ON MIXING ENHANCEMENT AND SOUND EMISSION OF HIGH SPEED JET FLOWS

Mohammed Khalil Ibrahim Khalil
In The Name of Allah
The Most Beneficent And Most Merciful
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by

Mohammed Khalil Ibrahim Khalil

B.Sc., Cairo University, 1991
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ABSTRACT

Mixing enhancement by the use of passive control in a form vortex generators of vane-type tabs were placed at diametrically opposite locations along the circumference of a circular supersonic jet nozzle exit. A nozzle with a design Mach number of 1.33 was used for the present investigation. The effects of the vane-type tab on both flow field and acoustic field were studied experimentally. Two configurations of vane-type tabs, i.e., stationary and free-to-rotate, were studied for different values of the nozzle pressure ratio, which is defined as the ratio of the jet total pressure to the ambient pressure. Pitot probe surveys show that the vane-type tabs generate streamwise vortices, or tip vortices, which spread the jet flow more than the baseline jet in terms of the decay of centerline velocity. Acoustic measurements show that the overall sound pressure level, OASPL, was significantly reduced up to 10 dB and screech tones were also suppressed compared with the baseline jet with a thrust penalty of about 1.5 ∼ 2% per vane. Parameters related with the vane-type tabs are protrusion height into the flow, vane angle, and the number of vanes. Parametric study has been systematically performed as well as the comparison with others previously published tabs configuration.
Another approach to enhance the mixing is proposed, which is based on lateral steady/unsteady injection of an array of micro-jets, placed along the circumference of the nozzle exit of a primary jet at equal azimuthal angle intervals. This was studied experimentally to clarify characteristics of mixing and noise of compressible primary jets. Specifically, two modes of the micro-jets were investigated: symmetric and asymmetric, or flapping, injections. Fully-expanded and under-expanded primary jets exhausted from a convergent, or sonic, nozzle are considered in the present study. The unsteady micro-jets were injected at a Strouhal number, \( St \), of 0.16, based on the nozzle exit diameter and the velocity at the nozzle exit, which is close to one of sub-harmonics of the most amplified Strouhal number, for two cases of total unsteady mass injection: 4% and 6% of primary jet mass flow rate. Results of the mean flow field showed that the antisymmetric injection has a higher spreading rate than the steady and unsteady axisymmetric injections in terms of centerline velocity decay. Even with 50% reduction in the unsteady mass injection, the antisymmetric mode was excited, which persisted even downstream of the potential core region. Those results were confirmed from a linear stability analysis of a fully-expanded jet, which showed that antisymmetric modes for natural disturbances are more unstable in the downstream region than the corresponding axisymmetric modes. Moreover, reduction in the radiated noise was observed in the case of steady axisymmetric injection. Thus, these results suggest that these micro-jets have the potential for future use as a device for shear flow control.

Finally, the linear stability analysis is applied to investigate the onset of the transition in shear layer of compressible jet. Specifically we apply the conventional \( e^N \) method to predict transition in the free shear layer flow. Mean velocity profiles of a compressible circular jet were experimentally obtained at different downstream locations, which were used as an input data for the linear stability calculation code. The growth rates of disturbances given by the linear stability analysis were integrated to obtain an envelope
curve. On the other hand, the transition was detected experimentally by the use of three techniques: 1) oil flow visualization, 2) microphone measurement of pressure fluctuations, 3) hot wire anemometer measurement of velocity fluctuations. The results from both the microphone and the hot-wire anemometer measurements showed that the transition starts at about 1.333 $D$ from the nozzle exit, where $D$ is the nozzle diameter, which is near the “apparent origin” in the fully developed jet flow. Correlation between spatial stability analysis and experimental results showed that the value of the factor $N$ in the $e^N$ method is about 3.5 for axisymmetric ($n = 0$) and 2.8 for asymmetric ($n = 1$) disturbance modes in the present compressible jet. The author believes that this length play an important role in the screech tone generation mechanism because the shear layer, up to transition point, has higher receptivity to environmental disturbances than turbulent shear layer flow.
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\(A\) : root mean square value of disturbances

\(a\) : speed of sound

\(c\) : chord length of vane-type tab

\(C_1, C_2\) : complex constants

\(C_g\) : group velocity

\(c_p\) : specific heat at constant pressure

\(c_v\) : specific heat at constant volume

\(D\) : nozzle diameter

\(D_t\) : nozzle throat diameter

\(e\) : internal energy

\(f\) : injection frequency of micro jets

\(f_s\) : screech frequency

\(i\) : \(\sqrt{-1}\)

\(I_n\) : modified Bessel’s function of first kind of order \(n\)
\( K_n \): modified Bessel’s function of second kind of order \( n \)

\( M \): jet Mach number

\( M_D \): nozzle design Mach number

\( M_j \): fully expanded jet Mach number; Mach number a jet will have when isentropically expanded to ambient pressure

\( N \): factor of \( e^N \) method

\( n \): azimuthal wave number of disturbance; one of jet instability modes

\( N_v \): number of vane-type tabs

\( p \): pressure

\( P_t \): jet total pressure, or chamber pressure

\( P_{t2} \): total pressure after normal shock wave

\( Pr \): Prandtl number

\( R_{0.5} \): jet radius, that is defined as the radial length from the axis to the location where the velocity becomes half of the centerline velocity; i.e., \( U(R_{0.5}) = U_c/2 \)

\( Re \): Reynolds number

\( St \): Strouhal number, \( fD/U \)

\( T \): temperature

\( t \): time

\( T_t \): jet total temperature, or chamber temperature
$t_v$ : thickness of vane-type tab

$U$ : velocity

$v$ : fluctuating quantity such as velocity, pressure, density, and temperature

$v_x, v_r, v_\phi$ : axial, radial, and angular velocity components

$w$ : tab protrusion length

$x, r, \phi$ : axial, radial, and azimuthal coordinates of cylindrical coordinates

$x, y, z$ : Cartesian coordinates

$\alpha$ : $\alpha = \alpha_r + i\alpha_i$

$\alpha_i$ : axial spatial growth rate of disturbance

$\alpha_r$ : axial wave number of disturbance

$\alpha_v$ : vane-type tab angle measured from the jet axis

$\bar{R}$ : the specific gas constant

$\gamma$ : specific heat ratio, $\gamma = c_p/c_v$

$\Omega$ : driving motor’s rotational speed in revolution per minute

$\omega$ : radian frequency of disturbance

$\psi$ : sound emission angle measured from the downstream jet axis

$\rho$ : density

$\tau$ : period of injection cycle

$\theta$ : momentum thickness
\( \phi_j \): injected mass fraction that is defined as the ratio of injected mass to primary jet mass flow rate

\( P_{\mu} \): total pressure of micro jet, or micro-chamber pressure

**Subscripts**

0 : initial value

\( j \): station number in the axial direction

\( k \): station number in the radial direction

\( i \): imaginary part

\( o \): jet centerline

\( r \): real part

\( t2 \): total pressure after normal shock wave

\( \infty \): ambient or in the far field

**Superscripts**

\( - \): time average

\( ' \): fluctuating component

\( \sim \): amplitude of fluctuation component

**Abbreviations**

AIAA : American Institute of Aeronautics and Astronautics

CAA  : computational aeroacoustics
CFD : computational fluid dynamics
DNS : direct numerical simulation
EPNdb : effective perceived noise levels
FFT : fast Fourier transform
GPIB : general purpose interface bus
HPF : high pass-filter
JSASS : Japan Society for Aeronautics and Space Science
JSFM : Japan Society of Fluid Mechanics
JSME : Japan Society of Mechanical Engineers
LES : large eddy simulation
MEMS : micro electro-mechanical systems
NASA : National Aeronautics and Space Administration
NPR : nozzle pressure ratio
NPR$_\mu$ : nozzle pressure ratio of microjets
OASPL : overall sound pressure level
RANS : Reynolds average Navier Stockes
SPL : sound pressure level
Chapter 1

Introduction

1.1 Research Background

Since the first commercial turbojets were introduced into service following World War II, jet noise levels from subsonic transports have been reduced substantially. This decrease in jet noise represents the combined outcome of extensive research into jet noise suppression and increased engine bypass ratios, which led to reduced jet velocity and, hence, to lower noise intensity as shown in Fig 1.1. The developments of the latter in the engine cycle also produced greater propulsive efficiency and reduced fuel burn. Thus, engine noise has been reduced simultaneously with greater fuel economy. However, jet noise still a significant noise source during full power takeoff, especially for derivative growth airplanes that require larger levels of thrust by using the same engine. In fact, engine system noise studies [1] indicate that, at both take off and landing operation the jet noise have a considerable contribution to the total flyover noise signature even if noise suppression due to acoustic liners is included, as shown in Fig. 1.2. Therefore, continued advances in jet noise reduction are necessary if the long-term goal of 20 dB reduction in total aircraft noise is to be met. Jet noise has also been a major stumbling block in the development of supersonic transports, in addition to being a hindrance to the widespread
1.1. RESEARCH BACKGROUND

Normalized to 100,000 lb thrust
Noise levels are for airplane/engine configurations at time of initial service

First generation turbofan

Second generation turbofan

Figure 1.1: History of sideline noise levels at time of aircraft certification (adapted from ref. [2]).

acceptance of jet powered general aviation aircraft [2].

1.1.1 Sound Generation Mechanisms in a Supersonic Jet

Flow characteristics of supersonic jets are different depending on whether the pressure at the nozzle exit is greater than (underexpanded), less than (overexpanded), or equal to the ambient pressure surrounding the jet (fully expanded). Unless a supersonic jet issuing from a convergent-divergent nozzle is operated very close to the nozzle design condition, its noise spectrum invariably consists of discrete and broadband components. (see Fig. 1.3 which is based on ref. [3]) The discrete components are commonly referred to as screech tones or feedback tones as illustrated by the small shaded areas in Fig. 1.3. For imperfectly expanded supersonic jets with rather strong shock cells, the screech tone is often accompanied by its harmonics. In some cases such as underexpanded jets from a convergent nozzles, as many as four harmonics have been observed. The screech
component disappears when the jet is perfectly expanded. For perfectly expanded jets the noise spectrum is made up of a broad, smooth peak, as illustrated by the peaks of the lower four curves of Fig. 1.3. This broadband noise component is generated by the turbulence in the mixing layers of the jet. For this reason it is called the turbulent mixing noise. If the ratio of the chamber to ambient pressure of the jet is changed so that the jet is operated in an off-design condition, then experiments show that additional broadband noise would be emitted. This noise component, which owes its origin to the presence of a quasi-periodic shock cell structure inside the jet plume, is known as broadband shock-associated noise, or simply shock-associated noise as illustrated by the large shaded areas of Fig 1.3. The dominant part of the broadband shock-associated noise is comprised of spectral peak with a relatively narrow half width. One of the most peculiar characteristics of broadband shock-associated noise is that the frequency of the
spectral peak is a function of the direction of radiation. The spectral peak frequency becomes lower near the jet inlet direction and increases monotonically toward the jet flow direction. Recently, after a careful examination of all available narrow-band shock associated noise spectra, it was pointed out that the fundamental screech tone frequency is always smaller than the frequencies of broadband shock noise [4]. As a matter of fact, the screech tone frequency is a very reliable indicator of the lower limit of the shock-associate noise spectrum [5].

Experimental data indicate that the noise is generated by different mechanisms. These mechanisms are associated with the complex flow of a supersonic jet shown in Fig. 1.4. One group of sound-generating mechanisms in the supersonic jet is due to the interaction of flow disturbances with shock waves. The elements of this group that model the sound generated by the interaction of flow disturbances with shock waves are illustrated in Fig 1.4. These elements are 1) the interaction of a vortex with a plane shock wave and 2) the interaction of a plane sound wave with a shock. Other sound generating mechanisms are not directly associated with the presence of shock waves, they are also illustrated in Fig 1.4. One is due to a wavy-plume structure that is modeled by flow past a wavy wall. This models the sound component generated by large scale structures within the jet plume. Clearly, Mach waves are expected to be generated by flow past a wavy wall. In the model, these Mach waves are assumed to be analogous to the eddy Mach wave radiated from supersonic jets. The vortex-vortex is another interaction mechanism of sound generation. This model is presented in Fig 1.4 where the interaction of turbulent structures is illustrated within the jet. This is believed to be responsible for the background noise of supersonic jet flows [6].

It is well known that the noise from supersonic jets is dominated by large-scale unsteady flow structures. Hence, there has been some success achieved in noise prediction
Figure 1.3: Jet noise spectra for a convergent nozzle at subsonic and supersonic velocities; angle from downstream jet axis 80°. Based on data presented in Ref. [3].
by using wave instability models [5]. For overexpanded and underexpanded jets, interaction of these large-scale structures with the shock cells leads to an additional source of broadband noise. Propagating disturbance upstream with ensuing receptivity by the shear layer near the nozzle exit lip can make a feedback loop, which leads to an additional source of noise, a resonant screech tone at a selected frequency.

In contrast to the supersonic case, the underlying physics of noise generation at subsonic jet speeds is not well understood. In principle, The Lighthill’s acoustic analogy [7] provides an elegant means to relate the unsteadiness of the jet to the far-field sound. Unfortunately, predictive use of the acoustic analogy is based on the presence of detailed turbulent statistics (i.e., fourth-order space-time velocity correlations), which are difficult to measure or compute. Typically, various ad hoc assumptions have been made to enable jet noise predictions in an engineering context [8]. The current belief is that, even though large-scale energetic flow structures dominate the mean-flow development and

Figure 1.4: Fundamental sound-generating mechanisms of supersonic jet flow [6].
also control the rate of energy transfer to small scales of turbulence, the dominant source of jet noise is associated with somewhat smaller scales of turbulence. However, given the tremendous difficulty in either measuring or computing the small-scale structures, there has been no direct evidence to support this belief.

1.1.1.1 Experimental Approach

Clearly, the capability to measure, process, and extract the missing statistical information will represent a major breakthrough toward improved understanding, prediction, and hence reduction of subsonic jet noise. A breakthrough experiment of this kind represents a daunting task at present, requiring significant further advances in instrumentation technology both in terms of hardware development and its implementation in the environment of high-speed, heated jets. Coupled with concomitant breakthroughs in actuator and sensor technologies, this would likely enable the development of a range of active control strategies and improved jet noise prediction methods. In general, present jet noise predictions are based on an experimental databank, with theoretical methods used to guide interpolation and, in some cases, extrapolation of the measured data. The new approach would facilitate a primary role for (physics based) computational techniques that have been calibrated against experimental databases [2].

1.1.1.2 Computational Approach

Crucial to the above development would be the ability to perform numerical simulations of jet noise. During the past decade, there have been several exciting developments across a broad spectrum of prediction methodologies for jet noise. These developments include engineering methods based on a steady RANS solution for the jet mixing region [9] as well as numerical techniques based on direct numerical simulation (DNS) and large
eddy simulation (LES) computation coupled with an accurate noise propagation scheme such as some form of Lighthill’s acoustic analogy or DNS itself [10, 11, 12]. Good progress is currently being made with CAA techniques, such as involving screech simulations, that were able to match many of the features observed experimentally [13].

Currently, CFD methods are well-positioned for predicting the mean flow field in the turbulent flow of static circular jets at high Reynolds numbers over a range of jet exhaust Mach numbers and exit temperatures, as well as in jets of more complex geometry. However, for the purpose of noise prediction, further work is required to provide both steady and unsteady predictions under flight conditions, which would include the aerodynamic and acoustic effects of interference between the jet and the aircraft surfaces (i.e., scattering of jet noise by the wing trailing edge). Overall, it is important to extend calculations to provide good resolution of sound wave in the frequency range, i.e., full scale from 2 to 15 kHz, to provide accurate data for the prediction of aircraft noise in units of EPNdB. Predictions for takeoff and landing need to cover a range of altitudes and attack angles in the flight plane, and to cover combinations of jet and flight speeds. Of particular importance are the spectra at various angles in the flight plane and for non-axisymmetric jets at different azimuthal angles. This presents a great challenge for laboratory measurements and computer simulations. This objective could not be completed with the use of steady RANS flow solutions only. Nevertheless, the goal is a fast flow solver that provides the mean flow data as well as distributions of the turbulent kinetic energy and the length scale of the energy containing eddies, combined with a filter function or the equivalent for the prediction of spectra and the SPL.

Because these approaches focus on elucidating the underlying physics of jet noise, they will also become the foundation for further development of effective active noise control strategies. Thus long-range research has to combine continued development of noise
reduction strategies based on physical reasoning, experimental tests, and computational prediction development. As discussed below, a few concepts have been proposed (and even validated) in this regard.

1.1.2 Recent Studies on the Control of Jet Flow and its Noise

1.1.2.1 Passive Concepts

Currently established passive concepts for jet noise reduction include those based on geometric modifications such as multielement suppressors, ejectors, and annular plugs as well as aerothermodynamic concepts involving an inverted velocity or temperature profile [8]. With the exception of the inverted temperature profile, which serves as a thermal acoustic shield by refracting sound away from an observer or via multiple reflections within the jet, all of the above concepts reduce noise through a combination of enhanced mixing (i.e., rapid velocity decay, which reduces low-frequency noise), reduced characteristic jet dimension, or increased mean shear near the nozzle exit plane (which increases the high frequency noise that is more easily attenuated via atmospheric absorption). Also, long duct mixed flow nacelles can add extra area for acoustic treatment application. While the acoustic merit of these concepts has been demonstrated, design tradeoffs against potential aerodynamic penalty and implementation issues, such as effect on cruise performance has to be considered. Most of these ideas might have some limit in actual application.

An alternative class of passive devices for jet noise reduction involves trailing edge devices such as tabs, shown in Fig. 1.5, (small protrusions at the trailing edge of a nozzle) that enhance mixing by introducing axial vorticity into the jet. For high subsonic jets at the model scale, tabs have shown reductions in peak noise levels; however, this is accompanied by an increase in high frequency noise. When calculated in an integrated perceived noise level, which is relevant for full scale application, this result shows an
increase in noise [14]. On the other hand, for supersonic jets, tabs (or roughness at the nozzle exit plane) are known to be effective in reducing the screech noise from imperfectly expanded supersonic jets by disrupting the feedback loop responsible for such cycles [15]. Other aspects regarding understanding, prediction, and reduction of screech are thoroughly discussed by Raman [16].

Another trailing edge device that has been developed in recent years is the chevron, which introduces longitudinal vorticity in the shear layer. Chevron devices, shown in Fig. 1.5, have been shown to increase mixing [17] and to reduce noise by up to 2.5 EPNdB with minimal loss in performance [18]. Considerable work will be required to optimize the performance, i.e., aerodynamics, and acoustics of the chevron. To do that, the fundamental of the chevron mechanism should be understood, which will aid that optimization.

Recently a concept has been proposed for passive reduction of jet noise. That is the suspension of a flexible filament in the jet-mixing region [19]. It is relatively easy to implement and has been shown to be quite effective in reducing both screech tones and broadband shock noise, as shown in Fig 1.6. Other findings (albeit of a preliminary nature) suggest that filament suspension may not be as effective at subsonic jet speeds. Specifically, Simonich et al. [14] found that reductions of 1 to 2 dB in the low- to mid-frequency range were accompanied by similar increases in the high-frequency range, leading to higher perceived noise levels for a few cases. Based on the mixed results obtained thus far, this approach needs further investigation. For instance, the variability of noise reduction levels with filament characteristics (such as type, length, and diameter) indicates that further work is necessary not only to clarify the physical role of the filament in noise suppression, but also to optimize this concept more completely. Near-term investigations should focus on optimization efforts in the subsonic regime, using a com-
Figure 1.5: Passive control of jet noise suppression: a) Separate flow exhaust nozzle (baseline); b) Nozzle with alternating chevron \cite{17}; c) Chevron-tab nozzle \cite{14}; d) Internally mixed flow exhaust nozzle \cite{8}.
1.1. RESEARCH BACKGROUND

A novel application of active noise reduction aims to use water injection in small quantities to reduce supersonic jet noise [20]. This approach is predicated on being able to transfer a part of the turbulent kinetic energy in the jet to water droplets so as to reduce far-field noise. This particular mechanism is quite different from that of previous investigations [21] where extremely small water droplets (order of 1 micron) were used to attenuate fan tone noise through evaporation effects. Water injection in massive quantities has been routinely used for the reduction of jet noise from launch vehicles [22]. For the rocket noise application, similar reductions have been obtained using the alternative technique of directing the exhaust gas into a body of water [23].

Due to the inherent need for water supply, transition of the water injection concept to real exhaust systems will be contingent upon tradeoffs between the quantity of water required and the extent of noise benefit derived. Nevertheless, coordinated analytical and experimental research on this topic will add to the current knowledge of source and mechanism responsible for jet noise, and also lead to the development of a practical suppression concept.

Papamoschou et al. [24] has experimentally demonstrated directional noise suppression from high-speed jets using an asymmetric parallel secondary flow around a primary jet. The secondary flow suppressed Mach wave radiation directed towards the ground, leaving unaltered the Mach waves propagated toward the heavens, as shown in Fig. 1.7. An eccentric nozzle configuration with a Mach 1.5 inner flow and a Mach 1.0 outer flow
a) Baseline jet (without filament).

b) Jet with a 3 diameters long filament.

c) Spectra of sound pressure level (SPL).

Figure 1.6: Noise Reduction of underexpanded supersonic jet by a flexible filaments [19].
produced noise suppression superior to that from concentric arrangements or from the fully-mixed equivalent jet.

Active control of jet noise, via unsteady actuators mounted near the nozzle exit, is an attractive option from the standpoint of maintaining an optimal aerodynamic performance under a wide range of operating conditions. Despite its demonstrated ability to increase jet mixing, active jet flow control has generally proved unsuccessful at reducing noise and has actually increased far-field noise in some cases [25]. Active control of jet noise is clearly in its infancy and would require significant advances in actuation systems, control algorithms, and measurement techniques in order to realize its powerful potential. Glow discharge devices [26], Helmholtz resonators [27], MEMS [28], and fluidic injection [29, 25] (see Figs. 1.8, 1.9, and 1.10) are all actuation systems that indicate promise in controlling jet flow. In addition to addressing technical issues relevant to these forms of control, there is a general need to examine the robustness of such devices in the harsh environment of high-temperature jet exhaust. Due to limited access for sensors, closed loop jet noise control poses significant challenges. Since ongoing work on optical sensors may, however, remedy the need for remote flow diagnostics, there is a need to assess the relative noise reduction potential of closed loop control (in comparison to open loop techniques) in laboratory experiments.

1.2 Objectives of the Present Thesis

As suggested by the title of the present thesis, the main objective is to develop a passive and/or active control technique to enhance underexpanded supersonic jet mixing and reduces its associate noise emission. Prediction of related issues to jet flow or jet noise field is considered also in the present thesis.
a) Baseline jet, emitting Mach waves.

b) Asymmetric secondary flow eliminates downstream Mach Waves.

Figure 1.7: Effect of secondary flow on noise and mean flow of high-speed jet [24].
1.2. OBJECTIVES OF THE PRESENT THESIS

Passive control technique in the form of free-to-rotate vane-type tabs has been proposed. Measurements of the flow field, acoustic field, and thrust penalty of the proposed free-to-rotate vane-type tabs have been performed. Comparisons with similar stationary vane-type tabs as well as other well documented types of tabs (simple or rectangular tabs and delta tabs) are also considered in the present thesis.

Toward achieving and active flow control scheme, steady and unsteady lateral injection of micro jets actuator is proposed. The objective is to study the effects of a different types of micro jets actuation on sonic and supersonic underexpanded jets. Unsteady asymmetric actuation or flapping actuation, unsteady axisymmetric actuation, and steady axisymmetric actuation with different injected mass fraction are studied, and their effects on the flow and noise field will be reported. A theoretical interpretation of the results based on linear stability theory will be made.

It is believed that the jet screech tones in underexpanded supersonic jets are caused by an initial flow disturbance, which issue from the nozzle exit and propagates downstream where it interacts with the shock cell system to generate noise. The resulting noise is
Figure 1.9: Control of supersonic impinging jet flows using microjets: a) Schematic of the experimental arrangement; b) Instantaneous shadowgraph images, no control; c) Instantaneous shadowgraph images, with microjets control [28].
assumed to radiate upstream outside the jet flow to produce another disturbance as it intersects the nozzle lip where the jet boundary layer is laminar and has high receptivity to external disturbance.

It is believe that the free shear layer is highly receptive to external disturbance up to the beginning of turbulent shear layer. Changing the characteristics of the free shear layers at nozzle lip has a great effect on the amplitude of the screech tones [30]. Successful reduction of the amplitude of screech tone has been attained through the introduction of small protrusion (tabs) into the jet at the nozzle lip (or roughness at the nozzle exit plane). It is reported that these tabs make a free shear layer thicker, and thereby inhibit the growth rate of the instability waves associated with the screech phenomena [31, 15]. One of the main objectives of the present study is to predict the length from the nozzle lip to the transition point of the free shear layer of a jet. The author believes that this length is one of the main elements of the feedback loop of the screech noise generation mechanism. So, an attempt has been made to predict the transition location of the free shear layer of high-speed sonic jet by using $e^N$ method.
1.3 Organization of the Thesis

The present thesis consists of six chapters. In chapter 1, general background, motivation, and objectives of this study are discussed.

In Chapter 2 the high speed jet facility, employed instrumentations and the experimental methodology are described.

In chapter 3 a passive control is investigated by using vortex generators in the form of a vane-type tabs placed along the circumference of a circular supersonic jet nozzle exit at diametrically opposite locations. A nozzle of a design Mach number of 1.33 is used in this investigation. The effect of the tabs on both flow field and the acoustic field are studied experimentally. Two configurations of tabs, i.e., stationary and free-to-rotate, are studied for different values of the nozzle pressure ratio defined as the ratio of the jet total pressure to the ambient pressure. Comparison between the proposed tabs and the well documented type of tabs (simple or rectangular and delta tabs), having the same area blockage, have been reported.

In chapter 4 lateral steady/unsteady injection of micro-jets, that are placed along the circumference of the nozzle exit at equal azimuthal angle intervals, is investigated. Characteristics of jet mixing and noise production are discussed. Specifically, two modes of the micro-jet injection are investigated: axisymmetric and antisymmetric, or flapping. Fully expanded and under expanded primary jets exhausted from a convergent, or sonic, nozzle are considered. The unsteady micro-jets are periodically injected with a nondimensional frequency (Strouhal number, St), of 0.16, which is defined on the base of the nozzle exit diameter and the velocity at the nozzle exit. This value is close to one of sub-harmonics of the most amplified Strouhal number, for two cases of total unsteady mass injection: 4% and 6% of primary jet mass flow rate.
In chapter 5 the relation between the spatial linear stability of disturbances and the onset of transition in compressible jet shear layer is investigated. An attempt is made to apply the conventional $e^N$ method to predict the transition in the free shear layer flow of a compressible jet. The mean velocity profile is experimentally obtained at different downstream locations. These profiles are used as input data for the linear stability analysis. Results from this analysis is, the growth rate of unstable disturbances are then integrated to obtain an envelope curve. On the other hand, the transition is detected experimentally by using three techniques: 1) oil flow visualization, 2) microphone measurement for pressure fluctuations, and 3) hot wire anemometer measurement for velocity fluctuations. The results from both the microphone and the hot-wire anemometer measurements are used to determine the $N$ factor.

The thesis will be concluded with some remarks in chapter 6
Chapter 2

Experimental Facility and Method

2.1 High Speed Jet Facility

Experiments were conducted an open jet facility at the Department of Aerospace Engineering, Nagoya University, as shown in Fig. 2.1. Two nozzles are employed in the present study to produce the primary jet flow. One nozzle is a supersonic nozzle of an exit diameter of 7.8 mm and a throat diameter of 7.5 mm. The design Mach number $M_D$, is 1.33. Another is a convergent (sonic) nozzle with an exit diameter of 7.5 mm. The nozzle is attached to a cylindrical plenum chamber that has a diameter of 220 mm and a length of 400 mm. High-pressure air is supplied from a tank with a volume of $12 \text{ m}^3$ stored at a pressure of $12 \text{ kgf/cm}^2$ that is connected to the plenum chamber via a 1 inch inner diameter high pressure pipe. A high precision pressure regulator and a solenoid valve were used in order to control the pressure of the plenum chamber within a 0.25% accuracy. The solenoid valve is opened and closed via a signal comes from the data acquisition computer.

The data acquisition computer is a celeron 460 MHz Machine which is equipped with a 16 channel AD (analog to digital) board (Micro-Science ADM 682PCI) and a GPIB board
2.1. HIGH SPEED JET FACILITY

Figure 2.1: Detailed schematic of experimental facility at Aerospace Engineering Department, Nagoya university

(general purpose interface bus). The AD board can digitize and record at a sampling rate of 125 kHz. It is connected to a 6 channel strain amplifier (KYOWA DPM-6H) which is in turn connected to various pressure transducers and load cells. The GPIB is used for interfacing with the FFT analyzer. This controls and collects data from FFT analyzer.

For automatic flow-field measurements, a three-dimensional mechanical traverse with three stepping motors is employed. These motors are connected to a hardware driver (Oriental motor UDX5114), which in turn is connected to NEC 486DX computer equipped with an interface board. Both the data acquisition computer and traverse-controlled computer are serially connected for automatic flow-field measurements. Figure 2.2 shows several photographs of the facility.
Figure 2.2: Photographs of the experimental facility: a) Instrumentations; b) PC-controlled mechanical traverse; c) Microphone, directivity arc and anechoic chamber; d) High precision pressure regulator, air filter and intermediate tank; e) Plenum chamber over three load cells.
2.2 Sound Measurements

Two microphones are employed for the sound pressure level (SPL) measurements. One is a RION UC-29 1/4 inch condenser microphone that has a maximum resolution frequency of 100kHz and a maximum SPL of 160 dB. Another is ONO-SOKKI LA-5110 1/2 inch condenser microphone that has a maximum resolution frequency of 20kHz and a maximum SPL of 130 dB. The microphone can be traversed along an arc that is placed at a distance of $100D$ from the nozzle exit, where $D$ is the diameter of the nozzle exit. The angle from the jet axis can be varied from $15^\circ$ to $110^\circ$. Most of the acoustic results presented in this study were obtained from acoustic data collected at a sampling rate of 100 kHz. The spectrum of sound pressure level is obtained by using a fast Fourier transform program. At the final stage of experiments, a two channel multi-purpose FFT analyzer (ONO-SOKKI CF-5210) is employed.

The outside surfaces of the plenum chamber and other bodies placed in the near field were covered with two layers of an acoustically absorbent, 6 mm thick polyurethane foam, to reduce the reflection of sound from those objects. Although acoustic measurements were performed in a laboratory that was not acoustically lined, overall features of the emitted noise were well represented by the spectra, therefore that relative comparisons regarding the effects of tabs and/or micro-jets actuation suppose to be valid. In the present measurements, the background noise was 47 dB in A-weighted SPL. During the final phase of experiments an anechoic room was built. A comparison between the background noise at nozzle exit with and without the anechoic room is shown in Fig. 2.3. It is clearly seen that low frequency noise has effect in the absence of the anechoic room.
Figure 2.3: Background noise level at nozzle exit with and without anechoic chamber.

(a) SPL spectra.

(b) OASPL at nozzle exit.
2.3 Pressure Measurements

Most of the flow field data were obtained by using a standard Pitot probe with an outer diameter of 0.6 mm. This probe is connected to a KYOWA PA-5KB pressure transducer that has a maximum sampling frequency of 800 Hz and a limit pressure of $5 \text{ kgf/cm}^2$. A short dual-cone static pressure probe designed by Pinckney [32] at NASA Langely and used previously by Norum and Seiner [33, 34] was employed for static pressure measurements in the case of under-expanded jet. The probe size was scaled down to fit it to the present nozzle geometry and to reduce the interference of the probe with the jet flow. The probe was calibrated with a supersonic wind tunnel. The calibration results showed that the static pressure could be obtained with an error of 3%. The result of calibration is shown in Fig. 2.4 along with the detailed geometry of the probe as well as the computational results by using the characteristics method. This static probe is connected to a KYOWA PGM-2KC pressure transducer that has a maximum sampling frequency of 24,000 Hz and a limit pressure of $2 \text{ kgf/cm}^2$. Since the static pressure rises sharply downstream of a shock, the static probe can be used to examine the spacing and strength of the shock cell system. Despite errors caused by the probe intrusion, the measured shock cell spacings showed good agreement with those determined from Schlieren pictures.

The Mach number distribution along the jet axis in the case of under-expanded jet was obtained by measuring both Pitot and static pressures, and using the Rayleigh’s supersonic Pitot formula. On the other hand, in the case of fully-expanded jet, the static pressure is constant and equal to the ambient pressure. Therefore, the centerline distribution of Mach number can be calculated directly from the Pitot probe measurement alone by assuming an isentropic flow.
a) Geometrical shape and wind tunnel calibration of the static pressure probe.

b) Computation of the static pressure field around the probe by using the characteristics method.

Figure 2.4: Comparison between measured and computed static pressure at Mach 1.5 by using the static pressure probe.
2.4 Jet Thrust Measurements

The jet thrust was measured with a one-component force measurement system fabricated especially for the present facility. In this system a plenum chamber is mounted on an equally triangular plate, which is in turn mounted on three load cells (KYOWA LUB-30K) with a maximum of 800 Hz sampling frequency and a limit of 30 kgf. The flow was routed via a single flexible tube with a diameter of 25.4 mm directed axially from the bottom of the plenum chamber. First of all, by plugging the exit of the plenum chamber, the forces produced when the feeding tubes became taut were measured for different values of the nozzle pressure ratio, $NPR$, which is defined as the ratio of the jet total pressure to the ambient pressure. These zero flow forces were subtracted from the forces measured with jet flow in order to obtain the actual thrust [35].

2.5 Other Measurements

An ONO SOKKI HT-4100 digital tachometer, which can measure a maximum of 20,000 revolutions per minute, was employed to measure the rotational frequency of micro-orifices in Chapter 4. The velocity fluctuations in Chapter 5 were measured by using TSI I-probe of 5 $\mu$m diameter wire alone with KANOMAX linearizer.

Throughout data acquisition, transducer zero errors were monitored before each run. The plenum pressure and ambient pressure were checked and the data were normalized according to current conditions.

2.6 Flow Visualization

The flow was also visualized for under-expanded jet by using a conventional two-mirror Töpler Schlieren technique with a pulsed light source of 10 $\mu$s duration.
Chapter 3

Mixing Enhancement and Noise Reduction by Passive Control

3.1 Introduction

Over the past twenty years, many researchers have investigated the effects of delta tabs and simple or rectangular tabs on both circular and rectangular jets [31, 36, 37, 35] attempting to enhance jet mixing and reduce jet noise. The tab is a small protrusion placed at the jet nozzle exit that produces streamwise vortices. These streamwise vortices can have a profound impact on the jet spreading and jet noise. The tabs generate a pair of counter-rotating vortices with the direction of rotation to be opposite to that expected from the wrapping of the boundary layer [37]. In their studies, the jet cross sectional shape could be distorted in a variety of ways, depending on the number and placement of the tabs.

It has been known for a long time that the tab can eliminate screech noise from a supersonic jet [38, 39]. The screech is a loud discrete tone emitted by a supersonic jet when operated under off-design conditions. The noise produced by coherent turbulent
structures goes upstream through stationary shocks and expansions formed in the jet flow implanting new disturbances into the shear layer at the nozzle lip. Thus established feedback loop yields the screech tone. The noise from shock-containing supersonic jets is composed of two components [31]. The first component is screech tone. The second one is broadband in nature. It is dominated by jet mixing noise at lower frequencies and shock-associated noise at higher frequencies. It was reported that these tabs make a mixing layer thicker, and thereby inhibit the growth rate of instability waves associated with the screech phenomena [31, 15].

The main subject in this chapter is to propose tabs in a form of freely rotating vane-type tabs. Their effect on round jet flow, which is a fundamental flow element of many applications, and its emitted sound as well as the associated thrust penalty are investigated experimentally. The tabs are supported by special bearings that allow tabs to rotate freely around the jet axis. The jet flow generates aerodynamic forces on the part of the vane, which is inside the jet. This force causes the vanes to rotate around the axis.

Vane-type tabs generate single trailing vortex. When the vanes are placed at diametrically opposite locations along the circumference of the nozzle exit such that the vortex produced by each vane has the same sense of rotation, then the torque produced by each vane are summed up. This configuration, which is thereafter referred to as the “free-to-rotate vane tabs”, is shown schematically in Fig. 3.1a. Stationary vane tabs shown in Fig. 3.1b are also investigated for comparison.

Only cold jet, the total temperature of which is the same as the room temperature, is considered in this study. Furthermore, only time-averaged flow field characteristics are addressed, where detailed flow-field data are obtained mainly through Pitot probe surveys. Acoustic measurements and the associated thrust loss are also reported. Parametric
study has been performed for the parameters, which are related to the vane-type tab system: protrusion height into a flow, vane angle, and the number of vanes. Comparison between the present tabs and the well documented type of tabs (simple or rectangular and delta tabs), having the same area blockage and examined for the same operating conditions, have been reported.

3.2 Nozzle and Tab Geometry

The vane-type tab configuration employed in the present study has a thickness to chord ratio, \( t_v/c \), of 0.33. The protrusion height is denoted by \( w \), and the nozzle exit diameter by \( D \). The supersonic nozzle has an exit diameter of 7.8 mm and a throat diameter of 7.5 mm with a design Mach number, \( M_D \), of 1.33. The ratio \( w/D \) is referred to as the protrusion ratio. The cases with \( w/D = 6.4\% \) and 12.8\% are studied. The area blockage due to tabs (% area blockage = \( (\text{area}_{\text{without tab}} - \text{area}_{\text{with tab}})/\text{area}_{\text{without tab}} \times 100 \)), is a function of the protrusion ratio, \( w/D \), and vane angle, \( \alpha_v \), which is given by the following equation:

\[
\text{Area Blockage}\% = \frac{\text{Area}_{\text{without tab}} - \text{Area}_{\text{with tab}}}{\text{Area}_{\text{without tab}}} \times 100
\]

\[
= \frac{2N_v\bar{C}}{\pi} \left[ \frac{\varepsilon^3}{8C^2} - \frac{\varepsilon}{3} + \frac{2w}{D} \right]
\]

\[
\approx \frac{4N_v\bar{C}(w/D)}{\pi} \times 100 \quad (D \gg c)
\]  

(3.1)

where
3.2. NOZZLE AND TAB GEOMETRY

a) Free-to-rotate vanes.

b) Stationary vanes.

Figure 3.1: Schematic of vane-type tab configurations.
This function is illustrated in Fig. 3.2 for two values of $w/D$ ratio. The area blockage due to each vane is about $1.5 \sim 2\%$ for the case $w/D = 6.4\%$ and $\alpha_v = 30^\circ$. The vane-type tabs were hand-made, and thus the dimensions quoted are not precise. Furthermore, since the vane angle was set by eye, some differences were likely to occur in experiments. This causes some minor differences that appear in certain repeated data.
3.3 Results

The notation $M_j$ is used to denote the fully expanded Mach number. It is uniquely related to the pressure ratio, $P_t/p_\infty$ which is also called the nozzle pressure ratio or NPR, through the following equation:

$$M_j = \left\{ \left[ \left( \frac{P_t}{P_\infty} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \frac{2}{\gamma - 1} \right\}^{0.5} \tag{3.2}$$

where $P_t$ is the chamber pressure and $p_\infty$ is the ambient pressure. Most of the results presented here are for $P_t/p_\infty = 5$, which corresponds to $M_j = 1.71$; this is a typical underexpanded supersonic jet.

The Mach number in the plume of an underexpanded supersonic jet can become actually higher than $M_j$. The pressure as well as the Mach number overshoots or undershoots the value in the fully expanded condition as the flow passes through the shock cell system. Note that $M_j$ is nothing more than the exit Mach number of a fictitious nozzle, at whose exit the flow expands up to the given NPR. In other words, it is an average jet Mach number in the region where the flow oscillates along the jet axis.

3.3.1 Jet Spreading

The parameters related with jet spreading are the vane configuration, i.e., free-to-rotate or stationary, the number of vanes, the protrusion height, and the vane angle from the jet axis. The effects due to these parameters are discussed in what follows.

3.3.1.1 Effects of the Number of Vanes

A large increase in jet spreading due to the influence of tabs is indicated in Fig. 3.3
for both stationary and free-to-rotate, where \( w/D = 6.4\% \) and \( \alpha_v = 30^\circ \). The measured pressure in supersonic flow regions represents the stagnation pressure \( P_{t2} \), which is the pressure just downstream of the standing bow shock produced by a Pitot probe itself.

The results are characterized by wavy patterns of pressure distributions due to shock / expansion structure inside the jet. The tabs can alter and weaken the shock / expansion structures dramatically. The jet centerline stagnation pressure and thus the Mach number are found to decay much faster than those of the baseline jet, which suggests an increase in jet spreading. The effect is most pronounced with four tabs in both configurations, whereas the least effect is seen with a single tab. The free-to-rotate tabs are found to make jet decay faster than the corresponding stationary tabs. The maximum observed rotational frequency for the case of four vanes was about 1,047 Hz. The free-to-rotate vanes generate streamwise vortices with a rotation frequency equal to the frequency of vane rotation unlike “steady-state” vortices produced by stationary vanes. It is known that the interaction of these vortices with the jet has a profound impact on the jet spreading [36]. The rotation of streamwise vortices around the jet axis was found to have more effect on the jet spreading than “steady-state” vortices have, as shown previously.

Mach number distributions in a cross-sectional plane were measured with a Pitot tube for \( M_j = 1.71 \). These data were taken in the downstream region where the flow was subsonic everywhere and the static pressure had been relaxed to the ambient pressure [37]. The Mach number was calculated from the Pitot tube measurement. Figure 3.4 shows the Mach number contours at \( x/D = 10 \) for the baseline jet, four stationary vanes, and four free-to-rotate vanes, with \( w/D = 6.4\% \) and \( \alpha_v = 30^\circ \). The enormous effects of tabs on the jet spread can be readily appreciated.
a) Stationary vanes.

b) Free-to-rotate vanes.

Figure 3.3: Effect of vane number on centerline stagnation pressure with $w/D = 6.4\%$ at $\alpha_v = 30^\circ$ and $M_j = 1.71$. 
Figure 3.4: Mach number contours at $x/D = 10$ for 4 vanes, where $w/D = 6.4\%$ and $\alpha_v = 30^\circ$. 
3.3. RESULTS

3.3.1.2 Effects of Tab Protrusion Length, $w$

Figures 3.5 and 3.6 show pressure distributions along the jet axis for both stationary and free-to-rotate vanes, respectively, where the number of vanes and the $w/D$ ratio are varied at $\alpha_v = 30^\circ$. It is seen from the figures that increase in the $w/D$ ratio is found to spread the jet; however, this is also associated with decrease in nozzle thrust due to increase in area blockage. The effect is most pronounced in the stationary two vanes case with $w/D = 12.8\%$. It should be noted that the distributions along the centerline do not fully represent actual jet spreading. Figure 3.7 shows the Mach number contours at $x/D = 5$ for both stationary and free-to-rotate cases with two vane tabs. From Fig. 3.7a it is observed that two stationary vane-type tabs essentially bifurcate the jet. Zaman et al. [35] has observed the similar effect by using two delta tabs for a sonic nozzle.

3.3.1.3 Effects of Vane Angle, $\alpha_v$

Figures 3.8 shows the effects of vane angle, $\alpha_v$, on the centerline stagnation pressure for both stationary and free-to-rotate cases with four vanes at $w/D = 6.4\%$. Increasing the vane angle is found to spread the jet, especially for the free-to-rotate case. However, increase in vane angle may result in a thrust loss due to increase in area blockage, as shown in Fig. 3.2.

3.3.1.4 Effects Vane-Induced Vortex Orientation

Up to this point, the orientation of all vortices induced by vanes is the same for stationary vane-tabs. The effect of different orientations is studied in this subsection for stationary case. Specifically, four cases of orientation have been studied for four vanes. The cases are shown in Fig. 3.9. Figure 3.10 presents the centerline stagnation pressure
a) $w/D = 6.4\%$.

b) $w/D = 12.8\%$.

Figure 3.5: Effect of $w/D$ and the number of vanes on centerline stagnation pressure for stationary vanes at $\alpha_v = 30^\circ$ and $M_j = 1.71$. 
a) $w/D = 6.4\%$.

b) $w/D = 12.8\%$.

Figure 3.6: Effect of $w/D$ and the number of vanes on centerline stagnation pressure for free-to-rotate vane at $\alpha_v = 30^\circ$ and $M_j = 1.71$. 
distribution of these four cases as well as the baseline jet. In all cases the jet centerline stagnation pressure and thus the Mach number decays much faster than those of the baseline jet, which suggests an increase in jet spreading. As shown from this figure the effect is most pronounced with the case 4 arrangements, whereas the least effect shows the case 1. The cross sectional Mach number contours are seen in Fig. 3.11 for each case at \( x/D = 5 \). There is a preferred spreading axis, which is clearly seen in this figure. The spreading axis is located between the two vanes, which produce a pair of counter-rotating vortices.

The answer to the question of why the case 4 has the largest effect compared to other cases is similar to that given by Zaman [36] for the delta tab. In the case 4 each pair of vanes, which make a pair of counter rotating streamwise vortices, act like a single delta tab. Zaman [36] stated that “the overall jet spreading caused by these vortices can be explained by the same reasoning that explains axis switching [40]. Owing to mutual induction, the two pairs first move towards the jet centerline. (This is shown schematically in Fig 3.12a.) The inward motion, however, is restricted as the two pairs approach each other, and shortly downstream, they rearrange to form two ‘out-flow’ pairs. The latter
3.3. RESULTS

Figure 3.8: Effect of vane angle, $\alpha_v$, on centerline stagnation pressure with 4 vanes at $w/D = 6.4\%$ and $M_j = 1.71$.

a) Stationary vanes.

b) Free-to-rotate.
Figure 3.9: Arrangements of vane-induced vortices for 4 vanes.

pairs eject jet core fluid while vortex induced motion propels them laterally away from the jet axis. This is what causes the rapid axis switching as well as the larger lateral spreading of the jet (as shown in Fig. 3.12b.)” Figure 3.13 represents vortex dynamics for the cases, which is similar to Fig. 3.12b based on Zaman [36] explanation.

3.3.2 Flow Visualization

Figures 3.14a to 3.14e show spark Schlieren photographs of the flow fields for the baseline jet and vane-type tabbed jets. In all photographs, the vane angle is $\alpha_v = 30^\circ$ and the jet Mach number $M_j = 1.71$. Figure 3.14a shows the baseline jet photograph, which is characterized by a large sinuous deformation of the jet. This is a characteristic of strongly screeching jet surface. Seven shock cells are observed, which is in close agreement with the number of shock cells observed by the Pitot-probe survey shown in Fig 3.3. In Figs 3.14b to 3.14e, enhancement of jet spreading is seen. The shock/expansion structures are weakened drastically when vane-type tabs are employed here.

3.3.3 Jet Noise

The far field noise spectra for four vanes with $w/D = 6.4\%$ and $12.8\%$ are shown
3.3. RESULTS

Figure 3.10: Effect of vane-induced vortices arrangement, on centerline stagnation pressure for stationary vane with 4 vanes at $\alpha_v = 30^\circ$ and $M_j = 1.71$.

for both stationary and free-to-rotate cases in Figs. 3.15 and 3.16, respectively. The microphone was placed at a distance of $100D$ and at an angle of $45^\circ$ from the jet axis. The baseline jet spectra are also presented in the figures, where screech components clearly appear. Its fundamental frequency is about 12.7 kHz, or the corresponding Strouhal number, $St$, of about 0.24, which is in close agreement with observations by a previous researcher [33]. The SPL of screech tone and the broadband noise have been reduced over an audible frequency range. On the other hand, the SPL for the higher frequencies is almost unaffected by the vanes; rather, it is slightly increased due to mixing enhancement. Thus the vane-type tabs proposed in this study can completely eliminate the screech component.

Figure 3.17 shows comparison of the present data with Tam’s predictions [4] given by
a) Baseline jet

b) Case 1

c) Case 2
d) Case 3
e) Case 4

Figure 3.11: Mach number contours at $x/D = 5$ for stationary vane with 4 vanes at $\alpha_v = 30^\circ$, $w/D = 6.4\%$ and $M_j = 1.71$. 
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Figure 3.12: Schematic of streamwise vorticity distribution for the delta tab cases and their effect on jet spreading (adapted from ref. [36]); a) in-flow pairs; b) out-flow pairs.

Figure 3.13: Schematic of streamwise vorticity distribution for the different vane-type tab arrangements according to their induced vortex orientation.
Figure 3.14: Schileren photographs at $M_j = 1.71$: a) Baseline jet; b) Four stationary vanes with $w/D = 6.4\%$ and $\alpha_v = 30^\circ$; c) Four stationary vanes with $w/D = 12.8\%$ and $\alpha_v = 30^\circ$; d) Four free-to-rotate vanes with $w/D = 6.4\%$ and $\alpha_v = 30^\circ$; e) Four free-to-rotate vanes with $w/D = 12.8\%$ and $\alpha_v = 30^\circ$. 
\[ \frac{f_s D_j}{U_j} = \frac{0.67}{(M_j - 1)^{0.5}} \left[ 1 + \frac{0.7M_j}{1 + \frac{\gamma - 1}{2} M_j^2} \right]^{0.5} \left( \frac{T_i}{T_\infty} \right)^{0.5} \]  

Since the Tam’s formula accounts for the helical mode only, in jets where symmetric or toroidal modes are present, primarily at \( NPR < 3 \), the data deviate from the Tam’s curve. As \( NPR \) increases and consequently \( M_j \) increases, the screech frequency decreases because of increase in shock cell spacing, which causes an increase in screech wave length, that is, a decrease in screech frequency.

Figure 3.18 shows the variation of the overall sound pressure level, OASPL, with \( NPR \) for both stationary and free-to-rotate cases with four vanes. The microphone is set at 100\( D \) and 45\( ^\circ \) from the downstream jet axis. For the baseline jet, the OASPL under a complete expansion condition (\( NPR \approx 3.2 \)) is lower than both over-expanded and under-expanded [41] jets. OASPLs are reduced by about 10 dB in both cases for \( NPR > 3.2 \) (underexpanded jet) when the protrusion ratio \( w/D = 6.4\% \). Zaman et al. [35] observed 5 dB reduction in OASPL by using delta tabs and a sonic nozzle with approximately the same area blockage as the present tabs. Increase in \( w/D \) has a slight effect on OASPL reduction (1-2 dB). In the free-to-rotate case \( w/D = 12.8\% \) has approximately the same OASPL level as the case of \( w/D = 6.4\% \) for high \( NPR \). This may be due to the noise produced from vane rotation speed for different \( w/D \).

Figure 3.19 shows the variation of OASPL with vane angle for two cases with four vanes. The data in this figure were measured by the microphone placed at \( r = 100D \) and \( \psi = 30^\circ \) from the downstream jet axis. The OASPL decreases approximately linearly until \( \alpha_v = 30^\circ \) and then levels off or slightly increases again. At \( \alpha_v = 30^\circ \) a maximum noise
a) \( w/D = 6.4\% \).

b) \( w/D = 12.8\% \).

Figure 3.15: SPL spectrum for stationary vanes with 4 vanes at \( \alpha_v = 30^\circ \) and \( M_j = 1.71 \).
3.3. RESULTS

Figure 3.16: SPL spectrum for free-to-rotate vanes with 4 vanes at $\alpha_v = 30^\circ$ and $M_j = 1.71$.

a) $w/D = 6.4\%$.

b) $w/D = 12.8\%$. 
reduction of about 10 dB is observed for stationary vanes with \( w/D = 6.4\% \). Regarding the effect of vane rotation on the OASPL, the reduction is about 12 dB for \( w/D = 6.4\% \).

Directivities characteristics of the OASPL measured along an arc of \( r = 100D \) are shown in Fig. 3.20 for both stationary and free-to-rotate of four vanes with \( \alpha_v = 30^\circ \) as well as the baseline jet. The maximum reductions in OASPL are observed in the range from \( \psi = 15^\circ \) to \( 55^\circ \) from the jet axis. Use of the present vanes produces a maximum value of OASPL at about \( 35^\circ \).

### 3.3.4 Jet Thrust

The thrust variations as a function of the nozzle pressure ratio, \( NPR \) and the pro-
Figure 3.18: Overall sound pressure, OASPL, variation with NPR with four vanes, $\alpha_v = 30^\circ$ and $M_j = 1.71$. 

(a) Stationary vanes.

(b) Free-to-rotate vanes.
Figure 3.19: Variation of overall sound pressure level, OASPL, with vane angle, $\alpha_v$, at $M_j = 1.71$.

The baseline jet has good agreement with the isentropic prediction in the underex-
3.3. RESULTS

<table>
<thead>
<tr>
<th></th>
<th>Baseline jet</th>
<th>w/D = 6.4 %</th>
<th>w/D = 12.8 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle, $\psi$ [deg.]</td>
<td></td>
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<tr>
<td>Jet</td>
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Figure 3.20: OASPL directivity arc at $r = 100D$ form the nozzle exit with $\alpha_v = 30^\circ$ and $M_j = 1.71$. 

a) Stationary vanes.

b) Free-to-rotate vanes.
panded region, i.e., NPR ≥ 3, which ensures the accuracy of the present measurements. Regarding the baseline jet with no-tab, the thrust loss increases with w/D. Similar results are obtained for the free-to-rotate cases, as shown in Fig. 3.21b. It is also seen that at higher values of NPR, the thrust loss due to free-to-rotate vanes is greater than that of the corresponding stationary case. For NPR = 5 (Mj = 1.71) and four stationary vane tabs with w/D = 6.4%, the loss is 6.9% of the baseline jet thrust. The corresponding free-to-rotate loss is 7.6%. These thrust losses are much less than those due to delta tabs [35] which are about 12%.

In order to take into account the reduction in thrust expected from the mere flow blockage, Fig. 3.22 shows the comparison in the thrust of the baseline jet for two nozzles A and B. The nozzle A is that considered in the present study. Nozzle B has the same design Mach number as the nozzle A, but its exit area is equal to that of the tabbed nozzle A equipped, with four vanes, αv = 30°, and w/D = 6.4%. Results of tabbed jet in stationary case with four vanes, αv = 30°, and w/D = 6.4% are also presented in the figures. The tabbed jet has about 1% thrust loss per one vane compared with the baseline jet of nozzle B. Figure 3.23 shows the variations of OASPL for the baseline jets with nozzle A and B, and the tabbed jet: the stationary case, four vanes, αv = 30°, and w/D = 6.4%. The data in this figure were measured by the microphone placed at r = 100D and ψ = 30° from the downstream jet axis. Constant thrust lines are also shown. For the same thrust, the tabbed jet has a lower OASPL than the baseline jets.

### 3.4 Comparison between Vane-Type, Simple, and Delta tabs

In the previous sections the effect of vane-type tabs on both flow and acoustic fields
3.4. COMPARISON BETWEEN VANE-TYPE, SIMPLE, AND DELTA TABS

a) Stationary vanes.

b) Free-to-rotate vanes.

Figure 3.21: Jet thrust versus NPR at different values of $w/D$. 
were studied [42]. The results showed an increase in the spreading and a reduction in the noise. In this section, the effects of vane-type tabs on both flow and acoustic fields are compared experimentally with known types of tabs, e.g., triangle or delta tabs and simple or rectangular tabs. The associated performance penalty, that is, thrust loss, is reported. A vane-type tab is characterized by producing a single tip vortex rather than a pair of counter-rotating vortices produced by a delta tab or simple tabs. The present investigation is concerned with round jet, which is a fundamental flow element of many applications. Only cold jet, whose total temperature is the same as the room temperature, are considered in this study. Furthermore, only time-averaged flow field characteristics are addressed, where detailed flow-field data are obtained mainly through Pitot probe surveys.
3.4. COMPARISON BETWEEN VANE-TYPE, SIMPLE, AND DELTA TABS

Figure 3.23: OASPL variation with NPR for nozzle A, nozzle B, stationary vanes, and free-to-rotate vanes. Vanes cases have $w/D=6.4\%$ and $\alpha_v = 30^\circ$ and used for nozzle A only. (Exit area of nozzle B = 92% exit area of nozzle A). Solid lines illustrate the constant thrust lines for the different nozzles.
3.4.1 Tabs Geometry

The delta and simple tab shapes employed here in this study are shown schematically in Fig. 3.24. For vane-type tabs the thickness to chord ratio of the vane, \( t_v/c \), is 0.33 (see Fig. 3.1). Four tabs were used for each configuration. The area blockage was kept within 1.5 \( \sim \) 2\% per tab for each configuration. For vane-type tabs, vane angle \( \alpha_v = 30^\circ \) was kept constant during this investigation, that corresponds to maximum reduction in the OASPL [42]. Two types of delta tabs are considered in the present investigations as well as simple tabs. The delta tabs considered here have triangular shapes with the base on the nozzle wall and the apex leaning downstream at about \( \theta = 45^\circ \) (Delta-45) from the nozzle exit plane. Another type with \( \theta = 0^\circ \) (Delta-00) is also investigated in the present study, as shown in Fig. 3.24. The jet is exhausted from supersonic nozzle has an exit diameter, \( D \), of 7.8 mm and a throat diameter, \( D_t \), of 7.5 mm with a design Mach number, \( M_D \), of 1.33.
3.4. COMPARISON BETWEEN VANE-TYPE, SIMPLE, AND DELTA TABS

3.4.2 Results of Comparison

3.4.2.1 Jet Spreading

A large increase in jet spreading under the influence of tabs is indicated in Fig. 3.25. The measured pressure in supersonic flow regions represents the Pitot pressure $P_{t2}$, which is the pressure just downstream of the standing bow shock produced by a Pitot probe itself.

The results are characterized by wavy patterns of pressure due to a standing shock-expansion structure inside the jet. The tabs can alter and weaken the shock-expansion structure dramatically. The jet centerline stagnation pressure and thus the Mach number are found to decay much faster than those of the baseline jet, which suggests an increase in jet spreading. The effect is most pronounced for simple tabs, whereas the least effect is seen with delta-00 tabs. It should be noted that the distributions along the centerline are not fully representative of actual jet spreading [35], because the tabs might bifurcate the jet locally [35, 42] as mentioned previously in section 3.3.

Figure 3.26 shows the radial Pitot pressure distribution at two different $x/D$ stations, specifically at $x/D = 10$ and $x/D = 15$. Simple and delta-45 tabs make a jet decay much faster than other types of tab.

Figure 3.27 shows Mach number contours in the $y-z$ plane at $x/D = 5$ for different tab types as well as the baseline jet. These data were taken far enough downstream in such a way that the flow was subsonic everywhere and the static pressure had been relaxed to the ambient pressure [37]. A pronounced distortion of the jet is seen under the influence of tabs, especially those, which produce two counter rotating vortices. They stretch the jet shear layer, and consequently increase spreading as shown previously schematically in Fig. 3.12 and Fig. 3.13. In Fig. 3.27b, case 4 of vane type tabs shows an increase
Figure 3.25: Effect of tab configuration on centerline stagnation pressure distribution at $M_j = 1.71$ and 4 tabs in all cases, and for vane-type tabs $w/D = 6.4\%$, $\alpha_v = 30^\circ$.

in the axis pass between the counter-rotating vortices much higher than the axis passing between a pair of co-rotating vortices.

3.4.2.2 Flow Visualization

Figure 3.28 shows the spark Schlieren photographs of the flow fields for the baseline jet and different tab types. Figure 3.28a shows the baselines jet photograph, which is marked by a large sinuous oscillation of the jet. This is characteristic of strongly screeching jet. The pattern including up to seven shock cells is observed from the Schlieren photograph which is in close agreement with the number of shock cells observed by Pitot-probe survey shown in Fig 3.25. These photographs confirm to some extend the measured Pitot pressure mentioned previously in Fig. 3.25.
3.4. COMPARISON BETWEEN VANE-TYPE, SIMPLE, AND DELTA TABS

Figure 3.26: Effect of tab configuration on radial stagnation pressure distribution at $M_j = 1.71$, 4 tabs in all cases, and for vane-type tabs $w/D = 6.4\%$, $\alpha_v = 30^\circ$.

a) $x/D = 15$.

b) $x/D = 10$.
Figure 3.27: Mach number contours at $x/D = 5$ and $M_j = 1.71$ : a) Baseline jet; b) Stationary vane-type tabs; c) Free-to-rotate vane-type tabs; d) Simple tabs; e) Delta-00 tabs; f) Delta-45 tabs. For each case 4 tabs are used, and for vane-type tabs $\alpha_v = 30^\circ$, $w/D = 6.4\%$. 
3.4. COMPARISON BETWEEN VANE-TYPE, SIMPLE, AND DELTA TABS

Figure 3.28: Schlieren photographs at $M_j = 1.71$: a) Baseline jet; b) Stationary vane-type tabs; c) Free-to-rotate vane-type tabs; d) Simple tabs; e) Delta-00 tabs; f) Delta-45 tabs. For each case 4 tabs are used, and for vane-type tabs $\alpha_v = 30^\circ$, $w/D = 6.4\%$. 
3.4.2.3 Jet Noise

The far field noise spectra for different tab types are shown Fig. 3.29, where a microphone is placed at a distance of $100D$ and at an angle of $45^\circ$ from the jet axis. The baseline jet spectra are also presented in the figures and are clearly characterized by screech components. Its fundamental component is about 12.7 kHz, or the corresponding Strouhal number, $St$, of about 0.24, which is in close agreement with observations by previous researchers [4]. All tab types can completely eliminate the screech component.

In addition to the screech component, the broadband levels are also being reduced over an audible frequency range. On the other hand, higher frequencies are almost unaffected by the influence of tabs, rather slightly increased.

The characteristics of OASPL directivity along an arc of $r=100D$, with the nozzle exit as a center, are shown in Fig. 3.30 for all cases as well as the baseline jet. The maximum reductions in OASPL are in the range from $\psi = 15^\circ$ to $50^\circ$ from the jet axis. All tab types have OASPL peak values at about $35^\circ$. Figure 3.32 summarizes the reduction in OASPL from the baseline jet for different tab types at an angle of $35^\circ$.

3.4.2.4 Jet Thrust

The thrust variations as a function of the nozzle pressure ratio, $NPR$, for all tab types are shown in Fig. 3.31. Also shown in the figures is an isentropic prediction given by Eq.(3.4). Figure 3.32 shows the percentage thrust penalty per tab from the measured thrust of the baseline jet at $M_j = 1.71$. Delta-45 has a thrust penalty of about 2.3% the same order of magnitude measured by Zaman et. al. [35] of about 3% penalty of ideal thrust by using delta-45 tabs and sonic nozzle. Simple tab has 1.99% loss of measured baseline jet. Both simple and delta-45 have higher loss compared with other tab types.
3.4. COMPARISON BETWEEN VANE-TYPE, SIMPLE, AND DELTA TABS

Figure 3.29: SPL spectra for different type of tab cases with 4 tabs used in each case and $M_j = 1.71$: a) Stationary vane-type tabs; b) Free-to-rotate vane-type tabs; c) Simple tabs; d) Delta-00 tabs; e) Delta-45 tabs. For each case 4 tabs are used, and for vane-type tabs $\alpha_v = 30^\circ$, $w/D = 6.4\%$. 
Figure 3.30: OASPL directivity arc at $r = 100D$ form the nozzle exit: ○, baseline jet; ◊, stationary vane-type tabs; ●, free-to-rotate vane-type tabs; □, simple tabs; △, delta-00 tabs; ▲, delta-45 tabs. For each case 4 tabs are used and $M_j = 1.71$, and for vane-type tabs $\alpha_v = 30^\circ$, $w/D = 6.4\%$. 
3.5 Concluding Remarks

A vortex generator configuration, referred to as vane-type tab is studied, in two different regimes: free-to-rotate and stationary. Results of this study are summarized as follows.

- The jet plume cross section maintains an axisymmetric shape for the free-to-rotate case, while it is often non-axisymmetric for the stationary case, depending on the number of vanes and their azimuthal locations.

- The centerline stagnation pressure and hence the Mach number for the case of free-to-rotate vanes are found to be decayed much faster than those of the stationary vane case.

Figure 3.31: Jet thrust versus NPR: “—”, isentropic prediction; ○, baseline jet; ◊, stationary vane-type tabs; ●, free-to-rotate vane-type tabs; □, simple tabs; △, delta-00 tabs; ▲, delta-45 tabs. For each case 4 tabs are used, and for vane-type tabs $\alpha_v = 30^\circ$, $w/D = 6.4\%$. 
In the case of two stationary vanes with $w/D = 12.8\%$, a jet is bifurcated into two parts. This jet bifurcation does not occur for the rotating case.

The vane tabs can effectively reduce the noise in underexpanded and overexpanded jet regimes.

From the centerline stagnation pressure data, the case 4 of vane-type tabs has the weakest shock/expansion structure compared with other cases. Since this case has two pairs of counter rotating vortices, it is similar to the case with two delta tabs.

A reduction of OASPL of about 10 dB is observed for four stationary vanes with $w/D = 6.4\%$ and a vane angle of $30^\circ$. In this case, the thrust loss is about $1.5 \sim 2\%$ per vane comparison with the baseline jet.

For the same thrust and exit area, tabbed jets have a lower noise level than the baseline jet.
• By increasing the ratio of \( w/D \), the thrust penalty increases, and the noise reduction increases consisting of about 12 dB for \( w/D = 12.8\% \).

• In general, stationary vanes have a lower thrust penalty than free-to-rotate vanes except for low NPR values with \( w/D = 6.4\% \).

• From spectrum results, the SPL of the audible frequency range (up to 20 kHz) is reduced due to the vane tabs which seems to be associated with shock associated noise. The higher frequencies, which are greater than 20 kHz, seem to be unaffected or increase due to mixing enhancement by the vane tabs.

• Different vortex generator configurations were studied and their effects on data of engineering interests such as thrust loss, increase of mixing and reduction in noise are evaluated.

• It became clear here that vane-type tabs and free-to rotate tabs have a reasonable reduction in OASPL with acceptable thrust loss compared with other tab types.
Chapter 4

Mixing Enhancement of
Compressible Jets by Using
Unsteady Micro-Jet Actuators

4.1 Introduction

There are many technological applications regarding jet mixing enhancement. For example, enhanced jet mixing can reduce temperature on in-plume aerodynamic surfaces, which provides greater flexibility in the choice of materials for their construction [43]. Similarly, the mixing efficiency of fuel jets in combustors is an important factor in their overall performance, where reductions in their size and weight will be possible if the mixing can be improved.

Jets and mixing layers are characterized by their mean-velocity profiles with an inflection point that causes inviscid instabilities. The primary mechanism of these instabilities is the so-called vortical induction, where viscosity plays a minor role, i.e., damping. Major goals for such flow control are mixing enhancement and noise suppression. In general,
two kinds of control, active and passive, can be considered. Examples of the passive control are tabs that are located at the nozzle exit [35], crown-shaped nozzle [44], and various other tailorings of the nozzle exit [36]. Passive control is attractive because in many cases it entails only simple design modifications. On the other hand, active control, where nozzle conditions are continuously updated through a feedback loop, has greater flexibility, and therefore greater potentials to change the jet flow.

Previous studies on shear layer dynamics and jet mixing control have often employed acoustic drivers because such drivers can produce almost axisymmetric and/or azimuthal modes at any desired frequency and amplitude. These acoustic drivers are closely related to the physics of shear layer flow. However, they are not suitable for controlling flows of practical interests because of their limitations in weight, power, and maintenance. In addition, as background noise and turbulence level increase with Mach number, the amplitude of excitation by the acoustic drivers would be insufficient to bring about a large change in the mean flow. Thus, any control by using low-amplitude excitation is not practical in many flows of engineering interest [45].

Recently, various active control techniques using blowing/suction have been proposed for enhancing flow mixing. Raman [45] exploited oscillating miniature fluidic jets to enhance mixing. Davis [46] used radial blowing from a pair of steady jets and obtained a variable control by adjusting the degree of penetration of the control jets into the main jet flow. Smith and Glezer [47] have vectored and mixed a jet flow by using synthetic jet, or zero mass flow devices that require only electrical power and no fluid addition. A pulsed jet version of this concept was applied to the exhaust of a full-scale engine [25]. The pulsing jets induced the first azimuthal mode of the jet shear layer. The effect of enhanced mixing was seen in the substantially reduced temperature of the potential flow core along the engine centerline. Computational fluid dynamic (CFD) simulations
of synthetic jets, or jets in general, have been attempted by Cain et al. [48] using an unsteady Reynolds-averaged Navier-Stockes code, and by Freund [49] using a direct numerical simulation code. These simulations verified the experimentally observed jet behaviors [25]. Nedungadi et al. [50] have investigated the higher antisymmetric modes of an array of synthetic jets and their effects on primary jet mixing and noise generation using CFD simulations.

In the present work, an experimental investigation of shear flow excitation using controllable unsteady microjets was conducted. The main motivation underlying this idea is to enhance jet mixing by exciting the jet shear layer using azimuthally distributed microjets. In the present study, the microjets were placed along the circumference of the nozzle exit of a primary jet at equal intervals and were directed toward the primary jet axis. Unsteady axisymmetric and antisymmetric, or flapping, actuation modes, as well as steady axisymmetric modes are studied for both fully expanded and underexpanded primary jets.

4.2 Microjets Actuator

4.2.1 Axisymmetric Actuation

The arrangement of a single microjet is shown in Fig. 4.1. Figure 4.2 is a photograph of the actuator attached to the primary jet nozzle, where the system of 12 stationary pressure tubes is connected to the micro-chamber. The unsteady microjets actuator employed here consists of the following parts: 1) a circular micro-orifice plate supported by bearings, 2) stationary pressure tubes connected to a micro-chamber, and 3) a driving motor. The micro-orifice plate has 36 orifices that are distributed along the circumference at 10-deg intervals. Each orifice has a diameter of 0.5 mm. The air supply tube
4.2. MICROJETS ACTUATOR

Figure 4.1: Detailed schematic diagram of a single microjet relative to primary jet.

The system consists of 12 tubes that are placed at 30-deg intervals. These tubes are stationary, whereas the micro-orifice plate is allowed to rotate freely around the primary jet axis. A speed-controlled motor is employed to rotate the micro-orifices with a maximum rotational speed of 12,000 revolutions per minute (rpm). During the rotation, at the moment when the micro-orifices are aligned with the stationary air tubes, microjets are emitted toward the primary jet axis.

Figure 4.3a shows injection of the microjets during one period, $\tau$, in axisymmetric actuation mode. By changing the rotational frequency of the micro-orifices, an arbitrary injection frequency can be realized according to the following formula:

$$f = 36 \times \frac{\Omega}{60}$$  \hspace{1cm} (4.1)

where $f$ is the injection frequency in hertz (Hz) and $\Omega$ is the rotational speed of motor, which is the same as that of the micro-orifices, in revolutions per minute (rpm). Moreover, the amplitude of the unsteady microjet flow can be controlled by varying the micro-chamber pressure ratio, $P_{\mu}/p_\infty$. 
To examine characteristics of the unsteady microjet flow, a Pitot probe was placed, facing the exit of micronozzle at $P_\mu/p_\infty = 2.50$ and $\Omega = 1000$ rpm. Figure 4.4a shows the time response of the Pitot probe. It is clearly seen that the signal shown in Fig. 4.4a includes a high-frequency component that is superimposed on a low-frequency component. These two frequencies are also shown in the spectrum of a time signal in Fig. 4.4b. The high frequency is in agreement with that computed by Eq. (4.1), that is, the injection frequency of microjets, whereas the low frequency is close to the rotational frequency of the micro-orifices circular plate. This low frequency seems to be caused by less accurate fabrication. That is, there may be slight differences in elevation between the micro-orifice holes, which could in turn appear as the time-varying signal of the rotational speed.

The injected mass fraction, $\varphi_j$, can be defined as the ratio of the injected mass to the mass flow rate of the primary jet flow. This ratio is a function of microjets pressure ratio, $NPR_\mu$, and primary jet nozzle pressure ratio, $NPR$. Figure 4.5a shows the theoretical relation between $NPR_\mu$ and $NPR$ with the injected mass fraction as a parameter, which was derived from the continuity equation for an isentropic flow. When $\varphi_j$ and $NPR$
a) Axisymmetric actuation.

b) Antisymmetric, or flapping, actuation.

Figure 4.3: Cross-sectional view of microjets at primary jet nozzle exit during a complete injection cycle for both axisymmetric and antisymmetric modes.
a) Time response of pressure.

b) Spectrum.

Figure 4.4: Unsteady injection characteristic due to single micro-jet with micro-jet pressure ratio 2.50 and micro-orifices rotational speed 1000 rpm.
are specified, the corresponding $NPR_{\mu}$ can be calculated. This computed $NPR_{\mu}$ will give the theoretical injected mass fraction. Although the actual injected mass fraction is considered to be less than the theoretical one, no attempt was made to measure the actual injected mass fraction. This loss in the injected mass fraction seems to occur because of a leakage due to a clearance between the micro-orifice plate and the air supply tubes, which is needed to reduce excessive friction at a high-speed rotation of micro-orifices.

### 4.2.2 Asymmetric, or Flapping, Actuation

The antisymmetric mode was created by shifting the locations of six of the overall air supply tubes by five degrees along the circumference. This would cause the two arrays of six microjets to be emitted at two different instants, as shown in Fig. 4.3b. Figure 4.5b shows the theoretical relation between $NPR_{\mu}$ and $NPR$ for different values of the injected mass fraction.

Note that, although it was intended to excite only a single mode, in actual experiments, several modes may be excited due to the finite number of microjets, the accuracy of actuator fabrication, and the transition process from closing and opening the microjets.

### 4.3 Results and Discussion

Two values of $M_j$ were considered here: $M_j = 1.00$ for a fully expanded jet and $M_j = 1.36$ for an underexpanded jet, which correspond to $NPR = 1.89$ and 3.00, respectively. The corresponding Reynolds numbers are $Re = 1.5 \times 10^5$ and $1.4 \times 10^5$, respectively, which are based on the nozzle exit diameter and the primary jet velocity at the nozzle exit. Experimental conditions for the present study are listed in Table 4.1, which consists of two cases for fully expanded jet and one case for underexpanded jet. The unsteady microjets were injected at a frequency of 6300 Hz, which corresponds to a Strouhal
a) Axisymmetric actuation.

b) Antisymmetric, or flapping, actuation.

Figure 4.5: Relation between primary jet, NPR, microjets pressure ratios, NPR\(_\mu\), and mass fraction, \(\varphi_j\), for both axisymmetric and asymmetric injections.
4.3. RESULTS AND DISCUSSION

Table 4.1: Experimental Conditions

<table>
<thead>
<tr>
<th>Actuation Mode</th>
<th>Fully-Expanded Jet 1</th>
<th>Fully-Expanded Jet 2</th>
<th>Under-Expanded Jet</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varphi_j$</td>
<td>NPR$_{\mu}$</td>
<td>NPR</td>
</tr>
<tr>
<td>Baseline Jet</td>
<td>0.00</td>
<td>1.00</td>
<td>1.89</td>
</tr>
<tr>
<td>Steady Injection</td>
<td>0.04</td>
<td>1.50</td>
<td>1.89</td>
</tr>
<tr>
<td>Unsteady Axisymmetric Injection</td>
<td>0.04</td>
<td>1.50</td>
<td>1.89</td>
</tr>
<tr>
<td>Unsteady Flapping Injection Case 1</td>
<td>0.04</td>
<td>2.80</td>
<td>1.89</td>
</tr>
<tr>
<td>Unsteady Flapping Injection Case 2</td>
<td>0.04</td>
<td>1.50</td>
<td>1.89</td>
</tr>
</tbody>
</table>

number, $St$, of 0.16, based on the nozzle exit diameter and the velocity at the nozzle exit. This is close to one of subharmonics of the most amplified Strouhal number [29]. As listed in table 4.1, there are two flapping injections; in case 1 the theoretical injected mass flow rate is the same as those of both axisymmetric and steady injections, whereas in case 2, it is reduced by 50%. The second case was chosen here, because the mean flowfield was strongly affected in preliminary tests.

4.3.1 Mean Flow Fields

4.3.1.1 Fully Expanded Jet

Figure 4.6a shows the centerline Mach number distribution of fully expanded sonic jet for various injection modes at $St = 0.16$ and $\varphi_j = 0.04$, as well as that of the baseline jet. From Fig. 4.6a, it is clearly seen that the centerline Mach in the cases with injection decays much faster than that of the baseline jet, leading to an increase in jet spreading and jet mixing.

The unsteady injections have higher spreading rates than the steady injection with case 1 of flapping injection, showing more dramatic change than case 2. The case 2 of unsteady injection is particularly interesting because even with a 50% reduction in the injected mass flow rate it produces a higher spreading rate than the axisymmetric injection. As will be mentioned more in detail in section 4.4, this is because antisymmetric disturbances are more unstable than axisymmetric ones, that is, the growth rate due to
flapping actuation is higher than that due to axisymmetric one. To see whether or not
the flapping actuation bifurcates the jet, radial distributions of Mach number at several
downstream locations were measured. The results are shown in Fig. 4.6b. As seen from
this figure, the bifurcation does not take place in the present flows and the Mach number
distributions shown in Fig. 4.6a well represent the actual jet spreading. Figure 4.7 shows
results similar to Fig. 4.6, but for a higher value of injected mass fraction, \( \varphi_j = 0.06 \).
We can see an increase in spreading rate as well as a reduction in jet potential core length,
compared with the case of \( \varphi_j = 0.04 \) shown in Fig. 4.6.

Figure 4.8 shows the axial distribution of jet radius for two values of injected mass
fraction: \( \varphi_j = 0.04 \) and 0.06. The jet radius is defined as the length from the axis to the
location where the velocity becomes half the centerline velocity [26]. As shown in Fig.
4.8, the jet radius increases in the downstream direction, where the case 1 of flapping
mode has the highest rate.

4.3.1.2 Underexpanded Jet

To evaluate the actuator performance for a jet plume with a shock cell structure, the
microjet actuation was applied to an underexpanded sonic jet with \( M_j = 1.36 \), which
corresponds to \( NPR = 3.00 \). Figure 4.9 shows Mach number distributions along the
centerline for steady and unsteady axisymmetric modes, unsteady flapping modes, and
the baseline jet. Here \( St = 0.16 \) and \( \varphi_j = 0.04 \). The distributions were obtained first by
measuring Pitot and static pressures along the jet centerline, and then applying Rayleigh’s
supersonic Pitot formula. The results obtained are characterized by a wavy pattern of
the Mach number due to the shock/expansion cellular structure inside the jet. The effect
of microjet actuation on the distribution is qualitatively similar to that of fully expanded
jet with the same injected mass fraction. Reduction in the potential core length and
shock strength due to the microjet actuation are clearly seen in Fig. 4.9. The potential
4.3. RESULTS AND DISCUSSION

a) Centerline Mach number distribution.

b) Radial Mach number distribution at different axial locations:
i) $x/D = 1$, ii) 3, iii) 5, iv) 7, and v) 9.

Figure 4.6: Mach number distributions of fully-expanded jet with $M_j = 1.00$, sonic jet: $\bigcirc$, baseline jet; $\bigtriangleup$, axisymmetric steady mode; $\bullet$, unsteady axisymmetric mode; $\blacksquare$, unsteady flapping mode 1; $\square$, unsteady flapping mode 2; $St = 0.16$ and $\varphi_j = 0.04$ ($\varphi_j = 0.02$ flapping mode 2).
a) Centerline Mach number distribution.

b) Radial Mach number distribution at different axial locations:
i) $x/D = 1$, ii) 3, iii) 5, iv) 7, and v) 9.

Figure 4.7: Mach number distributions of fully-expanded jet with $M_j = 1.00$, sonic jet: $\bigcirc$, baseline jet; $\bigtriangleup$, axisymmetric steady mode; $\bullet$, unsteady axisymmetric mode; $\blacksquare$, unsteady flapping mode 1; $\square$, unsteady flapping mode 2; $St = 0.16$ and $\varphi_j = 0.06$ ($\varphi_j = 0.03$ flapping mode 2).
4.3. RESULTS AND DISCUSSION

Figure 4.8: Axial distribution of normalized jet radius, $R_{0.5}/D$, of fully-expanded jet, $M_j = 1$: ○, baseline jet; △, axisymmetric steady mode; •, unsteady axisymmetric mode; ■, unsteady flapping mode 1; □, unsteady flapping mode 2; $St = 0.16$. 

a) $\varphi_j = 0.04$. 

b) $\varphi_j = 0.06$. 

$R_{0.5}/D$ versus $x/D$ for different values of $\varphi_j$. The symbols represent different modes of oscillation: baseline jet (○), axisymmetric steady mode (△), unsteady axisymmetric mode (•), unsteady flapping mode 1 (■), and unsteady flapping mode 2 (□). 

$St = 0.16$.
core length is one of frequently used parameters to quantify the mixing characteristics of a jet [51]. This length is defined as the distance measured in the axial direction from the nozzle exit to the location where the inner edge of the ring-shaped jet shear layer surface merges to a point at the axis. This length can be determined rather easily in the case of fully expanded jet, where the centerline Mach number remains constant until the merging point, and then it decreases monotonically toward the downstream. In this study, the downstream end of the potential core was defined as the location where the centerline Mach number drops to 5% below the nozzle exit Mach number.

The potential core length is difficult to define in the case of underexpanded jet because of the presence of a shock cell structure causing significant oscillations in the centerline Mach number distribution. To overcome this difficulty, an average centerline Mach number was calculated by locally averaging the Mach number within each periodic shock cell. The potential core in the present study is then defined as the point where this locally averaged centerline Mach number begins to monotonically decrease below the averaged centerline Mach number measured in the upstream. Specifically, this averaged Mach number was obtained by averaging the Mach number distributions in the first four shock cells, which is close to that computed by Eq. (3.2).

Based on this definition, the reduction of the potential core length in the case 1 of unsteady flapping mode was 41% and 56% for underexpanded and fully expanded jets, respectively, for the same value of the injected mass fraction. Other actuation cases also showed similar trends of reduction in jet potential core length for both underexpanded and fully expanded jets, which are summarized in Fig. 4.10. Radial distributions of Mach number at several downstream locations are presented in Fig. 4.11. This shows that the jet bifurcation does not take place and that the Mach number distributions indicated in Fig. 4.9 represent the actual jet spreading.
Figure 4.9: Mach number distribution of under-expanded jet with $M_j = 1.36$: a) Baseline jet; b) Axisymmetric steady mode; c) Axisymmetric mode; d) Flapping mode 2; e) Flapping mode 1; $St = 0.16$ and $\varphi_j = 0.04$ ($\varphi_j = 0.02$ flapping mode 2).
Figure 4.10: Potential core length normalized by baseline jet potential core length for both fully-expanded and under-expanded jets at $St = 0.16$ for unsteady injection.

Figure 4.11: Radial Mach number distribution at different axial locations: a) $x/D = 2$, b) 3, c) 5, d) 7 and e) 10.
For under-expanded jet with $M_j = 1.36$: ○, baseline jet; △, axisymmetric steady mode; ●, unsteady axisymmetric mode; ■, unsteady flapping mode 1; □, unsteady flapping mode 2; $St = 0.16$ and $\varphi_j = 0.04$ ($\varphi_j = 0.02$ flapping mode 2).
Figure 4.12: Axial distribution of normalized jet radius, $R_{0.5}/D$, for under-expanded jet with $M_j = 1.36$: $\bigcirc$, baseline jet; $\triangle$, axisymmetric steady mode; $\bullet$, unsteady axisymmetric mode; $\blacksquare$, unsteady flapping mode 1; $\square$, unsteady flapping mode 2; $St = 0.16$ and $\varphi_j = 0.04$ ($\varphi_j = 0.02$ flapping mode 2).

Figure 4.12 shows axial distributions of the jet radius. The two flapping modes have comparable effects on this parameter, which is different from the case of fully expanded jet. As will be shown later, an increase in jet Mach number has a stabilizing effect on the fully expanded jets. As seen from the results of Fig. 4.12, the presence of a shock cell structure inside the jet plume has an opposite, destabilizing effect, especially for antisymmetric disturbances. The underexpanded jet considered here was visualized by a Schlieren method, the results of which are shown in Fig. 4.13. Note that, although they represent instantaneous flow fields, these photographs confirm, to some extent, the measured mean flow fields shown in Fig. 4.9.

### 4.4 Linear Stability Analysis

Conventional linear stability calculations are performed for the fully expanded jet considered in this study. In this analysis, the main objectives are 1) qualitative understanding
Figure 4.13: Schlieren pictures of under-expanded jet with $M_j = 1.36$: a) Base-line jet; b) Axisymmetric steady mode; c) Axisymmetric mode; d) Flapping mode 2; e) Flapping mode 1, $St = 0.16$ and $\varphi_j = 0.04$ ($\varphi_j = 0.02$ flapping mode 2).
of how the disturbances grow downstream; 2) the effect of azimuthal disturbance mode on the stability, that is, the difference between axisymmetric and antisymmetric modes; and 3) the role of compressibility for stability. Only results will be presented here. For details of the calculation procedure, the reader is referred to appendix A or to Michalke [52].

The linear theory deals with small perturbations and linearized equations, so that it can only predict the initial, local growth of small perturbations. However, some general insight into the development of large-scale structure in the jet can be speculated by examining the linear solution. A hyperbolic tangent velocity profile of the following form was used for the base flow:

$$\frac{U(r)}{U_o} = 0.5 \left[ 1 + \tanh \left\{ 0.25 \frac{R_{0.5}}{\theta} \left( \frac{R_{0.5}}{r} - \frac{r}{R_{0.5}} \right) \right\} \right]$$  \hspace{1cm} (4.2)

where $\theta$ is the momentum thickness that is defined as

$$\theta = \int_{0}^{\infty} \left( \frac{\rho(r)}{\rho_o} \right) \left( \frac{U(r)}{U_o} \right) \left( 1 - \frac{U(r)}{U_o} \right) dr$$  \hspace{1cm} (4.3)

Michalke [52], Chan and Leong [53], Morris [54], Crigthon and Gaster [55], and Plaschko [56] have employed this profile for their jet stability analyses.

The temperature is related to the velocity by the Busemann-Crocco law which is valid for a boundary-layer flow with a constant pressure and $Pr = 1$.

$$\frac{T(r)}{T_o} = \frac{T_\infty}{T_o} + \left( 1 - \frac{T_\infty}{T_o} \right) \frac{U(r)}{U_o} + \frac{\gamma - 1}{2} M_o^2 \frac{U(r)}{U_o} \left( 1 - \frac{U(r)}{U_o} \right)$$  \hspace{1cm} (4.4)
where $T_\infty = T(\infty)$ is the ambient temperature.

In the present calculations, where spatially growing disturbances are considered, $\omega$ is taken to be real, and $n$ is integer, whereas $\alpha$ is complex: $\alpha = \alpha_r + i\alpha_i$ [52]. The disturbances will be damped if $\alpha_i > 0$, neutral if $\alpha_i = 0$, and amplified if $\alpha_i < 0$.

### 4.4.1 Linear Stability Results

Figure 4.14 shows the normalized axial velocity distribution in radial direction at six locations. Along with experimental data obtained in this study, the hyperbolic-tangent velocity profile of Eq. (4.3) is presented in this figure, assuming that the jet flow is isothermal, that is, $T_\infty/T_o = 1$. Note that $R_{0.5}/\theta$ in Eq. (4.3) is needed for the hyperbolic-tangent velocity profile at a given axial location. The least-square fitting were used to determine the axial variation of $\theta/R_{0.5}$, the results of which is shown in Fig. 4.15. The fitted straight line has the following form:

$$\theta/R_{0.5} = 0.060(x/D) + 0.015 \quad (4.5)$$

This relation is similar to the relation proposed by Crigthon and Gaster [55] and Plaschko [56] except for the constant value. This constant seems to depend on flow conditions at the nozzle exit.

Figure 4.16 presents the axial growth rate $\alpha_i$, normalized by the momentum thickness, $\theta$, versus the radian frequency, $\omega$, normalized by the momentum thickness and jet velocity, at six locations in the jet potential core region. The growth rates of disturbances for both zeroth (axisymmetric) mode with $n = 0$ and first (antisymmetric) mode with $n = 1$ are shown in Fig. 4.16. As we go downstream, the antisymmetric disturbances become more
4.4. LINEAR STABILITY ANALYSIS

Figure 4.14: Radial velocity distribution inside jet potential core at different axial locations: •, experimental data; —, hyperbolic tangent velocity profile. a) $x/D = 0$, b) 1, c) 2, d) 3, e) 4 and f) 5.

Figure 4.15: Axial variation of ratio of momentum thickness to jet radius, $\theta/R_{0.5}$, in the jet potential core region: •, present experimental data; —, present fitting; ···, Crighton and Gaster [55] fitting.
unstable than the axisymmetric ones. Note that this analysis is based on the local mean
velocities of the baseline jet; therefore, the change in the mean velocity due to the growth
of disturbances is neglected in the present analysis. Furthermore, this analysis assumes
that the disturbances are small, which is not exactly applicable to microjet actuation like
the present case. Therefore, the reason why the antisymmetric, or flapping, actuation
is more effective than the axisymmetric one with regard to mixing can be deduced only
qualitatively from the results of stability analysis. In any case, it is clear from these results
that the antisymmetric disturbances produced by the present antisymmetric actuation
are more amplified in the downstream, which will change the mean flow more effectively
than the axisymmetric actuation.

To see the effects of compressibility on jet stability, similar calculations were performed
for the case of incompressible flow, that is, $M_o = 0$, regarding both axisymmetric and
antisymmetric modes, where the flow at $x/D = 3$ was selected as a base flow. The results
are shown in Fig. 4.17, along with the compressible flow data at the same location of
$x/D$. It turns out that the compressibility of jet flow has a stabilizing effect because
the compressible flow is less unstable than the corresponding incompressible one. This
will give an explanation for the effects of the actuation employed here on both the fully
expanded and underexpanded jets, mentioned earlier.

Note that in this linear stability analysis the pressure field is held constant inside
the jet plume. This is not the case for underexpanded jet with the shock cell structure.
There, the pressure is no longer constant inside the plume, but overshoots or undershoots
the ambient value. This pressure field seems to play an important role in destabilizing
the jet flow, which can explain why the underexpanded jet with the case 2 of flapping
mode, which has a reduced mass flow injection, has a comparable effect with the case
1, as shown in Fig. 4.12. This tendency was not seen in the cases of fully expanded
Figure 4.16: Spatial growth rate, $-\alpha_i$, vs. frequency, $\omega$, of both axisymmetric and antisymmetric disturbances at various axial locations in the region of jet potential core at $M_o = 1$: “−”, axisymmetric; “−”, antisymmetric. 
a) $x/D = 0$, b) 1, c) 2, d) 3, e) 4, and f) 5.
jet, as shown in Fig. 4.8, with regard to the axial distribution of jet radius. In short, it is suggested that formation of a shock cell structure has a strong destabilizing effect, especially for antisymmetric disturbances.

### 4.5 Jet Noise

The acoustic field responds strongly to the frequency of actuation and its harmonics when some external excitation is applied to a jet. Figures 4.18 and 4.19 show results of sound spectra for both fully expanded and under expanded jets, respectively. A microphone is located in the acoustic field at an angle of 45 deg from the jet axis and 100D away from the nozzle exit. The underexpanded baseline jet is clearly characterized by a screech tone with a frequency of 15 kHz (see Fig. 4.19a). Comparison of the results from the excited jets with those from the baseline jets indicates that natural and excited disturbances radiate noise in the downstream direction by a similar mechanism. Moreover, the results support the viewpoint that organized flow disturbances are directly responsi-
ble for a major portion of downstream noise radiation produced by jet [56]. However, a reduction in the radiated noise was observed only in the steady injection cases for both fully expanded and under expanded jets. This reduction seems to be due to mixing enhancement in a passive way such as in the case of vortex generators or tabs placed at the nozzle exit [35].

Figure 4.20 shows one-third octave band spectra in the axisymmetric steady injection cases shown in Figs. 4.18b and 4.19b, for both fully expanded and underexpanded jets at three different emission angles from the jet axis: $\psi = 30, 60, \text{ and } 90 \text{ deg}$. Results for the baseline jets are also presented Fig. 4.20 for comparison. The effect of actuation is clearly recognized as a reduction in SPL over most of the frequency range of the spectra, although the high frequency portion is slightly increased. Mixing enhancement due to the steady actuation increases the high frequency portion of the spectra. A similar effect due to tabs was reported in ref. [35]. The peak SPL at an emission angle of 30 deg is reduced by about 2-3 dB, whereas it does not change or slightly increases at the other two emission angles, 60 and 90 deg. Figure 4.21 shows directivity arcs for these cases, where a maximum reduction in the overall sound pressure level, OASPL, is about 3 to 5 dB for both fully expanded and underexpanded jets.

4.6 Concluding Remarks

Lateral steady or unsteady injection of an array of microjets placed along the circumference of the nozzle exit of a primary jet was experimentally studied to examine the characteristics of mixing and noise in compressible primary jets. Specifically, unsteady axisymmetric and flapping injection modes were employed in addition to steady axisymmetric injection. The unsteady injections were performed at an injection Strouhal number, $St$, of 0.16, based on the nozzle diameter and the primary jet velocity at the
Figure 4.18: Noise spectrum of fully-expanded jet with $M_j = 1.00$: a) Baseline jet, b) Axisymmetric steady mode, c) Axisymmetric unsteady mode, d) Flapping mode 2, e) Flapping mode 1, $St = 0.16$ and $\varphi_j = 0.04$ ($\varphi_j = 0.02$ flapping mode 2).
4.6. CONCLUDING REMARKS

Figure 4.19: Noise spectrum of under-expanded jet with $M_j = 1.36$: a) Baseline jet, b) Axisymmetric steady mode, c) Axisymmetric unsteady mode, d) Flapping mode 2, e) Flapping mode 1, $St = 0.16$ and $\phi_j = 0.04$ ($\phi_j = 0.02$ flapping mode 2).
a) Fully expanded with $M_j = 1.00$.

b) Underexpanded jet with $M_j = 1.36$.

Figure 4.20: 1/3-octave band SPL spectra at different emission angles: “—”, baseline jet and “·· ·”, axisymmetric steady injection, $\varphi_j = 0.04$. 
4.6. CONCLUDING REMARKS

a) Fully expanded with $M_j = 1.00$.

b) Underexpanded jet with $M_j = 1.36$.

Figure 4.21: OASPL directivity arc: $\bigcirc$, baseline jet; $\triangle$, axisymmetric steady injection and $\varphi_j = 0.04$. 
nozzle exit, which is close to one of the subharmonics of the most amplified Strouhal number obtained theoretically, for two cases of the total unsteady mass injection: 4% and 6% of the primary jet mass flow rate.

The results regarding the mean flowfield showed that the flapping injection has a higher spreading rate than the steady or unsteady axisymmetric injection with regard to the decay of jet centerline velocity. Even when the unsteady mass injection is reduced by 50%, the antisymmetric mode grew and persisted further downstream, that is, even in the flow downstream of the potential core region, compared with the case of axisymmetric mode. These results were qualitatively confirmed by performing a linear stability analysis for the fully expanded jet, which showed that the antisymmetric mode of natural disturbances is more unstable than the axisymmetric one in the downstream region.

In the case of underexpanded jet, the presence of a shock cell structure inside the jet plume has a strong destabilizing effect, especially for antisymmetric disturbances. This was confirmed by comparable effects of two different flapping actuations regarding mass injection on the downstream evolution of jet radius.

The radiated noise was reduced in the case of steady axisymmetric injection actuation, whereas it was increased in other cases of unsteady injection.
Chapter 5

Correlation between Linear Stability Theory and Transition of Compressible Jet Shear Layer

5.1 Introduction

It is believed that the jet screech tones in underexpanded supersonic jets are caused by an initial flow disturbance, which issue from the nozzle exit and propagates downstream where it interacts with the shock cell system to generate noise. The resulting noise is assumed to radiate upstream outside the jet flow to produce another disturbance as it intersects the nozzle lip where the jet boundary layer is laminar and has high receptivity to external disturbance.

The free shear layer is highly receptive to external disturbance up to the beginning of turbulent shear layer. Changing the characteristics of the free shear layers at nozzle lip has a great effect on the amplitude of the screech tones [30]. Successful reduction of the amplitude of screech tone has been attained through the introduction of small protrusion (tabs) into the jet at the nozzle lip (or roughness at the nozzle exit plane). It is reported
that these tabs make a free shear layer thicker, and thereby inhibit the growth rate of the instability waves associated with the screech phenomena [31, 15]. The main objective of the present chapter is to predict the length from the nozzle lip to the transition point of the free shear layer of a jet. The author believes that this length is one of the main elements of the feedback loop of the screech noise generation mechanism. So, an attempt has been made to predict the transition location of the free shear layer of high-speed sonic jet by using $e^N$ method.

The problem of transition from a laminar to turbulent flow is of great practical interest and has a wide range of engineering applications. The first stage of a transition process is the boundary layer receptivity. This receptivity is a mechanism that enables environmental disturbances to enter a laminar boundary layer, or laminar shear layer, and to generate unstable waves [57]. When the amplitude of these environmental disturbances is small, the linear amplification or growth of those unstable waves proceeds to the second stage of the transition process, which can be explained by the linear stability theory. The third stage occurs when the unstable waves reach a finite amplitude; their behaviors begin to deviate from the prediction by the linear theory, where wave/wave interactions and nonlinear evolution occur, leading to a turbulent flow [58].

Even if we assume that the linear stability theory can be valid up to the onset of transition, its major shortcoming is that it cannot predict the location of transition. What the linear stability theory can do is to compute the amplitude ratio, i.e., the amplification rate between two locations only, where the absolute values of these amplitudes remain to be unknown. If it is assumed that the transition occurs when the amplitude of the most unstable disturbances reaches a prescribed threshold, it is easily understood that the linear theory alone is unable to predict the transition location. However, in spite of this negative situation, transition predictions have to be made for actual problems. This can be achieved by adding a somewhat empirical ingredient to the linear theory, which
is the basis for the $e^N$ method.

The $e^N$ method was first developed independently by Smith and Gamberoni [59] and by Van Ingen [60] in 1956 to predict the transition location for wall-bounded shear flows. Today the $e^N$ method is still a state-of-the-art transition prediction design tool for wall-bounded shear flows encountered in industrial applications. As far as the author knows, however, no attempt has been made to develop an analogy of the $e^N$ transition-prediction method for free shear flows such as jet and wake. This is a main objective of the present study, where attention is paid to circular jets as free shear flows.

Jets and mixing layers are characterized by their mean-velocity profiles with an inflection point that causes inviscid instabilities. The primary mechanism of these instabilities is vortical induction, where viscosity plays only a role of damping. Thus jet flows at high Reynolds numbers can be well described by the inviscid flow equations, i.e., the Euler equations. Therefore, the linearized form of these equations considered in the linear stability analysis will be meaningful.

### 5.2 Linear Stability Theory

The principle of the linear stability theory is to introduce small sinusoidal disturbances into the linearized inviscid Euler equations in order to compute unstable frequencies. Here, it is assumed that any fluctuating quantity $\Delta'$ such as velocity, pressure, density, and temperature, is expressed by

$$\Delta' = \hat{\Delta}(r)e^{in\phi+i(\alpha x - \omega t)}$$  \hspace{1cm} (5.1)

The complex amplitude function $\hat{\Delta}$ depends on $r$ only, and $n$ is an integer that represents
the $n$-th azimuthal wave number. Basically, $\alpha$ and $\omega$ are complex values.

Since the fluctuation quantities are very small, the quadratic terms regarding these quantities will be neglected in the inviscid equations. It is also assumed here that the mean flow quantities do not change significantly over a wavelength of disturbances; therefore $\bar{U}$, which is the mean velocity component in the $x$ direction, as well as temperature $\bar{T}$ and density $\bar{\rho}$, are a function of $r$. The mean pressure $\bar{p}$ is assumed to be constant in this study, which corresponds to the case for a fully expanded jet.

This parallel basic flow approximation implies that the stability at a particular location of $x$ is determined by local conditions; in other words, it is not affected by quantities in other regions. This leads to a system of ordinary differential equations for amplitude function $\Delta(r)$, which includes four equations for incompressible flows, i.e., the continuity equation and the $r, \phi, x$ momentum equations, and two equations of energy and state are needed for compressible flow in addition to them. These stability equations can be combined to obtain a single equation for the pressure amplitude, $\tilde{p}$ [52] (see Appendix A).

\[
\frac{\partial^2 \tilde{p}}{\partial r^2} + \left[ \frac{1}{r} - \frac{1}{W} \frac{\partial W}{\partial r} \right] \frac{\partial \tilde{p}}{\partial r} - \left[ \alpha^2 (1 - M_o^2 W) + \frac{n^2}{r^2} \right] \tilde{p} = 0
\]  

(5.2)

where

\[
W = \left[ \frac{\bar{U}(r) - \frac{\omega}{\alpha} \bar{U}_o}{U_o} \right]^2 \bar{T}(r) \frac{1}{T_o}
\]  

(5.3)

When $r$ tends to 0 (jet axis), or $\infty$ (ambient fluid), the quantity $W$ of Eq. (5.3) approaches a constant value, and the asymptotic solutions to Eq. (5.2) are given by the modified
Bessel functions, $I_n$ and $K_n$, of order $n$. Since the pressure disturbance that $\tilde{p}(0)$ is bounded and $\tilde{p}(\infty)$ must go to zero, we obtain the following asymptotic relations: for $r \to 0$

$$\tilde{p}(r) = C_1 I_n(\alpha \sqrt{1 - M_o^2 W(0)}r)$$

(5.4)

and for $r \to \infty$

$$\tilde{p}(r) = C_2 K_n(\alpha \sqrt{1 - M_o^2 W(\infty)}r)$$

(5.5)

Thus an eigenvalue problem with the complex eigenvalue $\alpha$ has to be solved for given values of $U(r)/U_o, T(r)/T_o, M_o, \omega$ and $n$ that are satisfy the following dispersion relation.

$$F(\alpha, n, \omega; M_o) = 0$$

(5.6)

This relation can be calculated explicitly for some simple cases. For more realistic velocity profiles, the result is obtained by numerically integrating Eq. (5.2) for values of $\alpha$, which are iteratively changed until the boundary conditions (5.4) and (5.5) are satisfied.

### 5.2.1 Temporal and Spatial Theories

When we consider temporal instability growth, $\alpha$ is assumed to be real and $\omega$ complex: $\omega = \omega_r + i\omega_i$. Eq. (5.1) then takes the following form:
\[ \Delta' = \tilde{\Delta}(r)e^{\omega_i t}e^{i(\alpha x - \omega_r t)} \]  

(5.7)

Depending on the sign of the temporal amplification rate \(\omega_i\), the disturbances are damped for \(\omega_i < 0\), amplified for \(\omega_i > 0\), and neutral for \(\omega_i = 0\). \(\omega_r\) represents the frequency and \(\alpha\) the wavenumber in the \(x\) direction.

On the other hand, when we consider spatial instability growth, \(\omega\) is assumed to be real and \(\alpha\) becomes complex: \(\alpha = \alpha_r + i\alpha_i\). As a result, Eq. (5.1) is expressed by:

\[ \Delta' = \tilde{\Delta}(r)e^{-\alpha_i x}e^{i(\alpha r x - \omega t)} \]  

(5.8)

In this case, the fluctuation can be amplified for \(\alpha_i < 0\), neutral for \(\alpha_i = 0\), and damped for \(\alpha_i > 0\) in the \(x\) direction.

From these definitions, it is clear that any eigenvalue problem involves three real parameters; that is, they are \((\alpha, \omega_i, \omega_r)\) for temporal growth, and \((\alpha_r, \alpha_i, \omega)\) for spatial growth.

The mean velocity and temperature profiles have to be given as input data for the above eigenvalue problem. The latter can be made to be related to the former by using the Busemann-Crocco law that is valid for a boundary layer flow with a constant pressure and \(Pr = 1\) (see Eq. (4.4)).

In addition to these values, other inputs are \(M_o, T_o\), and \(n\). One of real value parameters mentioned above also has to be given to complete the problem, which is \(\alpha\) for temporal growth and \(\omega\) for spatial growth. Two remaining parameters will be obtained
as eigenvalues as a result of calculation, as well as disturbance amplitude profiles as eigenfunctions.

In the present analysis the velocity profile was given as the following hyperbolic-tangent form, as Eq.(4.2).

In general, in compressible flows the mean flow density, $\bar{\rho}(r)$, also changes with the location. For an ideal gas, the equation of state gives the following relation between the density and temperature for a constant pressure field.

$$\frac{T(r)}{T_o} = \left[\frac{\rho(r)}{\rho_o}\right]^{-1} \quad (5.9)$$

5.2.2 Relation between Spatial and Temporal Instability Theories

It is possible to convert the temporal stability problem to the spatial one by using the concept of group velocity that represents the velocity at which energy propagates in a conservative system. In the temporal instability growth theory, the group velocity, $C_g$, is defined by [61, 62]:

$$C_g = \frac{\partial \omega_i}{\partial \alpha} \quad (5.10)$$

It can be demonstrated that the temporal growth rate, $\omega_i$, can be converted to a spatial growth rate, $-\alpha_i$, by the following relation:
\[-\alpha_i = \frac{\omega_i}{C_g}\]  

(5.11)

In principle, the relation (5.11) is only valid for small values of \(\omega_i\), i.e., in the neighborhood of a neutral point.

### 5.2.3 Physical Disturbances (Wave Amplitude)

The physical disturbances correspond to the real parts of Eq. (5.1). When the spatial growth theory is applied to axisymmetric disturbances with \(n = 0\), the physical disturbance, \(\Delta'_{\text{physical}}\), is expressed by:

\[
\Delta'_{\text{physical}} = |\Delta(r)|e^{-\alpha_i x}\cos(\alpha_i \tau - \omega t + \varphi(r))
\]

(5.12)

where

\[
|\Delta| = \sqrt{\Delta_r^2 + \Delta_i^2}, \quad \cos\varphi = \Delta_r/|\Delta|, \quad \text{and} \quad \sin\varphi = \Delta_i/|\Delta|
\]

(5.13)

\(\tilde{\Delta}'_{\text{physical}}\) is the root mean square value of \(\Delta'_{\text{physical}}\), which is obtained by integrating Eq. (5.12) over one temporal period:

\[
\tilde{\Delta}'_{\text{physical}} = \frac{1}{\sqrt{2}}|\Delta(r)|e^{-\alpha_i x}
\]

(5.14)

If \(A\) represents the magnitude of \(\tilde{\Delta}'_{\text{physical}}\), we have:
\[ \frac{1}{A} \frac{dA}{dx} = -\alpha_i \]  

(5.15)

This equation is valid for any radial location \( r \) \([58, 63]\).

### 5.3 Principle of the \( e^N \) Method

The principle of this method is described here for two dimensional incompressible boundary layer flows. A laminar basic flow is specified, which can be either computed or measured, so that the mean velocity profiles are available at a number of streamwise locations \( x_j \). Then the local stability analysis for this basic flow is performed at each location \( x_j \) to determine the amplification, or growth rate, for locally unstable disturbances. These calculations enable us to obtain a stability diagram that shows a range of amplified Tollmien-Shlichting waves as a function of streamwise location, which is shown in the upper part of Fig. 5.1.

Here we consider a wave that propagates downstream with a physical frequency \( f_1 \). Figure 5.1 shows that this wave travels first through a stable region; that is, it can be damped up to \( x = x_o \). Then it is amplified up to \( x = x_1 \), and damped after that. At any location with \( x > x_o \), the wave amplitude \( A \) can be related to the amplitude \( A_o \) at the neutral point \( x_o \) by integrating the Eq. (5.15). In the framework of the spatial instability growth theory, this is expressed by

\[ \frac{A}{A_o} = exp \left\{ \int_{x_o}^{x} -\alpha_i dx \right\}, \quad \text{or} \quad \ln \left( \frac{A}{A_o} \right) = \int_{x_o}^{x} -\alpha_i dx \]  

(5.16)
where $A_o$ is usually called the initial disturbance amplitude. This value is linked to the surrounding disturbance environment through some receptivity mechanism. The streamwise variation of the natural logarithm of $A/A_o$ is plotted in the lower part of Fig. 5.1 for the frequency $f_1$ as well as other frequencies such as $f_2$ and $f_3$. It is obvious from Eq. (5.15) that $\ln(A/A_o) = 0$ at $x = x_o$ and that the slopes of the curves vanish at $x = x_o$ and $x = x_1$. The dashed line in Fig. 5.1 represents an envelope for these curves, which is called the factor of $N$ [58].

\[
N = \max_f \left[ \ln \left( \frac{A}{A_o} \right) \right] \tag{5.17}
\]

At each streamwise location of $x$, $N$ represents the maximum amplification factor of the
disturbances[58].

So far, the only assumption is use of the linear stability theory. Unfortunately, this is not sufficient to determine the transition location. Therefore if it is predicted with this method, an additional assumption is needed, which was suggested independently by Smith and Gamberoni [59] and by Van Ingen [60]. They collected many experimental transition data and found that the $N$ factor attains nearly a constant value between 7 and 9 at the measured transition point. This means that a transition occurs when the amplitude of most amplified Tollmien-Schlichting waves attains the value from $e^7$ ($= 1,097$) to $e^9$ ($= 8,103$) times as large as its initial amplitude, $A_o$.

Here, we summarize the procedure to determine the transition location, which consists of three steps:

1. Calculate or measure the laminar velocity profile at each downstream station.

2. Calculate stability conditions for these profiles.

3. Integrate the local growth rate in order to define an envelope curve. The transition location is then easily determined if a threshold value of the $N$ factor is specified.

Since nonlinear mechanisms and receptivity are not considered in this method, it has shortcomings [58]. However, the $e^N$ method is certainly the most popular technique used today as a practical transition prediction tool.

The objective of the present study is to use stability calculations to predict the transition location of a compressible jet’s free shear layer by means of the $e^N$ method, and determine the threshold value of $N$ for the present jet flow.
5.4 Results and Discussion

5.4.1 Mean Flow Field of Compressible Jet

In the present study the mean flow field was determined experimentally. The jet is issued from a convergent nozzle having an exit diameter, $D$, of 7.5 mm. $M_j$ is commonly used to denote the fully-expanded Mach number, which is one of the jet’s characteristics. It has a unique relation with the nozzle pressure ratio, $NPR$, which is defined by $P_t/p_\infty$ by the following equation:

$$M_j = \left\{ \left[ \left( \frac{P_t}{P_\infty} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \frac{2}{\gamma - 1} \right\}^{0.5} \quad (5.18)$$

The jet considered in the present study is a sonic jet, i.e., $M_j = 1.00$, which corresponds to $NPR = 1.89$. The corresponding Reynolds number is $Re = 1.5 \times 10^5$, which is based on the nozzle exit diameter and the primary jet velocity.

Figure 5.2 shows the distribution of centerline Mach number, which was obtained only by Pitot probe measurement. The Mach number was calculated by using the isentropic flow relation of Eq. (5.18), where $P_t$ in this case represents the local Pitot pressure.

The potential core length has been determined the same way as described in chapter 4, section 3.1.2. Thus, we have a potential core length of about $5D$ for the present jet. The vertical lines shown in Fig. 5.2 represent the axial locations at which the axial velocity component was measured along each radial coordinate.

Figure 5.3 shows the centerline distribution of $T_0/T_\infty$ ratio. This ratio was calculated from the centerline Mach number distribution by using an isentropic relation. This parameter is indispensable for calculating the temperature profile used in stability analysis, based on Busemann-Crocco’s law of Eq. (4.4).
The normalized radial Mach number distributions at 25 axial locations from the nozzle exit up to \( x/D = 6.667 \) are shown in Fig. 5.4. In addition to experimental data obtained in this study, the least square fitted hyperbolic-tangent velocity profile of Eq. (4.2) is also presented in this figure. In this fitting method, for a given hyperbolic tangent velocity profile: \( M_{\text{Approx.}}(r_k/R_{0.5}, R_{0.5}/\theta) \) and experimental data: \( M_{\text{Exp.}}(r_k/R_{0.5}) \), where \( M \) is Mach number, the optimal value of \( R_{0.5}/\theta \) is obtained as the minimum value of the following term:

\[
F = \sum_{k=1}^{m} \left[ M_{\text{Exp.}}(r_k/R_{0.5}) - M_{\text{Approx.}}(r_k/R_{0.5}, R_{0.5}/\theta) \right]^2
\]  

(5.19)

where \( m \) is the number of experimental data points.

For example, at \( x/D = 0.933 \), Fig. 5.5a shows the variation of \( F \) defined by Eq. (5.19) with \( R_{0.5}/\theta \). It is clearly seen from this figure that the optimal value of \( R_{0.5}/\theta \) is about 17. Figure 5.5b shows comparison of radial distribution between the experimental data
and hyperbolic tangent velocity profiles with three different values of $R_{0.5}/\theta$ including the optimal value.

As a result of this fitting, we can calculate the variation of $\theta/R_{0.5}$ in the axial direction, the result of which is shown in Fig. 5.6 as well as the data by Crigthon and Gaster [55] for comparison. The fitted straight line has the same slope as that of Crigthon and Gaster [55], but a different value of the constant. This constant seems to depend on jet initial conditions at the nozzle exit. Note that the value of $R_{0.5}/\theta$ at each location is needed to decide the hyperbolic-tangent velocity profile presented in Fig. 5.4.

Figure 5.7 shows the axial distribution of jet radius that is defined as the length from the axis to the location where the velocity becomes half the centerline velocity at each station [26]. There are two distinct axial regions seen in the figure; one is a potential core region, and the other is a fully developed jet region. The best linear fitting lines as well as these experimental data are presented in the figure. The fully developed region seems to grow from an “apparent origin” located at about $x/D = 1.333$. 

Figure 5.3: Jet centerline temperature distribution normalized by the ambient temperature.
Figure 5.4: Radial Mach number distribution at different axial locations: ◦; experimental data, and “—”; hyperbolic tangent velocity profile:
a) \(x/D = 0.000\), b) 0.133, c) 0.267, d) 0.400, e) 0.533, f) 0.667, g) 0.800, h) 0.933, i) 1.067, j) 1.200, k) 1.333, l) 1.600, m) 1.867, n) 2.133, o) 2.400, p) 2.667, q) 2.933, r) 3.200, s) 3.467, t) 3.733, u) 4.000, v) 4.667, w) 5.333, x) 6.000, y) 6.667.
5.4. RESULTS AND DISCUSSION

5.4.2 Linear Stability Calculations of Jet

Inviscid spatial stability analysis has been performed for the present jet flow. The jet linear stability code employed here was first validated by comparing it with previously published computations by Michalke [52], for the hyperbolic-tangent velocity profile of Eq. (4.2), with $R_{0.5}/\theta = 6.25$, $M_o = 1.2$ and $T_o/T_\infty = 1$, which is an isothermal flow, the results of which are shown in Fig. 5.8. This figure presents axial growth rate, $-\alpha_i$, normalized by the momentum thickness versus radian frequency, $\omega$, normalized by the momentum thickness and centerline jet velocity. The present computations have reasonable agreement with Michalke’s results for both axisymmetric ($n = 0$) and asymmetric ($n = 1$) disturbance modes.

Results of stability calculation based on the velocity profiles presented in Fig. 5.4 are shown in Fig. 5.9 for axisymmetric disturbances with $n = 0$. The quantities are non-dimensionalized by the local momentum thickness and local centerline jet velocity,
Figure 5.6: Axial variation of ratio of momentum thickness to jet radius, $\theta/R_{0.5}$, in the jet potential core region: $\circ$, present experimental data; “—”, present fitting; “···”, Crighton and Gaster [55] fitting.

Figure 5.7: Axial variation of jet radius, $R_{0.5}$: $\bullet$, experimental data; “—”, best linear fitting.
and the corresponding dimensional results are shown in Fig. 5.10. We can clearly see a neutral stability curve in this figure, where in the downstream a single neutral frequency exists at each axial location.

The thin axisymmetric shear layer is unstable for a large number of discrete azimuthal modes [56], but a fully developed jet is unstable for the first azimuthal mode, *i.e.*, \( n = 1 \), downstream of the termination of the potential core [64]. Cohen and Wygnanski [65] showed that as \( R_{0.5}/\theta \) decreases, the importance of higher azimuthal modes (\( n \geq 2 \)) relatively diminishes, and that only the first azimuthal (\( n = 1 \)) and the axisymmetric (\( n = 0 \)) modes remains amplified at the end of the potential core. Therefore, in the present study, only these two modes will be considered, whose calculated growth rates are illustrated in Fig. 5.11 for six axial locations. As we go downstream, it turns out that the antisymmetric disturbances become more unstable than the axisymmetric ones.
a) Stability curves at different axial locations.

b) Growth rate contours.

Figure 5.9: Nondimensional stability curves at different axial locations for axisymmetric disturbances mode ($n = 0$).
5.4. RESULTS AND DISCUSSION

a) Stability curves at different axial locations.

b) Growth rate and neutral stability curve.

Figure 5.10: Dimensional stability curves at different axial locations for axisymmetric disturbance mode ($n=0$).
5.4.3 Transition Detection

Transition starts when turbulent structures or spots first appear in the laminar shear layer. Under natural conditions, the spots originate in a more or less random fashion. Once created, they are swept along in the mean flow, growing axially and laterally, and finally cover the entire surface. The transition region is defined as a region where the turbulent spots grow, overlap, and begin to form a turbulent shear layer. When a hot wire is placed in this region, both the turbulent spots and laminar flow will successively appear in the recorded fluctuation data. This is referred to as the intermittency phenomenon [58]. In this study, three methods are employed to detect the transition location of a jet shear layer flow under consideration, which will be described in the following.
5.4. RESULTS AND DISCUSSION

5.4.3.1 Oil Flow Visualization

Any visualization technique using Schlieren photography cannot be employed for transition detection of the jet flow, because the density variations are too low to detect the change in flow behavior inside the jet plume. The smoke visualization cannot be used either, because the main flow velocity is too large to observe. To overcome these difficulties, we attempted oil flow visualization. We placed a thin plate painted with oil and with a thickness of 0.5 mm, in a meridian plane of the jet, as shown in Fig. 5.12a, and a video camera captured the time evolution of the oil flow on the surface during the experiment.

Although the plate is very thin compared with the nozzle exit diameter, the effect of its intrusion inside the jet plume on the transition location will exist. The results of the oil flow visualization are shown in Fig. 5.12b. Turbulent spots can be clearly recognized as white spots near the jet boundary, whose origin lies between $1D$ and $2D$. As mentioned earlier, the present oil flow results might have some differences from a real jet flow. One important issue in this visualization is to determine the jet boundary. Since the accuracy of pressure transducers is poor in low speed regions, it is difficult to determine the jet boundary only by Pitot pressure measurements. The determination of the jet boundary will be very helpful in specifying the transition location by using a microphone or a hot-wire anemometer, which will be discussed.

5.4.3.2 Pressure Fluctuations Measured by Microphone

One of the methods to detect the transition location is use of a microphone to measure the pressure fluctuations of a jet flow. In the present method the microphone was moved along a line parallel to the jet boundary determined by the oil flow visualization mentioned above. Specifically, the line was away from the jet boundary by $0.8D$, as shown in Fig. 5.13. The microphone at this location can catch only sound waves with wavelengths less
a) Experimental arrangement for oil flow visualization.

b) Jet boundary visualized by oil flow.

Figure 5.12: Oil flow visualization.
5.4. RESULTS AND DISCUSSION

than 0.8D, produced by the jet shear layer. This means that the microphone signal has to be high-pass-filtered, the lowest frequency of which has a wavelength of 0.8D. The resulting signals are shown in Fig. 5.14 at different downstream locations from the nozzle exit. In this figure, regular peak patterns start to appear from approximately \( x \approx 1.6D \), whose location is considered to be the onset of the turbulent spots, or transition.

5.4.3.3 Velocity Fluctuations Measured by Hot Wire

Transition detection by using a hot wire anemometer is one of well-documented methods of measuring velocity fluctuations [58]. In the present experiment, a hot wire was traversed along a line parallel to the jet boundary, which was away from it by 0.2D in order to avoid a hot wire damage due to the high-speed jet flow. The velocity fluctuations measured by the hot wire at various downstream locations are shown in Fig. 5.15. We
Figure 5.14: HPF microphone signals at different axial locations.
can clearly see that the turbulent spots begin to appear at approximately \( x = 1.333D \).
This is not so much different from \( 1.6D \) by the microphone mentioned above, and seems
to be also in reasonable agreement with the transition location between \( x = 1D \) and
\( 2D \) due to the oil flow visualization. It is very interesting to notice that the transition
location determined by the hot wire anemometer is very close to the “apparent origin”
of the fully developed jet flow, as shown in Fig. 5.7.

5.4.4 Envelope Curve

The spatial growth rate, \( -\alpha_i \), has been integrated based on Eq. (5.16) to obtain an
envelope curve similar to that of a boundary layer flow presented in Fig. 5.1. The lower
limit of the integration is the nozzle exit, \( i.e., x = 0 \). The results of spatial growth rate
integration for a selected set of frequencies for axisymmetric disturbance modes \( (n = 0) \)
are shown in Fig. 5.16a. These integration curves represent the streamwise variations of
the natural logarithm of \( A/A_o \), and the thick solid line is the envelope of these curves,
which is referred to as the factor of \( N [58] \).

Based on both microphone and hot-wire measurements, the transition location seems
to be between \( x = 1.333D \) and \( x = 1.6D \). The microphone has a diameter of about 0.125
inch, which might affect its resolution regarding the transition location. Therefore the hot
wire measurement seems to be more accurate than the microphone measurement in terms
of transition location. Hence, \( x = 1.333D \) is considered as the onset of transition region in
this study. The corresponding \( N \) factor is about 3.5 for axisymmetric disturbance modes
\( (n = 0) \), as shown in Fig. 5.16a. Figure 5.16b shows the same results for first azimuthal
disturbance modes \( (n = 1) \). In this case, the factor \( N \) becomes about 2.8. That is, this
difference in the value of \( N \) suggests that the axisymmetric disturbances can be more
amplified in this region than the asymmetric ones can. This is consistent with the results
Figure 5.15: Velocity fluctuations measured by hot wire at different axial locations.
of Cohen and Wygnanski [65], who showed that first azimuthal disturbance modes is more amplified than the axisymmetric disturbance modes near the end of the potential core only. Therefore the transition prediction based on stability computations of axisymmetric disturbance modes should be more accurate than the asymmetric ones. The natural logarithm of \( A/A_o \) for the first azimuthal disturbance modes with low frequencies is higher in magnitude than that of the axisymmetric disturbance modes. This is confirmed because the antisymmetric \((n = 1)\) disturbances are more unstable than the axisymmetric \((n = 0)\) disturbances at low frequencies, as presented earlier in Fig. 5.11.

For boundary layer flow the transition location was correlated with the value of \( N \) between 7 and 9. This variation in the \( N \) factor was found to be due to a variation in free-stream turbulence level [63]. An increase in this level will decrease the transition Reynolds number, and consequently the \( N \) factor will have a low value and vice versa.

No attempt has been made for the present study to investigate the effect of jet upstream turbulence level on \( N \) factor and the transition location. From the value of \( N \) factor based on stability computation of the axisymmetric disturbance modes, a transition is believed to occur when the amplitude of most amplified waves becomes \( e^{3.5} (= 33) \) times as large as its initial amplitude, \( A_o \). On the other hand, in a boundary layer flow, where the low limit of \( N \) factor is 7, the transition occurs when the amplitude of most amplified waves becomes \( e^7 (= 1,097) \) times as large as its initial amplitude, \( A_o \). Comparison of these values of \( N \) factor asserts that the boundary layer flow is more stable than the free shear flow.

5.5 Concluding Remarks

A correlation between linear stability calculations of a compressible jet flow and the onset of its transition location has been established. The jet mean velocity profiles were
a) Axisymmetric disturbance modes \((n=0)\).

b) Asymmetric disturbance mode \((n=1)\).

Figure 5.16: Envelope curve for instability growth.
obtained experimentally at different downstream locations. The stability calculations were performed for these profiles and integrated in order to obtain an envelope curve. The transition was observed experimentally by oil flow visualization, microphone measurement for pressure fluctuations, and a hot wire measurement for velocity fluctuations. Results from both the microphone and hot wire measurements determined the transition location to be about $x = 1.333D$, where $D$ is the nozzle exit diameter. This is very close to the location of the “apparent origin” in the fully developed jet flow. It was found that the value of $N$ factor is about 3.5 for axisymmetric ($n = 0$) disturbance mode, and 2.8 for asymmetric ($n = 1$) disturbance mode. This difference in the value of $N$ suggests that axisymmetric disturbances can be more amplified in this upstream region than asymmetric ones. Therefore a prediction based on stability computation of axisymmetric disturbance mode should be more accurate than that of asymmetric one. Further work is still required to investigate the effect of jet upstream turbulence level on the value of $N$ factor as well as the transition location.
Chapter 6

Conclusions

Mixing enhancement by the use of passive control in the form of vortex generator configuration referred to as vane-type tab in two different configurations: free-to-rotate and stationary has been proposed, and it was confirmed that these vanes are quite effective in influencing jet evolution and mixing. The results are summarized as follows.

- The jet plume cross section maintains an axisymmetric shape for the free-to-rotate case, while it is often non-axisymmetric for the stationary case, depending on the number of vanes and their azimuthal locations.

- The centerline stagnation pressure and hence the Mach number for the case of free-to-rotate vanes are found to be decayed faster than the stationary vane case.

- In the case of two stationary vanes with $w/D = 12.8\%$, a jet is bifurcated into two parts. This jet bifurcation does not occur for the rotating case.

- The vane tabs can effectively reduce the noise in underexpanded and overexpanded jets regimes.

- From the centerline stagnation pressure data, case 4 of vane-type tabs has the weakest shock/expansion structure compared with other vane-type tabs cases. Since
this case has two pair of counter rotating vortices, it is similar to jet tabbed with two delta tabs.

- A jet noise reduction of about 10 dB is observed for four stationary vanes with \( w/D = 6.4\% \) and a vane angle of 30°, where there is a thrust loss of about 1.5 ~ 2% per vane compared with the baseline jet.

- For the same thrust and exit area, the tabbed jet have lower noise level than the baseline jet.

- By increasing the ratio of \( w/D \), thrust penalty increases and noise is more reduced, \textit{i.e.}, about 12 dB for \( w/D = 12.8\% \).

- Generally, stationary vanes have a lower thrust penalty than free-to-rotate vanes except for low \( NPR \) values with \( w/D = 6.4\% \).

- From spectrum results, in the audible frequency range (up to 20 kHz) noise is reduced due to the vane tabs which seems to be associated with shock associated noise. The higher frequencies, which are greater than 20 kHz, seem to be unaffected and/or increase due to mixing enhancement by the vane tabs.

- Different vortex generator configurations were studied, and their effects on data of engineering interest such as thrust loss, increase of mixing, and reduction in noise were evaluated. It became clear here that vane-type tabs and free-to rotate tabs have a reasonable reduction in OASPL with acceptable thrust loss compared with other tab types.

Lateral steady or unsteady injection of an array of microjets placed along the circumference of the nozzle exit of a primary jet was experimentally studied to examine
the characteristics of mixing and noise in compressible primary jets. Specifically, unsteady axisymmetric and flapping injection modes were employed in addition to steady axisymmetric injection. The unsteady injections were performed at an injection Strouhal number, $St$, of 0.16, based on the nozzle diameter and the primary jet velocity at nozzle exit, which is close to one of subharmonics of the most amplified Strouhal number obtained theoretically, for two cases of the total unsteady mass injection: 4% and 6% of primary jet mass flow rate.

The results of mean flowfield showed that the flapping injection has a higher spreading rate than the steady or unsteady axisymmetric injection with regard to the decay of jet centerline velocity. Even when the unsteady mass injection is reduced by 50%, the antisymmetric mode grew and persisted further downstream, that is, even in the flow downstream of the potential core region, compared with the case of axisymmetric mode with a full unsteady mass injection. These results were qualitatively confirmed by performing a linear stability analysis for a fully expanded jet, which showed that the antisymmetric mode of natural disturbances is more unstable than the axisymmetric one in the downstream region.

In the case of underexpanded jet, the presence of the shock cell structure inside the jet plume has a strong destabilizing effect, especially for antisymmetric disturbances. This was confirmed by comparable effects of two different flapping actuations on the evolution of jet radius in the downstream direction. The radiated noise was reduced in the case of steady axisymmetric injection actuation, whereas it was increased in other cases of unsteady injection.

Finally, the correlation between linear stability calculations of a compressible jet flow and the onset of transition location has been established. The spatial stability theory was considered in the present study. The jet mean velocity profiles were obtained exper-
mentally at different downstream locations. The stability calculations were performed for these profiles and integrated in order to obtain an envelope curve. The transition was observed experimentally by oil flow visualization, a microphone measurement for pressure fluctuations, and a hot wire measurement for velocity fluctuations. Results from both the microphone and hot wire measurements determined the transition location to be about $x = 1.333D$, where $D$ is the nozzle exit diameter. This is very close to the location of the “apparent origin” in the fully developed jet flow. It was found that the value of $N$ factor is about 3.5 for axisymmetric ($n = 0$) disturbance mode, and 2.8 for asymmetric ($n = 1$) disturbance mode. This difference in the value of $N$ suggests that axisymmetric disturbances can be more amplified in this upstream region than the asymmetric ones. Therefore the prediction method based on stability computation of axisymmetric disturbance mode should be more accurate than that of asymmetric one. Further work is still required to investigate the effect of jet upstream turbulence level on the value of $N$ factor as well as the transition location.
Bibliography


Appendix A

Linear Stability Analysis of Inviscid Compressible Circular Jets

A.1 The basic jet flow

An axisymmetric jet flow originates when a fluid under the action of over-pressure is flowing through a circular nozzle into a field containing a fluid at rest. We assume that there is no swirl in the flow, and that the fluid of the jet is the same as the ambient fluid. If the Reynolds number of the flow is large, then close to the jet origin the axial velocity component is large compared with the radial component, i.e., the jet flow is approximately parallel. In order to investigate the jet instability, it is therefore acceptable to assume that the undisturbed basic jet flow is parallel, i.e., the jet velocity vector has only an axial component $\bar{U}$.

Since the basic jet flow is axisymmetric, we use a cylindrical coordinate system $(x,r,\phi)$ with the x-axis being the jet axis. The continuity equation is satisfied, if we have $\bar{U} = \bar{U}(r)$, and the basic density distribution is $\bar{\rho} = \bar{\rho}(r)$. In the parallel jet flow the pressure $\bar{P} = P_\infty$ is constant for large Reynolds numbers where $P_\infty$ is the pressure in the ambient
fluid. We assume additionally that we have a thermally ideal gas with constant heat capacities, and that the Prandtl number is unity. Then for large Reynolds numbers we can assume that the disturbance motion is not essentially influenced by diffusive processes caused by viscosity, heat conduction and dissipation. Hence the corresponding equations for the problem are continuity and Euler equation and the energy equation.

A suitable normalization of the velocity profile is based on the centerline velocity $\bar{U}(0) = U_o$ and the jet radius $R$ where $\bar{U}(R) = U_o/2$ so that

$$\bar{U}(r)/U_o = U(r/R)$$  \hspace{1cm} (A.1)

with $U(0) = 1$ and $U(1) = 1/2$. The parallel flow approximation is not an exact solution of the Navier-Stockes equation, but only an approximate one for large Reynolds number. In the present study we will use the hyperbolic tangent velocity profile proposed by Mickalke (1984)[52]

$$U(r) = 0.5 \left[1 + \text{tanh} \left(0.25 \frac{R}{\theta} \left( \frac{R}{r} - \frac{r}{R} \right) \right) \right]$$  \hspace{1cm} (A.2)

where $\theta$ is the momentum boundary layer thickness

For compressible flow of homogeneous fluid the density $\bar{\rho}(r)$ and the absolute temperature $\bar{T}(r)$ are generally not constant. For ideal gas the equation of state yield for constant pressure

$$\frac{\bar{T}(r)}{T_o} = \frac{\rho_o}{\bar{\rho}(r)}$$  \hspace{1cm} (A.3)
It is convenient to relate the temperature field to the velocity field by means of
**Busmann-Crocco Law** which is valid for a boundary layer flow with Prandtl number \( Pr = 1 \) and constant pressure

\[
\frac{T(r)}{T_o} = \frac{T_\infty}{T_o} + \left[ 1 - \frac{T_\infty}{T_o} \right] U(r) + \frac{\gamma - 1}{2} M^2 U(r)(1 - U(r)) \tag{A.4}
\]

where \( T_\infty \) is the ambient temperature, \( \gamma \) is the specific heat ratio and \( M = U_o/a_o \) is the jet Mach number with \( a_o \) being the sound speed at the jet temperature \( T_o \).

### A.2 Basic equations

Consider the continuity, Euler, and energy equations in cylindrical coordinate

**Continuity equation**

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial \left( \rho v_r r \right)}{\partial r} + \frac{1}{r} \frac{\partial \left( \rho v_\phi \right)}{\partial \phi} + \frac{\partial \left( \rho v_x \right)}{\partial x} = 0 \tag{A.5}
\]

**Momentum equations**

**\( r \)-direction**

\[
\rho \left[ \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\phi}{r} \frac{\partial v_r}{\partial \phi} + v_x \frac{\partial v_r}{\partial x} - \frac{v_\phi^2}{r} \right] = -\frac{\partial p}{\partial r} \tag{A.6}
\]

**\( \phi \)-direction**
A.3. LINEARIZED DISTURBANCE EQUATIONS

We suppose here that a steady jet with velocity $\vec{U}$, having components $\bar{U}(r), 0, 0$ relative to cylindrical coordinates $x, r, \phi$, pressure $\bar{P}$, density $\bar{\rho}(r)$, and temperature $\bar{T}(r)$ is subject to a small disturbance about these values as follows:

\[ \rho \left[ \frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\phi}{r} \frac{\partial v_\phi}{\partial \phi} + v_x \frac{\partial v_\phi}{\partial x} + \frac{v_r v_\phi}{r} \right] = -\frac{1}{r} \frac{\partial p}{\partial \phi} \]  \hspace{1cm} (A.7)

$x$-direction

\[ \rho \left[ \frac{\partial v_x}{\partial t} + v_r \frac{\partial v_x}{\partial r} + \frac{v_\phi}{r} \frac{\partial v_x}{\partial \phi} + v_x \frac{\partial v_x}{\partial x} \right] = -\frac{\partial p}{\partial x} \]  \hspace{1cm} (A.8)

Energy equation

\[ \rho \left[ \frac{\partial e}{\partial t} + v_r \frac{\partial e}{\partial r} + \frac{v_\phi}{r} \frac{\partial e}{\partial \phi} + v_x \frac{\partial e}{\partial x} \right] = -\rho \left[ 1 \frac{\partial (rv_r)}{\partial r} + \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_x}{\partial x} \right] \]  \hspace{1cm} (A.9)

Equation of state

\[ p = \bar{R} \rho T \]  \hspace{1cm} (A.10)

where $e = c_v T$ is the internal energy

A.3 Linearized disturbance equations

We suppose here that a steady jet with velocity $\vec{U}$, having components $\bar{U}(r), 0, 0$ relative to cylindrical coordinates $x, r, \phi$, pressure $\bar{P}$, density $\bar{\rho}(r)$, and temperature $\bar{T}(r)$ is subject to a small disturbance about these values as follows:
\[
\begin{align*}
v_x &= \bar{U}(r) + v'_x, \\
v_r &= v'_r, \\
v_\phi &= v'_\phi, \\
P &= \bar{P} + p', \\
\rho &= \bar{\rho}(r) + \rho', \\
T &= \bar{T}(r) + T'
\end{align*}
\] (A.11)

By introducing (A.11) into (A.5) to (A.10) and assuming small disturbance quantities so that the high order terms can be neglected. The linearized inviscid disturbance equations for a compressible flow with constant specific heats are given by

Linearized continuity equation

\[
\frac{\partial \rho'}{\partial t} + \bar{U} \frac{\partial \rho'}{\partial x} = -\bar{\rho} \left[ \frac{\partial v'_x}{\partial x} + \frac{1}{r} \frac{\partial (rv'_r)}{\partial r} + \frac{1}{r} \frac{\partial (v'_\phi)}{\partial \phi} \right] - v'_r \frac{\partial \bar{\rho}}{\partial r}
\] (A.12)

Linearized momentum equations

\textit{r-direction}

\[
-\bar{\rho} \left[ \frac{\partial v'_x}{\partial t} + \bar{U} \frac{\partial v'_x}{\partial x} \right] = -\frac{\partial p'}{\partial r}
\] (A.13)

\textit{\phi-direction}

\[
\bar{\rho} \left[ \frac{\partial v'_\phi}{\partial t} + \bar{U} \frac{\partial v'_\phi}{\partial x} \right] = -\frac{1}{r} \frac{\partial p'}{\partial \phi}
\] (A.14)
x-direction

\[
\hat{\rho} \left[ \frac{\partial v'_x}{\partial t} + \bar{U} \frac{\partial v'_z}{\partial x} + v'_r \frac{\partial \bar{U}}{\partial r} \right] = -\frac{\partial p'}{\partial x}
\]  
(A.15)

Linearized energy equation

\[
c_{p} \hat{\rho} \left[ \frac{\partial T'}{\partial t} + \bar{U} \frac{\partial T'}{\partial x} + v'_r \frac{\partial \bar{T}}{\partial r} \right] = -\bar{P} \left[ \frac{\partial v'_r}{\partial r} + \frac{v'_r}{r} + \frac{1}{r} \frac{\partial v'_\phi}{\partial \phi} + \frac{\partial v'_x}{\partial x} \right]
\]  
(A.16)

Linearized equation of state

\[
\frac{P'}{\bar{P}} = \frac{\rho'}{\bar{\rho}} + \frac{T'}{\bar{T}}
\]  
(A.17)

the disturbance quantities may be written as follows

\[
\begin{align*}
v'_x &= \tilde{v}_x(r)e^{in\phi+ianx-i\omega t} \\
v'_r &= \tilde{v}_r(r)e^{in\phi+ianx-i\omega t} \\
v'_\phi &= \tilde{v}_\phi(r)e^{in\phi+ianx-i\omega t} \\
p' &= \tilde{p}(r)e^{in\phi+ianx-i\omega t} \\
T' &= \tilde{T}(r)e^{in\phi+ianx-i\omega t} \\
\rho' &= \tilde{\rho}(r)e^{in\phi+ianx-i\omega t}
\end{align*}
\]  
(A.18)

where \(n\) is the integer azimuthal wave number, \(\alpha\) is the wave number, and \(\omega\) is the radian frequency. Generally \(\alpha\) and \(\omega\) are complex, and \(n\) is real positive. By introducing the
above disturbances, the linearized disturbance equations became:

Continuity equation

\[ i\alpha (U - \frac{\omega}{\alpha}) \ddot{\rho} = -\dot{\rho} \left[ i(\ddot{v}_x \alpha + \ddot{v}_\phi n) + \frac{\ddot{v}_r}{r} + \frac{\partial \ddot{v}_r}{\partial r} \right] - \dot{v}_r \frac{\partial \ddot{\rho}}{\partial r} \]  \hspace{1cm} (A.19)

Momentum equations

\( r \)-direction

\[ \bar{\rho} i\alpha (U - \frac{\omega}{\alpha}) \ddot{v}_r = -\frac{\partial \ddot{\rho}}{\partial r} \]  \hspace{1cm} (A.20)

\( \phi \)-direction

\[ \bar{\rho} \alpha (U - \frac{\omega}{\alpha}) \ddot{v}_\phi = -\ddot{p}_n \frac{r}{r} \]  \hspace{1cm} (A.21)

\( x \)-direction

\[ \bar{\rho} \left[ (U - \frac{\omega}{\alpha}) \ddot{v}_x + \ddot{v}_r \frac{\partial U}{\partial r} \right] = -\ddot{p}_\alpha \]  \hspace{1cm} (A.22)

Energy equation

\[ i\alpha (U - \frac{\omega}{\alpha}) \ddot{T} + \ddot{v}_r \frac{\partial \ddot{T}}{\partial r} = -\ddot{T}(\gamma - 1) \left[ i(\ddot{v}_x \alpha + \ddot{v}_\phi n) + \frac{\ddot{v}_r}{r} + \frac{\partial \ddot{v}_r}{\partial r} \right] \]  \hspace{1cm} (A.23)

Equation of state
A.4 Eigenvalue equation

Elimination of \( \tilde{\rho}, \tilde{v}_\phi, \tilde{v}_x \) and \( \tilde{T} \) in equation (A.19), (A.21), (A.22), (A.23) and (A.24) with the substitution about \( \tilde{v}_r \) from equation (A.20). The disturbance equations can be reduced to a single one for the pressure amplitude equation \( \tilde{p}(r) \):

\[
\frac{\partial^2 \tilde{p}}{\partial r^2} + \left[ \frac{1}{r} - \frac{1}{W} \frac{\partial W}{\partial r} \right] \frac{\partial \tilde{p}}{\partial r} - \left[ \alpha^2 (1 - M^2 W) + \frac{n^2}{r^2} \right] \tilde{p} = 0
\]

where

\[
W = \left[ \frac{\tilde{U}(r) - \frac{\omega}{a}}{U_o} \right]^2 \left/ \frac{\tilde{T}(r)}{T_o} \right.
\]

and

\[
M = \frac{U_o}{a_o}, \quad a_o = \sqrt{\gamma R T_o}
\]

The boundary conditions to be satisfied by the pressure disturbance are:

\[
\tilde{p}(0) = \text{finite}, \quad \tilde{p}(\infty) = 0
\]

Hence and eigenvalue problem is posed, namely that, for a given value frequency \( \omega \), the
eigenvalue $\alpha$ has to be found which leads to an eigenfunction $\tilde{p}(r)$ of (A.25) satisfying the boundary conditions (A.28).

A.5 Asymptotic solution and boundary condition

For $r \to 0$ (jet axis) and $r \to \infty$ (ambient fluid) both $\bar{U}(r)$ and $\bar{T}(r)$ approach constant values therefore the function $W$ also approaches a constant values and its derivative, $\frac{\partial W}{\partial r} \to 0$. Hence equation (A.25) can be written as:

$$\frac{\partial^2 \tilde{p}}{\partial r^2} + \left[\frac{1}{r} \frac{\partial \tilde{p}}{\partial r} - \frac{\alpha^2(1 - M^2W) + n^2}{r^2}\right] \tilde{p} = 0$$  \hspace{1cm} (A.29)

Which is the modified Bessel’s equation with general solution given by:

$$\tilde{p}(r) = C_1 I_n(\alpha \sqrt{1 - M^2W(r)}r) + C_2 K_n(\alpha \sqrt{1 - M^2W(r)}r)$$  \hspace{1cm} (A.30)

Where $I_n$ and $K_n$ are the modified Bessel functions of first and second kind of order $n$ respectively, $C_1$ and $C_2$ are arbitrary complex constants. Due to the asymptotic behavior of the modified Bessel functions which shows that $I_n \to \infty$ as $r \to \infty$ and $K_n \to \infty$ as $r \to 0$. The boundary condition of the pressure disturbance can be written as:

$$\tilde{p}(r \to 0) = C_1 I_n(\alpha \sqrt{1 - M^2W(0)}r)$$  \hspace{1cm} (A.31)
A.6. SPATIAL AND TEMPORAL INSTABILITY MODES

\[ \tilde{p}(r \to \infty) = C_2 K_n(\alpha \sqrt{1 - M^2 W(\infty)} r) \]  

\hspace{1cm} (A.32)

A.6 Spatial and temporal instability modes

Generally, \( \alpha \) and \( \omega \) are complex and \( \alpha_r \) (real part of \( \alpha \)) is the wave number, \( \omega_r \) (the real part of \( \omega \)) the frequency, and \( \alpha_i, \omega_i \) (the imaginary parts of \( \alpha \) and \( \omega \) respectively) the amplification or growth rates. (\( \omega_i > 0 \) denotes temporal amplified disturbance in time, while \( \alpha_i < 0 \) denotes spatially amplified disturbances in the positive direction of \( x \)). The two cases of most interest are those in which \( \alpha \) or \( \omega \) is wholly real. In the first case, when \( \alpha \) is real and \( \omega \) is complex we have a time-dependent amplification system (temporal stability analysis) and in the second case, when \( \omega \) is real and \( \alpha \) is complex we have a spatially dependent system (spatial stability analysis). We can summarize the two cases as follow:

Temporal stability analysis

\[ \alpha = \alpha_r, \quad \omega = \omega_r + i \omega_i \]  

\hspace{1cm} (A.33)

Spatial stability analysis

\[ \alpha = \alpha_r + i \alpha_i, \quad \omega = \omega_r \]  

\hspace{1cm} (A.34)

However, from a physical point of view, spatially growing disturbance are more appropriate, as suggested by Gaster (1962)[62]. Results of stability theory with spatially growing disturbance by Michalke (1984)[52] showed better agreement with experimental results. Gaster (1962)[62] showed that the spatially growth rate is related to the time growth rate
by the group velocity and this relation can be written as:

\[ \alpha_i = -\frac{\omega_i}{C_g} \]

(A.35)

where \( C_g \) is the group velocity and is given by:

\[ C_g = \frac{\partial \omega_r}{\partial \alpha_r} \]

(A.36)

A.7 Method of solution

As already mentioned, an eigenvalue problem for complex eigenvalue \( \alpha \) (spatial stability analysis) has to be solved for a given \( \bar{U}(r)/U_o, \bar{T}(r)/T_o, M, \omega \) and \( n \). For this the differential equation (A.25) can be integrated numerically for a chosen value of \( \alpha \). It has to be varied until the boundary condition (A.31) and (A.32) are satisfied. In this way only eigenvalues of amplified disturbances \( (\alpha_i < 0) \) could be found.
## Research Achievements

### Journal Papers

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<td>1</td>
<td>Suppression of Supersonic Jet Noise by Free-to-Rotate Vane-Type Tabs.</td>
<td>JSASS 13th International Session in 37th Aircraft Symposium, 13-15 October 1999, Ota-Ku Sangyo Plaza, Tokyo, Japan.</td>
<td>IBRAHIM, M. K. and NAKAMURA, Y.</td>
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