FORMULA SHEET

Time Harmonic Maxwell's Equations:

$\nabla\times\underline{E} = -j\omega\mu\underline{H}$ $\quad \nabla\cdot\underline{D} = \rho_v$ $\quad \underline{D} = \varepsilon\underline{E}$

$\nabla\times\underline{H} = j\omega\varepsilon\underline{E}$ $\quad \nabla\cdot\underline{B} = 0$ $\quad \underline{B} = \mu\underline{H}$

Universal Constants:

$\varepsilon_o = \frac{10^{-9}}{36\,\pi}$ F/m $\quad \mu_o = 4\pi\times10^{-7}$ H/m

$c = 1/\sqrt{\mu_o\varepsilon_o} = 3\times10^8$ m/s

Uniform Plane Wave in Lossless Media:

$$\nabla^2\underline{E} + \omega^2\mu\,\varepsilon\,\underline{E} = 0$$

$\underline{E} = \underline{E_o}e^{-j\beta\;\underline{u_p}\cdot\underline{r}}$ $\quad \underline{H} = \frac{1}{\eta}\,\underline{u_p}\times\underline{E}$

$\beta = \omega\sqrt{\mu\varepsilon}$ $\quad \lambda = 2\pi/\beta$

$\eta = \sqrt{\mu/\varepsilon}$ $\quad u_{ph} = 1/\sqrt{\mu\varepsilon}$

Propagation in Lossy Media (General):

$\underline{E} = \underline{E_o}e^{-\gamma\;\underline{u_p}\cdot\underline{r}}$ $\quad \gamma = \alpha + j\beta = j\mathrm{k_C}$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right)^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right)^{1/2}$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon(1 - j\sigma/\omega\varepsilon)}}$$

Low Loss Dielectric

$$\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta \approx \omega\sqrt{\mu\varepsilon}\left(1 + \frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)$$

$$\eta_c \approx \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right)$$

Good Conductors:

$$\alpha = \beta \approx \sqrt{\sigma\mu\pi f} = \frac{1}{\delta}$$

$$\eta_c \approx (1+j)\frac{1}{\sigma\delta}$$

$$u_{ph} \approx \sqrt{\frac{2\omega}{\mu\sigma}}$$

Reflection & Transmission (Oblique 1 -> 2)

$\theta_i = \theta_r$ $\quad \beta_1\sin\theta_i = \beta_2\sin\theta_t$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\mathrm{T}_\perp = \frac{2\,\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\mathrm{T}_\parallel = \frac{2\,\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

Reflection & Transmission (Normal 1 -> 2)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\mathrm{T} = 1 + \Gamma = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$Z_{in}(d) = \eta_1\frac{\eta_2 + j\eta_1\tan(\beta_1 d)}{\eta_1 + j\eta_2\tan(\beta_1 d)}$$

Critical & Brewster Angles:

$$\sin\theta_c = \sqrt{\varepsilon_2/\varepsilon_1}$$

$$\sin^2\theta_{B\perp} = \frac{1 - \mu_1\varepsilon_2/\mu_2\varepsilon_1}{1 - (\mu_1/\mu_2)^2} = \frac{1 - (\eta_1/\eta_2)^2}{1 - (\mu_1/\mu_2)^2}$$

$$\sin^2\theta_{B\parallel} = \frac{1 - \mu_2\varepsilon_1/\mu_1\varepsilon_2}{1 - (\varepsilon_1/\varepsilon_2)^2} = \frac{1 - (\eta_2/\eta_1)^2}{1 - (\varepsilon_1/\varepsilon_2)^2}$$