FORMULA SHEET

Time Harmonic Maxwell's Equations:

$\nabla \times \underline{E} = -j\omega\mu\underline{H}$ $\quad \nabla \cdot \underline{D} = \rho_v$ $\quad \underline{D} = \varepsilon\underline{E}$

$\nabla \times \underline{H} = j\omega\varepsilon\underline{E}$ $\quad \nabla \cdot \underline{B} = 0$ $\quad \underline{B} = \mu\underline{H}$

Universal Constants:

$\varepsilon_o = \frac{10^{-9}}{36\,\pi}$ F/m $\qquad \mu_o = 4\pi\times10^{-7}$ H/m

$c = 1/\sqrt{\mu_o\varepsilon_o} = 3\times10^8$ m/s

Uniform Plane Wave in Lossless Media:

$$\nabla^2\underline{E} + \omega^2\mu\,\varepsilon\,\underline{E} = 0$$

$\underline{E} = \underline{E_o}e^{-j\beta\,\underline{u_p}\cdot\underline{r}}$ $\qquad \underline{H} = \frac{1}{\eta}\,\underline{u_p}\times\underline{E}$

$\beta = \omega\sqrt{\mu\varepsilon}$ $\qquad \lambda = 2\pi/\beta$

$\eta = \sqrt{\mu/\varepsilon}$ $\qquad u_{ph} = 1/\sqrt{\mu\varepsilon}$

Propagation in Lossy Media (General):

$\underline{E} = \underline{E_o}e^{-\gamma\,\underline{u_p}\cdot\underline{r}}$ $\qquad \gamma = \alpha + j\beta$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right)^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right)^{1/2}$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon(1-j\sigma/\omega\varepsilon)}}$$

Low Loss Dielectric

$$\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta \approx \omega\sqrt{\mu\varepsilon}\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)$$

$$\eta_c \approx \sqrt{\frac{\mu}{\varepsilon}}\left(1+j\,\frac{\sigma}{2\omega\varepsilon}\right)$$

Good Conductors:

$$\alpha = \beta \approx \sqrt{\sigma\mu\pi f} = \frac{1}{\delta}$$

$$\eta_c \approx (1+j)\frac{1}{\sigma\delta}$$

$$u_{ph} \approx \sqrt{\frac{2\omega}{\mu\sigma}}$$

General T.L. Equations

$$V(z) = V_o^+e^{-\gamma z} + V_o^-e^{\gamma z}$$

$$I(z)\,Z_o = V_o^+e^{-\gamma z} - V_o^-e^{\gamma z}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma_L|e^{j\theta_\Gamma}$$

$$S = \left|\frac{V_{max}}{V_{min}}\right| = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$Z_{in} = Z_o\frac{Z_L + Z_o\tanh(\gamma\ell)}{Z_o + Z_L\tanh(\gamma\ell)}$$

$$\Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o}$$

Distributed Parameters of T.L.:

Parameter	Parallel Plate	Coaxial	Units
R	$\frac{2}{w}\sqrt{\frac{\pi f\mu_c}{\sigma_c}}$	$\frac{1}{2\pi\sigma_c\delta}\left(\frac{1}{a}+\frac{1}{b}\right)$	Ω/m
L	$\frac{\mu d}{w}$	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	H/m
G	$\frac{\sigma_d w}{d}$	$\frac{2\pi\sigma_d}{\ln(b/a)}$	$(\Omega m)^{-1}$
C	$\frac{\varepsilon w}{d}$	$\frac{2\pi\varepsilon}{\ln(b/a)}$	F/m

Voltage & Current Waves:

$$V(z') = \frac{I_L}{2}(Z_L + Z_o)e^{\gamma z'}\left[1+\Gamma_L\;e^{-2\gamma z'}\right]$$

$$= \frac{V_g Z_o}{Z_g + Z_o}e^{-\gamma(\ell - z')}\frac{1+\Gamma_L\;e^{-2\gamma z'}}{1-\Gamma_g\Gamma_L\;e^{-2\gamma\ell}}$$

$$I(z') = \frac{I_L}{2Z_o}(Z_L + Z_o)e^{\gamma z'}\left[1-\Gamma_L\;e^{-2\gamma z'}\right]$$

$$= \frac{V_g}{Z_g + Z_o}e^{-\gamma(\ell - z')}\frac{1-\Gamma_L\;e^{-2\gamma z'}}{1-\Gamma_g\Gamma_L\;e^{-2\gamma\ell}}$$