Novel Closed-Form Exact Expressions and Asymptotic Analysis for the Symbol Error Rate of Single and Multiple-Branch MRC and EGC Receivers over α-μ Fading

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Abstract

In this paper, we present a novel framework for deriving closed-form exact expressions for the symbol error rate (SER) of α-μ fading channels assuming single-branch as well as equal-gain combining and maximal-ratio combining receivers considering most of the commonly used modulation schemes. The proposed framework is based on Mellin-transform and the SER expressions are given in terms of the univariate and multivariate Fox H-functions, which have been recently extensively used in the literature. The proposed framework has the following advantages over previous ones: first, it is straightforward and general, therefore it allows deriving the exact SER expressions for cases untreated before in the literature, second, it enables direct derivation for the asymptotic expressions of the SER for high average signal-to-noise ratios. To validate the obtained expressions, we compare the results of the special case of Nakagami-m fading channel to those reported in the literature. Furthermore, Monte Carlo simulations are conducted and their results are shown to perfectly match the analytic expressions. Finally, the obtained asymptotic expressions for all the studied modulation schemes and diversity receivers are shown to match the behavior of their corresponding exact values for a wide range of the SNR values that are of practical interest.

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Index Terms

Symbol error rate, α-μ fading, asymptotic analysis, Fox H-function, diversity systems.

I. INTRODUCTION

Performance evaluation of digital modulation schemes over fading channels has been a long-studied problem in the field of wireless communications. Lots of performance metrics have been traditionally used to quantify the performance of such systems. This includes the symbol or bit error rate (SER/BER), the outage probability (OP) or the amount of fading (AoF), among many others. The probability of error, whether in the form of SER or BER, has probably been the most sought after metric among those mentioned above [1]. However, it is probably one of the most challenging to obtain, especially in closed form. This is particularly true for the rather complicated recently introduced generalized fading models such as the α-μ [2], the κ-μ and the η-μ models [3]. The classical way of obtaining the probability of error for a specific modulation scheme has always been through averaging the conditional probability of error on a specific signal-to-noise ratio (SNR) over the distribution of the SNR. As simple as it sounds, the process rarely results in tractable integrals that lead to closed-form expressions and thus its usability from a practical point of view has been limited to simple fading models such as the Rayleigh distribution. Therefore, alternative frameworks have been proposed for deriving closed-form expressions of the SER. One of the most popular approaches is presented in the seminal works by Alouini *et al* in [4] and [5], who have laid the foundation of what is commonly known as the moment-generating function (MGF) approach. This approach has been successfully applied to the Rayleigh, the Nakagami-m, the Rice, and the Nakagami-q fading models (see [1, Ch. 8] and references therein). However, it requires performing some tricky integrations for moderately complicated fading distributions such as the case of Nakagami-m. Moreover, when applied to rather complicated generalized fading models such as the α-μ one, this approach usually fails in providing closed-form expressions. Additionally, it can not be straightforwardly used to estimate the asymptotic behavior of the SER for large values of the average SNR, which is a useful metric for performance evaluation.

On another related front, diversity receivers have long been used as an effective way to mitigate the detrimental effect of fading. Within the last couple of years, lots of works in the literature discussed the performance of diversity receivers within the context of α-μ fading. In particular, the problem of performance analysis of selection combining (SC) diversity over α-μ fading in the presence of co-channel interference has been discussed in [6]. In [7], we derived expressions for the OP of dual branch SC and switch-and-stay combining (SSC) diversity in the presence of a single dominant independent co-channel interferer with a minimum desired signal power constraint considering this fading model. Also, the authors in [8] studied the signal-to-interference ratio (SIR-) based SC diversity system assuming correlated branches. Among the most famous diversity techniques that have been widely studied are the maximal-ratio combining (MRC) and equal-gain combining (EGC) [1]. The OP when using EGC and MRC diversity assuming α-μ fading has been studied in [9] and [10] using the moment-matching method where approximate expressions were obtained. In [11], Aalo *et al* derived expressions for the SER of EGC, MRC as well as SC diversity receivers over the generalized-Gamma (GG) fading channel, which is an alternative representation of the α-μ fading at hand. The expressions obtained in this work, in spite of being exact, are not given in closed form. Instead, they are in the form of numerical integrations where the integrand consists of the product or the sum of multiple Fox H-functions [12] or Meijer G-function [13, Sec. 9.3], depending on the type of diversity used. The expressions are therefore not very convenient to deal with. The work in [14] presents closed-form expressions for the moments of the output SNR of the generalized selection combining $\text{GSC}(2, L)$ receiver when operating over the GG fading channel as well as infinite-series representations for the MGF and the cumulative distribution function (CDF) of the output SNR. The BER and OP have also been evaluated numerically and no closed-form expressions were reported. Also, in [15], closed-form upper bounds for the outage probability as well as the BER of the EGC receiver over the GG fading channel have been derived. Although the results are quite neat, the validity of the bounds is restricted to rational values of α and are, in fact, quite loose at high SNRs especially for small values of the parameter μ. Another interesting work by Samimi and Azmi [16] uses a well-known infinite series representation for the SER of the EGC receiver where the series terms are shown to be complicated functions of Meijer's G-functions. The authors then propose a method for approximating the series terms in order to make evaluating the ASER more numerically tractable. Finally, in [17], the authors evaluate the SER of the EGC and MRC receivers over the GG fading channel using the MGF-based approach, where again the expressions are given in terms of complicated numerical integrations and no closed-form expressions are given.

Thus motivated, in this paper, we provide the following contributions to the SER analysis of single and multiple-branch receivers operating over fading channels in general and the α-μ fading in particular. First, we propose an alternative simple and straightforward framework for evaluating the SER. The framework could be used for any fading model but, in this paper, it will be specifically tailored to the α-μ case as a commonly used generalized fading model. The basic idea of the proposed framework is to use the Mellin transform to express the SER in the form of a Mellin-Barnes integral [18]. Fortunately, special functions represented by Mellin-Barnes integrals, such as the generalized Hypergeometric function, the Meijer G-function, and the Fox H-function, have been extensively studied since the beginning of the twentieth century (for a survey on those functions and their properties, the reader is referred to [12], [19], [20]). Therefore, we shall be able to obtain *Exact closed-from* expressions for the SER of different modulation techniques such as coherent binary frequency shift keying (CBFSK), M-ary amplitude shift keying (M-ary ASK), M-ary phase shift keying (M-ary PSK), M-ary quadrature amplitude modulation (M-QAM), differential binary phase shift keying (DBPSK), and non-coherent M-ary frequency shift keying (NC M-ary FSK). This virtually represents all the modulation schemes that are in use in practice. The obtained closed-form expressions are novel[1] and are given in terms of the univariate and multivariate Fox H-functions[2]. Furthermore, they are valid for any values of α and μ. This is unlike the few expressions available in the literature for some scattered cases, which usually assume some constraints on the values of α and/or μ (e.g., [26]). We confirm the validity of the obtained expressions by studying the Nakagami-m fading model as a special case of the α-μ one and showing that the obtained expressions indeed reduce to those previously reported in the literature. Moreover, Monte Carlo simulation results are presented and are shown to perfectly match the analytic expressions.

The second contribution of this paper is the study of the asymptotic behavior of the SER at high average SNRs for all the previously mentioned modulation schemes. With the exception of the contribution in [27], where the authors provided asymptotic SER expressions *only* for M-ary PSK over α-μ fading, the literature is really lacking of such investigations over this channel model. We believe that asymptotic expansions have a very important practical value as they provide an indication of the rate of the change of the SER with respect to the average SNR. Thanks to our proposed Mellin-based framework for deriving the SER, the derivation of the asymptotic expressions is very straightforward. The obtained asymptotic expressions are much simpler than the closed-form ones and can be easily

[1] Results similar to those in this work for ***some binary*** modulation schemes have been reported in [21], however, the authors assumed rational values for α in order to represent the results in terms of the Meijer G-function rather than the Fox H-function and be able to evaluate the expressions numerically.

[2] The Fox H-function has been extensively used in the literature and a MATHEMATICA® implementation of the univariate version has been reported in [22] and [23] while a MATLAB® implementation for the bivariate version has been given in [24] and [25].

evaluated to quantify the system performance in the high-SNR regime. Moreover, and unlike numerical integration techniques, they are not subject to numerical underflow even when the exact SER is very small. The third contribution of this work is generalizing the above results to the case of multiple-branch EGC and MRC diversity receivers operating over α-μ fading. This includes both the exact as well as the asymptotic analysis.

The rest of the paper is organized as follows: the following section discusses the methodology used to obtain the closed-form expressions for the probability of error. In Section III, we study the Nakagami-m fading model as a special case and use its results to confirm the validity of the obtained expressions. In Section IV, we present expressions for the asymptotic behavior of the probability of error in the high-SNR regime. Extension of the work to the EGC and MRC receivers case is carried out in Section V. Numerical results and discussions are then presented in Section VI before the paper is finally concluded in Section VII. Some relevant appendices then follow.

II. Symbol Error Rate for the Single-branch Receiver over α-μ Fading with Linear Modulations

We first define the *unconditional* SER as the expectation of the *conditional* SER with respect to the SNR, i.e.,

$$P_e = \int_{\zeta=0}^{\infty} P(\text{error}|\zeta) f_\zeta(\zeta) d\zeta, \tag{1}$$

where ζ is the instantaneous SNR, P_e is the unconditional SER, $P(\text{error}|\zeta)$ represents the conditional SER and $f_\zeta(\zeta)$ is the probability density function (PDF) of the SNR. It is interesting to note here that for some widely-used modulation schemes such as the M-ary PSK, a closed-form expression for the conditional SER is actually not available. This is unlike its derivative with respect to ζ. Therefore, we propose the following alternative form of the SER:

$$P_e = -\int_{\zeta=0}^{\infty} \frac{dP(\text{error}|\zeta)}{d\zeta} F_\zeta(\zeta) d\zeta, \tag{2}$$

where $F_\zeta(\zeta)$ is the cumulative distribution function (CDF) of the SNR. This form is directly derived by applying integration by parts to (1) and noting that $F_\zeta(0) = 0$ and $\lim_{\zeta\to\infty} P(\text{error}|\zeta) = 0$. Thus, we may use either (1) or (2) to derive the SER depending on the modulation scheme.

Our proposed framework for deriving the SER is based on expressing the SER as a Mellin-Barnes integral. The resultant expression is easily represented in terms of a generic special function such as the Fox-H function or the Meijer-G function. Depending on the specific values of the parameters, the obtained expression may even reduce to simpler functions as in the case of Nakagami-m fading. In our analysis, we will make a frequent use of the following definition of the Mellin-transform [28]:

Definition 1. *Given a continuous function $f(\zeta)$ defined on the domain $\zeta \in [0,\infty[$, then its Mellin transform is defined as*

$$f^*(s) \equiv \mathcal{M}\{f(\zeta)\} = \int_0^\infty f(\zeta)\zeta^{s-1}d\zeta. \tag{3}$$

The Mellin transform exists as long as the integral $\int_{\zeta=0}^{\infty} |f(\zeta)|\zeta^{k-1}d\zeta$ is bounded for some $k > 0$. Moreover, the inversion of the Mellin transform is accomplished by means of the inversion integral

$$f(\zeta) \equiv \mathcal{M}^{-1}\{f^*(s)\} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} f^*(s)\zeta^{-s}ds \tag{4}$$

for some $c > k$.

Usually the Mellin-transform of a function is valid only for complex values satisfying inequalities of the form $a_1 < \Re\{s\} < a_2$ where the constants a_1 and a_2 depend on the function $f(\zeta)$ itself. This domain of analyticity is called the *strip of definition* of the Mellin-transform [28]. Now, the SER is expressed in the form of a Mellin-Barnes integral thanks to the following theorem:

Theorem 1. *Consider a general fading channel where the received SNR PDF is $f_\zeta(\zeta)$ and its CDF is $F_\zeta(\zeta)$. Suppose their Mellin transforms are $f^*(z)$ and $F^*(z)$, respectively. If the Mellin transform of $P(error|\zeta)$ exists, then the unconditional SER for a single-branch receiver is given by any of the following equivalent forms:*

$$P_e = \frac{1}{2\pi i}\int_{c+1-i\infty}^{c+1+i\infty} f^*(1-z)h^*(z)dz \tag{5a}$$

$$= \frac{1}{2\pi i}\int_{c+1-i\infty}^{c+1+i\infty} F^*(-z)g^*(z)dz \tag{5b}$$

where $h^*(z)$ is the Mellin transform of $h(\zeta) \equiv P(error|\zeta)$, $g^*(z)$ is the Mellin transform of $g(\zeta) \equiv -\zeta\frac{d}{d\zeta}P(error|\zeta)$, and the constant c is such that $-c$ lies in the strip of definition of $f^*(z)$ and $1+c$ lies in the strip of definition of $h^*(z)$.

Proof. Since $f_\zeta(\zeta)$ is a PDF, it is absolutely integrable and it is guaranteed to have a Mellin-transform,

$f^*(z)$. Hence, we may apply the Parseval relation of the Mellin transform [28, Eq. (2.31)] to (1) yielding

$$P_e = \frac{1}{2\pi i}\int_{s=c'-i\infty}^{c'+i\infty} f^*(s)h^*(1-s)ds \tag{6}$$

where the constant c' lies in the strip of definition of $f^*(s)$ and $1-c'$ lies in the strip of definition of $h(1-s)$. Applying the change of variables $z'=-s$, we obtain

$$P_e = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} f^*(-z')h^*(z'+1)dz', \tag{7}$$

where $c=-c'$. Equation (5a) is derived straightforwardly by applying the change of variables $z=z'+1$ to (7). Finally, (5b) is obtained by noting that $F(\zeta)=\int_0^\zeta f_\zeta(u)du$ and using [13, Eqs. (17.42.2.ii) and (17.42.3.i)] to get $g^*(z)=zh^*(z)$ and $F^*(z)=-f^*(z+1)/z$. Hence, $f^*(1-z)=zF^*(z)$ and (5b) directly follows from (5a). It can also be derived by applying the Parseval relation of the Mellin transform to (2). □

In this paper, we customize Theorem 1 to the case of α-μ fading. Since it is easier to work with the derivative of the conditional SER, $\frac{d}{d\zeta}P(\text{error}|\zeta)$, for most of the considered modulation schemes, we prefer to use (5b) over (5a) in deriving closed-form expression of the unconditional SER, P_e[3]. The CDF of the SNR of α-μ fading channels is given by[4]

$$F_\zeta(\zeta) = \frac{\gamma\left(\mu, \mu\left(\frac{\zeta}{\hat{\zeta}}\right)^\alpha\right)}{\Gamma(\mu)} \tag{8}$$

where $\alpha>0$ is an arbitrary parameter, $\hat{\zeta}=\left(\mathbb{E}\left\{\zeta^\alpha\right\}\right)^{1/\alpha}$, $\mu>0$ is the inverse of the normalized variance of ζ^α given by $\mu=\frac{\mathbb{E}^2(\zeta^\alpha)}{\mathbb{E}(\zeta^{2\alpha})-\mathbb{E}^2(\zeta^\alpha)}$, and $\mathbb{E}(.)$ denotes the expectation operator. Also, $\Gamma(.)$ is the complete Gamma function, and $\gamma(.)$ is the lower incomplete gamma function [13, Sec. 8.310]. It is straightforward to show that the Mellin transform of $F_\zeta(\zeta)$ is given by

$$F^*(z) = -\frac{1}{z\Gamma(\mu)}\left(\frac{\mu^{1/\alpha}}{\hat{\zeta}}\right)^{-z}\Gamma\left(\mu+\frac{z}{\alpha}\right), \qquad -\alpha\mu<\Re\{z\}<0. \tag{9}$$

Hence, substituting (9) into (5b), we obtain the following expression for the SER of α-μ fading

[3] In fact, both (5a) and (5b) are equally difficult for deriving the SER of the DBPSK and NC M-ary FSK modulation schemes but we prefer to use (5b) in order to maintain a consistent analysis over all modulation schemes.

[4] It is worth mentioning here that when the channel suffers from α-μ fading with parameters (α,μ), the SNR, which is proportional to the square of the fading envelope, will also be α-μ distributed but with parameters $(\frac{\alpha}{2},\mu)$. In this paper, we assume that the SNR directly follows an α-μ distribution with parameters (α,μ).

TABLE I
SUMMARY OF THE DERIVATIVE $-\frac{d}{d\zeta}P(\text{ERROR}|\zeta)$ FOR THE DIFFERENT MODULATION SCHEMES CONSIDERED.

Modulation Scheme	$-\frac{d}{d\zeta}P(\text{error}\vert\zeta)$
CBFSK	$\frac{1}{2\sqrt{2\pi\zeta}}e^{-\zeta/2}$
M-ary ASK	$\frac{\sqrt{3}(M-1)}{M\sqrt{\pi(M^2-1)\zeta}}e^{-3\zeta/(M^2-1)}$
M-ary PSK	$\frac{1}{\sqrt{\pi\zeta}}\sin(\pi/M)\left[\frac{1}{2}+Q'\left(\sqrt{2\zeta}\cos(\pi/M)\right)\right]e^{-\sin^2(\pi/M)\zeta}$
M-QAM	$\frac{\sqrt{M}-1}{\sqrt{M\zeta}}\sqrt{\frac{6}{(M-1)\pi}}\left[\frac{1}{\sqrt{M}}+2\frac{\sqrt{M}-1}{\sqrt{M}}Q'\left(\sqrt{\frac{3\zeta}{M-1}}\right)\right]e^{-3\zeta/2(M-1)}$
DBPSK	$\frac{1}{2}e^{-\zeta}$
NC M-ary FSK	$\sum_{n=1}^{M-1}(-1)^{n+1}\binom{M-1}{n}\frac{n}{(n+1)^2}e^{-n\zeta/(n+1)}$

channels:

$$P_e = \frac{1}{\Gamma(\mu)}\frac{1}{2\pi i}\int_{z=c_1-i\infty}^{c_1+i\infty}\frac{1}{z}\left(\frac{\mu^{1/\alpha}}{\hat{\zeta}}\right)^z\Gamma\left(\mu-\frac{z}{\alpha}\right)g^*(z)dz \tag{10}$$

where c_1 is a real constant such that $0 < c_1 < \alpha\mu$. Based on the result above, closed-form expressions for the unconditional SER can be directly obtained as a Mellin-Barnes type integral if one can derive a closed-form expression for $g^*(z)$ (or $h^*(z)$) in Theorem 1. Depending on the value of the parameters, the obtained expression can be either directly represented in terms of the Fox-H function or reduced to simpler ones such as the Meijer-G function and the hypergeometric function as will be shown later in the sequel. Another important advantage of the expression in Theorem 1 is that it enables a direct derivation of the asymptotic expressions for P_e when $\hat{\zeta}$ is large as will be detailed in Section IV.

Hence, our main problem now reduces to finding closed-from expressions for $g^*(z)$ in Theorem 1, which is the Mellin transform of $-\zeta\frac{d}{d\zeta}P(\text{error}|\zeta)$. Fortunately, this is possible for the above-mentioned modulation schemes where the derivatives of the conditional SER are summarized in Table I. In this table, $Q'(.)$ is the complement of the Gaussian-Q function[5] defined by

$$Q'(z) \equiv \int_0^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt, \tag{11}$$

and the derivatives provided are directly obtained from the corresponding closed-form expressions for the conditional probability of error provided in [1, Ch. 8] with the exception of the M-ary PSK case, which we have proved previously in [29, Appendix 1]. Generally, as seen from Table I, it is noted that $dP(\text{error}|\zeta)/d\zeta$ contains one or more of the factors $e^{-b\zeta}$, $e^{-b\zeta}/\sqrt{\zeta}$, and $e^{-b\zeta}Q'(\sqrt{a\zeta})/\sqrt{\zeta}$ where a and b are positive constants. Thus, $g^*(z)$ is a linear combination of the following terms: $\mathcal{M}\{\zeta \exp(-b\zeta)\}$, $\mathcal{M}\{\sqrt{\zeta} \exp(-b\zeta)\}$ and $\mathcal{M}\{\sqrt{\zeta} \exp(-b\zeta)\}Q'(\sqrt{a\zeta})$. Accordingly, using Theorem 1, we derive the expressions summarized in Table II for the unconditional SER. In this table, we have

$$\mathcal{I}_0(b) = \frac{1}{2\pi i \Gamma(\mu)} \int_{z=c_1-i\infty}^{c_1+i\infty} \left(\frac{\mu^{1/\alpha}}{\hat{\zeta}}\right)^z \Gamma\left(\mu - \frac{z}{\alpha}\right) \frac{1}{z} \mathcal{M}\left\{\zeta e^{-b\zeta}\right\} dz, \tag{12a}$$

$$\mathcal{I}_1(b) = \frac{1}{2\pi i \Gamma(\mu)} \int_{z=c_1-i\infty}^{c_1+i\infty} \left(\frac{\mu^{1/\alpha}}{\hat{\zeta}}\right)^z \Gamma\left(\mu - \frac{z}{\alpha}\right) \frac{1}{z} \mathcal{M}\left\{\sqrt{\zeta} e^{-b\zeta}\right\} dz, \tag{12b}$$

$$\mathcal{I}_2(a,b) = \frac{1}{2\pi i \Gamma(\mu)} \int_{z=c_1-i\infty}^{c_1+i\infty} \left(\frac{\mu^{1/\alpha}}{\hat{\zeta}}\right)^z \Gamma\left(\mu - \frac{z}{\alpha}\right) \frac{1}{z} \mathcal{M}\left\{\sqrt{\zeta} Q'(\sqrt{a\zeta}) e^{-b\zeta}\right\} dz. \tag{12c}$$

And the required Mellin-transforms can be derived straightforwardly by substituting the corresponding function in (3) and making use of the definition of the gamma function yielding

$$\mathcal{M}\left\{\zeta e^{-b\zeta}\right\} = b^{-(z+1)}\Gamma(z+1), \tag{13a}$$

$$\mathcal{M}\left\{\sqrt{\zeta} e^{-b\zeta}\right\} = b^{-(z+1/2)}\Gamma\left(z+\frac{1}{2}\right), \tag{13b}$$

$$\mathcal{M}\left\{\sqrt{\zeta} e^{-b\zeta} Q'(\sqrt{a\zeta})\right\} = \frac{1}{2\sqrt{\pi}} \frac{1}{2\pi i} \int_{s=c_2-i\infty}^{c_2+i\infty} \frac{(a/2)^s b^{-(z+s+1/2)}}{s} \Gamma\left(\frac{1}{2}-s\right) \Gamma\left(z+s+\frac{1}{2}\right) ds \tag{13c}$$

where c_2 is a real constant such that $0 < c_2 < 1/2$. For deriving (13c), we made use of the relation between the Gaussian-Q function and the incomplete Gamma function in [13, Eq. (8.359.4)]. Thus, and after some manipulations, we obtain the following final expressions for $\mathcal{I}_0(b)$, $\mathcal{I}_1(b)$, and $\mathcal{I}_2(a,b)$:

[5] In fact, it is also possible to use the ordinary Gaussian-Q function in our analysis. However, the complementary function is more convenient because its Mellin transform is a bit simpler.

TABLE II
SUMMARY OF THE P_e FOR THE DIFFERENT MODULATION SCHEMES CONSIDERED IN THIS PAPER.

Modulation Scheme	P_e
CBFSK	$\frac{1}{2\sqrt{2\pi}}\mathcal{I}_1\left(\frac{1}{2}\right)$
M-ary ASK	$\frac{\sqrt{3}(M-1)}{M\sqrt{\pi(M^2-1)}}\mathcal{I}_1\left(\frac{3}{M^2-1}\right)$
M-ary PSK	$\frac{\sin(\pi/M)}{\sqrt{\pi}}\left[\frac{1}{2}\mathcal{I}_1\left(\sin^2(\pi/M)\right)+\mathcal{I}_2\left(2\cos^2(\pi/M),\sin^2(\pi/M)\right)\right]$
M-QAM	$\frac{\sqrt{M}-1}{\sqrt{M}}\sqrt{\frac{6}{\pi(M-1)}}\left[\frac{1}{\sqrt{M}}\mathcal{I}_1\left(\frac{3}{2(M-1)}\right)+2\frac{\sqrt{M}-1}{\sqrt{M}}\mathcal{I}_2\left(\frac{3}{M-1},\frac{3}{2(M-1)}\right)\right]$
DBPSK	$\frac{1}{2}\mathcal{I}_0(1)$
NC M-ary FSK	$\sum_{n=1}^{M-1}(-1)^{n+1}\binom{M-1}{n}\frac{n}{(n+1)^2}\mathcal{I}_0\left(\frac{n}{n+1}\right)$

$$\mathcal{I}_0(b)=\frac{1}{b\Gamma(\mu)}H_{1,1}^{1,1}\left(\frac{\mu^{1/\alpha}}{b\hat{\zeta}}\middle|\begin{matrix}(1,1)\\(\mu,1/\alpha)\end{matrix}\right), \tag{14a}$$

$$\mathcal{I}_1(b)=\frac{1}{\sqrt{b}\Gamma(\mu)}H_{2,2}^{1,2}\left(\frac{\mu^{1/\alpha}}{b\hat{\zeta}}\middle|\begin{matrix}(1/2,1),(1,1)\\(\mu,1/\alpha),(0,1)\end{matrix}\right), \tag{14b}$$

$$\mathcal{I}_2(a,b)=\frac{1}{2\sqrt{\pi b}\Gamma(\mu)}H_{1,0;1,2;1,2}^{0,1;1,1;1,1}\left(\frac{a}{2b},\frac{\mu^{1/\alpha}}{b\hat{\zeta}}\middle|\begin{matrix}(1/2;1,1) & (1,1) & (1,1)\\ - & (1/2,1),(0,1) & (\mu,1/\alpha),(0,1)\end{matrix}\right) \tag{14c}$$

where $H_{p,q}^{m,b}(\xi)$ is the univariate Fox-H function defined by [12, Eq. (1.2)] and $H_{p,q;p_1,q_1;\ldots;p_L,q_L}^{0,n;m_1,n_1;\ldots;m_L,n_L}(\xi_1,\ldots,\xi_L)$ is the multivariate H-function defined by [12, Eq. (A.1)]. Finally, substituting Eqs. (14) into the expressions in Table II, we obtain the final closed-form expressions for the SER for the single-branch receiver over α-μ fading as summarized in Table III. It is worth mentioning here that the obtained expressions for M-ary ASK, M-ary PSK, M-QAM and NC M-ary FSK, are all novel and have never been reported before in the literature. Also, the result for CBFSK is identical to [21, Eq. (10)]. The expressions are valid for any combination of α and μ without any restrictions and they accommodate other fading models as the Weibull and Nakagami-m ones as special cases. We will discuss the special case of Nakagami-m in details in the following section.

One important note is due here; we highly believe that the proposed methodology will be applicable

TABLE III

FINAL FORM OF THE P_e FOR THE DIFFERENT MODULATION SCHEMES CONSIDERED IN THIS PAPER.

Modulation Scheme	P_e
CBFSK	$\frac{1}{2\sqrt{\pi}\Gamma(\mu)} H_{2,2}^{1,2}\left(\frac{2\mu^{1/\alpha}}{\hat{\zeta}}\middle\vert\begin{matrix}(1/2,1),(1,1)\\(\mu,1/\alpha),(0,1)\end{matrix}\right)$
M-ary ASK	$\frac{M-1}{M\sqrt{\pi}\Gamma(\mu)} H_{2,2}^{1,2}\left(\frac{(M^2-1)\mu^{1/\alpha}}{3\hat{\zeta}}\middle\vert\begin{matrix}(1/2,1),(1,1)\\(\mu,1/\alpha),(0,1)\end{matrix}\right)$
M-ary PSK	$\frac{1}{2\Gamma(\mu)\sqrt{\pi}}\left[H_{2,2}^{1,2}\left(\frac{\mu^{1/\alpha}}{\sin^2(\pi/M)\hat{\zeta}}\middle\vert\begin{matrix}(1/2,1),(1,1)\\(\mu,1\alpha),(0,1)\end{matrix}\right) + \frac{1}{\sqrt{\pi}} H_{0,1;1,2;1,2}^{1,0;1,1;1,1}\left(\cot^2(\pi/M), \frac{\mu^{1/\alpha}}{\sin^2(\pi/M)\hat{\zeta}}\middle\vert\begin{matrix}(1/2;1,1); & (1,1); & (1,1)\\ -; & (1/2,1),(0,1); & (\mu,1/\alpha),(0,1)\end{matrix}\right)\right]$
M-QAM	$\frac{2(\sqrt{M}-1)}{M\sqrt{\pi}\Gamma(\mu)}\left[H_{2,2}^{1,2}\left(\frac{2(M-1)\mu^{1/\alpha}}{3\hat{\zeta}}\middle\vert\begin{matrix}(1/2,1),(1,1)\\(\mu,1\alpha),(0,1)\end{matrix}\right) + \frac{\sqrt{M}-1}{\sqrt{\pi}} H_{0,1;1,2;1,2}^{1,0;1,1;1,1}\left(1, \frac{2(M-1)\mu^{1/\alpha}}{3\hat{\zeta}}\middle\vert\begin{matrix}(1/2;1,1); & (1,1); & (1,1)\\ -; & (1/2,1),(0,1); & (\mu,1/\alpha),(0,1)\end{matrix}\right)\right]$
DBPSK	$\frac{1}{2\Gamma(\mu)} H_{1,1}^{1,1}\left(\frac{\mu^{1/\alpha}}{\hat{\zeta}}\middle\vert\begin{matrix}(1,1)\\(\mu,1/\alpha)\end{matrix}\right)$
NC M-ary FSK	$\frac{1}{\Gamma(\mu)}\sum_{n=1}^{M-1}(-1)^{n+1}\binom{M-1}{n}\frac{1}{n+1} H_{1,1}^{1,1}\left(\frac{(n+1)\mu^{1/\alpha}}{n\hat{\zeta}}\middle\vert\begin{matrix}(1,1)\\(\mu,1/\alpha)\end{matrix}\right)$

to more general fading distributions such as the extended generalized-$\mathcal{K}$ (EGK) fading distribution [30] or even the more general family of the H-function distribution [31], [32][6]. The reason for such a belief is that the Mellin transforms of those distributions, $f^*(z)$ and $F^*(z)$, exist and are in the form of a ratio between products of complex gamma functions; see [12, Chapter 2, Eq. (2.8)] and note that the EGK fading distribution is a special case of the H-function distribution (upon using [33, Theorem 6.1] with some arrangement). Hence, according to Theorem 1 and noting that $h^*(z)$ and $g^*(z)$ are independent of the fading distribution, we can easily deduce that the SER will still be a linear combination of some functions $\mathcal{I}_0(b)$, $\mathcal{I}_1(b)$, and $\mathcal{I}_2(a,b)$, which are a bit different from those defined in (14). The main difference, however, is that the term $\Gamma(\mu - z/\alpha)$ in (14) will be replaced by a ratio of products of complex Gamma functions. Hence, the functions $\mathcal{I}_0(b)$, $\mathcal{I}_1(b)$, and $\mathcal{I}_2(a,b)$ are the inverse Mellin transform of a ratio of products of complex Gamma functions and thus they can be represented in terms of *higher order* Fox-H functions.

[6]It is worth mentioning here that results pertaining to the performance of some ***binary*** modulation schemes over the H-function distribution of families have been already presented in [32].

III. A Special Case: Nakagami-m Fading

It is of interest to see if the SER expressions of the single-branch receiver over α-μ fading reduce to those of Nakagami-m fading by setting $\alpha = 1$ and $\mu = m$[7]. In Appendix A, we shall prove that, for the Nakagami-m fading, the expressions for $\mathcal{I}_0(b)$, $\mathcal{I}_1(b)$, and $\mathcal{I}_2(a,b)$ reduce to

$$\mathcal{I}_0(b) = \frac{1}{b}\phi(b), \tag{15a}$$

$$\mathcal{I}_1(b) = \frac{\Gamma(m+1/2)}{\sqrt{b}\Gamma(m+1)}\phi(b)_2F_1\left(m, \frac{1}{2}; m+1; (\phi(b))^{1/m}\right), \tag{15b}$$

$$\mathcal{I}_2(a,b) = \frac{1}{\sqrt{\pi b}}\frac{\phi(b)}{\sqrt{1+2b/a}}F_1\left(\frac{1}{2}; m, \frac{1}{2}-m; \frac{3}{2}; \frac{(\phi(b))^{1/m}}{1+2b/a}, \frac{1}{1+2b/a}\right) \tag{15c}$$

where $\phi(s) = (1+s\hat{\zeta}/m)^{-m}$ is the MGF of the Nakagami-m distribution, ${}_2F_1(a,b;c;z)$ is the Gauss hypergeometric function [13, Eq. (9.100)] and $F_1(a;b,b';c,c',x,y)$ is the Appell series [34, Eq. (5.7.6)]. Substituting (15) into the results in Table II, we obtain the final expressions for the unconditional SER for Nakagami-m fading channels as summarized in Table IV.

First of all, we see that the probability of error expressions for CBFSK and M-ary PSK reported in the table are identical to Eqs. (5) and (10) in [35], respectively. In addition, the expression for DBPSK is identical to [13, Eq. (8.183)] and that for NC M-ary FSK reduces to [13, Eq. (8.161)] after some simple manipulations. Also, the expression for M-ary ASK is reducible to the one in [1, Eq. (8.105b)] after using [13, Eq. (9.131)]. Finally, for the M-QAM case, the following expression is provided in [35, Eq. (12)] for Nakagami-m fading:

$$\begin{aligned} P_e = {} & \frac{2}{\sqrt{\pi}}\frac{\sqrt{M}-1}{\sqrt{M}}\phi\left(\frac{3}{2(M-1)}\right)\frac{\Gamma(m+1/2)}{\Gamma(m+1)}{}_2F_1\left(m, \frac{1}{2}; m+1; \left(1+\frac{3\hat{\zeta}}{2m(M-1)}\right)^{-1}\right) \\ & - \frac{2}{\pi(2m+1)}\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)^2 \phi\left(\frac{3}{(M-1)}\right)F_1\left(1; m, 1; m+\frac{3}{2}; \frac{1+3\hat{\zeta}/2(M-1)m}{1+3\hat{\zeta}/(M-1)m}; \frac{1}{2}\right). \end{aligned} \tag{16}$$

In Appendix B, we show that this expression is indeed equivalent to that provided in Table IV. We now switch our attention to the asymptotic analysis for the probability of error.

[7] Note that the Nakagami-m fading scenario is obtained by setting $\alpha = 1$ and not $\alpha = 2$ because we started our analysis with the SNR being α-μ distributed with parameters (α, μ) and not with parameters $(\frac{\alpha}{2}, \mu)$.

TABLE IV

THE P_e EXPRESSIONS FOR THE DIFFERENT MODULATION SCHEMES CONSIDERED IN THIS PAPER ASSUMING THE NAKAGAMI-m FADING MODEL AS A SPECIAL CASE.

Modulation Scheme	P_e
CBFSK	$\frac{1}{2\sqrt{\pi}}\frac{\Gamma(m+1/2)}{\Gamma(m+1)}\phi\left(\frac{1}{2}\right){}_2F_1\left(m,\frac{1}{2};m+1;\left(1+\frac{\hat{\zeta}}{2m}\right)^{-1}\right)$
M-ary ASK	$\frac{M-1}{M\sqrt{\pi}}\frac{\Gamma(m+1/2)}{\Gamma(m+1)}\phi\left(\frac{3}{M^2-1}\right){}_2F_1\left(m,\frac{1}{2};m+1;\left(1+\frac{3\hat{\zeta}}{m(M^2-1)}\right)^{-1}\right)$
M-ary PSK	$\frac{\phi\left(\sin^2(\pi/M)\right)}{\sqrt{\pi}}\left[\frac{\Gamma(m+1/2)}{2\Gamma(m+1)}{}_2F_1\left(m,\frac{1}{2};m+1;\left(1+\frac{\hat{\zeta}\sin^2(\pi/M)}{m}\right)^{-1}\right)+\frac{\cos(\pi/M)}{\sqrt{\pi}}F_1\left(\frac{1}{2};m,\frac{1}{2}-m;\frac{3}{2};\frac{\cos^2(\pi/M)}{1+2\zeta\sin^2\hat{(}\pi/M)},\cos^2(\pi/M)\right)\right]$
M-QAM	$\sqrt{\frac{2}{\pi}}\frac{\sqrt{M}-1}{\sqrt{M}}\phi\left(\frac{3}{2(M-1)}\right)\times\left[\sqrt{\frac{2}{M}}\frac{\Gamma(m+1/2)}{\Gamma(m+1)}{}_2F_1\left(m,\frac{1}{2};m+1;\left(1+\frac{3\hat{\zeta}}{2m(M-1)}\right)^{-1}\right)+\frac{2}{\sqrt{\pi}}\frac{\sqrt{M}-1}{\sqrt{M}}F_1\left(\frac{1}{2};m,\frac{1}{2}-m;\frac{3}{2};\left(2+\frac{3\hat{\zeta}}{m(M-1)}\right)^{-1};\frac{1}{2}\right)\right]$
DBPSK	$\frac{1}{2}\phi(1)$
NC M-ary FSK	$\sum_{n=1}^{M-1}(-1)^{n+1}\binom{M-1}{n}\frac{1}{n+1}\phi\left(\frac{n}{n+1}\right)$

IV. Asymptotic Analysis of the Probability of Error

It is of practical interest to derive asymptotic expansions of the SER for very large values of the average SNR, $\hat{\zeta}$[8]. This is achieved by investigating the asymptotic behavior of the integrals $\mathcal{I}_0(b)$, $\mathcal{I}_1(b)$, and $\mathcal{I}_2(a,b)$ when the average SNR, $\hat{\zeta}$ is large. Towards that end, asymptotic expressions can be easily derived using the complex residue theorem. Generally, all the poles of the complex integrands in (14) are simple. Moreover, the poles may be divided into two sets: those lying to the left of the contour of integration and those lying to the right of it, the latter being given by $z=\alpha(\mu+k)$ where

[8] We refer to $\hat{\zeta}$ as the average SNR though it is not equal to the ordinary mean of the SNR $E\{\zeta\}$ for $\alpha\neq 1$. However, it does serve as an intuitive measure of the most likely value of the SNR even if $\alpha\neq 1$.

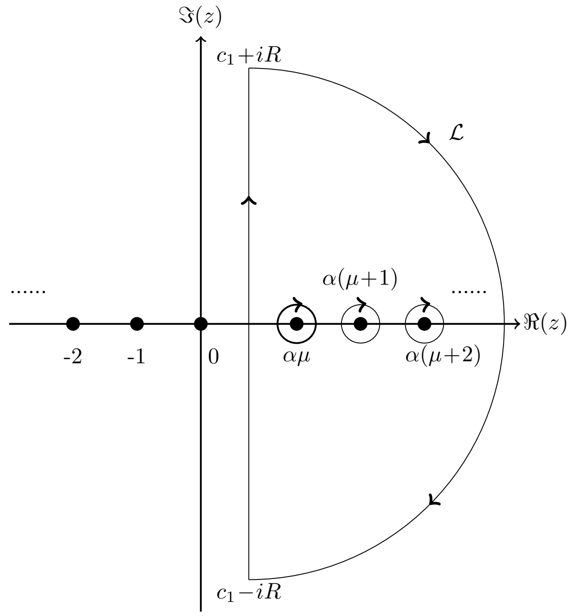

Fig. 1. Path of the complex contour integration used to evaluate $\mathcal{I}_0(b)$. When $\hat{\zeta}$ is sufficiently large and as R tends to ∞, the integration over the half circle in the path $\mathcal{L}$ will be negligible. Hence, the integration over $\mathcal{L}$ will be equivalent to the integration from $c_1 - i\infty$ to $c_2 + i\infty$. The asymptotic expansion is obtained by considering only the pole at $z = \alpha\mu$ encircled by a thick line.

k is an integer. An illustration for the case of $\mathcal{I}_0(b)$ is shown in Fig. 1. According to [36, Theorem 1.7 and Theorem 1.11], asymptotic expansions of the Fox-H function can be obtained by evaluating the residue of the complex integration at the poles closest to the contour of integration. Moreover, for large values of $\hat{\zeta}$, we should consider the poles lying to the right of the contour. Thus, we determine the asymptotic expansion by evaluating the residue at $z = \alpha\mu$. Taking into consideration that the value of the residue of $\Gamma(\mu - z/\alpha)$ at $z = \alpha\mu$ is $-\alpha$, we derive the following asymptotic expansions for $\mathcal{I}_0(b)$ and $\mathcal{I}_1(b)$ when $\hat{\zeta}$ is considerably large:

$$\mathcal{I}_0(b) \sim \frac{\mu^\mu \Gamma(1+\alpha\mu)}{b\Gamma(\mu+1)} (b\hat{\zeta})^{-\alpha\mu}, \quad \mathcal{I}_1(b) \sim \frac{\mu^\mu \Gamma(\frac{1}{2}+\alpha\mu)}{\sqrt{b}\Gamma(\mu+1)} (b\hat{\zeta})^{-\alpha\mu}. \tag{17}$$

For $\mathcal{I}_2(a,b)$, we first use the fact that [13, Eq. (9.113)]

$${}_2F_1(A,B;C;z) = \frac{\Gamma(C)}{\Gamma(A)\Gamma(B)} \frac{1}{2\pi i} \int_{z=c-i\infty}^{c+i\infty} \frac{\Gamma(s+A)\Gamma(s+B)\Gamma(-s)}{\Gamma(s+C)} (-z)^s ds$$

where the path of integration separates the poles of $\Gamma(s+A)$ and $\Gamma(s+B)$ from the poles of $\Gamma(-s)$. Hence, and after simple manipulations, we can write $\mathcal{I}_2(a,b)$ as

$$\mathcal{I}_2(a,b) = \frac{\sqrt{a}}{b\sqrt{2\pi}\Gamma(\mu)(2\pi i)} \int_{z=c_1-i\infty}^{c_1+i\infty} \left(\frac{\mu^{1/\alpha}}{b\hat{\zeta}}\right)^z \Gamma(\mu - z/\alpha)\Gamma(z){}_2F_1\left(\frac{1}{2}, z+1; \frac{3}{2}; -\frac{a}{2b}\right) dz. \tag{18}$$

Noting that the Gaussian hypergeometric function ${}_2F_1(a,b;c;z)$ is an entire function with respect to b, the asymptotic expansion for large $\hat{\zeta}$ can be written as

$$\mathcal{I}_2(a,b) \sim \frac{\sqrt{a}\mu^{\mu}\Gamma(\alpha\mu+1)}{b\sqrt{2\pi}\Gamma(\mu+1)} {}_2F_1\left(\frac{1}{2}, 1+\alpha\mu; \frac{3}{2}; -\frac{a}{2b}\right) (b\hat{\zeta})^{-\alpha\mu}. \tag{19}$$

Based on the above obtained asymptotic expansions for $\mathcal{I}_0(b)$, $\mathcal{I}_1(b)$ and $\mathcal{I}_2(a,b)$, the asymptotic expressions for the SER for high average SNR detailed in Table V directly follow. We notice from the table that all the asymptotic expansions can be given in the form $P_e \propto \hat{\zeta}^{-\alpha\mu}$ where the proportionality constant depends on the modulation scheme. Thus, for high $\hat{\zeta}$, it is straightforward to compare different modulation schemes by simply comparing the proportionality constants of their asymptotic expansions.

V. Extension to Multi-branch MRC and EGC Diversity Receivers

The above analysis can be directly extended to the case of MRC and EGC diversity receivers operating over independent α-μ fading channels. Assume that the diversity receiver combines signals coming from L branches and let the instantaneous SNR at the l^{th} branch be ζ_l, which we assume to follow the α-μ distribution with parameters α_l and μ_l. In this context, we assume that the instantaneous SNRs at the L-branches ζ_l, $i = 1, \ldots, L$ are independent and non-identically distributed (i.n.i.d.). The unconditional SER can thus be represented by the following integral:

$$P_e = \int_{\zeta_c=0}^{\infty} P(\text{error}|\zeta_c) f_{\zeta_c}(\zeta_c) d\zeta_c \tag{20}$$

where ζ_c is the SNR at the combiner output, which is a function of the SNRs ζ_l, $l = 1, \ldots, L$ and $f_{\zeta_c}(\zeta_c)$ is its PDF. Generally, it is very difficult to obtain a closed-form expression for the PDF (or the CDF) of the output of either the EGC or MRC diversity receiver. Therefore, the above expression is not much useful in deriving a closed-form expression for the unconditional SER. Alternatively, we can express the conditional SER directly in terms of $\zeta_1, \ldots, \zeta_L$. Hence, the unconditional SER is simply

TABLE V
ASYMPTOTIC EXPRESSIONS FOR P_e FOR THE DIFFERENT MODULATION SCHEMES OVER α-μ FADING.

Modulation Scheme	Asymptotic P_e
CBFSK	$\frac{\mu^\mu\Gamma(\frac{1}{2}+\alpha\mu)}{2\sqrt{\pi}\Gamma(1+\mu)}\left(\frac{\hat{\zeta}}{2}\right)^{-\alpha\mu}$
M-ary ASK	$\frac{(M-1)\mu^\mu\Gamma(\frac{1}{2}+\alpha\mu)}{M\sqrt{\pi}\Gamma(1+\mu)}\left(\frac{3\hat{\zeta}}{M^2-1}\right)^{-\alpha\mu}$
M-ary PSK	$\frac{\mu^\mu}{\Gamma(1+\mu)\sqrt{\pi}}\left(\hat{\zeta}\sin^2(\pi/M)\right)^{-\alpha\mu} \times\left[\frac{1}{2}\Gamma\left(\frac{1}{2}+\alpha\mu\right)+\frac{\cot(\pi/M)}{\sqrt{\pi}}\Gamma(1+\alpha\mu){}_2F_1\left(\frac{1}{2},1+\alpha\mu;\frac{3}{2};-\cot^2\left(\frac{\pi}{M}\right)\right)\right]$
M-QAM	$\frac{2\mu^\mu}{\Gamma(1+\mu)\sqrt{\pi}}\frac{\sqrt{M}-1}{\sqrt{M}}\left(\frac{3\hat{\zeta}}{2(M-1)}\right)^{-\alpha\mu} \times\left[\sqrt{\frac{1}{M}}\Gamma\left(\frac{1}{2}+\alpha\mu\right)+2\frac{\sqrt{M}-1}{\sqrt{M\pi}}\Gamma(1+\alpha\mu){}_2F_1\left(\frac{1}{2},1+\alpha\mu;\frac{3}{2};-1\right)\right]$
DBPSK	$\frac{\mu^\mu\Gamma(1+\alpha\mu)}{2\Gamma(1+\mu)}\hat{\zeta}^{-\alpha\mu}$
NC M-ary FSK	$\sum_{n=1}^{M-1}(-1)^{n+1}\binom{M-1}{n}\frac{\mu^\mu\Gamma(1+\alpha\mu)}{(n+1)\Gamma(1+\mu)}\left(\frac{n\hat{\zeta}}{n+1}\right)^{-\alpha\mu}$

the expectation of the conditional SER with respect to the joint distribution of $\zeta_1,\ldots,\zeta_L$, i.e.,

$$P_e=\mathbb{E}\left\{P(\text{error}|\zeta_1,\ldots,\zeta_L)\right\}=\int_{\zeta_1=0}^{\infty}\cdots\int_{\zeta_L=0}^{\infty}P(\text{error}|\zeta_1,\ldots,\zeta_L)f_{\zeta_1,\ldots,\zeta_L}(\zeta_1,\ldots,\zeta_L)d\zeta_1\ldots d\zeta_L. \tag{21}$$

Since the SNRs $\zeta_1,\ldots,\zeta_L$ are statistically independent, their joint PDF is simply the product of their individual PDFs. Denoting $\boldsymbol{\zeta}=\begin{bmatrix}\zeta_1 & \ldots & \zeta_L\end{bmatrix}^{\mathrm{T}}$, the unconditional SER is given by the following L-dimensional integration:

$$P_e=\int_{\boldsymbol{\zeta}}P(\text{error}|\boldsymbol{\zeta})\left(\prod_{l=1}^{L}f_{\zeta_l}(\zeta_l)\right)d\boldsymbol{\zeta} \tag{22}$$

where $f_{\zeta_l}(\zeta_l)$ is the PDF of ζ_l and $\int_{\boldsymbol{\zeta}}$ is a shorthand for $\int_{\zeta_1=0}^{\infty}\cdots\int_{\zeta_L=0}^{\infty}$. Similar to the case of single branch communication and some modulation schemes such as the PSK, we need to work with the derivative of the SER since, unlike the SER itself, it has a closed-form expression. In this case, we need to apply the by-parts integration rule to only one of the variables, say ζ_1, as stated in the following lemma:

Lemma 1. *The average SER of a digital modulation scheme operating over L i.n.i.d. faded branches could be written as:*

$$P_e = -\int_{\boldsymbol{\zeta}} \frac{\partial}{\partial \zeta_1} P(\mathit{error}|\boldsymbol{\zeta}) F_{\zeta_1}(\zeta_1) \left(\prod_{l=2}^{L} f_{\zeta_l}(\zeta_l) \right) d\boldsymbol{\zeta}. \tag{23}$$

Proof. The SER can be represented by the following integral:

$$P_e = \int_{\zeta_2=0}^{\infty} \cdots \int_{\zeta_L=0}^{\infty} \int_{\zeta_1=0}^{\infty} P(\text{error}|\boldsymbol{\zeta}) f_{\zeta_1}(\zeta_1) d\zeta_1 \left(\prod_{l=2}^{L} f_{\zeta_l}(\zeta_l) \right) d\zeta_L \ldots d\zeta_2. \tag{24}$$

Applying integration by parts to the innermost integration yields

$$\begin{aligned} P_e &= \int_{\zeta_2=0}^{\infty} \cdots \int_{\zeta_L=0}^{\infty} \left[P(\text{error}|\boldsymbol{\zeta}) F_{\zeta_1}(\zeta_1) \Big|_{\zeta_1=0}^{\infty} - \int_{\zeta_1=0}^{\infty} \frac{\partial}{\partial \zeta_1} P(\text{error}|\boldsymbol{\zeta}) F_{\zeta_1}(\zeta_1) d\zeta_1 \right] \left(\prod_{l=2}^{L} f_{\zeta_l}(\zeta_l) \right) d\zeta_L \ldots d\zeta_2 \\ &= -\int_{\boldsymbol{\zeta}} \frac{\partial}{\partial \zeta_1} P(\text{error}|\boldsymbol{\zeta}) F_{\zeta_1}(\zeta_1) \left(\prod_{l=2}^{L} f_{\zeta_l}(\zeta_l) \right) d\boldsymbol{\zeta} \end{aligned} \tag{25}$$

where the last line follows from the fact that $F_{\zeta_1}(0) = 0$ and $\lim_{\zeta_1 \to \infty} P(\text{error}|\boldsymbol{\zeta}) = 0$. □

We may now use either (22) or (23) to derive the required SER. In order to express the SER in terms of the multivariate H-function, we need to represent the SER as a Mellin-Barnes integral type as shown before in the single-branch case. A generalization of Theorem 1 is thus provided in the following theorem:

Theorem 2. *The unconditional SER for a diversity receiver combining L statistically independent signals can be represented by one of the following equivalent L-fold complex integrations.*

$$P_e = \frac{1}{(2\pi i)^L} \int_{\mathbf{z}} \left(\prod_{l=1}^{L} f^*(1-z_l) \right) h^*(\mathbf{z}) d\mathbf{z} \tag{26a}$$

$$= \frac{1}{(2\pi i)^L} \int_{\mathbf{z}} \left(\prod_{l=1}^{L} z_l F^*(-z_l) \right) h^*(\mathbf{z}) d\mathbf{z} \tag{26b}$$

where $\mathbf{z} = \begin{bmatrix} z_1 & \ldots z_L \end{bmatrix}$ and the integral sign $\int_{\mathbf{z}}$ is a shorthand for $\int_{z_1=c_1-i\infty}^{c_1+i\infty} \cdots \int_{z_L=c_L-i\infty}^{c_L+i\infty}$ where the constants $\{c_l\}$ are real and chosen properly. The function $h^*(\mathbf{z})$ is the L-dimensional Mellin-transform *of $P(error|\boldsymbol{\zeta})$ defined by*

$$h^*(\mathbf{z}) \equiv \mathcal{M}_L\left\{P(error|\boldsymbol{\zeta})\right\} = \int_{\boldsymbol{\zeta}} P(error|\boldsymbol{\zeta}) \left(\prod_{l=1}^{L} \zeta_l^{z_l-1}\right) d\boldsymbol{\zeta} \tag{27a}$$

$$= \frac{1}{z_1}\mathcal{M}_L\left\{-\zeta_1 \frac{\partial}{\partial \zeta_1} P(error|\boldsymbol{\zeta})\right\} \tag{27b}$$

Proof. The proof is direct generalization to that of Theorem 1 with the use of the Parseval's relation for the multidimensional Mellin transform [37, Chapter 3, Theorem 3.14]. Therefore, it is omitted. □

Applying the special case of independent branches with α-μ fading, we have the following important corollary.

Corollary 1. *The unconditional SER for a diversity receiver combining L statistically independent signals that are α-μ faded can be represented by the following L-fold complex integration:*

$$P_e = \frac{1}{(2\pi i)^L \prod_{l=1}^{L} \Gamma(\mu_l)} \int_{\mathbf{z}} \Lambda(\mathbf{z}) h^*(\mathbf{z}) d\mathbf{z} \tag{28}$$

where $\mathbf{z} = \begin{bmatrix} z_1 & \ldots z_L \end{bmatrix}$ and the integral sign $\int_{\mathbf{z}}$ is a shorthand for $\int_{z_1=c_1-i\infty}^{c_1+i\infty} \cdots \int_{z_L=c_L-i\infty}^{c_L+i\infty}$ where the constants $\{c_l\}$ are real and satisfy $0 < c_l < \alpha_l \mu_l$, $l = 1, \ldots, L$. The function $\Lambda(\mathbf{z})$ is given by

$$\Lambda(\mathbf{z}) = \prod_{l=1}^{L} \left(\frac{\mu_l^{1/\alpha_l}}{\hat{\zeta}_l}\right)^{z_l} \Gamma\left(\mu_l - \frac{z_l}{\alpha_l}\right). \tag{29}$$

Proof. For α-μ fading channels, the Mellin transform of the CDFs $F_{\zeta_l}(\zeta_l)$, $l = 1, \ldots, L$ are given by

$$F^*(z_l) = -\frac{1}{z_l \Gamma(\mu_l)} \left(\frac{\mu^{1/\alpha}}{\hat{\zeta}_l}\right)^{-z_l} \Gamma\left(\mu + \frac{z_l}{\alpha}\right), \qquad -\alpha\mu < \Re\{z_l\} < 0 \tag{30}$$

Substituting (30) into (26b) and making some arrangements, the corollary is proved. □

According to Theorem 2, the procedure for deriving closed-form expressions for the SER is a direct extension to that followed in the single-branch case and it goes as follows: first, we write the conditional SER (or its derivative) as a direct function of $\zeta_1, \ldots, \zeta_L$, second, we derive its L-dimensional Mellin transform $h^*(\mathbf{z})$ as defined in Theorem 2. Finally, we plug the obtained expression for $h^*(\mathbf{z})$ in (28) in order to express the SER as a Mellin-Barnes integral and consequently in terms of the multivariate H-function. For CBFSK, M-ary ASK, M-ary PSK, and QAM modulation, it is easier to work with the derivative of the conditional SER. Hence, we use (27b) for deriving $h^*(\mathbf{z})$. For these modulation schemes, the derivative of the conditional SER is a linear combination of one or more of the functions in the first column of Table X in Appendix C, to which we refer as the *basic functions*, $g(\boldsymbol{\zeta})$. In this table, we also list the Mellin transform of those basic functions as well as the corresponding $\mathcal{I}$-functions defined by

$$\mathcal{I} \equiv \frac{1}{(2\pi i)^L \prod_{l=1}^{L} \Gamma(\mu_l)} \int_{\mathbf{z}} \Lambda(\mathbf{z}) \frac{1}{z_1} \mathcal{M}_L\{g(\boldsymbol{\zeta})\} d\mathbf{z}. \tag{31}$$

It is straightforward to prove that, for MRC diversity, the final expression for the SER will be that provided in Table II with the functions $\mathcal{I}_1(\cdot)$ and $\mathcal{I}_2(\cdot)$ replaced by the functions $\mathcal{I}_{1,MRC}(\cdot)$ and $\mathcal{I}_{2,MRC}(\cdot)$ in Table X. Similarly, for the EGC diversity, we use the functions $\mathcal{I}_{1,EGC}(\cdot)$ and $\mathcal{I}_{2,EGC}(\cdot)$ given in Table X in place of $\mathcal{I}_1(\cdot)$ and $\mathcal{I}_2(\cdot)$ to obtain the final expressions for the SER.

On the other hand, we found that for DBPSK and NC M-ary FSK modulation, it is easier to derive $h^*(\mathbf{z})$ using the conditional SER itself, i.e., using (27a). For these modulation schemes, the basic functions and their Mellin transforms are listed in Table XI in Appendix C where the $\mathcal{I}$-functions are given by

$$\mathcal{I} \equiv \frac{1}{(2\pi i)^L \prod_{l=1}^{L} \Gamma(\mu_l)} \int_{\mathbf{z}} \Lambda(\mathbf{z}) \mathcal{M}_L\{g(\boldsymbol{\zeta})\} d\mathbf{z}. \tag{32}$$

The unconditional SER is thus given by

$$\text{DBPSK: } P_e = \frac{1}{2}\mathcal{I}_{0,c}(1), \text{ NC } M\text{-ary FSK: } P_e \quad = \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{n+1} \mathcal{I}_{0,c}\left(\frac{n}{n+1}\right) \tag{33}$$

where $\mathcal{I}_{0,c}(\cdot)$ depends on the type of the combining scheme used. The final expressions for the SER for different modulation schemes and MRC diversity are summarized in Table VI and those for EGC diversity are in Table VII.

Finally, in order to obtain the asymptotic expressions for the SER of the MRC diversity and EGC diversity, we first derive the asymptotic expansions of $\mathcal{I}_{0,MRC}(b)$, $\mathcal{I}_{1,MRC}(b)$, $\mathcal{I}_{2,MRC}(a, b)$, $\mathcal{I}_{0,EGC}(b)$, $\mathcal{I}_{1,EGC}(b)$, and $\mathcal{I}_{2,EGMRC}(a, b)$ using the complex residue theorem as done before in Section IV. Doing

TABLE VI

FINAL FORM OF THE P_e FOR THE DIFFERENT MODULATION SCHEMES CONSIDERED IN THIS PAPER WHEN MRC DIVERSITY IS APPLIED.

Modulation	P_e
CBFSK	$\frac{1}{2\sqrt{\pi}\prod_{l=1}^{L}\Gamma(\mu_l)} H_{1,1;1,1;\ldots;1,1}^{0,1;1,1;\ldots;1,1}\left(\begin{array}{c} 2\mu_1^{1/\alpha_1}/\hat{\zeta}_1 \\ \vdots \\ 2\mu_L^{1/\alpha_L}/\hat{\zeta}_L \end{array}\middle\| \begin{array}{cccc} (1/2;1,\ldots;1) & (1,1) & \ldots & (1,1) \\ (0;1,\ldots,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)$
M-ary ASK	$\frac{M-1}{M\sqrt{\pi}\prod_{l=1}^{L}\Gamma(\mu_l)} H_{1,1;1,1;\ldots;1,1}^{0,1;1,1;\ldots;1,1}\left(\begin{array}{c} (M^2-1)\mu_1^{1/\alpha_1}/3\hat{\zeta}_1 \\ \vdots \\ (M^2-1)\mu_L^{1/\alpha_L}/3\hat{\zeta}_L \end{array}\middle\| \begin{array}{cccc} (1/2;1,\ldots;1) & (1,1) & \ldots & (1,1) \\ (0;1,\ldots,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)$
M-PSK	$\frac{1}{2\sqrt{\pi}\prod_{l=1}^{L}\Gamma(\mu_l)}\left[H_{1,1;1,1;\ldots;1,1}^{0,1;1,1;\ldots;1,1}\left(\begin{array}{c} \mu_1^{1/\alpha_1}/\sin^2(\pi/M)\hat{\zeta}_1 \\ \vdots \\ \mu_L^{1/\alpha_L}/\sin^2(\pi/M)\hat{\zeta}_L \end{array}\middle\| \begin{array}{cccc} (1/2;1,\ldots;1) & (1,1) & \ldots & (1,1) \\ (0;1,\ldots,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)\right.$ $\left. +\frac{1}{\sqrt{\pi}} H_{1,1;1,2;1,1;\ldots;1,1}^{0,1;1,1;1,1;\ldots;1,1}\left(\begin{array}{c} \cot^2(\pi/M) \\ \mu_1^{1/\alpha_1}/\sin^2(\pi/M)\hat{\zeta}_1 \\ \vdots \\ \mu_L^{1/\alpha_L}/\sin^2(\pi/M)\hat{\zeta}_L \end{array}\middle\| \begin{array}{ccccc} (1/2;1,1,\ldots,1) & (1,1) & (1,1) & \ldots & (1,1) \\ (0;0,1,\ldots,1) & (1/2,1),(0,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)\right]$
M-QAM	$\frac{2(\sqrt{M}-1)}{M\sqrt{\pi}\prod_{l=1}^{L}\Gamma(\mu_l)}\left[H_{1,1;1,1;\ldots;1,1}^{0,1;1,1;\ldots;1,1}\left(\begin{array}{c} 2(M-1)\mu_1^{1/\alpha_1}/3\hat{\zeta}_1 \\ \vdots \\ 2(M-1)\mu_L^{1/\alpha_L}/3\hat{\zeta}_L \end{array}\middle\| \begin{array}{cccc} (1/2;1,\ldots;1) & (1,1) & \ldots & (1,1) \\ (0;1,\ldots,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)\right.$ $\left. +\frac{\sqrt{M}-1}{\sqrt{\pi}} H_{1,1;1,2;1,1;\ldots;1,1}^{0,1;1,1;1,1;\ldots;1,1}\left(\begin{array}{c} 1 \\ 2(M-1)\mu_1^{1/\alpha_1}/3\hat{\zeta}_1 \\ \vdots \\ 2(M-1)\mu_1^{1/\alpha_L}/3\hat{\zeta}_L \end{array}\middle\| \begin{array}{ccccc} (1/2;1,1,\ldots,1) & (1,1) & (1,1) & \ldots & (1,1) \\ (0;0,1,\ldots,1) & (1/2,1),(0,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)\right]$
DBPSK	$\frac{1}{2}\prod_{l=1}^{L}\frac{1}{\Gamma(\mu_l)} H_{1,1}^{1,1}\left(\frac{\mu_l^{1/\alpha_l}}{\hat{\zeta}_l}\middle\| \begin{array}{c} (1,1) \\ (\mu_l,1/\alpha_l) \end{array}\right)$
NCFSK	$\sum_{n=1}^{M-1}\frac{(-1)^{n+1}}{n+1}\binom{M-1}{n}\prod_{l=1}^{L}\frac{1}{\Gamma(\mu_l)} H_{1,1}^{1,1}\left(\frac{(n+1)\mu_l^{1/\alpha_l}}{n\hat{\zeta}_l}\middle\| \begin{array}{c} (1,1) \\ (\mu_l,1/\alpha_l) \end{array}\right)$

so, we have the following asymptotic expressions for MRC diversity

$$\mathcal{I}_{0,MRC}(b) \sim \prod_{l=1}^{L}\frac{\Gamma(1+\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}(b\hat{\zeta}_l)^{-\alpha_l\mu_l}, \tag{34a}$$

$$\mathcal{I}_{1,MRC}(b) \sim \frac{\Gamma(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l)}{\sqrt{b}\Gamma(1+\sum_{l=1}^{L}\alpha_l\mu_l)}\prod_{l=1}^{L}\frac{\Gamma(1+\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}(b\hat{\zeta}_l)^{-\alpha_l\mu_l}, \tag{34b}$$

$$\mathcal{I}_{2,MRC}(a,b) \sim \frac{\sqrt{a}}{b\sqrt{2\pi}}{}_2F_1\left(1+\sum_{l=1}^{L}\alpha_l\mu_l;\frac{1}{2};\frac{3}{2};-\frac{a}{2b}\right)\prod_{l=1}^{L}\frac{\Gamma(1+\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}(b\hat{\zeta}_l)^{-\alpha_l\mu_l}, \tag{34c}$$

TABLE VII
FINAL FORM OF THE P_e FOR THE DIFFERENT MODULATION SCHEMES CONSIDERED IN THIS PAPER WHEN EGC DIVERSITY IS APPLIED.

Modulation	P_e
CBFSK	$\frac{2^{L-1}}{\prod_{l=1}^{L}\Gamma(\mu_l)} H^{0,0;1,1;\ldots;1,1}_{0,1;1,1;\ldots;1,1}\left(\begin{array}{c} L\mu_1^{1/\alpha_1}/2\hat{\zeta}_1 \\ \vdots \\ L\mu_L^{1/\alpha_L}/2\hat{\zeta}_L \end{array}\middle\vert\begin{array}{cccc} - & (1,2) & \ldots & (1,2) \\ (0;1,\ldots,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)$
M-ary ASK	$\frac{2^{L}(M-1)}{M\prod_{l=1}^{L}\Gamma(\mu_l)} H^{0,0;1,1;\ldots;1,1}_{0,1;1,1;\ldots;1,1}\left(\begin{array}{c} L(M^2-1)\mu_1^{1/\alpha_1}/12\hat{\zeta}_1 \\ \vdots \\ L(M^2-1)\mu_L^{1/\alpha_L}/12\hat{\zeta}_L \end{array}\middle\vert\begin{array}{cccc} - & (1,2) & \ldots & (1,2) \\ (0;1,\ldots,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)$
M-ary PSK	$\frac{2^{L-1}}{\prod_{l=1}^{L}\Gamma(\mu_l)}\left[H^{0,0;1,1;\ldots;1,1}_{0,1;1,1;\ldots;1,1}\left(\begin{array}{c} L\mu_1^{1/\alpha_1}/4\sin^2(\pi/M)\hat{\zeta}_1 \\ \vdots \\ L\mu_L^{1/\alpha_L}/4\sin^2(\pi/M)\hat{\zeta}_L \end{array}\middle\vert\begin{array}{cccc} - & (1,2) & \ldots & (1,2) \\ (0;1,\ldots,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)\right.$ $\left.+\frac{1}{\pi}H^{0,1;1,1;1,1;\ldots;1,1}_{1,1;1,2;1,1;\ldots;1,1}\left(\begin{array}{c} \cot^2(\pi/M) \\ L\mu_1^{1/\alpha_1}/\sin^2(\pi/M)\hat{\zeta}_1 \\ \vdots \\ L\mu_L^{1/\alpha_L}/\sin^2(\pi/M)\hat{\zeta}_L \end{array}\middle\vert\begin{array}{ccccc} (1/2;1,1,\ldots,1) & (1,1) & (1,2) & \ldots & (1,2) \\ (0;0,2,\ldots,2) & (1/2,1),(0,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)\right]$
M-QAM	$\frac{2^{L+1}(\sqrt{M}-1)}{M\prod_{l=1}^{L}\Gamma(\mu_l)}\left[H^{0,0;1,1;\ldots;1,1}_{0,1;1,1;\ldots;1,1}\left(\begin{array}{c} L(M-1)\mu_1^{1/\alpha_1}/6\hat{\zeta}_1 \\ \vdots \\ L(M-1)\mu_L^{1/\alpha_L}/6\hat{\zeta}_L \end{array}\middle\vert\begin{array}{cccc} - & (1,2) & \ldots & (1,2) \\ (0;1,\ldots,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)\right.$ $\left.+\frac{\sqrt{M}-1}{\pi}H^{0,1;1,1;1,1;\ldots;1,1}_{1,1;1,2;1,1;\ldots;1,1}\left(\begin{array}{c} 1 \\ 2L(M-1)\mu_1^{1/\alpha_1}/3\hat{\zeta}_1 \\ \vdots \\ 2L(M-1)\mu_L^{1/\alpha_L}/3\hat{\zeta}_L \end{array}\middle\vert\begin{array}{ccccc} (1/2;1,1,\ldots,1) & (1,1) & (1,2) & \ldots & (1,2) \\ (0;0,2,\ldots,2) & (1/2,1),(0,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)\right]$
DBPSK	$\frac{2^{L-1}\sqrt{\pi}}{\prod_{l=1}^{L}\Gamma(\mu_l)} H^{0,0;1,1;\ldots;1,1}_{0,1;1,1;\ldots;1,1}\left(\begin{array}{c} L\mu_1^{1/\alpha_1}/4\hat{\zeta}_1 \\ \vdots \\ L\mu_L^{1/\alpha_L}/4\hat{\zeta}_L \end{array}\middle\vert\begin{array}{cccc} - & (1,2) & \ldots & (1,2) \\ (1/2;1,\ldots,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)$
NCFSK	$\frac{2^{L}\sqrt{\pi}}{\prod_{l=1}^{L}\Gamma(\mu_l)}\sum_{n=1}^{M-1}\frac{(-1)^{n+1}}{n+1}\binom{M-1}{n} H^{0,0;1,1;\ldots;1,1}_{0,1;1,1;\ldots;1,1}\left(\begin{array}{c} L(n+1)\mu_1^{1/\alpha_1}/4n\hat{\zeta}_1 \\ \vdots \\ L(n+1)\mu_L^{1/\alpha_L}/4n\hat{\zeta}_L \end{array}\middle\vert\begin{array}{cccc} - & (1,2) & \ldots & (1,2) \\ (1/2;1,\ldots,1) & (\mu_1,1/\alpha_1) & \ldots & (\mu_L,1/\alpha_L) \end{array}\right)$

and the following expressions for the EGC diversity

$$\mathcal{I}_{0,EGC}(b) \sim \frac{\sqrt{\pi}}{\Gamma(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l)}\prod_{l=1}^{L}\frac{\Gamma(1+2\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{4b\hat{\zeta}_l}{L}\right)^{-\alpha_l\mu_l}, \tag{35a}$$

$$\mathcal{I}_{1,EGC}(b) \sim \frac{\sqrt{\pi}}{\sqrt{b}\Gamma(1+\sum_{l=1}^{L}\alpha_l\mu_l)}\prod_{l=1}^{L}\frac{\Gamma(1+2\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{4b\hat{\zeta}_l}{L}\right)^{-\alpha_l\mu_l}, \tag{35b}$$

$$\mathcal{I}_{2,EGC}(a,b) \sim \frac{\sqrt{a}}{b\sqrt{2}}\frac{{}_2F_1\left(1+\sum_{l=1}^{L}\alpha_l\mu_l,\frac{1}{2};\frac{3}{2};-\frac{a}{2b}\right)}{\Gamma\left(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l\right)}\prod_{l=1}^{L}\frac{\Gamma(1+2\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{4b\hat{\zeta}_l}{L}\right)^{-\alpha_l\mu_l}. \tag{35c}$$

Substituting (34) into the expressions in Table II, we obtain the asymptotic expressions in Table VIII. Similarly, the asymptotic expressions of the SER with EGC are obtained by substituting (35) into

TABLE VIII
ASYMPTOTIC EXPRESSIONS FOR P_e FOR THE DIFFERENT MODULATION SCHEMES OVER α-μ FADING WITH MRC DIVERSITY.

Modulation	Asymptotic P_e
CBFSK	$\frac{\Gamma(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l)}{2\sqrt{\pi}\Gamma(1+\sum_{l=1}^{L}\alpha_l\mu_l)}\prod_{l=1}^{L}\frac{\Gamma(1+\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{\hat{\zeta}_l}{2}\right)^{-\alpha_l\mu_l}$
M-ary ASK	$\frac{(M-1)\Gamma(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l)}{M\sqrt{\pi}\Gamma(1+\sum_{l=1}^{L}\alpha_l\mu_l)}\prod_{l=1}^{L}\frac{\Gamma(1+\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{3\hat{\zeta}_l}{M^2-1}\right)^{-\alpha_l\mu_l}$
M-ary PSK	$\frac{1}{2\sqrt{\pi}}\prod_{l=1}^{L}\frac{\Gamma(1+\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}(\sin^2(\pi/M)\hat{\zeta}_l)^{-\alpha_l\mu_l} \times\left[\frac{\Gamma\left(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l\right)}{\Gamma\left(1+\sum_{l=1}^{L}\alpha_l\mu_l\right)}+\frac{2\cot(\pi/M)}{\sqrt{\pi}}{}_2F_1\left(\frac{1}{2},1+\sum_{l=1}^{L}\alpha_l\mu_l;\frac{3}{2};-\cot^2\left(\frac{\pi}{M}\right)\right)\right]$
M-QAM	$\frac{2(\sqrt{M}-1)}{M\sqrt{\pi}}\prod_{l=1}^{L}\frac{\Gamma(1+\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{3\hat{\zeta}_l}{2(M-1)}\right)^{-\alpha_l\mu_l} \times\left[\frac{\Gamma\left(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l\right)}{\Gamma\left(1+\sum_{l=1}^{L}\alpha_l\mu_l\right)}+2\frac{\sqrt{M}-1}{\sqrt{\pi}}{}_2F_1\left(\frac{1}{2},1+\sum_{l=1}^{L}\alpha_l\mu_l;\frac{3}{2};-1\right)\right]$
DBPSK	$\frac{1}{2}\prod_{l=1}^{L}\frac{\Gamma(1+\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\hat{\zeta}_l^{-\alpha_l\mu_l}$
NCFSK	$\sum_{n=1}^{M-1}\frac{(-1)^{n+1}}{n+1}\binom{M-1}{n}\prod_{l=1}^{L}\frac{\Gamma(1+\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{n\hat{\zeta}_l}{n+1}\right)^{-\alpha_l\mu_l}$

the expressions in Table II and are given in Table IX. Similar to single-branch communication, the asymptotic SER is proportional to $\prod_{l=1}^{L}\hat{\zeta}_l^{-\alpha_l\mu_l}$ where the proportionality constant depends on both the modulation scheme and the type of the diversity receiver.

VI. NUMERICAL RESULTS

In this section, we compare the exact expressions of the SER to their corresponding asymptotic expansions for single-branch communication as well as MRC and EGC diversity receivers. First, we investigate the single-branch case in Figs. 2, 3, and 4 where we plot the expressions for a group of binary, 4-, 8- and 16-symbols based systems. In these results, we consider different combinations of α and μ. The solid lines represent the exact expressions while the dashed ones represent the asymptotic behavior of the SER in the high average SNR regime. Also, the markers denote results obtained via

TABLE IX
ASYMPTOTIC EXPRESSIONS FOR P_e FOR THE DIFFERENT MODULATION SCHEMES OVER α-μ FADING WITH EGC DIVERSITY.

Modulation	Asymptotic P_e
CBFSK	$\frac{1}{2\Gamma\left(1+\sum_{l=1}^{L}\alpha_l\mu_l\right)}\prod_{l=1}^{L}\frac{\Gamma(1+2\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\alpha_l\mu_l^{\mu_l}\left(\frac{2\hat{\zeta}_l}{L}\right)^{-\alpha_l\mu_l}$
M-ary ASK	$\frac{M-1}{M\Gamma\left(1+\sum_{l=1}^{L}\alpha_l\mu_l\right)}\prod_{l=1}^{L}\frac{\Gamma(1+2\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\alpha_l\mu_l^{\mu_l}\left(\frac{12\hat{\zeta}_l}{(M^2-1)L}\right)^{-\alpha_l\mu_l}$
M-ary PSK	$\frac{1}{2}\prod_{l=1}^{L}\frac{\Gamma(1+2\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{4\sin^2(\pi/M)\hat{\zeta}_l}{L}\right)^{-\alpha_l\mu_l}$ $\times\left[\frac{1}{\Gamma\left(1+\sum_{l=1}^{L}\alpha_l\mu_l\right)}+\frac{2\cot(\pi/M)}{\sqrt{\pi}\Gamma\left(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l\right)}{}_2F_1\left(\frac{1}{2},1+\sum_{l=1}^{L}\alpha_l\mu_l;\frac{3}{2};-\cot^2\left(\frac{\pi}{M}\right)\right)\right]$
M-QAM	$\frac{2(\sqrt{M}-1)}{M}\prod_{l=1}^{L}\frac{\Gamma(1+2\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{6\hat{\zeta}_l}{(M-1)L}\right)^{-\alpha_l\mu_l}$ $\times\left[\frac{1}{\Gamma\left(1+\sum_{l=1}^{L}\alpha_l\mu_l\right)}+\frac{2(\sqrt{M}-1)}{\sqrt{\pi}\Gamma\left(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l\right)}{}_2F_1\left(\frac{1}{2},1+\sum_{l=1}^{L}\alpha_l\mu_l;\frac{3}{2};-1\right)\right]$
DBPSK	$\frac{\sqrt{\pi}}{2\Gamma\left(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l\right)}\prod_{l=1}^{L}\frac{\Gamma(1+2\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{4\hat{\zeta}_l}{L}\right)^{-\alpha_l\mu_l}$
NCFSK	$\frac{\sqrt{\pi}}{\Gamma\left(\frac{1}{2}+\sum_{l=1}^{L}\alpha_l\mu_l\right)}\sum_{n=1}^{M-1}\frac{(-1)^{n+1}}{n+1}\binom{M-1}{n}\prod_{l=1}^{L}\frac{\Gamma(1+2\alpha_l\mu_l)}{\Gamma(1+\mu_l)}\mu_l^{\mu_l}\left(\frac{4n\hat{\zeta}_l}{L(n+1)}\right)^{-\alpha_l\mu_l}$

Monte Carlo simulations, which were carried out via MATLAB$^{©}$, which was also used to implement the Fox-H function. The number of transmitted symbols used in the simulations is the maximum of 10^4 and $10^{[\log_{10}(50/P_{ae})]}$ where P_{ae} is the computed asymptotic value of the SER and $[.]$ denotes the rounding operation. The α-μ variates were generated either via the classical inverse CDF method or the method proposed in [38].

As seen from the figures, the simulation results perfectly match the results of the analytical expressions, which indeed proves the validity of the presented expressions in this paper. Moreover, the proposed asymptotic expressions in Table V seem to perform well for high values of SNRs, roughly when the exact value is less than 10^{-4}. Moreover, for high average SNR, the logarithm of the exact SER has almost a constant slope, which depends only on the values of α and μ and not on the modulation scheme used. This is totally consistent with our derived asymptotic expressions.

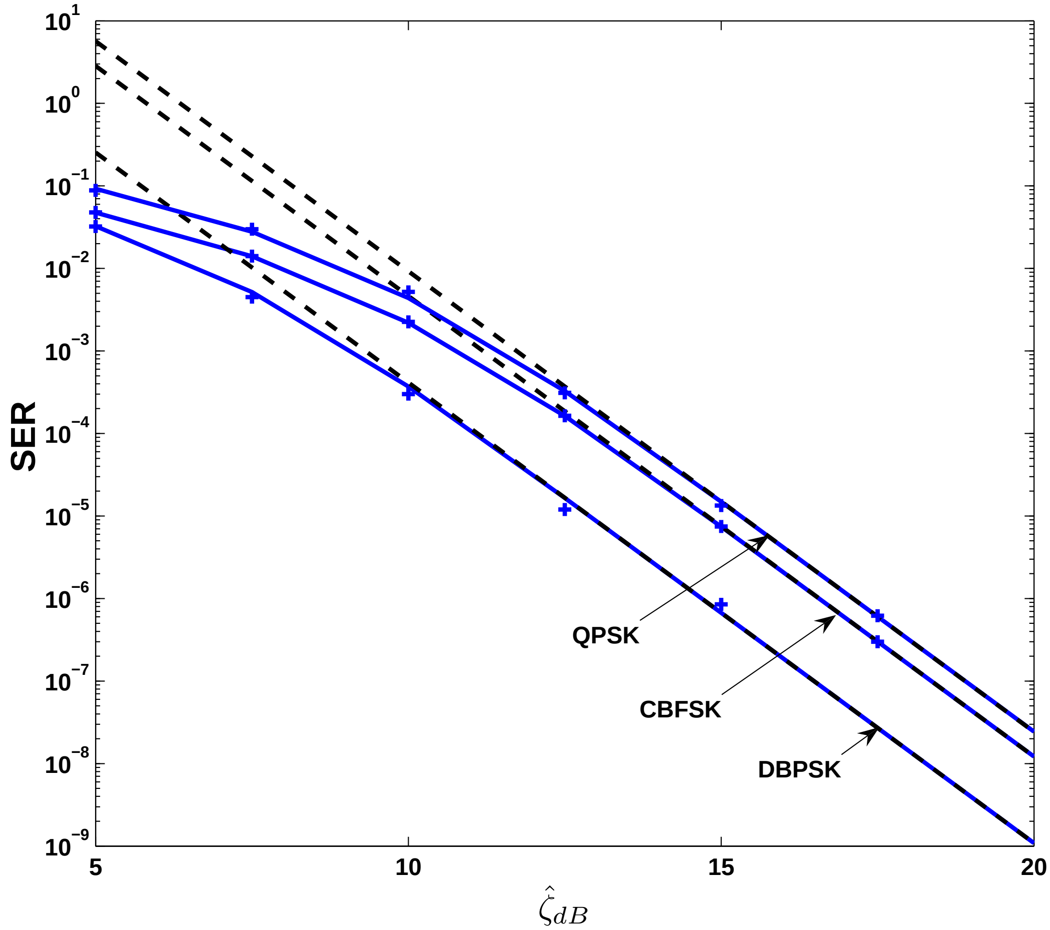

Fig. 2. Exact and asymptotic values of the SER of QPSK, CBFSK and DBPSK for $\alpha = 6.2$ and $\mu = 0.9$. The solid lines represent the exact expressions while the dashed ones represent the asymptotic behavior. Markers represent the results of Monte Carlo simulations.

We now switch our attention to the multiple-branch receiver case where we plot the SER results assuming identical branches with $\alpha = 2.5$ and $\mu = 1$ for EGC in Fig. 5 and for MRC in Fig. 6. In Fig. 5, we consider 16-NCFSK and 16-ASK modulation schemes while in Fig. 6, we consider 16-QAM and 16-PSK ones. In each figure, we consider both the dual-branch ($L = 2$) and the quad-branch ($L = 4$) cases. Similar to the single-branch case, it is evident that the asymptotic expressions provide an efficient tool to estimate the SER at high SNR values. Comparing to the case of single-branch communication in Fig. 4, we easily notice the substantial reduction of the SER obtained by employing diversity receivers. In all cases, the asymptotic expansion successfully tracks the decrease rate of the logarithm of the SER.

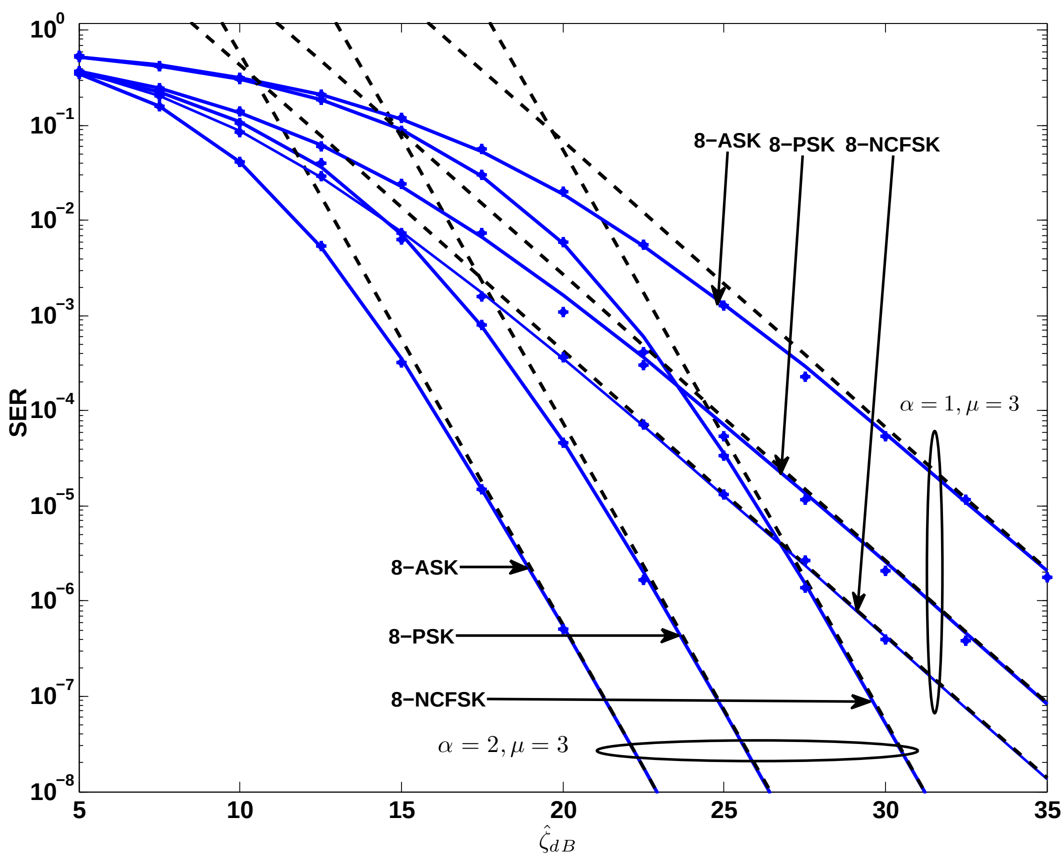

Fig. 3. Exact and asymptotic values of the SER of 8-ASK, 8-NCFSK and 8-PSK for $\alpha = 2$, $\mu = 3$ and $\alpha = 1$, $\mu = 3$. The solid lines represent the exact expressions while the dashed ones represent the asymptotic behavior. Markers represent the results of Monte Carlo simulations.

VII. Conclusions

In this paper, we have proposed a novel framework for deriving the SER of virtually all modulation schemes when communicating over fading channels. The framework was then applied to the α-μ fading case assuming both single-branch communication as well as EGC and MRC diversity receivers. The proposed framework has enabled us to easily derive exact expressions for the SER for the CBFSK, M-ary ASK, M-ary PSK, M-ary QAM, DBPSK, and NC M-ary FSK modulation schemes, which are given in terms of the univariate and the multivariate Fox-H function. Furthermore, we have shown that the obtained expressions reduce to the well-known results of Nakagami-m fading when we set $\alpha = 1$ and $\mu = m$.

Thanks to our proposed framework, we have also been able to directly derive asymptotic expressions for the SER for all the above-mentioned modulation schemes and diversity receivers. The obtained

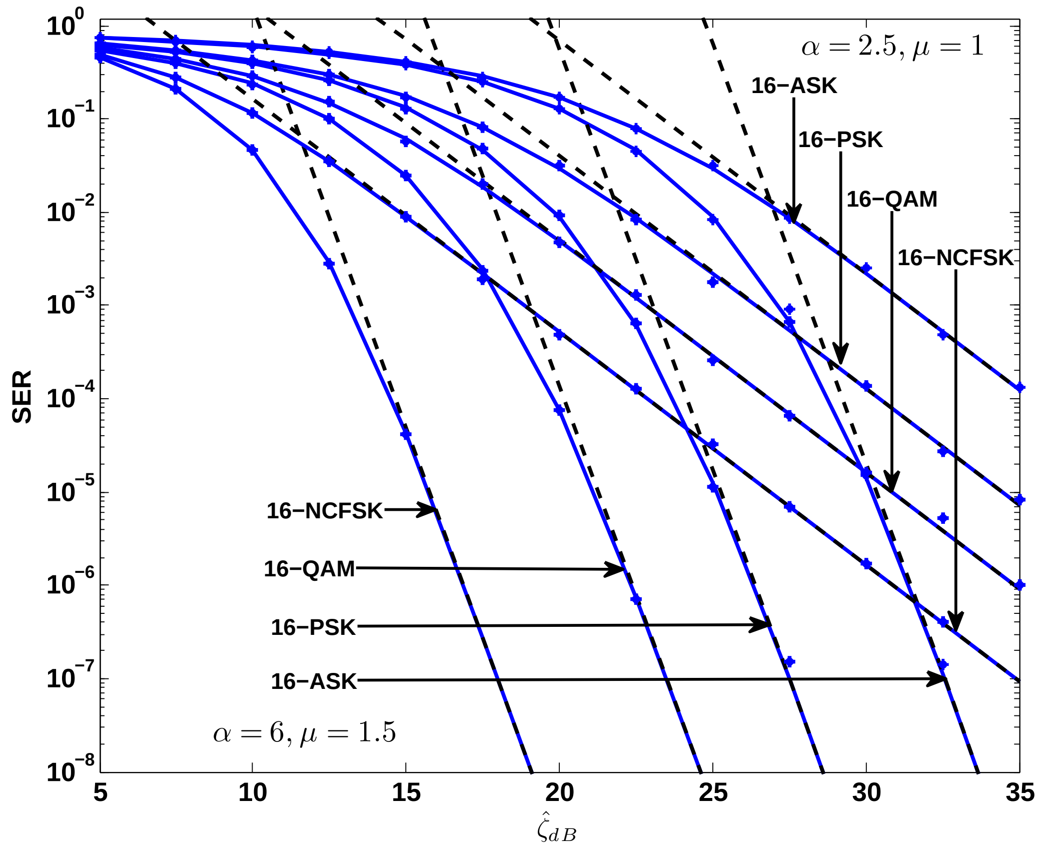

Fig. 4. Exact and asymptotic values of the SER of 16-ASK, 16-NCFSK, 16-PSK and 16-QAM $\alpha = 6$, $\mu = 1.5$ and $\alpha = 2.5$, $\mu = 1$. The solid lines represent the exact expressions while the dashed ones represent the asymptotic behavior. Markers represent the results of Monte Carlo simulations.

asymptotic expansions are not only very accurate for large values of the average SNR but also very stable to compute because closed-form expressions for their logarithms is available. Moreover, they are consistent with the fact that the rate of change of the logarithm of the SER primarily depends on the particular values of α and μ and not the modulation scheme. This statement has been verified in Section VI.

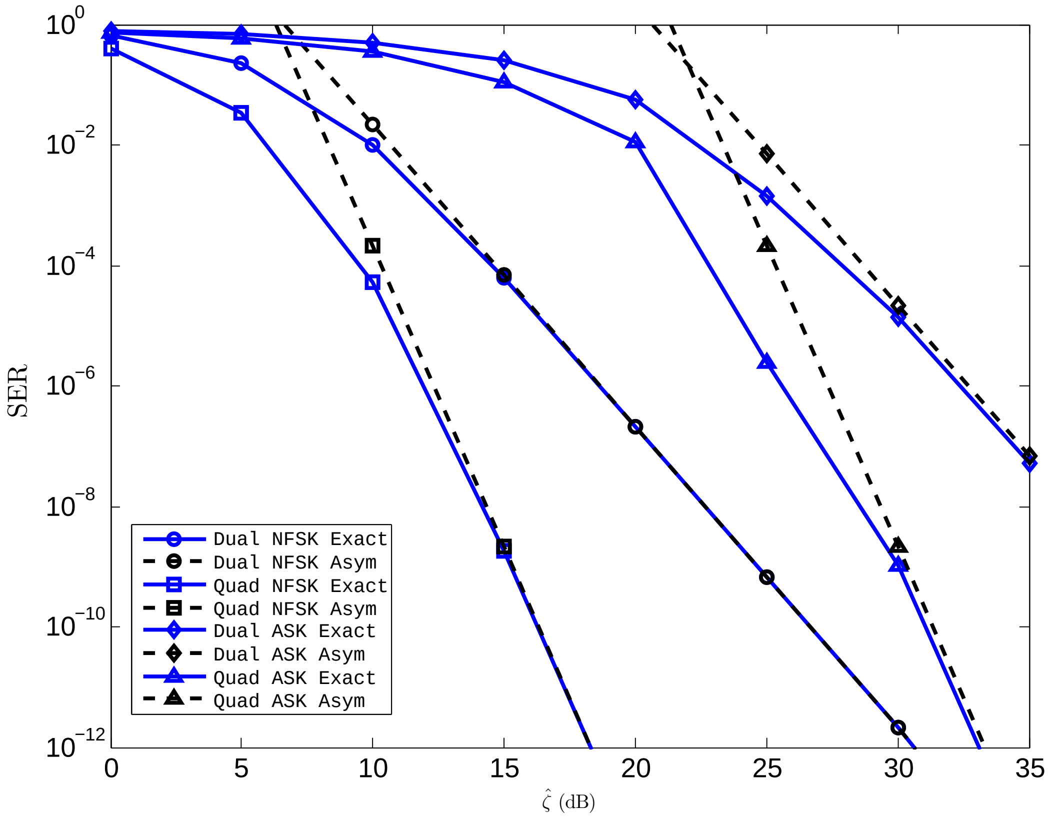

Fig. 5. Exact and asymptotic values of the SER for a dual- and quad-branch EGC receiver employing 16-NCFSK and 16-ASK with $\alpha = 2.5$ and $\mu = 1$.

APPENDIX A
EXPRESSIONS FOR $\mathcal{I}_0(b)$, $\mathcal{I}_1(b)$ AND $\mathcal{I}_2(a, b)$ FOR NAKAGAMI-m FADING

For the Nakagami-m distribution, we set $\alpha = 1$ and $\mu = m$ in (14). Applying the change of variables $z' = -z$ for $\mathcal{I}_0(b)$, we get

$$\mathcal{I}_0(b) = \frac{1}{b\Gamma(m)} \frac{1}{2\pi i} \int_{z'=-c_1-i\infty}^{-c_1+i\infty} \Gamma(m+z')\Gamma(-z') \left(\frac{b\hat{\zeta}}{m}\right)^{z'} dz' = \frac{1}{b}\left(1+\frac{b\hat{\zeta}}{m}\right)^{-m}$$

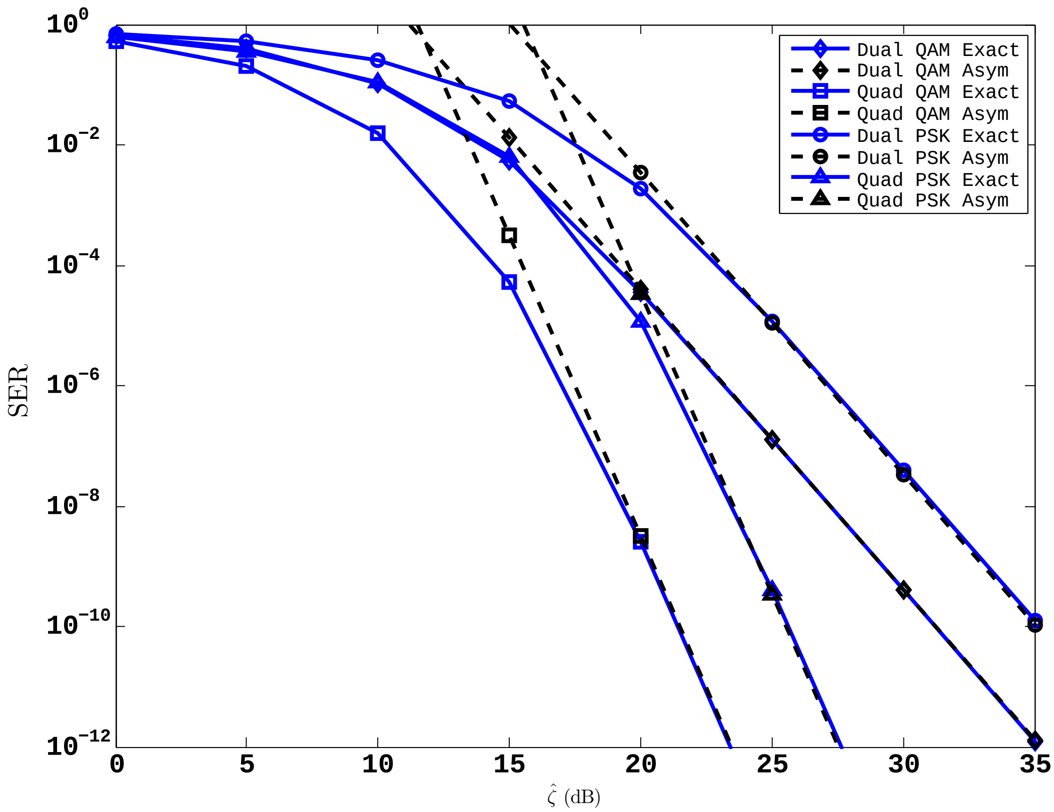

Fig. 6. Exact and asymptotic values of the SER for a dual- and quad-branch MRC receiver employing 16-QAM and 16-PSK with $\alpha = 2.5$ and $\mu = 1$.

where we used [13, Eq. (6.422.3)]. Similarly, we apply the change of variables $z = z' + m$ for $\mathcal{I}_1(b)$ yielding

$$\mathcal{I}_1(b) = \frac{1}{\sqrt{b}\Gamma(m)} \left(\frac{m}{b\hat{\zeta}}\right)^m \frac{1}{2\pi i} \int_{z'=c_1-m-i\infty}^{c_1-m+i\infty} \frac{\Gamma(z')\Gamma(z'+m+1/2)\Gamma(z'+m)}{\Gamma(z'+m+1)} \left(\frac{m}{b\hat{\zeta}}\right)^{z'} dz'$$
$$= \frac{\Gamma(m+1/2)}{\sqrt{b}\Gamma(m+1)} \left(1 + \frac{b\hat{\zeta}}{m}\right)^{-m} {}_2F_1\left(m, \frac{1}{2}; m+1; \frac{1}{1+b\hat{\zeta}/m}\right) \quad (36)$$

where we used [13, Eq. (9.113)] followed by [13, Eq. (9.131.1)]. Finally, we make the change of variables $z = z' + m$ and $s = s' + 1/2$ for $\mathcal{I}_2(a, b)$ and using [13, Eq. (9.185)] and [39, Eq. (39) and then (25)], we can prove that

$$\begin{aligned}\mathcal{I}_2(a,b) &= \frac{1}{2\sqrt{b\pi}\Gamma(m)}\left(\frac{m}{b\hat{\zeta}}\right)^m\left(\frac{a}{2b}\right)^{1/2}\frac{\Gamma(1/2)\Gamma(m)}{\Gamma(3/2)}F_2\left(m+1;\frac{1}{2},m;\frac{3}{2},m+1;-\frac{a}{2b},-\frac{m}{b\hat{\zeta}}\right)\\ &= \frac{1}{\sqrt{b\pi}}\left(\frac{m}{b\hat{\zeta}}\right)^m\left(\frac{a}{2b}\right)^{1/2}\left(1+\frac{m}{b\hat{\zeta}}\right)^{-m}F_1\left(\frac{1}{2};1,m;\frac{3}{2};-\frac{a}{2b},-\frac{a/2b}{1+m/b\hat{\zeta}}\right)\\ &= \frac{1}{\sqrt{b\pi}}\left(1+\frac{b\hat{\zeta}}{m}\right)^{-m}\left(1+\frac{2b}{a}\right)^{-1/2}F_1\left(\frac{1}{2};\frac{1}{2}-m,m;\frac{3}{2};\left(1+\frac{2b}{a}\right)^{-1},\left(1+\frac{b\hat{\zeta}}{m}\right)^{-1}\left(1+\frac{2b}{a}\right)^{-1}\right).\end{aligned} \tag{37}$$

APPENDIX B
THE PROOF OF THE EQUIVALENCE OF THE M-QAM EXPRESSION IN TABLE IV AND (16)

First, write $F_1\left(\frac{1}{2};m,\frac{1}{2}-m;\frac{3}{2};x,\frac{1}{2}\right)$ in its Euler integral form [34, Eq. (5.8.5)], i.e.,

$$\begin{aligned}F_1\left(\frac{1}{2};m,\frac{1}{2}-m;\frac{3}{2};x,\frac{1}{2}\right) &= \frac{1}{2}\int_{u=0}^{2}u^{-1/2}(1-ux)^{-m}\left(1-\frac{u}{2}\right)^{m-1/2}du\\ &\quad -\frac{1}{2}\int_{u=1}^{2}u^{-1/2}(1-ux)^{-m}\left(1-\frac{u}{2}\right)^{m-1/2}du\end{aligned} \tag{38}$$

Substituting $u=2v$ in the first integral and $u=2/(2-v)$ in the second, preforming some manipulations, and making use of the Euler integral form of the hyper-geometric function [13, Eq. (9.111)] and that of the Appell $F_1(\cdot)$ function, we easily obtain the following relation

$$\begin{aligned}F_1\left(\frac{1}{2};m,\frac{1}{2}-m;\frac{3}{2};x,\frac{1}{2}\right) &= \sqrt{\frac{\pi}{2}}\frac{\Gamma(m+1/2)}{\Gamma(m+1)}{}_2F_1\left(\frac{1}{2},m;m+1;2x\right)\\ &\quad -\frac{2^{-(m+1/2)}(1-x)^{-m}}{2m+1}F_1\left(1;m,1;\frac{3}{2};\frac{1}{2(1-x)};\frac{1}{2}\right)\end{aligned} \tag{39}$$

Substituting (39) into the M-QAM expression in Table IV with $x=(2+3\hat{\zeta}/m(M-1))^{-1}$ proves the required equivalence.

APPENDIX C
BASIC FUNCTIONS FOR THE SER FOR THE DIFFERENT MODULATION SCHEMES

TABLE X
BASIC FUNCTIONS USED FOR SER OF CBFSK, M-ARY ASK, M-ARY PSK, QAM MODULATION, THEIR MELLIN TRANSFORMS AND THE CORRESPONDING $\mathcal{I}$-FUNCTIONS. c_2 IS A REAL CONSTANT SUCH THAT $0 < c_2 < 1/2$.

$g(\boldsymbol{\zeta})$	Mellin Transform and corresponding $\mathcal{I}$-function
$\frac{\zeta_1 e^{-b\zeta_{MRC}}}{\sqrt{\zeta_{MRC}}}$	$\mathcal{M}_L\left\{g(\boldsymbol{\zeta})\right\} = z_1 \frac{\Gamma\left(\sum_{l=1}^{L} z_l + \frac{1}{2}\right)}{\sqrt{b}\Gamma\left(\sum_{l=1}^{L} z_l + 1\right)} \prod_{l=1}^{L} b^{-z_l}\Gamma(z_l),$ $\mathcal{I}_{1,MRC}(b) = \frac{1}{\sqrt{b}\prod_{l=1}^{L}\Gamma(\mu_l)} H_{1,1;1,1;\ldots;1,1}^{0,1;1,1;\ldots;1,1}\left(\begin{matrix} \mu_1^{1/\alpha_1}/b\hat{\zeta}_1 \\ \vdots \\ \mu_L^{1/\alpha_L}/b\hat{\zeta}_L \end{matrix}\,\middle\vert\,\begin{matrix} (1/2;1,\ldots;1) & (1,1) & \ldots & (1,1) \\ (0;1,\ldots,1) & (\mu_1, 1/\alpha_1) & \ldots & (\mu_L, 1/\alpha_L) \end{matrix}\right)$
$\frac{\zeta_1 e^{-b\zeta_{MRC}} Q'(\sqrt{a\zeta_{MRC}})}{\sqrt{\zeta_{MRC}}}$	$\mathcal{M}_L\{g(\boldsymbol{\zeta})\} = z_1 \frac{\prod_{l=1}^{L} b^{-z_l}\Gamma(z_l)}{2\sqrt{b\pi}\Gamma\left(\sum_{l=1}^{L} z_l + 1\right)} \frac{1}{2\pi i}\int_{s=c_2-i\infty}^{c_2+i\infty} \frac{\Gamma\left(\frac{1}{2}-s\right)\Gamma\left(s+\sum_{l=1}^{L} z_l + \frac{1}{2}\right)}{s}\left(\frac{a}{2b}\right)^s ds,$ $\mathcal{I}_{2,MRC}(a,b) =$ $\frac{1}{2\sqrt{b\pi}\prod_{l=1}^{L}\Gamma(\mu_l)} H_{1,1;1,2;1,1;\ldots;1,1}^{0,1;1,1;1,1;\ldots;1,1}\left(\begin{matrix} a/2b \\ \mu_1^{1/\alpha_1}/b\hat{\zeta}_1 \\ \vdots \\ \mu_L^{1/\alpha_L}/b\hat{\zeta}_L \end{matrix}\,\middle\vert\,\begin{matrix} (1/2;1,1,\ldots,1) & (1,1) & (1,1) & \ldots & (1,1) \\ (0;0,1,\ldots,1) & (1/2,1),(0,1) & (\mu_1, 1/\alpha_1) & \ldots & (\mu_L, 1/\alpha_L) \end{matrix}\right)$
$\sqrt{\frac{\zeta_1}{L}} e^{-b\zeta_{EGC}}$	$\mathcal{M}_L\{g(\boldsymbol{\zeta})\} = z_1 \frac{2^L\sqrt{\pi}}{\sqrt{b}\Gamma\left(\sum_{l=1}^{L} z_l + 1\right)} \prod_{l=1}^{L} (4b/L)^{-z_l}\Gamma(2z_l),$ $\mathcal{I}_{1,EGC}(b) = \frac{2^L\sqrt{\pi}}{\sqrt{b}\prod_{l=1}^{L}\Gamma(\mu_l)} H_{0,1;1,1;\ldots;1,1}^{0,0;1,1;\ldots;1,1}\left(\begin{matrix} L\mu_1^{1/\alpha_1}/4b\hat{\zeta}_1 \\ \vdots \\ L\mu_L^{1/\alpha_L}/4b\hat{\zeta}_L \end{matrix}\,\middle\vert\,\begin{matrix} - & (1,2) & \ldots & (1,2) \\ (0;1,\ldots,1) & (\mu_1, 1/\alpha_1) & \ldots & (\mu_L, 1/\alpha_L) \end{matrix}\right)$
$\sqrt{\frac{\zeta_1}{L}} e^{-b\zeta_{EGC}} Q'(\sqrt{a\zeta_{EGC}})$	$\mathcal{M}_L\{g(\boldsymbol{\zeta})\} = z_1 \frac{2^{L-1}\prod_{l=1}^{L}\Gamma(2z_l)(b/L)^{-z_l}}{\sqrt{b\pi}\Gamma\left(2\sum_{l=1}^{L} z_l + 1\right)} \frac{1}{2\pi i}\int_{s=c_2-i\infty}^{c_2+i\infty} \frac{\Gamma\left(\frac{1}{2}-s\right)\Gamma\left(s+\sum_{l=1}^{L} z_l + \frac{1}{2}\right)}{s}\left(\frac{a}{2b}\right)^s ds,$ $\mathcal{I}_{2,EGC}(a,b) =$ $\frac{2^{L-1}}{\sqrt{b\pi}\prod_{l=1}^{L}\Gamma(\mu_l)} H_{1,1;1,2;1,1;\ldots;1,1}^{0,1;1,1;1,1;\ldots;1,1}\left(\begin{matrix} a/2b \\ L\mu_1^{1/\alpha_1}/b\hat{\zeta}_1 \\ \vdots \\ L\mu_L^{1/\alpha_L}/b\hat{\zeta}_L \end{matrix}\,\middle\vert\,\begin{matrix} (1/2;1,1,\ldots,1) & (1,1) & (1,2) & \ldots & (1,2) \\ (0;0,2,\ldots,2) & (1/2,1),(0,1) & (\mu_1, 1/\alpha_1) & \ldots & (\mu_L, 1/\alpha_L) \end{matrix}\right)$

TABLE XI
BASIC FUNCTIONS USED FOR DBPSK AND NC M-ARY FSK MODULATION, THEIR MELLIN TRANSFORMS AND THE CORRESPONDING $\mathcal{I}$-FUNCTIONS.

$g(\boldsymbol{\zeta})$	Mellin Transform and corresponding $\mathcal{I}$-function
$e^{-b\zeta_{MRC}}$	$\mathcal{M}_L\{g(\boldsymbol{\zeta})\} = \prod_{l=1}^{L} b^{-z_l}\Gamma(z_l),$ $\mathcal{I}_{0,MRC}(b) = \prod_{l=1}^{L} \frac{1}{\Gamma(\mu_l)} H_{1,1}^{1,1}\left(\frac{\mu_l^{1/\alpha_l}}{b\hat{\zeta}_l}\,\middle\vert\,\begin{matrix} (1,1) \\ (\mu_l, 1/\alpha_l) \end{matrix}\right)$
$e^{-b\zeta_{EGC}}$	$\mathcal{M}_L\{g(\boldsymbol{\zeta})\} = \frac{2^L\sqrt{\pi}}{\Gamma\left(\sum_{l=1}^{L} z_l + \frac{1}{2}\right)} \prod_{l=1}^{L} (4b/L)^{-z_l}\Gamma(2z_l),$ $\mathcal{I}_{0,EGC}(b) = \frac{2^L\sqrt{\pi}}{\prod_{l=1}^{L}\Gamma(\mu_l)} H_{0,1;1,1;\ldots;1,1}^{0,0;1,1;\ldots;1,1}\left(\begin{matrix} L\mu_1^{1/\alpha_1}/4b\hat{\zeta}_1 \\ \vdots \\ L\mu_L^{1/\alpha_L}/4b\hat{\zeta}_L \end{matrix}\,\middle\vert\,\begin{matrix} - & (1,2) & \ldots & (1,2) \\ (1/2;1,\ldots,1) & (\mu_1, 1/\alpha_1) & \ldots & (\mu_L, 1/\alpha_L) \end{matrix}\right)$

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