

FORMULA SHEET

Time Harmonic Maxwell's Equations:

$$\begin{aligned}\underline{\nabla} \times \underline{E} &= -j\omega\mu\underline{H} & \underline{\nabla} \cdot \underline{D} &= \rho_v & \underline{D} &= \varepsilon\underline{E} \\ \underline{\nabla} \times \underline{H} &= j\omega\varepsilon\underline{E} & \underline{\nabla} \cdot \underline{B} &= 0 & \underline{B} &= \mu\underline{H}\end{aligned}$$

Uniform Plane Wave in Lossless Media:

$$\begin{aligned}\nabla^2 \underline{E} + \omega^2 \mu \varepsilon \underline{E} &= 0 \\ \underline{E} &= \underline{E}_0 e^{-j\beta \underline{u}_p \cdot \underline{r}} & \underline{H} &= \frac{1}{\eta} \underline{u}_p \times \underline{E} \\ \beta &= \omega \sqrt{\mu \varepsilon} & \lambda &= 2\pi / \beta \\ \eta &= \sqrt{\mu / \varepsilon} & u_{ph} &= 1 / \sqrt{\mu \varepsilon}\end{aligned}$$

Universal Constants:

$$\begin{aligned}\varepsilon_0 &= \frac{10^{-9}}{36\pi} \text{ F/m} & \mu_0 &= 4\pi \times 10^{-7} \text{ H/m} \\ c &= 1 / \sqrt{\mu_0 \varepsilon_0} = 3 \times 10^8 \text{ m/s}\end{aligned}$$

Propagation in Lossy Media (General):

$$\begin{aligned}\underline{E} &= \underline{E}_0 e^{-\gamma \underline{u}_p \cdot \underline{r}} & \gamma &= \alpha + j\beta = jk_c \\ \alpha &= \omega \sqrt{\frac{\mu \varepsilon}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon} \right)^2} - 1 \right)^{1/2} \\ \beta &= \omega \sqrt{\frac{\mu \varepsilon}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon} \right)^2} + 1 \right)^{1/2} \\ \eta_c &= \sqrt{\frac{\mu}{\varepsilon(1 - j\sigma/\omega\varepsilon)}}\end{aligned}$$

Low Loss Dielectric

$$\begin{aligned}\alpha &\approx \frac{\sigma}{2} \sqrt{\frac{\mu}{\varepsilon}} \\ \beta &\approx \omega \sqrt{\mu \varepsilon} \left(1 + \frac{1}{8} \left(\frac{\sigma}{\omega \varepsilon} \right)^2 \right) \\ \eta_c &\approx \sqrt{\frac{\mu}{\varepsilon}} \left(1 + j \frac{\sigma}{2\omega \varepsilon} \right)\end{aligned}$$

Good Conductors:

$$\begin{aligned}\alpha &= \beta \approx \sqrt{\sigma \mu \pi f} = \frac{1}{\delta} \\ \eta_c &\approx (1 + j) \frac{1}{\sigma \delta} \\ u_{ph} &\approx \sqrt{\frac{2\omega}{\mu \sigma}}\end{aligned}$$

Reflection & Transmission (Normal 1 -> 2)

$$\begin{aligned}\Gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \\ T &= 1 + \Gamma = \frac{2\eta_2}{\eta_2 + \eta_1} \\ S &= \frac{1 + |\Gamma|}{1 - |\Gamma|} \\ Z_{in}(d) &= \eta_1 \frac{\eta_2 + j\eta_1 \tan(\beta_1 d)}{\eta_1 + j\eta_2 \tan(\beta_1 d)}\end{aligned}$$

Reflection & Transmission (Oblique 1 -> 2)

$$\begin{aligned}\theta_i &= \theta_r & \beta_1 \sin \theta_i &= \beta_2 \sin \theta_t \\ \Gamma_{\perp} &= \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \\ T_{\perp} &= \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \\ \Gamma_{\parallel} &= \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} \\ T_{\parallel} &= \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}\end{aligned}$$

Critical & Brewster Angles:

$$\begin{aligned}\sin \theta_c &= \sqrt{\varepsilon_2 / \varepsilon_1} \\ \sin^2 \theta_{B\perp} &= \frac{1 - \mu_1 \varepsilon_2 / \mu_2 \varepsilon_1}{1 - (\mu_1 / \mu_2)^2} = \frac{1 - (\eta_1 / \eta_2)^2}{1 - (\mu_1 / \mu_2)^2} \\ \sin^2 \theta_{B\parallel} &= \frac{1 - \mu_2 \varepsilon_1 / \mu_1 \varepsilon_2}{1 - (\varepsilon_1 / \varepsilon_2)^2} = \frac{1 - (\eta_2 / \eta_1)^2}{1 - (\varepsilon_1 / \varepsilon_2)^2}\end{aligned}$$