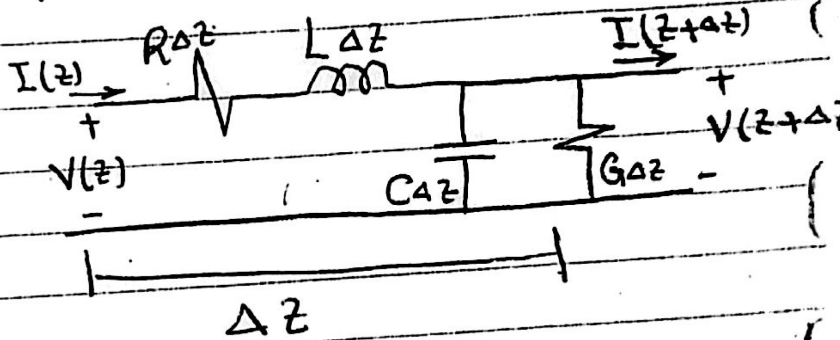
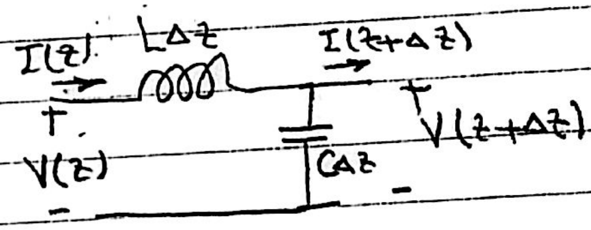
* Distrubited Parametars of T.L:

For a unit length of T.L (Δz), we will find that it has some parametars (C, L, G, R).

R: due to losses in cond.
G: " " " " dielec.



For a loss less T.L
$R=0,\ G=0$

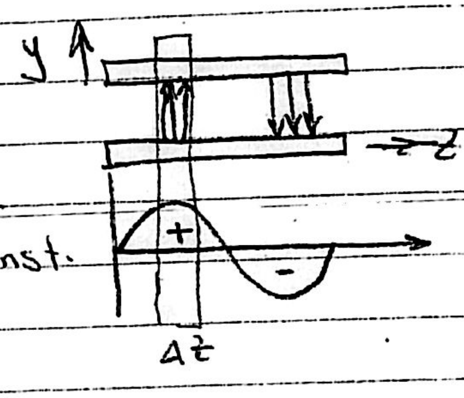


1st Calculation of C

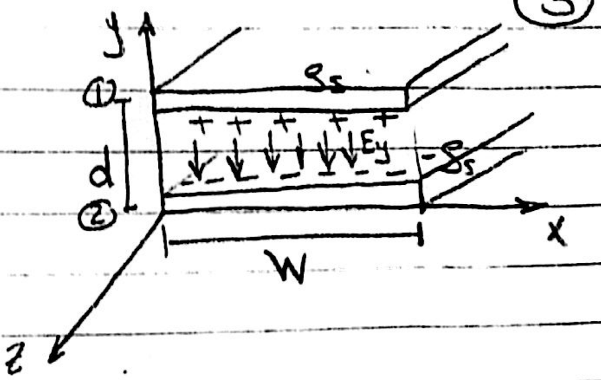
$$C = \frac{Q_1}{V_{12}}$$

To get Q, V we will use electrostatic model (why??).

We know $E = E_0 e^{-j\beta z}$
$E \propto z$ harmonically
we will take only Δz from T.L
so we can consider that E.f is const.
(electrostatic).



1st we will assume ρ_s on surfaces.



$$Q_1 = \int \rho_s \, dS \rightarrow dx\,dz$$

unit element (Δz

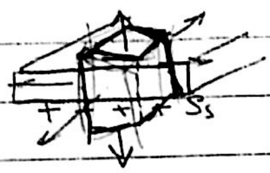
$$= \int_0^W \rho_s \, dx \cdot \Delta z$$

$$= \boxed{\rho_s \cdot W \cdot \Delta z}$$

$$V_{12} = \int_1^2 \underline{E} \cdot d\underline{l}$$

→ To Calculate $\underline{E}$ we will use Gauss Theory.

$$\oint_{S_1} \underline{D} \cdot d\underline{s} = \int \rho_s \, ds$$



only lower surface give integ. value

$$\int_0^{\Delta z}\int_0^W \epsilon E_y \, dx\,dz\,(-a_y) = \rho_s W \cdot \Delta z$$

$$-\epsilon E_y \cancel{W} \cancel{\Delta z} = \rho_s \cancel{W} \cdot \cancel{\Delta z} \Rightarrow \boxed{E_y = \frac{-\rho_s}{\epsilon}}$$

$$V_{12} = \int_d^0 \frac{-\rho_s}{\epsilon} \, dy$$

→ any path from (1→2) we choose easiest one.

$$= \boxed{\frac{\rho_s}{\epsilon} d}$$

$$C = \frac{Q_1}{V} = \frac{\cancel{\rho_s} \cdot W \cdot \Delta z}{\cancel{\rho_s} \cdot d/\epsilon} = \epsilon \cdot \frac{W}{d} \cdot \Delta z$$

$$\boxed{C' = \frac{C}{\Delta z} = \epsilon \cdot \frac{W}{d}} \quad (F/m)$$

<u>To Calculate L</u>

We take in lec. $L \cdot C = \mu \epsilon \Rightarrow \boxed{L = \mu \cdot \frac{d}{W}}$

<u>also,</u>

$$L = \frac{\Lambda}{I} = \frac{\text{Flux}}{\text{Current}}.$$

$$\Lambda = \int \underline{B} \cdot d\underline{S} = \int_0^d \mu H_x \, dy \, \Delta z$$

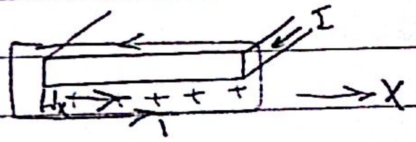
$\oint B \cdot ds = 0$

open loop

closed loop

$$= \mu H_x d \cdot \Delta z$$

$$I = \oint \underline{H} \cdot d\underline{l}$$



$$= \int_0^W H_x \, dx = H_x \cdot W$$

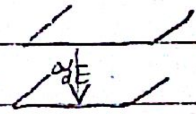
$R = \frac{L}{\sigma A} =$

$$L = \frac{\mu \cancel{H_x} d}{\cancel{H_x} \cdot W} \Delta z = \mu \cdot \frac{d}{W} \Delta z \Rightarrow$$

$$\boxed{L' = \frac{L}{\Delta z} = \mu \cdot \frac{d}{W}} \quad (H/m)$$

J_d (A/m²)

$$G = \frac{I}{V} = \frac{\sigma_d \cancel{E_y} \cdot W \cdot \Delta z}{\cancel{E_y} \cdot d} = \sigma_d \cdot \frac{W}{d} \cdot \Delta z$$



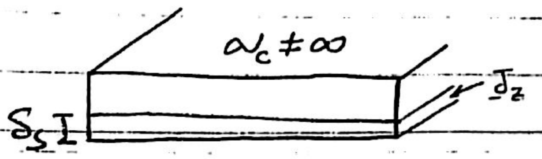
$$I = \int \underline{J_d} \cdot d\underline{S} = \int_0^W \sigma_d E_y \, dx \cdot \Delta z = \sigma_d E_y \cdot W \Delta z$$

$\underline{a_y}$ $dx\,dz\,\underline{a_y}$

$$\boxed{G' = \frac{G}{\Delta z} = \sigma_d \cdot \frac{W}{d}}$$ (S/m or $\Omega^{-1}m^{-1}$) Conductance. (5)

To Calculate R

due to $\sigma_c \neq \infty$



Due to good Conductor so surface current will move in small depth inside conductor Called "Skin depth"

$$R = \frac{L}{\sigma_c A}$$ ⇒ L: length that current move through

A: Cross section Area that current cross it.

$$R = \frac{\Delta z}{\sigma_c (W \cdot \delta_s)}$$

$R = \frac{1}{\sigma A}$ → $W \cdot \delta_s$

$$\boxed{R'_u = \frac{R}{\Delta z} = \frac{1}{\sigma_c \delta_s W}}$$ Ω/m

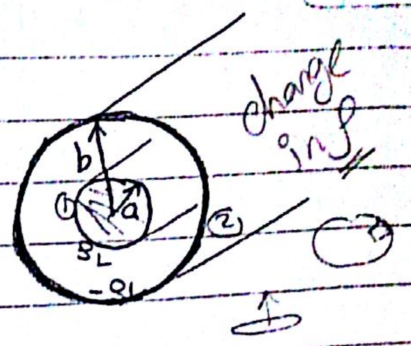
↳ for only one plate so total R

$$\boxed{R'_{tot} = \frac{2R_s}{W}}$$ $R_s = \frac{1}{\sigma_c \delta_s}$ = Sheet resistance

$\sqrt{\mu \pi f \sigma}$

$$\delta_s = \frac{1}{\sqrt{\pi f \mu \sigma_c}} , \quad R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}}$$

For Coaxial Cable:



$$① \quad C = \frac{Q}{V_{12}}$$

$$Q = \int \rho_L \, dz = \rho_L \Delta z$$

$$V_{12} = \int_1^2 \underline{E} \cdot d\underline{l} \qquad d\underline{l} = d\rho \, \underline{a}_\rho + \rho \, d\phi \, \underline{a}_\phi + dz \, \underline{a}_z$$

Gauss law to calc $\underline{E}$

$$\oint \underline{D} \cdot d\underline{s} = \int \rho_L \, dl \qquad d\underline{s} = \rho \, d\phi \, dz \, \underline{a}_\rho + d\rho \, dz \, \underline{a}_\phi + \rho \, d\rho \, d\phi \, \underline{a}_z$$

$$\int_0^{2\pi} \epsilon E_\rho \cdot \rho \, d\phi \, \cancel{\Delta z} = \rho_L \cancel{\Delta z}$$

$$\boxed{E_\rho = \frac{\rho_L}{2\pi\epsilon\rho}}$$

$$V_{12} = \int_a^b \frac{\rho_L}{2\pi\epsilon\rho} \, d\rho = \frac{\rho_L}{2\pi\epsilon} \ln\rho \Big|_a^b = \frac{\rho_L}{2\pi\epsilon} (\ln b - \ln a)$$

$$= \frac{\rho_L}{2\pi\epsilon} \ln(b/a)$$

$$C = \frac{\rho_L \Delta z}{\frac{\rho_L}{2\pi\epsilon} \ln(b/a)} = \frac{2\pi\epsilon}{\ln(b/a)} \Delta z$$

$$\boxed{C' = \frac{C}{\Delta z} = \frac{2\pi\epsilon}{\ln(b/a)}}$$

$$L'C' = \mu \epsilon$$

$$\boxed{L' = \mu \frac{\ln(b/a)}{2\pi}}$$

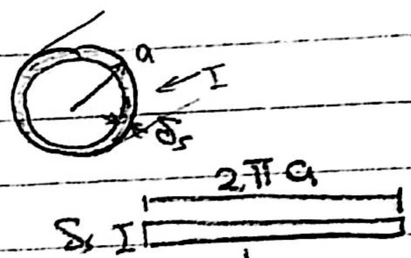
$$G'/C' = \frac{\sigma_d}{\epsilon}$$

$$\boxed{G' = \frac{2\pi \sigma_d}{\ln(b/a)}}$$

$$R' = R_{in}' + R_{out}'$$

$$R_{in}' = \frac{L}{\sigma_c A}$$

$$= \frac{\Delta z}{\sigma_c (\delta_s \cdot 2\pi a)}$$



The area that current cross.

$$R_{out} = \frac{\Delta z}{\sigma_c (\delta_s \cdot 2\pi b)}$$

$$R_{tot}' = \frac{1}{2\pi \sigma_c \delta_s} \left(\frac{1}{a} + \frac{1}{b} \right)$$

LOSSY PARALLEL-PLATE TRANSMISSION LINES

If the parallel-plate conductors have a very large but finite conductivity σ_c (which must not be confused with the conductivity σ of the dielectric medium), ohmic power will be dissipated in the plates. This necessitates the presence of a nonvanishing axial electric field $\mathbf{a}_z E_z$ at the plate surfaces, such that the average Poynting vector

$$\mathscr{P}_{\text{av}} = \mathbf{a}_y p_\sigma = \tfrac{1}{2}\mathscr{R}e(\mathbf{a}_z E_z \times \mathbf{a}_x H_x^*) \tag{9-24}$$

has a y-component and equals the average power per unit area dissipated in each of the conducting plates. (Obviously the cross product of $\mathbf{a}_y E_y$ and $\mathbf{a}_x H_x$ does not result in a y-component.)

We note in the calculation of the power loss in the plate conductors of a finite conductivity σ_c that a nonvanishing electric field $\mathbf{a}_z E_z$ must exist. The very existence of this axial electric field makes the wave along a lossy transmission line strictly not TEM. However, this axial component is ordinarily very small in comparison to the transverse component E_y. An estimate of their relative magnitudes can be made as follows:

$$\frac{|E_z|}{|E_y|} = \frac{|\eta_c H_x|}{|\eta H_x|} = \sqrt{\frac{\epsilon}{\mu}}\,|\eta_c|$$
$$= \sqrt{\frac{\omega\epsilon\mu_c}{\mu\sigma_c}} = \sqrt{\frac{\omega\epsilon}{\sigma_c}},$$

where Eq. (8–54) has been used. For copper plates $[\sigma_c = 5.80 \times 10^7 \text{ (S/m)}]$ in air $[\epsilon = \epsilon_0 = 10^{-9}/36\pi \text{ (F/m)}]$ at a frequency of 3 (GHz),

$$|E_z| \cong 5.3 \times 10^{-5}|E_y| \ll |E_y|.$$

Hence we retain the designation TEM as well as all its consequences. The introduction of a small E_z in the calculation of p_σ and R is considered a slight perturbation.