume the wave propagation direction to be in z-direction, we get:

$$\mathbf{E}(x, y, z, t) \Rightarrow \mathbf{E}(z, t) \quad \& \quad \frac{\partial}{\partial x} \; \& \; \frac{\partial}{\partial y} (\mathrm{E},\mathrm{H}) = 0$$

Solutions of wave equations in free space

If we assume the wave propagation direction to be in z-direction.

$$E\,(z,t) = E_x^+\, e^{j\,(\omega t\,-kz)}\,\underline{u}_x + E_y^+\, e^{j\,(\omega t\,-kz+\,\phi)}\,\underline{u}_y$$

$$= \underline{E}_o e^{j\,(\omega t\,-kz)}$$

where, $\underline{E}_o = E_x^+ \underline{u}_x + E_y^+\, e^{j\phi}\,\underline{u}_y$

K $\stackrel{\text{def}}{=}$ wavenumber = $\omega\sqrt{\mu\,\varepsilon}$

$\phi \stackrel{\text{def}}{=}$ phase difference between the x & y components

$$H\,(z,t) = \frac{1}{\eta_o}\left(E_x^+\, e^{j\,(\omega t\,-kz)}\,\underline{u}_y - E_y^+\, e^{j\,(\omega t\,-kz+\,\phi)}\,\underline{u}_x\right)$$

where, $\eta_o \stackrel{\text{def}}{=}$ *Intrinsic Impedance* = $\sqrt{\mu_o / \varepsilon_o}$

Solutions of wave equations in free space

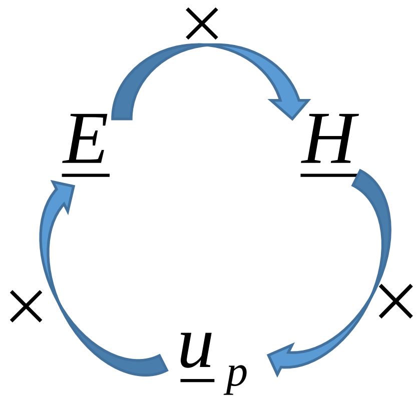
Generally, for a wave propagating in $\underline{u}_p$ direction.

$$E\,(R,t) = \underline{E}_o e^{j\,(\omega t\,-\underline{k}.\underline{R})}$$

where, $\underline{k} = k_x \underline{u}_x + k_y \underline{u}_y + k_z \underline{u}_z$

$$\underline{R} = x\,\underline{u}_x + y\,\underline{u}_y + z\,\underline{u}_z$$

$$H\,(R,t) = \frac{1}{\eta_o}(\,\underline{u}_p * \underline{E}_o)\,e^{j\,(\omega t\,-\underline{k}.\underline{R})}$$



Finally, $\underline{H}(R) = \dfrac{1}{\eta}\ \underline{u}_n \times \underline{E}(R)$ $\qquad \underline{E}(R) = \eta\ \ \underline{H}(R) \times \underline{u}_n$

Uniform Plane waves characteristics

Quantity	Type of medium	
	Free space($\mu=\mu_o$, $\varepsilon=\varepsilon_o$, $\sigma=0$)	**Lossless**($\mu=\mu_o\mu_r$, $\varepsilon=\varepsilon_o\varepsilon_r$, $\sigma=0$)
Phase constant (wave number)	$\omega\sqrt{\mu o\ \varepsilon o} = \frac{\omega}{co}$	$\omega\sqrt{\mu\ \varepsilon} = \frac{\omega\ \sqrt{\mu r\ \varepsilon r}}{co}$
Impedance	$\sqrt{\mu o\ /\ \varepsilon o} = \mathbf{120\pi}$	$\sqrt{\mu\ /\ \varepsilon} = \mathbf{120\pi}\sqrt{\mu r\ /\ \varepsilon r}$
Wavelength	$2\pi/K$	$2\pi/K$
Period	$2\pi/\omega$	$2\pi/\omega$
Phase velocity	Co	Co / $\sqrt{\mu r\ \varepsilon r}$

Polarization

Definition

The polarization of a uniform plane wave describes the time-varying behavior of the electric field intensity vector at a given point in space.

Types

1) Linear Polarization 2) Circular Polarization 3) Elliptical Polarization

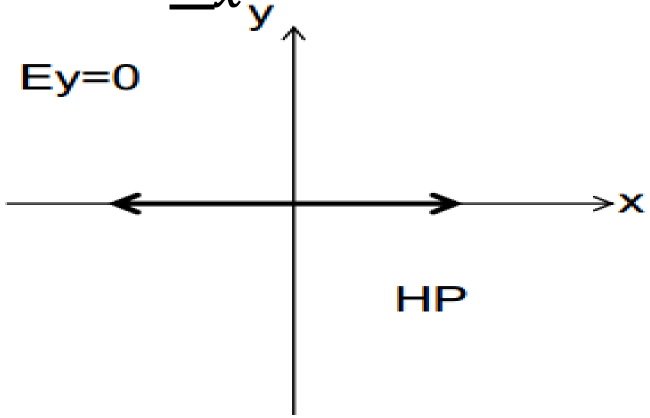
Note: To determine the polarization, you must write the electric field in the form of 2 components perpendicular to each other & perpendicular to the propagation direction.

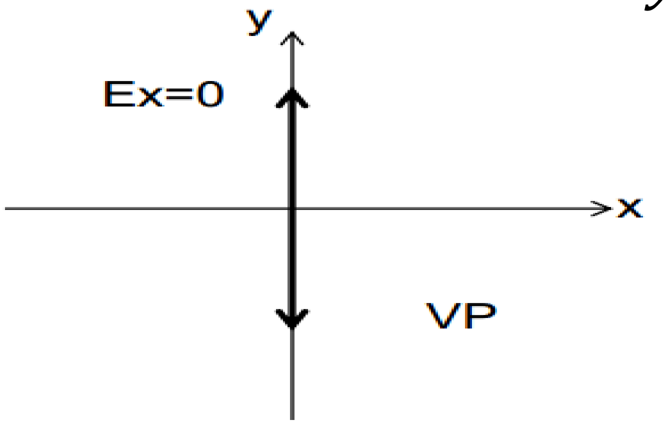
$$E\,(z,t) = E_x^+\, e^{j\,(\omega t\,-kz)}\,\underline{u}_x + E_y^+\, e^{j\,(\omega t\,-kz+\,\phi)}\,\underline{u}_y$$

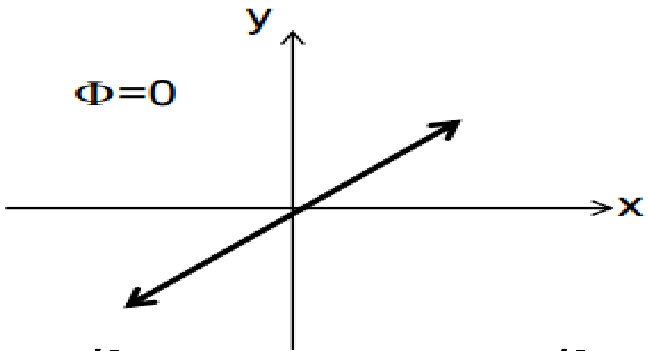
Linear Polarization

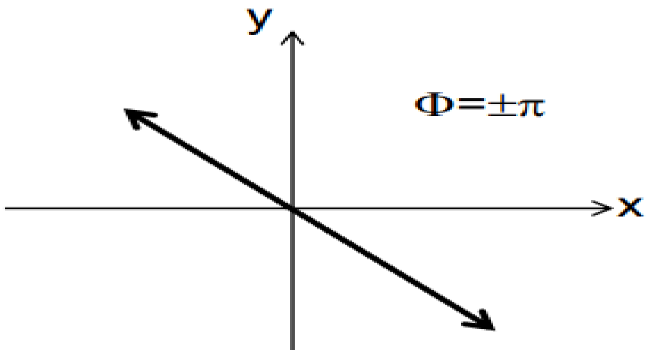
$$E\ (z) = E_x^+\ e^{-jkz}\ \underline{u}_x$$

$$E\ (z) = E_y^+\ e^{-j(kz-\phi)}\ \underline{u}_y$$







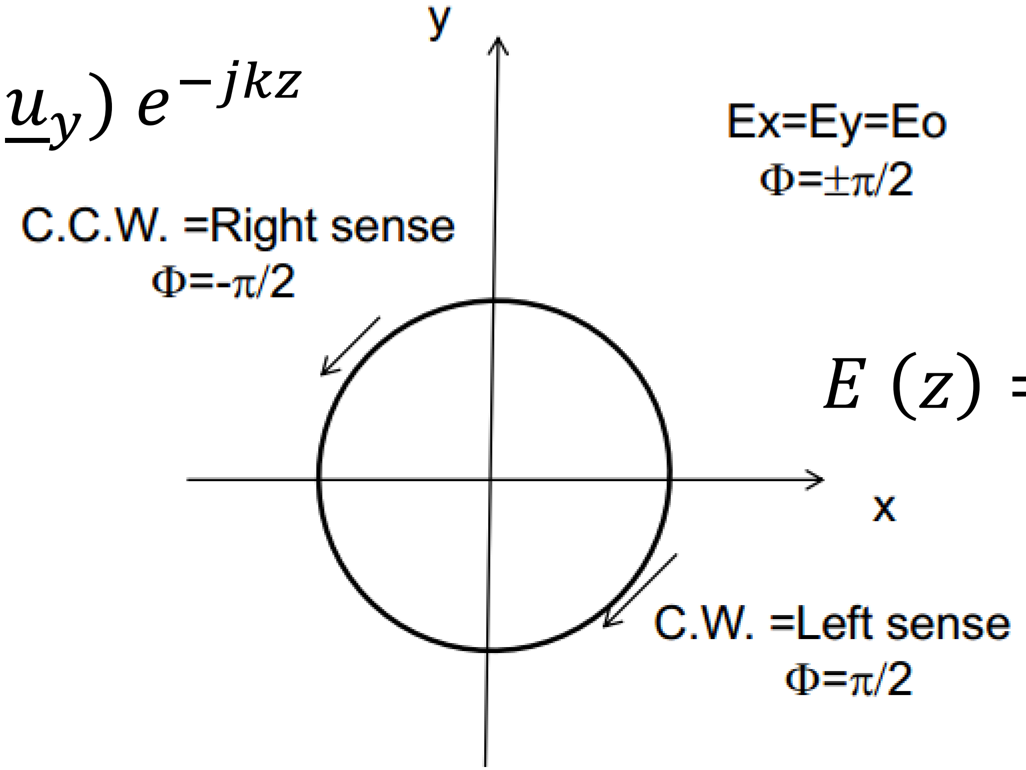


$$E\ (z) = E_x^+\ e^{-jkz}\ \underline{u}_x + E_y^+\ e^{-jkz}\ \underline{u}_y$$

$$E\ (z) = E_x^+\ e^{-jkz}\ \underline{u}_x + E_y^+\ e^{-j(kz\pm\pi)}\ \underline{u}_y$$

Circular Polarization

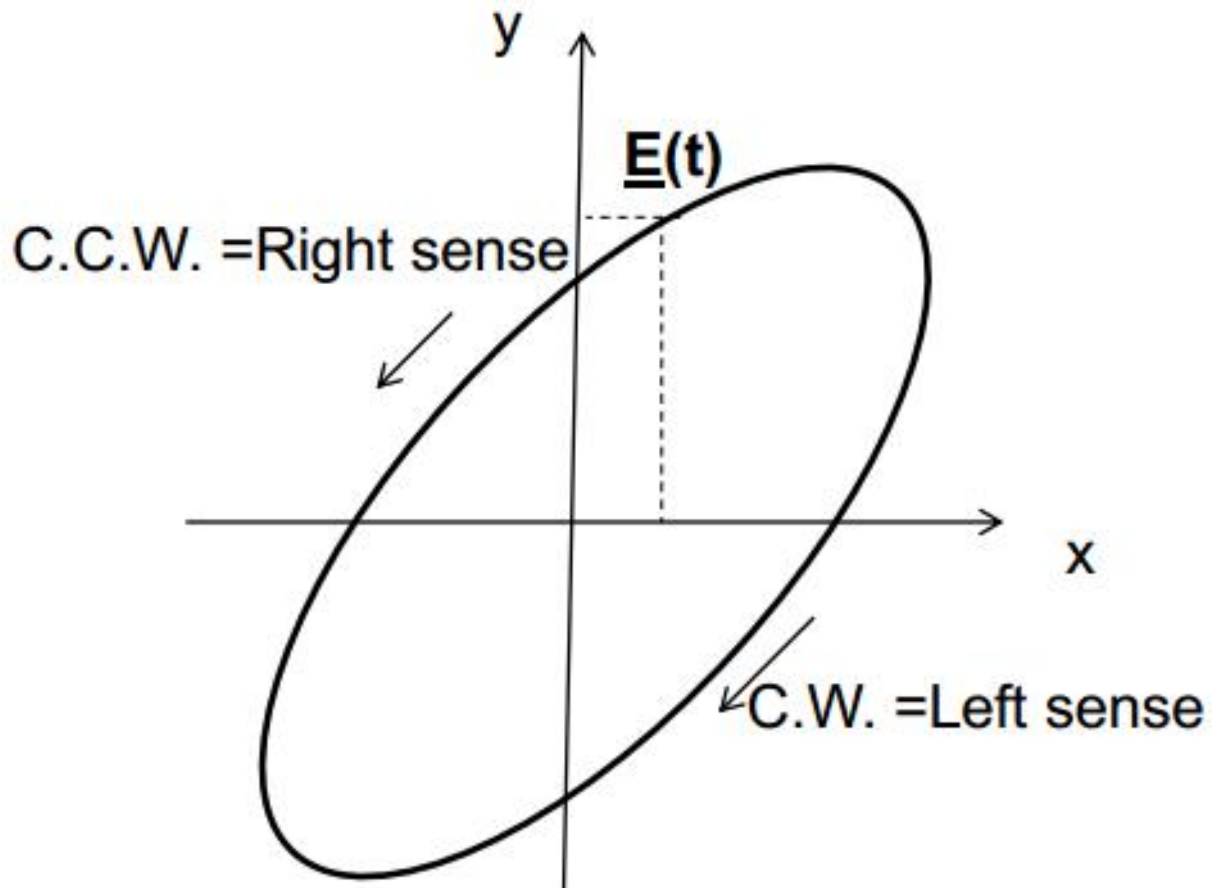
$$E\,(z) = (E_o\,\underline{u}_x - jE_0\,\underline{u}_y)\,e^{-jkz}$$



$$E\,(z) = (E_o\,\underline{u}_x + jE_0\,\underline{u}_y)\,e^{-jkz}$$

$$E\,(z) = E_o\,e^{-jkz}\,\underline{u}_x + E_0\,e^{-j(kz\pm\frac{\pi}{2})}\,\underline{u}_y$$

Elliptical Polarization



$$E\,(z) = E_x^+\, e^{-jkz}\, \underline{u}_x + E_y^+\, e^{-j(kz\pm\phi)}\, \underline{u}_y$$

Exercise I

1) **P.8–4** For a harmonic uniform plane wave propagating in a simple medium, both **E** and **H** vary in accordance with the factor $\exp(-j\mathbf{k}\cdot\mathbf{R})$ as indicated in Eq. (8–26). Show that the four Maxwell's equations for uniform plane wave in a source-free region reduce to the following:

$$\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H},$$

$$\mathbf{k} \times \mathbf{H} = -\omega\epsilon\mathbf{E},$$

$$\mathbf{k} \cdot \mathbf{E} = 0,$$

$$\mathbf{k} \cdot \mathbf{H} = 0.$$

Exercise I

2) **P.8–5** The instantaneous expression for the magnetic field intensity of a uniform plane wave propagating in the $+y$ direction in air is given by

$$\mathbf{H} = \mathbf{a}_z 4 \times 10^{-6} \cos\left(10^7 \pi t - k_0 y + \frac{\pi}{4}\right) \quad \text{(A/m)}.$$

a) Determine k_0 and the location where H_z vanishes at $t = 3$ (ms).
b) Write the instantaneous expression for $\mathbf{E}$.

3) **P.8–6** The E-field of a uniform plane wave propagating in a dielectric medium is given by

$$E(t, z) = \mathbf{a}_x 2 \cos(10^8 t - z/\sqrt{3}) - \mathbf{a}_y \sin(10^8 t - z/\sqrt{3}) \quad \text{(V/m)}.$$

a) Determine the frequency and wavelength of the wave.
b) What is the dielectric constant of the medium?
c) Describe the polarization of the wave.
d) Find the corresponding H-field.

(Assignment)

Extra Problems

1) Find complete expressions for $\underline{E}, \underline{H}$ of a uniform plane wave propagating in $\underline{u}_y$ direction in free space with $f = 400$ MHz , given that $\underline{E}$ has an instantaneous value of 250 mV/m at the point $y = 0.5$ m, $t = 0.2$ ns & in the direction of the vector $0.6\underline{u}_x - 0.8\underline{u}_z$.

2) A vertically polarized uniform plane wave in free space has a propagation vector $\underline{k_o} = 3\underline{u}_x + 4\underline{u}_y$ and $|E_o|_{rms} = 1$ mV/m . Find the wave frequency, hence $\underline{E}(t), \underline{H}(t)$

3) A uniform plane wave of $\lambda_o = 0.12$ m is propagating in unknown lossless material where its wavelength decreases to 8 cm, $|\underline{E}| = 50$ v/m and $|\underline{H}| = 0.1$ A/m. Find the frequency and ε_r, μ_r of this material.

Extra Problems Solutions

1)

Req $\underline{E}, \underline{H}, \underline{S}$

Answer

Propagation in +ve $y \Rightarrow \underline{E}(y) = \underline{E_0}\, e^{-jk_0 y}$

($\underline{E_0}$ perpendicular to y !)

where $\omega = 2\pi f$ & $k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi * 4 * 10^8}{3 * 10^8} = \frac{8\pi}{3}$ rad/m

$$\underline{\mathcal{E}}(y,t) = Re\{E(y)\, e^{j\omega t}\} = \underline{E_0} \cos(\omega t - k_0 y)$$

$$= \underline{E_0} \cos\left(2\pi * 4 \times 10^8 t - \frac{8\pi}{3} y\right)$$

Given $\underline{\mathcal{E}}(y,t)\big|_{y=0.5m,\ t=0.2ns} = \underline{E_0} \cos\left(8\pi * 10^8 * 0.2 * 10^{-9} - \frac{8\pi}{3} * 0.5\right) = 250(0.6\underline{u}_x - 0.8\underline{u}_z)$

$$\boxed{\underline{E_0} = -175.36\, \underline{u}_x + 233.81\, \underline{u}_z}$$

$$\therefore \underline{\mathcal{E}}(y,t) = (-175.36\, \underline{u}_x + 233.81\, \underline{u}_z) \cos\left(8\pi * 10^8 t - \frac{8\pi}{3} y\right) \text{ mV/m}$$

$$\underline{\mathcal{H}}(y,t) = \frac{1}{\eta_0}\, \underline{u}_y \times \underline{\mathcal{E}}(y,t)$$

$$= \frac{1}{120\pi}\, \underline{u}_y \times \left[(-175.36\, \underline{u}_x + 233.81\, \underline{u}_z)\right] \cos\left(8\pi * 10^8 t - \frac{8\pi}{3} y\right)$$

$$\boxed{\underline{\mathcal{H}}(y,t) = \frac{1}{120\pi}(175.36\, \underline{u}_z + 233.81\, \underline{u}_x) * \cos\left(8\pi * 10^8 t - \frac{8\pi}{3} y\right) \text{ mA/m}}$$

($\underline{u}_z \rightarrow \underline{u}_x \rightarrow \underline{u}_y$)

You may check your answer with $\underline{E} \cdot \underline{H} = 0$ (since $\underline{E} \perp \underline{H}$)

Extra Problems Solutions

2) Vertically polarized.

$\underline{K}_o = 3\,\underline{a}_x + 4\,\underline{a}_y$; $|E_o|_{rms} = 1\ mV/m$.

Find the wave freq, $E(t)$, $\underline{H}(t)$ & P_{av}.

freq :- $|K_o| = \sqrt{3^2+4^2} = 5\ rad/m$.

$$K_o = \frac{\omega}{c} = \frac{2\pi f}{c} \rightarrow f = \frac{K_o \cdot c}{2\pi}$$

$f =$ [illegible] 2.387×10^8 hertz.

$\underline{E}(t)$:- $\underline{E}(x,y) = |E_o|_{rms}\, e^{-j\underline{K}_o\cdot\underline{r}}\, \underline{a}_z\ mV_{rms}/m$.

$$= \sqrt{2} * |E_o|_{rms}\, e^{-j(3x+4y)}\, \underline{a}_z\ mV/m.$$

$$= \sqrt{2}\, e^{-j(3x+4y)}\, \underline{a}_z\ mV/m$$

$$\underline{\mathcal{E}}(x,y,t) = Re\{\underline{E}(x,y)\, e^{j\omega t}\}$$

$$= \underline{a}_z\, \sqrt{2}\, \cos(\omega t - 3x - 4y)\ mV/m.$$

$$\underline{\mathcal{H}}(x,y,t) = \frac{1}{\eta_o}\, \underline{a}_p \times \underline{\mathcal{E}}(x,y,t) \quad ; \quad \underline{a}_p = \frac{\underline{K}_o}{|K_o|} = 0.6\underline{a}_x + 0.8\underline{a}_y$$

$$\underline{\mathcal{H}}(x,y,t) = \frac{\sqrt{2}}{120\pi} \cos(\omega t - 3x - 4y)\, [(0.6\,\underline{a}_x + 0.8\,\underline{a}_y) \times \underline{a}_z]$$

$$= \frac{\sqrt{2}}{120\pi} \cos(\omega t - 3x - 4y)\, [0.8\,\underline{a}_x - 0.6\,\underline{a}_y]\ mA/m$$

Extra Problems Solutions

3)

Given :- uniform plane wave, lossless medium.

$\lambda_0 = 0.12\ m$ ← wavelength free space

$\lambda = 8\ cm$ ← wavelength in medium

$|E| = 50\ V/m$

$|H| = 0.1\ A/m$

Req :- frequency f

ε_r , μ_r of the material.

Solution :-

$$f = \frac{c_0}{\lambda_0} = \frac{3*10^8\ m/sec}{0.12\ m} = 2.5\ GHz \quad \rightarrow (1)$$

$$f = \frac{v_{ph}}{\lambda} = 2.5\ GHz \Rightarrow v_{ph} = f\lambda = 2.5\ GHz * 0.08\ m = 2*10^8\ m/sec$$

$$v_{ph} = \frac{c_0}{\sqrt{\mu_r \varepsilon_r}}$$

$$\sqrt{\mu_r \varepsilon_r} = \frac{c_0}{v_{ph}} = 1.5 \quad \rightarrow (2)$$

$$|\eta| = \left|\frac{|E|}{|H|}\right| = \sqrt{\frac{\mu}{\varepsilon}} = \frac{50}{0.1} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} = 500 \quad \rightarrow (3)$$

to get μ_r, ε_r solve (2), (3) eqn's

$$\left(\sqrt{\mu_r \varepsilon_r} = 1.5\right) * \left(\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{500}{120\pi}\right)$$

$$\mu_r = \frac{750}{120\pi} = 1.9894 \quad \text{unitless}$$

$$\varepsilon_r = 1.1309 \quad \text{unitless}$$

(unitless → no unit)