Experimental and Theoretical Investigations of Edge Tones in High Speed Jets

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Abstract
Experimental and theoretical investigations of the high speed jets impinge on a wedge-shaped edge have been conducted. Sonic circular and square jets of the same hydraulic diameter are examined in the present study. Edge-tones produced due to a wedge-shaped edge having different angles, specifically, 10°, 20°, 60° and 180°, have been investigated. Experiments reveal that several instability modes exist simultaneously in the microphone-captured acoustic signal. The minimum breadth is defined as the minimum distance of the edge from the nozzle exit for the first tone to be generated. Experiments showed that at small edge angle, the minimum breadth is approximately half of its equivalent two-dimension case. Semi-empirical frequency formula for the dominant tone is proposed based on the present experimental data for small edge angles. The theoretical results based on the vortex-sheet model proposed by Tam and Ahuja(1) showed that both helical and axisymmetric stability modes exist during jet impingement. The dominant tone usually has helical stability mode which is the well known edge-tone. Other tones are known as impinging-tones in the literatures and have axisymmetric stability modes. Finally, it has been shown that the experimentally obtained mean Strouhal numbers for helical and axisymmetric modes show good agreement with the Strouhal number of the least dispersive wave of the same mode calculated at various Mach numbers.

Key words: High-Speed Jet, Impingement-Tones, Edge-Tones, Vortex-Sheet Model

1. Introduction

Impinging tone is a tone of discrete sound produced when a jet of gas issuing from a slit or axisymmetric nozzle impinges on a flat plate placed at a short distance from the slit or the nozzle exit. Similarly, edge-tone can be considered as a special type of impinging tone and it is generated when the jet impinges wedge-shaped edge instead of a flat plate(2). These tones are due to self-sustained oscillations of the impinging shear layers. These oscillations are the source of flow noise and undesirable structural loading, and occur in a range of applications that includes transonic windtunnels(3), Short/Vertical Takeoff and Landing (S/VTOL) aircrafts (4)–(6), velocity probes(7), and pressure probes(8).

Self-sustained oscillations of the impinging shear layer are driven by a well known feedback mechanism. This feedback mechanism can be explained as follows: vortical instabilities in the shear layer at the nozzle exit are formed between the jet and ambient air. These instabilities are amplified as they convect downstream. When these instabilities encounter the solid wall, normal to the jet axis or wedge-shaped edge, they produce strong pressure fluctuations that propagate upstream as acoustic waves along the jet column or outside it. When these acoustic waves reach the nozzle exit, they further excite the shear-layer disturbances and resulting in the shedding of new vortices. The vortical instabilities and acoustic waves form the feedback loop. Figure 1 shows a schematic of the edge-tone and impinging-tone feedback
One of the main features of the impinging tones and edge-tone is the staging phenomenon. It has experimentally shown that as the distance between the wall and the jet is increased, the impingement tone frequency does not change continuously rather it changes discontinuously. The impingement tone frequency, $f_n$, is determined by the phase-lock principle which state that the sum of the time taken for the instability waves to propagate from the nozzle exit to the wall and the time taken for the feedback acoustic waves to propagate from the wall upstream inside or outside the jet to the nozzle exit must be equal to an integer multiple of the period of oscillation $f_n$. Using this principle, an expression for the impingement tone frequency, $f_n$, can be derived as follows

$$\frac{h}{\Lambda} + \frac{h}{\lambda} = n \quad n = 1, 2, 3, \ldots$$

(1)

where $h$ is the stand-off distance between the nozzle exit and the edge-tip or plate. $\Lambda$ and $\lambda$ are the wavelengths of downstream-propagating instability waves and upstream-propagating acoustic waves, respectively. The relation between $\Lambda$, $\lambda$ and $f_n$ can be written as follows

$$\Lambda = \frac{U_c}{f_n}$$

(2)

$$\lambda = \frac{a}{f_n}$$

(3)

Where $U_c$ and $a$ are the velocities of downstream-propagating instability waves and upstream-propagating acoustic waves, respectively. Substitute Eqs. (2) and (3) in Eq. (1), the following expression for $f_n$ can be written

$$f_n = \frac{nU_c}{h(1 + \frac{U_c}{a})}$$

(4)

The staging phenomenon referred to before is the abrupt change in the value of $n$, which leads to a discontinuous change in the impingement tone frequency as $h$ is increased. Usually
Eq. (4) is written in terms of Strouhal number, \( St \), as follows

\[
St = \frac{2R_j f_n}{U_j} = \frac{n \left( \frac{U_c}{U_j} \right)}{\frac{1}{2R_j} \left( 1 + \left( \frac{U_c}{U_j} \right) M_j \right)}, \tag{5}
\]

Where \( M_j = \frac{U_j}{a} \) is the jet Mach number, \( R_j \) is the jet radius and \( \frac{U_c}{U_j} \) is the ratio between the velocity of downstream-propagating instability waves and jet velocity. Experimental evidence showed that this ratio is constant \(^{(5)}\) and in the range from 0.55 and 0.7.

Equation (5) shows that the instability frequency can be changed in one of two ways, either changing the stand-off distance by keeping the Mach number constant or changing the Mach number by keeping the stand-off distance constant. Panicker and Raman\(^{(5)}\) pointed out that the change in the frequency over a realistic Mach number range while keeping the stand-off distance constant is much lower than the change of frequency over a realistic stand-off distance range while keeping the Mach number constant. This requires a greater frequency resolution to detect a frequency change from one Mach number to another at a constant stand-off distance.

Most of the reported experimental results of impinging tones are for axisymmetric jets and flat plates\(^{(1)}\), \(^{(5)}\), \(^{(11)}\). On the other side, most of the reported experimental results of edge-tones are for two-dimensional jets and wedge-shaped edges\(^{(10)}\), \(^{(12)}\). The two-dimensional jet considered in these studies usually rectangle jets of large aspect ratio\(^{(13)}\). Lepicovsky and Ahuja\(^{(8)}\) and Hussain and Zaman\(^{(7)}\) reported edge-tone results for a probe with axisymmetric jet.

Investigations of discrete tone generation mechanism of impinging jets by Marsh\(^{(14)}\), Wagner\(^{(15)}\), Neuwerth\(^{(16)}\), Umeda et al.\(^{(17)}\), and Tam and Ahuja\(^{(1)}\) revealed that in order to produce stable impinging tones, the Strouhal number of the instability frequency must match the Strouhal number of the Kelvin-Helmholtz instability in an axisymmetric free jet which is usually less than 0.7. Their studies also reveal that for subsonic jets the stability waves and the flow field associated with the feedback loop possess axial symmetry. It is well known that round jets can undergo axisymmetric as well as helical and flapping mode instability.

Tam and Ahuja\(^{(1)}\), on the basis of their numerical work using the vortex-sheet model for compressible inviscid jets, showed that the neutral solution or subsonic instabilities were responsible for the feedback mechanism observed in the case of impingement jets. These neutral solution, propagate along the jet column at subsonic speeds relative to the ambient. For subsonic jets, these neutral waves were shown to propagate upstream along the jet column. The neutral waves are neither amplified nor decayed as they propagate. Their work was very important because it answer a lot of questions for the jet impingement feedback mechanism. The Strouhal number mismatch theory developed by Tam and Ahuja\(^{(1)}\) showed that stable impingement tone could not be sustained below a Mach number of 0.65. Based on their study, it was found that for subsonic jets only the first axisymmetric mode of neutral stability has a Strouhal number which match those of Kelvin-Helmholtz instability wave of the jet flow. This is the reason why axisymmetric feedback resonance was observed in past experiments for these subsonic jets. In contrast to this, for supersonic jets both the first axisymmetric mode and the first helical mode satisfy the frequency matching condition and thus are possible modes of resonance. Their theoretical results were consistent with the experimental observation.

Powell\(^{(18)}\) showed the impingement tones of antisymmetric underexpanded jets could be divided into two classes. One, occurring in small plates where the plate size of the order of the nozzle diameter. The other occurring in large plates where the plate size greater than four nozzle diameters. Henderson and Powell\(^{(19)}\) showed that the impinging instability modes of a choked jet were classified as being axisymmetric or helical depending on the stand-off
distance, these modes could either coexist or the jet switch between these modes in a random fashion.

The objective of the present study is to characterize the impinging-tones and edge-tones due to high speed jets issuing from square and circular nozzles and impinge on two-dimensional wedge-shaped edge. There were considerable literature investigations of the edge-tone generation due to two-dimensional jets i.e., rectangular jets of high aspect ratio. In this study, the limiting case of the two-dimensional jets which is the square jet will be considered. Circular jet is also examined for comparison purposes. edge-tones produced due to a wedge-shaped edge having different angles, specifically, 10°, 20°, 60° and 180°, have been investigated. Edge angle of 180° is corresponding to jet-small flat plate impingement system which was investigated by Henderson and Powell[19]. The obtained results will be compared with the two-dimensional results reported in literature. Two jets will be considered in the present study, the first is the square jet and the second is a circular jet both have the same hydraulic diameter and sonic exit velocity, \( M_j = 1 \). The results will be divided into two parts. The first part will present and discuss the experimental results of jet-edge impingement system where most of the result are for square jet impingement. The second part of the paper will report the results of the vortex-sheet model for the circular jet and comparison with the experimental observation.

2. Experimental Setup

The open jet experimental facility is shown in Fig. 2. Two nozzles are employed in the present study to produce the primary jet flow. One nozzle is a convergent square nozzle and the other nozzle is a convergent circular nozzle. The exit hydraulic diameter of these nozzles is the same and equal to 8.88 mm. The nozzle is attached to a cylindrical plenum chamber that has a diameter of 220 mm and a length of 400 mm. High-pressure air is supplied from a tank with a volume of 12 \( m^3 \) stored at a pressure of 10 \( kgf/cm^2 \) that is connected to the plenum chamber via a 1 inch inner diameter high pressure pipe. A high precision pressure regulator and a solenoid valve were used in order to control the pressure of the plenum chamber within a 0.25% accuracy. The solenoid valve is opened and closed via a signal that comes from the data acquisition PC (DAQ PC) with a data acquisition board (National Instruments, PCI-6035E) and a GPIB board (general purpose interface bus). The latter is used as an interface with FFT analyzer, by which the data from the FFT analyzer are not only collected, but also controlled. The impinging object employed in this study is a wedge-shaped edge of thickness of 10 mm, length of 75 mm and a span of 150 mm as shown in Fig. 3. Four wedge-shaped edges of different angles were investigated in the present study, specifically \( \theta = 10°, 20°, 60° \) and 180°.

The DAQ PC is connected to a 6 channels DC strain amplifier (KYOWA DPM-6H) with a frequency response of 5 kHz, which is in turn connected to various pressure transducers and load cells. The flow field data were obtained by using a standard Pitot probe with an outer diameter of 0.6 mm. This probe is connected to a KYOWA PA-5KB pressure transducer that has a maximum sampling frequency of 800 Hz and a limit pressure of 5 \( kgf/cm^2 \).

The Pitot probe and the edge plate can be mounted on a three-dimensional mechanical traverse which could allow to change the stand-off distance between the nozzle exit and the Pitot probe or the edge-tip in precise and automated way. The three-dimensional mechanical traverse has three stepping motors. These motors are connected to three motor drivers (Oriental motor UDX5114), which in turn is connected to NEC 486DX computer equipped with an interface board. Both the data acquisition computer and traverse controlled computer are serially connected for automatic flow-field and acoustic field measurements, as shown in Fig. 2.

The Mach number distribution along the jet axis in the case of fully-expanded sonic jet, considered in this study, was obtained by measuring both Pitot pressure and the static
The centerline distribution of Mach number can be calculated directly from the Pitot probe measurements alone by assuming an isentropic flow.

Two microphones were employed for sound pressure level (SPL) measurements. The first microphone is a RION UC-29 1/4 inch condenser microphone that has a maximum frequency of 100 kHz and a maximum SPL of 164 dB. The second microphone is ONO-SOKKI LA-5110 1/2 inch condenser microphone that has a maximum frequency of 20 kHz and a maximum SPL of 130 dB. The microphone location is depicted in Fig. 3. Most of the acoustic data presented in this study were collected using the RION UC-29 1/4 inch microphone. The microphones are connected to both the two channels multi-purpose FFT analyzer (ONO-SOKKI CF-5210) and the DAQ PC via RS 232C serial port for SPL spectra and overall SPL (OASPL) measurements, respectively.

The outside surfaces of the plenum chamber and other bodies placed in the near field were covered with two layers of an acoustically absorbent, 6 mm thick polyurethane foam to reduce the sound reflected from those objects. In data acquisition, transducer zero errors were monitored before each run, and the plenum (chamber) and ambient pressures were also checked for each run, which were used to normalize measured data.

3. Results and Discussion

3.1. Jet Mean Flow-Field

Figure 4 shows the centerline Mach number distributions of the baseline jet for both circular and square nozzles. The potential core length is about 5D, where D is the hydraulic...
Fig. 4 Centerline Mach number distribution of the jet: • circular jet, □ square jet.

Fig. 5 Variation for the minimum breadth with Mach number for both two-dimensional and three-dimensional jets, the wedge angle, θ, is 20°.

diameter of the nozzle. The wedge-shape edge stand-off distance vary from the nozzle exit up to 10D as depicted by the gray area in this figure.

3.2. Characteristics of the Edge-Tone (Dominant-Tone)

One of the main feature of dominant-tone or edge-tone is the minimum breadth, which is the minimum distance, \( h_o \), required for the tone to be first generated. Figure 5 shows the variation of \( h_o \) with two exit Mach numbers for the square jet, as well as the results of Krothapalli et al.\(^{13}\) for rectangular jet having a width to height ratio of 16.7. The wedge-shaped edge angle is the same for both cases and equal to 20°. The minimum breadth shown in this figure is normalized by \( L \) which is the side length of the square nozzle, \( L = 8.88 \) mm. As shown in this figure, as the Mach number increases the minimum breadth increases slightly. The minimum breadth of the square jet is approximately half of the equivalent two-dimensional case.

Figures 6.a to 6.d show the narrow band frequency analysis of the acoustic wave captured by the microphone at different normalized stand-off distance between the nozzle exit and the edge-tip: specifically \( h/L = 1.5, 2.9, 3.5, 4.0, 4.5, 5.0, 6.0, 7.0, 7.7, \) and \( 9.0 \). The wedge-shaped edge angle is 20°. The frequencies at the peak Sound Pressure Level (SPL) in this figure represent the edge-tone frequencies. For a given exit Mach number, the edge-tone frequency decreases gradually as the stand-off distance increases over the minimum breath. This frequency continue to decrease until certain stand-off distance is reached where the edge-tone will suddenly jump to a new higher frequency. The process is repeated again as the stand-off distance increases and it is known as the "staging" phenomenon. The frequency can be described by Eq. (4) and the staging phenomenon is the result of an abrupt change in the value of \( n \) in this equation as the stand-off distance increases. The data indicate simultaneous existence of more than one stage at certain stand-off distance as shown in Fig. 6.f. Eight stages could be observed in the present data and they are shown in Fig. 7 for the dominant tone frequency. Generally, the number of observed stages depend on the experimental conditions.
Fig. 6  Sound pressure level spectra at different edge locations, \( h/L \): a) 1.5 , b) 2.9 , c) 3.5 , d) 4.0 , e) 4.5 , f) 5.0 , g) 6.0 , h) 7.0 , i) 7.7 , and j) 9.0.

Fig. 7  Edge-tone (dominant tone) frequency at different edge locations.

Fig. 8  Sound pressure level of the edge-tone (dominant tone) at different edge locations.

The Sound Pressure Levels (SPL) of the corresponding frequencies are shown in Fig. 8. The SPL distribution of stage frequencies during each stage shows a sharp rise followed by a drop during the end of stage. There is a peak SPL for each stage. This peak SPL increases until it reaches its maximum value around \( h/L = 3.8 \) then it decreases as the stand-off distance increases.
3.3. Effect of Edge Angle

Experiments were conducted to study the effect of wedge-shaped edge angle, $\theta$, (as depicted in Fig. 3 on the produced edge-tone. Four different wedge-shaped edge angles were investigated in this study; specifically $\theta = 10^\circ$, $20^\circ$, $60^\circ$, and $180^\circ$. Figure 9 shows the narrow band frequency analysis of the acoustic wave captured by the microphone where the stand-off distance between the nozzle exit and edge-tip, $h/L = 1.9$. For small edge angles, the edge-tone frequency was not observed as shown in Figs. 9.a and 9.b for edge angle $10^\circ$ and $20^\circ$, respectively. This was expected as the stand-off distance, $h/L$, is less than the minimum breadth, $h_o/L$, shown in Fig. 5. For the case of $\theta = 60^\circ$, three different peaks are observed in the spectra as shown in Fig. 9.c. The first peak is slightly above 10 kHz, the second peak is the dominant peak which was discussed in subsection (3.2) at 18 kHz and the third is nonharmonics in nature and it is around 23 kHz. These three identified peaks correspond to multiple instability frequencies due to the impingement of the jet with the wedge-shaped edge. These instabilities will be discussed in details in section (4). The fourth case of $\theta = 180^\circ$, only the edge-tone around 25 kHz is observed, as shown in Fig. 9.d. This observed instability frequency at the specific edge angle of $\theta = 180^\circ$ will be termed as impingement-tone frequency henceforth.

![Frequency spectra for different edge angles at $h/L = 1.9$: a) $\theta = 10^\circ$, b) $\theta = 20^\circ$, c) $\theta = 60^\circ$, and d) $\theta = 180^\circ$.](image)

Figure 9 Frequency spectra for different edge angles at $h/L = 1.9$: a) $\theta = 10^\circ$, b) $\theta = 20^\circ$, c) $\theta = 60^\circ$, and d) $\theta = 180^\circ$.

Figure 10 shows the narrow band frequency analysis similar to the data presented in Fig. 9 but at stand-off distance between the nozzle exit and edge-tip, $h/L = 3.2$. Surprisingly and in addition to the edge-tone presented earlier for $\theta = 20^\circ$, there is a another peak around 7.5 kHz observed for $\theta = 10^\circ$, $20^\circ$ and $60^\circ$, as shown in Figs. 10.a-10.c. The high frequency tones observed at 35 kHz is the harmonics of the edge-tones observed at 17.5 kHz. For $\theta = 180^\circ$, only the impingement-tone at 18 kHz is observed, as shown in Fig. 10.d. It should be stated here that the edge-tone characteristics presented in subsection (3.2) was limited to the dominant tone frequency, as shown in Figs. 6, 9, and 10. This tone was usually observed in the range of 15 kHz -20 kHz, as shown in Fig. 7.

Figure 11 shows the variation of the edge-tone frequencies with the change in the non-dimensional stand-off distance, $h/L$, for different wedge-shaped edge angle $\theta$. Three instability modes were identified at each $\theta$ and each mode underwent its staging phenomenon. For $\theta = 10^\circ$ and $20^\circ$, the instability modes observed at $h/L$ values which was greater than the
minimum breadth value shown in Fig. 5. The slope of the frequency of stage curve at a given instability mode stage changes as $-1/(hL)^2$; this explain the reduction in the slope of the frequency variation, plotted in Fig. 11, at high stand-off distances. Similar observation also reported by Panickar and Raman(5) and Krothapalli et al.(13). It can be seen that these variations are indeed restricted to a narrow range of frequencies for each mode as first reported by Neuwerth(16). The mean frequency for each mode is identified in Fig. 11 as horizontal dash line at $f_m$. These three distinct regimes identified in each sub-figure as $f_{m,1}$, $f_{m,2}$ and $f_{m,3}$. The values of the mean frequencies of each wedge-shaped edge angle, $\theta$, is printed in each

![Graph showing frequency spectra for different edge angles]

**Fig. 10** Frequency spectra for different edge angles at $h/L = 3.20$: a) $\theta = 10^\circ$, b) $\theta = 20^\circ$, c) $\theta = 60^\circ$, and d) $\theta = 180^\circ$.

![Graph showing edge-tone frequency at different edge locations]

**Fig. 11** Edge-tone frequency at different edge locations ($h/L$) and edge angles: a) $\theta = 10^\circ$, b) $\theta = 20^\circ$, c) $\theta = 60^\circ$, and d) $\theta = 180^\circ$. 
sub-figure.

The Sound Pressure Levels (SPL) of each instability mode are shown in Fig. 12 for each wedge-shaped angle, $\theta$. The results are similar to that presented in Fig. 8 and discussed in subsection (3.2).

![Fig. 12 Sound pressure level at different edge locations (h/L) and edge angles: a) $\theta = 10^\circ$, b) $\theta = 20^\circ$, c) $\theta = 60^\circ$, and d) $\theta = 180^\circ$.](image)

### 3.4. Frequency Formula

Powell\(^{(2)}\)\(^{(9)}\) wrote Eq. (1) in the following form

$$\frac{h}{\Lambda} + \frac{h}{\lambda} = n + p \quad n = 1, 2, 3, ... \quad (6)$$

where $p$ is a nonintegral value representing the possible phase difference at the edge tip between the downstream propagation and the upstream propagation. Powell proposed $p = \frac{1}{4}$ based on the hypothesis that the generation of the maximum pressure occurred as a vortex reached the edge tip. Nonomura et al.\(^{(20)}\) verified computationally the Powell's hypothesis. Their computational results showed that the phase lag $p$ is almost constant and in the range of $-0.5 < p < 0.5$, depending on the mode of the flow field, for a wide range of Reynolds number and jet Mach number parameters. It follows that Eq. (4) can be written as

$$f_n = \frac{(n + p)U_c}{h(1 + \frac{U_c}{a})} \quad (7)$$

$\frac{U_c}{a}$ is the ratio between the velocity of downstream-propagating instability waves and acoustic waves velocity. Experimental evidence showed that this ratio is constant and in the range from 0.55 and 0.7. In the present calculation for the frequency, $\frac{U_c}{a}$ was assumed equal to 0.62. For the present experimental data this was the value that best fit the theoretical calculations.

Figures 13.a and 13.b shows the plot of the dominant instability frequency for both square and circular nozzles, respectively. It should be noted here that the experimental data in Fig. 13.a is the same data that was plotted in Fig. 7 for the square nozzle. Both nozzles have the same hydraulic diameter of 8.88 mm. The predicted frequency using Eq. (7) is plotted in Figs. 13.a and 13.b as dotted and dashed lines for the observed eight stages. The dotted and dashed lines correspond to $p = 0$ and $p = \frac{1}{4}$, respectively. It is clearly shown that there are disagreements between the predicted frequency based on Eq. (7) and the experimental data.
Based on the previous results, it is warrant to propose another frequency formula for the present jet-edge impingement system. A semi-empirical frequency formula for the dominant instability frequency is proposed in this subsection. The equation can be written in an analogous form to Eq. (7). As explained earlier in section (1), the feedback loop of the edge-tones consists of the downstream-convected vortices initiated at the nozzle exit and the upstream-propagating pressure waves generated at the wedge-shaped edge. The proposed semi-empirical frequency formula, assumes that upstream-propagating waves can be considered as waves generated at an effective source point located at a distance $\Delta h$ from the edge tip, as shown in Fig. 1.a. The phase-locking condition will be satisfied for the new length of $h + \Delta h$ instead of $h$. Equation (7) can be written as

$$f_n = \frac{(n + \bar{\rho})U_c}{(h + \Delta h)(1 + \frac{\bar{\rho}}{n})},$$

where $\Delta h$ is a function in $n$ and $h$ and can be empirically obtained from the experimental data as follows

$$\Delta h = \frac{0.0290n - 5.7500h + 0.0073}{n + 6}.\quad(9)$$

Values for $\bar{\rho} = 0.25$ and $\frac{\bar{\rho}}{n} = 0.62$ are best fit the experimental data. The acoustic waves speed was measured and equal to 350 m/sec for the present experiments. The frequencies calculated from Eq. (8) is plotted as solid lines in Fig. 13 for both nozzles for the dominant instability frequency. There is a good agreement between the predicated frequency based on the proposed semi-empirical Eq. (8) and the experimental data.

![Figure 13](image)

**Fig. 13** Edge-tone (dominant-tone) variation with edge location ($h/L$ or $h/D$). Comparison with, – the proposed-semi-empirical feedback formula, ... feedback formula with $\rho = 0$, -- feedback formula with $\rho = 0.25$, ○ experimental data: a) square sonic jet and b) circular sonic jet.

Figure 14 shows similar results as Fig. 13 for different wedge-shaped edge angle. Again, there is good agreement with the experimental data only for the dominant frequencies for the present jet-edge impingement system which are in the range of 15-20 kHz. There are discrepancies between the predicted frequencies and the experimental data for other instability mode frequencies. It seems that the proposed frequency formula is not applicable to predict other instability mode frequencies compared with the dominant instability mode. The reason behind this was not clear.
4. Vortex Sheet Jet Model

In order to determine the neutral acoustic modes in the sonic impinging jet, the jet is modeled as uniform stream bounded by a vortex sheet. The model was originally proposed by Tam and Ahuja(1) for axisymmetric jet. According to this model, the frequency of the edge-tone and the characteristics of feedback loop is determined by the characteristics of the upstream propagating neutral waves and the downstream propagating instability waves of the jet. The overall characteristics of the loop are dictated by Kelvin-Helmholtz instability wave and the feedback neutral acoustics modes(1). It is known that the instability waves in the core region of the jet are restricted to a fairly narrow frequency band. Previous investigations(15) – (17), (21), both experimentally and theoretically, indicate that the Strouhal number range is limited to less than 0.7. Instability waves of higher Strouhal number are damped out quickly and would be unable to propagate far enough downstream to reach the nozzle exit to excite the feedback acoustic modes. It is well known that to produce stable edge-tones, the Strouhal number of the instability frequency must match the Strouhal number of the Kelvin-Helmholtz instability in axisymmetric jet which is restricted to the range of less than 0.7.

For underexpanded high-speed jets, where the shear-layer vortex interacts with shock wave produces a strong acoustic waves that propagate upstream. These waves will implant new disturbances into the shear layer at nozzle exit that travel downstream and grow, producing a jet instability. Repetition of this process results in a feedback loop, leading to the well known screech-tone(22). The vortex-shock interaction attributes to the generation of screech-tone by a mechanism similar to the edge-tone generation. Brassard et al.(23) and Jothi and Srinivasan(24) reported that the screech-tone frequency of the helical mode for a circular jet is approximately equal to that produced due to the square jet having the same hydraulic diameter. Figure 13 shows similar observations for the dominant edge-tone frequency for both circular and square jets. This frequency is due to helical mode instability as will be presented in subsection 4.3. Therefore, the subsequent analysis will be conducted for axisymmetric jet.
having the same hydraulic diameter of the present square jet using the model proposed by Tam and Ahuja(1). The results will be valid for at least the dominant instability mode frequency.

4.1. Formulation

An axisymmetric sonic jet having a diameter equal to the hydraulic diameter of the square jet is considered in present analysis. The jet velocity $U_j$ and radius $R_j$ bounded by a vortex-sheet as shown in Fig. 15. The jet is modeled in cylindrical coordinates given by $(x, \theta, r)$, which correspond the the axial, circumferential, and radial directions, respectively. Let $p_+$ and $p_-$ be the pressure associated with the disturbances outside and inside the jet, respectively, and $\zeta(x, \theta, t)$ be the radial displacement of the vortex sheet. Start from the linearized equation of motion of compressible flow(25)-(27), it can be shown that the governing equations and boundary conditions for $p_+$, $p_-$, and $\zeta$ are given by(1)

\[
\frac{1}{a_\infty^2} \frac{\partial^2 p_+}{\partial t^2} - \nabla^2 p_+ = 0, \quad r \geq R_j, \quad (10)
\]

\[
\frac{1}{a_j^2} \left( \frac{\partial}{\partial t} + U_j \frac{\partial}{\partial x} \right) p_- - \nabla^2 p_- = 0, \quad r \leq R_j, \quad (11)
\]

at $r = R_j$,

\[
p_+ = p_-. \quad (12)
\]

\[
\frac{\partial^2 \zeta}{\partial t^2} = -\frac{1}{\rho_\infty} \frac{\partial p_+}{\partial r}, \quad (13)
\]

\[
\left( \frac{\partial}{\partial t} + U_j \frac{\partial}{\partial x} \right) \zeta = -\frac{1}{\rho_j} \frac{\partial p_-}{\partial r}, \quad (14)
\]

at $r \rightarrow \infty$,

\[
p_+ \rightarrow \text{finite}. \quad (15)
\]

Where $\nabla^2$ indicates the Laplace operator. $a_\infty$ and $a_j$ are the speed of sound in the free stream and inside the jet, respectively. $\rho_\infty$ and $\rho_j$ are the density in the free stream and inside the jet, respectively.

In order to solve Eqs. (10) - (14), the normal model analysis is used and is given by

\[
\begin{bmatrix}
p_+(r, x, \theta, t) \\
p_-(r, x, \theta, t) \\
\zeta(r, x, \theta, t)
\end{bmatrix} =
\begin{bmatrix}
\hat{p}_+ \\
\hat{p}_- \\
\hat{\zeta}
\end{bmatrix} e^{i(kx + n\theta - \omega t)}. \quad (16)
\]

in Eq. (16), $k$ is the axial wavenumber, $n$ is the azimuthal wave (note that this $n$ is different from that used in Eqs. (1), (4), (5), (6), (7), (8) and (9) and to avoid confusion the staging wave number will henceforth be referred to as such) which can take on integral values corresponding to the instability mode, for axisymmetric mode $n = 0$ and for all helical modes.
$n \geq 1$. $\omega$ is the circular frequency ($\omega > 0$). $\hat{p}_-$ and $\hat{p}_+$ are the pressure eigenfunctions inside and outside the vortex sheet, respectively.

By substituting Eq. (16) into Eqs. (10) - (14) and eliminating $\hat{\zeta}$, it can be shown that the pressure eigenfunctions satisfy the following differential equations and boundary conditions:

\[
\frac{d^2 \hat{p}_+}{dr^2} + \frac{1}{r} \frac{d \hat{p}_+}{dr} - \frac{n^2}{r^2} \hat{p}_+ + \left[ \frac{\omega^2}{a_{\infty}^2} - k^2 \right] \hat{p}_+ = 0,
\]

\[
\frac{d^2 \hat{p}_-}{dr^2} + \frac{1}{r} \frac{d \hat{p}_-}{dr} - \frac{n^2}{r^2} \hat{p}_- + \left[ \frac{(\omega - U_j k)^2}{a_j^2} - k^2 \right] \hat{p}_- = 0.
\]

At $r = R_j$,

\[
p_+ = p_-,
\]

\[
\frac{1}{\rho_{\infty} \omega^2} \frac{d \hat{p}_+}{dr} = \frac{1}{\rho_j (\omega - U_j k)^2} \frac{d \hat{p}_-}{dr}.
\]

Since we are looking for neutral solution to the governing equations for which $\omega$ and $\alpha$ are both real. We use the solution given by Tam and Ahuja\(^{(1)}\) that expresses the dispersion relation, $D$, and the eigenfunctions in a form that only involves real functions. These functions are given as\(^{(5)}\)

\[
D(\omega, k) \equiv [\xi + J_n(\xi a - \alpha)] \frac{[K_{n-1}(\xi a - \alpha) + K_{n+1}(\xi a - \alpha)]}{K_n(\xi a - \alpha)} + \frac{C^2 |\xi - k|}{(a_{\infty} C / a_j - M_j)^2} [J_{n-1}(\xi a - \alpha) - J_{n+1}(\xi a - \alpha)] = 0,
\]

\[
\hat{p}_+ = \frac{J_n(\xi a - \alpha)}{K_n(\xi a - \alpha)} K_n(\xi a - \alpha) \frac{R_j}{R_j},
\]

\[
\hat{p}_- = J_n(\xi a - \alpha) \frac{J_n(\xi a - \alpha) R_j}{R_j}.
\]

where $J_n$ and $K_n$ are the Bessel function and the modified Bessel function of order $n$, respectively, $\alpha = k R_j$, $\xi = |C^2 - 1|^{1/2}$ and $C = \omega / a_{\infty}$ is the dimensionless phase velocity of the wave. Additionally, $\xi = (a_{\infty} C / a_j - M_j)^2 = 1|^{1/2}$ and $M_j = U_j / a_j$ is the jet Mach number.

4.2. Dispersion relation of neutral waves

At a given Mach number $M_j$, and for a given values of $\alpha$ and $n$, the eigenvalues $\omega$ are obtained by Eq. (21). Tam and Ahuja\(^{(1)}\) pointed out that the upstream propagating neutral wave modes are given by the roots of Eq. (21) along the negative real axis of complex $C$-plane laying between $C = -1$ and $C = (a_j / a_{\infty})(M_j - 1)$. This range of $C$ values implies that all the neutral acoustic mode instability waves for the subsonic impinging jet travel upstream. The solution method proposed by Vinoth and Rathakrishnan\(^{(11)}\) is employed to find the roots. In this method, Eq. (21) is solved, first by finding the approximate roots of the equation by grid search method, then using this approximate value as an initial guess for a root fining method, like Newton-Raphson iteration method, to find the accurate value of $\omega$. For this jet, the ratio of speed of sound in the jet to the speed of sound in the free stream depends only on the Mach number and this value is less than 1. The eigenfunction correspond to each eigenvalue is given by Eqs. (22) and (23). The entire set of eigenvalues can be characterized by azimuthal wavenumber, $n$ and a radial wave number, $m$. The neutral wave mode which correspond to the $n$th azimuthal mode and the $m$th radial mode will be designated by $(n, m)$ where $n = 0$ or 1 and $m = 1, 2, 3, 4, \ldots$ The number of a radial mode is characterized by the number of maxima of the eigenfunction, $|p|$. Figure 16 shows the eigenfunction of the axisymmetric neutral wave.
Fig. 16   Eigenfunction distribution of axisymmetric neutral wave mode for a sonic jet, $M_j = 1.0$, Strouhal number $St = 2fR_j/U_j$.

$(n = 0)$ at $M_j = 1.0$ for $(0,1), (0,2), (0,3), (0,4)$ and $(0,5)$ modes at different Strouhal numbers. For the $n = 0$ modes, the $|p|$ attains its maximum value at the jet axis.

Figure 17 shows the eigenfunction of the helical neutral wave ($n = 1$) at $M_j = 1.0$ for $(1,1), (1,2), (1,3), (1,4)$ and $(1,5)$ modes at different Strouhal numbers. For the $n = 1$ modes, the eigenfunction is zero at the jet axis. It is observed that all the pressure fluctuations of the wave are almost entirely confined inside the jet except the waves whose phase speed $C$ is close to -1. The results are consistent with the results of Tam and Ahuja(1) and Vinoth and Rathakrishnan(11).

The wavenumber-frequency relationships of axisymmetric mode ($n = 0$) and helical ($n = 1$) neutral waves of the first five modes ($m = 1−5$) for $M_j = 1.0$ are shown in Fig. 18 and Fig. 19, respectively. These represent the dispersion relations. The dispersion relation of each mode terminate along the straight line ($C = −1$) as the dimensionless wavenumber, $\alpha$, increases. The circle at the end of the dispersion relation denotes the cut-off Strouhal number of that mode.

Wagner,(15), Neuwerth,(16), Umeda et al.(17), and Tam and Ahuja(1) revealed that in order to produce stable impinging tones, the Strouhal number of the instability frequency must match the Strouhal number of the Kelvin-Helmholtz instability in an axisymmetric free jet which is usually less than 0.7. The neutral wave of axisymmetric first radial mode (0,1) completely lies within $St = 0.7$, as shown in Fig. 18. This implies that this mode can contribute to the self-excitation of edge-tone. The neutral wave of second axisymmetric radial mode (0,2) lies partially within $St = 0.7$. Because of this, this mode may not contribute to the self-excitation of edge-tone. On the other hand, only the helical first radial mode (1,1) completely lies within $St = 0.7$, as shown in Fig. 19. This implies that only the helical first radial
Fig. 18 Wavenumber-frequency relations of axisymmetric neutral wave for sonic jet, $M_j = 1.0$. The frequency range for which the Strouhal number ($St = 2fr_j/U_j$) of the waves is less than 0.7 is indicated by the gray color area. Only waves in this region have frequency which can match that of the Kelvin-Helmholtz instability waves of the jet flow.

Fig. 19 Wavenumber-frequency relations of helical neutral wave for sonic jet, $M_j = 1.0$.

Mode (1,1) can contribute to the self excitation of edge-tone. Higher radial modes for both axisymmetric and helical neutral waves don’t contribute to the self excitation of edge-tone.

4.3. Prediction of average Strouhal number

Form the previously presented experimental results, it is known that the Strouhal number of the edge-tones of a particular mode lies in a narrow range and therefore it is convenient to define a mean Strouhal number, irrespective of the nozzle-plate spacing. This mean Strouhal number has different values for different modes, as shown in Fig. 11. It is almost independent on the edge angle. Three mean Strouhal numbers can be calculated based on three mean impingement frequencies identified in Fig. 11 by horizontal dash-lines as $f_{m,1}$, $f_{m,2}$ and $f_{m,3}$. The first Strouhal number, $St_{m,1} = 0.271$, is based on $f_{m,1} = 9.625$ kHz, the second Strouhal number, $St_{m,2} = 0.497$, is based on $f_{m,2} = 17.663$ kHz, and the third Strouhal number $St_{m,3} = 0.703$, is based on $f_{m,3} = 25.000$ kHz. The length scale used in the Strouhal number computations is the jet diameter, $D_j$, and is equal to 8.88 mm. The feedback neutral stability waves are dispersive in nature. This means that different parts of a group of these waves tend to propagate with different velocities and disperse in space, as shown in Figs.18 and 19. For a stable feedback loop of edge-tone, the neutral wave dispersion must be minimum. This means that the feedback loop is tuned to a frequency range near that of the least dispersive neutral acoustic wave(1),(5),(11). Thus to calculate the average Strouhal number of a particular mode of the edge-tone, it is necessary to find the Strouhal number of the least dispersive neutral
acoustic wave of that mode. The least dispersive wave is given by the condition
\[ \frac{d^2 \omega}{dk^2} = 0. \]  \hfill (24)

Figure 20 shows the Strouhal number of the least dispersive upstream propagating neutral acoustic wave of axisymmetric (0,1) and (0,2), and helical (1,1) and (1,2) modes as function of jet Mach number. Plotted on this figure also are the average Strouhal number measured by Neuwerth(16), Wagner(15), Ho and Nosseir(21), Umeda et al.(17), Tam and Ahuja(1), Panickar and Raman(5) and those of the present experiments, \( S_{tm,1} = 0.271, S_{tm,2} = 0.497 \) and \( S_{tm,3} = 0.703 \). The average Strouhal number of the dominant edge-tone frequency due to the circular jet presented in Fig. 13.b is also plotted in this figure. There is a good agreement between the experimental and theoretical average Strouhal numbers. The axisymmetric (0,1) mode and helical mode (1,1) are already reported by Tam and Ahuja(1) and Panickar and Raman(5), respectively. The dominant edge-tones can be considered as the least dispersive upstream propagating neutral acoustic waves of helical (1,1) mode. For the least dispersive axisymmetric (0,2) mode, Strouhal number, \( S_{tm,3} = 0.703 \), is closed to the average Strouhal number. This mode is reported by Vinoth and Rathakrishnan(11). Note that there is no information about the wedge-shaped edge in the model. This implies that the self-sustained edge-tone depends on the intrinsic characteristics of the jet. The wedge-shaped edge has a role to select the upstream neutral wave for a given nozzle-edge spacing(11). These results extend the validity of Tam and Ahuja(1) model to the case where a sonic jet impinges on a wedge-shaped edge and this indirectly confirms that the major acoustic feedback path is inside the jet. Vinoth and Rathakrishnan(11) and Panickar and Raman(5) showed same results for jet impingement on plate and on plate with co-axial hole, respectively.

Tam and Ahuja(1) model can predict the edge-tone generated by a sonic jet impingement on a wedge-shaped edge with acceptable accuracy. Regardless of the mode, the feedback loop is completed by the least dispersive, intrinsic upstream propagating acoustic neutral wave inside the jet and the average edge-tone frequency is equal to the frequency of the least dispersive wave.

5. Conclusion

Experiments were conducted to study the effect of impingement of a sonic jet on a wedge-shaped edge and to study the effect of edge angle on the produced tones characteristic. The following findings can be made.

- Experiments showed that at small edge angle, the minimum breadth is approximately half of its equivalent two-dimension case.
The proposed semi-empirical frequency formula for the dominant frequency gives good prediction at small edge angles compared with the well know frequency formula proposed by Powell\(^{(2),(9)}\) for its equivalent two-dimensional case.

- Edge-tones were observed at all wedge-shaped edge angles. As the angle decreases the least dispersive upstream propagating neutral acoustic waves of axisymmetric mode tends to be less pronounced. However, as the edge angle increases both helical and axisymmetric modes were observed.
- The dominant edge-tones can be considered as the least dispersive upstream propagating neutral acoustic waves of helical \((1,1)\) mode.
- The average Strouhal numbers of all observed modes were less than or equal 0.7. This is in agreement with the finding of Tam and Ahuja\(^{(1)}\), i.e., for stable edge-tones, the Strouhal number of the instability frequency must match the Strouhal number of the Kelvin-Helmholtz instability in an axisymmetric free jet which is usually less than 0.7.
- The average Strouhal number of the instability frequency is predicated by the least dispersive upstream propagating acoustic neutral wave for the axisymmetric modes \((0,1)\) and \((0,2)\) as well as for the helical mode \((1,1)\).

References


