Secular Effect of Geomagnetic Field and Gravitational Waves on Earth's Satellite Orbits

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Abstract: In this work we study the perturbation and the change in the orbital elements due to the earth's magnetic field and the gravitational waves. The acceleration components are derived in the radial, transverse to it and normal to the orbital plane. The equation for the rates of variation of the elements are formed and solved to find the secular variation in the element for polar and equatorial satellites.

Key words: gravitational waves, Earth's magnetic field, perturbations, orbital mechanics.

1- Introduction

One of the forces acting on an artificial satellite is the Earth's magnetic field (Roy, 1982) [1]. If the satellite has metal in its construction the Earth's magnetic field induces eddy currents in the satellite, in addition, a slight retardation acts on the satellite. The changes in the orbit due to this force are small , in any real situation the effect of the small force is to cause small departures from Keplerian motion, but these deviation can hardly give rise to large-scale changes the effect of another force , like as in our work the force arising from gravitational waves . To gain concept of the concentration of charged particles in space that might lead to electromagnetic forces on a satellite, it is useful to estimate the flux and energy of those particles in the field of the Earth associated the aurorally zone, solar flares, solar winds, cosmic rays, and the inner and outer Van Allen radiation belts, and recently the gravitational waves. The question of the effect of such particles and the associated magnetic fields on satellite dynamics is yet to be resolved. As noted by (Bourdeau et al., 1961) the motion of a satellite through a magnetic field B, produce an induced potential that is a function of position on the satellite surface, that is, a function of a vector **R** from the satellite center to any point on the satellite surface [2]. The magnetic field strongly affects the motion of charged particles and the gravitational waves interact with magnetic field producing other effect on the motion of an orbiter in the Earth's gravitational field. The influence of the geo-magnetic field in satellite charging is quite important and should be taken into account in any detailed treatment [3]-[8]. In this work we investigate the perturbation on the elements of the satellite's orbit moving with velocity V through a magnetic field and existing of gravitational waves. The gravitational waves incident normally and propagated in the same direction of the magnetic field.

2- The Acceleration Components of Magnetic field

A force arises from the interaction of the Earth's magnetic field and any electric charge of an orbiting body given by (Gelying and Westerman, 1971) [9]

$$\mathbf{F}_e = q_e \, \mathbf{V} \times \mathbf{B} \tag{1}$$

Where q_e the charge is acquired by the satellite, **V** its velocity and **B** is the magnetic induction of the Earth's field. In MKS units

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{2}$$

Where the vector potential is

$$\mathbf{A} = \frac{\mu_0}{4\pi} \; \mathbf{M_e} \times \nabla\left(\frac{1}{r}\right) \tag{3}$$

$\mathbf{M_e}$ is the magnetic moment of the Earth, and μ_0 is the permeability of free space, carrying out the operations indicated in (1) through (3), we obtain

$$\frac{|\mathbf{F}_e|}{m} = q_e \frac{\mu_0 M_e}{4\pi m r^2} \frac{v}{r} \qquad (4)$$

Where m is the mass of satellite. As a first step to analyze effect of the magnetic field on the Kaplerian elements (a, e , i , ω, Ω ,τ), it is necessary to represent the acceleration exerted by the magnetic field on a satellite as a function of the radial, orthogonal, and normal, perturbing components (Baker, 1967) [10]. This can be done in terms of either the true anomaly *f*, or eccentric anomaly E. The resolving components of the disturbing force in the direction of the satellite radius-vector S, tangential to the orbit T and perpendicular to it W, are given as

$$S = q_e \frac{\mu_0 M_e}{4\pi m} \sqrt{\frac{\mu}{P^7}} \; e \sin f \; (1 + e \cos f)^3$$

$$T = q_e \frac{\mu_0 M_e}{4\pi m} \sqrt{\frac{\mu}{P^7}} \; (1 + e \cos f)^4$$

$$W = 0 \qquad (5)$$

Where $P = a(1 - e^2)$ is the parameter, a is the semi-major axis, e is the eccentricity of the orbit, μ is the Earth's gravitational constant and f is the true anomaly.

3- The Change of the Orbital Elements

According to Gauss form of Lagrange planetary equations under the action of the geomagnetic field and normal incident of gravitational waves the changes of the osculating elements for an elliptical orbit have the form

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} \{e(S_{mg} + S_{gw}) \cos f + \frac{p}{r} \; (T_{mg} + T_{gw})\}$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{n\,a} \{ \sin f (S_{mg} + S_{gw}) + (\cos f + \cos E) (T_{mg} + T_{gw})\}$$

$$\frac{di}{dt} = \frac{r \cos(f + \omega)}{n\,a^2\sqrt{1-e^2} \; \sin i} \; (W_{mg} + W_{gw})$$

$$\frac{d\Omega}{dt} = \frac{r \sin(f+\omega)}{n\, a^2\sqrt{1-e^2}\, \sin i} \quad (W_{mg} + W_{gw})$$

$$\frac{d\omega}{dt} = \frac{\sqrt{(1-e^2}}{nae} \left\{-\cos f \left(S_{mg} + S_{gw}\right) + \left(1 + \frac{r}{p}\right) \sin f \, \left(T_{mg} + T_{gw}\right)\right\} - \frac{r \sin(f+\omega)}{n\, a^2\sqrt{(1-e^2}} \cot i \, \left(W_{mg} + W_{gw}\right) \qquad (6)$$

Where (S_{mg} , T_{mg} , W_{mg}) and (S_{gw} , T_{gw} , W_{gw}) are referred to the acceleration components for the magnetic field and gravitational waves in the radial direction S , the transverse direction T at right angle to S in the orbital plane and perpendicular to the orbital plane W as in figure 1.

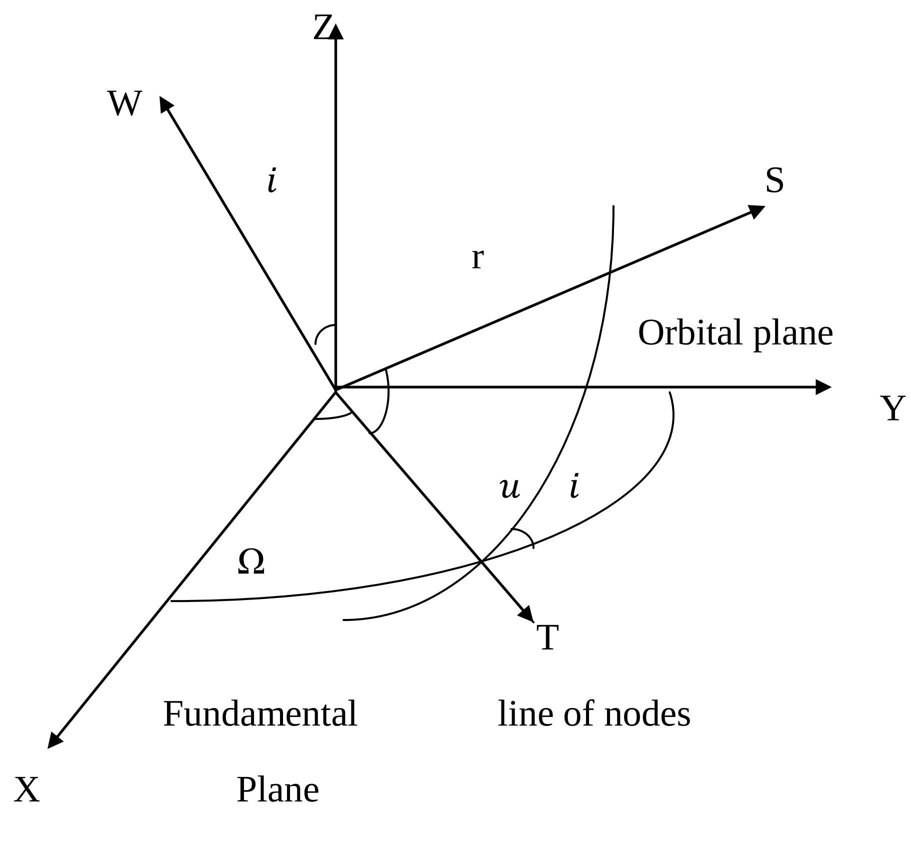

Fig. 1 The disturbing force of gravitational waves in (S, T, W) directions

Where $\Omega, \omega, u = (f + \omega)$ and i are the angles of longitude of ascending node, argument of perigee, true anomaly and inclination respectively. (S_{mg}, T_{mg}, W_{mg}) are the resolving acceleration components of geomagnetic force which derived in equation (5). The resolving acceleration components of gravitational waves force are

$$S_{gw} = P_x F_x + P_y F_y$$

$$T_{gw} = Q_x F_x + Q_y F_y$$

$$W_{gw} = W_x F_x + W_y F_y \tag{7}$$

Where

$$F_x = h_1 x + h_2 y$$

$$F_y = h_2 x - h_1 y$$

$$F_z = 0 \tag{8}$$

F_x, F_y and F_z are the components of the acceleration of normal ancient gravitational waves in (x, y, z) coordinates and

$$h_1 = \frac{1}{2}\frac{\partial^2 h_{11}}{\partial t^2} \quad ; \quad h_2 = \frac{1}{2}\frac{\partial^2 h_{12}}{\partial t^2} \tag{9}$$

$$h_{11} = h_+ \cos(n_g t + \alpha_1)$$

$$h_{12} = h_\times \cos(n_g t + \alpha_2) \tag{10}$$

Where n_g is the frequency of the wave, α_1 and α_2 are the phase difference, h_+ and $h_\times$ are the amplitude of the wave in the two orthogonal directions in the transverse plane [11]. Therefore

$$h_1 = -\frac{1}{2} n_g^2 h_+ \cos(n_g t + \alpha_1)$$

$$h_2 = -\frac{1}{2} n_g^2 h_\times \cos(n_g t + \alpha_2) \tag{11}$$

We have $P^{\wedge}, Q^{\wedge}$ and $W^{\wedge}$ are the unit vectors in the direction of **r**, normal to **r** in the orbital plane and normal to the orbital plane respectively,

$$\mathrm{P_x} = \cos\Omega \cos \mathrm{u} - \sin\Omega\, \mathrm{sinu} \cos i$$

$$\mathrm{P_y} = \sin\Omega \cos \mathrm{u} + \cos\Omega\, \mathrm{sinu} \cos i$$

$$\mathrm{P_z} = \sin \mathrm{u} \sin i$$

$$\mathrm{Q_x} = -\cos\Omega \sin \mathrm{u} - \sin\Omega\, \mathrm{cosu} \cos i$$

$$\mathrm{Q_y} = -\sin\Omega \sin \mathrm{u} + \cos\Omega\, \mathrm{cosu} \cos i$$

$$\mathrm{Q_z} = \cos \mathrm{u} \sin i$$

$$\mathrm{W_x} = \sin\Omega \sin i$$

$$\mathrm{W_y} = -\cos\Omega \sin i$$

$$\mathrm{W_z} = \cos i \qquad (12)$$

It follows that

$$x = r(\cos\Omega \cos \mathrm{u} - \sin\Omega\, \mathrm{sinu} \cos i)$$

$$y = r(\sin\Omega \cos \mathrm{u} + \cos\Omega\, \mathrm{sinu} \cos i)$$

$$\mathrm{z} = \sin \mathrm{u} \sin i \qquad (13)$$

Therefore from equation (8) to (13) equation (7) yield to

$$\mathrm{S_{gw}} = r\{A_1 + A_2 \cos 2(u) + A_3 \sin 2(u)\}$$

$$\mathrm{T_{gw}} = r\{-A_2 \sin 2(u) + A_3 \cos 2(u)\}$$

$$\mathrm{W_{gw}} = r\{C_1 \cos(u) + C_2 \sin(u)\} \sin i \qquad (14)$$

Where

$$A_1 = \frac{1}{2} sin^2 i \{h_1 \cos 2\Omega + h_2 \sin 2\Omega\}$$

$$A_2 = \frac{1+cos^2 i}{2} \{h_1 \cos 2\Omega + h_2 \sin 2\Omega\}$$

$$A_3 = \cos i\ \{-h_1 \sin 2\Omega + h_2 \cos 2\Omega\}$$

$$C_1 = h_1 \sin 2\Omega - h_2 \cos 2\Omega$$

$$C_2 = \cos i\ \{h_1 \cos 2\Omega + h_2 \sin 2\Omega\} \tag{15}$$

For the approximate integration of equations (6), we expand all the functions of the orbital coordinates on the right sides in series in powers of e. The coefficients in these series will be trigonometric functions of the mean anomaly

$$\mathrm{M} = n\,(t - \tau)$$

$$\cos kf = \cos k\mathrm{M} + ke[\cos(k+1)\,\mathrm{M} - \cos(k-1)\,\mathrm{M}]$$

$$\sin kf = \sin k\mathrm{M} + ke[\sin(k+1)\,\mathrm{M} - \sin(k-1)\,\mathrm{M}] \quad ; \quad K = 1,2,3,\ldots$$

$$\cos E = \cos \mathrm{M} + \frac{\mathrm{e}}{2}(\cos 2\,\mathrm{M} - 1)$$

$$r = a(1 - e\cos \mathrm{M}) \tag{16}$$

Where n the mean angular velocity of satellite, t the initial time of the motion , and τ is the time of perigee passage. Without loss of generality, we can take the satellite lies on the line of nodes at the initial time, so $\omega = \tau = 0$ and equations (6) become in the form

$$\frac{\mathrm{da}}{\mathrm{dt}} = \frac{2\mathrm{a}}{\mathrm{n}}\{\mathrm{T}_0 + \mathrm{e}(\mathrm{T}_1 + \mathrm{S}_0 \sin \mathrm{M} + \mathrm{T}_0 \cos \mathrm{M})\}$$

$$\frac{\mathrm{de}}{\mathrm{dt}} = \frac{1}{\mathrm{n}}\{\mathrm{S}_0 \sin \mathrm{M} + 2\mathrm{T}_0 \cos \mathrm{M} + \mathrm{e}[\mathrm{S}_1 \sin \mathrm{M} + 2\mathrm{T}_1 \cos \mathrm{M} + \mathrm{S}_0 \sin 2\mathrm{M} + \frac{3}{2}\mathrm{T}_0(\cos 2\,\mathrm{M} - 1)]\}$$

$$\frac{\mathrm{di}}{\mathrm{dt}} = \frac{\sin \mathrm{i}}{\mathrm{n}}\{\mathrm{W}_0 \cos \mathrm{M} + \mathrm{e}\,[\frac{\mathrm{W}_0}{2}(\cos 2\,\mathrm{M} - 3) + \mathrm{W}_1 \cos \mathrm{M}]\}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{dt}} = \frac{1}{\mathrm{n}}\{\mathrm{W}_0 \sin \mathrm{M} + \mathrm{e}\,[\mathrm{W}_1 \sin \mathrm{M} + \frac{1}{2}\mathrm{W}_0 \sin 2\mathrm{M}]\}$$

$$\frac{\mathrm{d}\omega}{\mathrm{dt}} = \frac{1}{\mathrm{ne}}\left\{-\mathrm{S}_0 \cos \mathrm{M} + 2\mathrm{T}_0 \sin \mathrm{M} + \mathrm{e}\,[\mathrm{S}_0 - \mathrm{S}_1 \cos \mathrm{M} + 2\mathrm{T}_1 \sin \mathrm{M} - \mathrm{S}_0 \cos 2\,\mathrm{M} + \frac{3}{2}\mathrm{T}_0 \sin 2\mathrm{M}]\right\} - \frac{\cos \mathrm{i}}{\mathrm{n}}\{\mathrm{W}_0 \sin \mathrm{M} + \mathrm{e}[\mathrm{W}_1 \sin \mathrm{M} + \frac{1}{2}\mathrm{W}_0 \sin 2\mathrm{M}]\} \tag{17}$$

Where

$$S_0 = A_1 + A_2 \cos 2M + A_3 \sin 2M + q_e \frac{\mu_{0\,M_e}}{4\pi m} \sqrt{\frac{\mu}{P^7}}\, e \sin M$$

$$S_1 = -A_1 \cos M + \frac{1}{2} A_2 (3 \cos 3M - 5 \cos M) + \frac{1}{2} A_3 (3 \sin 3M - 5 \sin M)$$

$$T_0 = -A_2 \sin 2M + A_3 \cos 2M + q_e \frac{\mu_{0\,M_e}}{4\pi m} \sqrt{\frac{\mu}{P^7}} (1 + 4e \cos M)$$

$$T_1 = -\frac{1}{2} A_2 (3 \sin 3M - 5 \sin M) + \frac{1}{2} A_3 (3 \cos 3M - 5 \cos M)$$

$$W_0 = C_1 \cos M + C_2 \sin M$$

$$W_1 = \frac{1}{2}\{C_1(\cos 2M - 3) + C_2 \sin 2M)\} \qquad (18)$$

Because of the smallness of the eccentricity e, it is of interest to consider such effects only for small powers of e. Therefore equations (17) is represented the first order approximations of equations (6) and the effect of geomagnetic and GW on the orbital elements. A general analysis of equations (17) shows that the semi-major axis a and the position of the orbital plane in space determined by the angles Ω and i, change for a wave frequency, wave amplitude and mass ratio of satellite $\frac{q}{m}$ due to the induction of Earth's magnetic field. For $i = \frac{\pi}{2}$, the plane of orbit of which is parallel to the direction of the gravitational waves and the direction of magnetic field given by the quantities $h_\times = 0$ and $\alpha_1 = \pm\frac{\pi}{2}$, in this case only the longitude of ascending node Ω changes and the shape of the orbit is constant. If the initial data for the orbit and the waves are defined by $i = \frac{\pi}{2}$, $\Omega = 0, h_\times = 0$ and $\alpha_1 = \pm\frac{\pi}{2}$ then the position of the orbital plane is constant in space and only the semi-major axis a changes without a change in the other parameters. For the initial data $i = \frac{\pi}{2}$, $h_\times = \alpha_1 = 0$ and $\Omega = \frac{\pi}{4}$ there is only a deviation of the orbital plane from the wave direction, determined by the angle i . For the case $i = h_\times = 0$ and $\alpha_1 = \pm\frac{\pi}{2}$ only eccentricity changes. In case $i = h_\times = 0$ and $\alpha_1 = 0$, this time only the angular distance ω changes.

4- Numerical Simulation

A numerical simulation is developed to test several of the above results. The simulation is a Runge – Kutta 4^{th} order integration of system defined by Equations (17) with respect to the mean anomaly M performed by MATHEMATICA V10. The simulation is valid for any orbit for charged satellite in a non-tilted dipole field with GW. The results of two different situations are presented here for polar and equatorial orbits. Considering the period of satellite is four hours and with semi-major 12600 km. The effects of the magnetic field and the gravitational waves are proportional to the charge to mass ratio $\frac{q}{m}$ and the frequency of the gravitational waves. Considering the wave's frequency n_g of the same order of magnitude of the magnetic field 10^{-9}, wave's amplitude of order 10^{-21} as coming from the bursts sources, and the mass ratio $\frac{q}{m} = 0.0393$ C /kg for satellite has mass 300 kg and q =117C .This means that with two parameters we determined the perturbations on the orbital motion and we can control on it. Also due to the interaction of GW with magnetic field the variation on the orbital element will periodic with the time.

Figure 2 displays the variation in the orbital plane due the perturbation of Ω in radian for polar orbits with inclination $i = 90^{\circ}$, $\omega = 0^{\circ}$ or 180° or $360^{\circ}$and $\Omega = \frac{\pi}{4}$.

Figure 3 displays the variation of the semi-major a in radians for the polar circular orbit with $\Omega = 0^{\circ}$, $h_{\times} = 0$ and $\alpha_1 = \pm\frac{\pi}{2}$.

Figure 4 displays the variation of the inclination i in radian for the polar circular orbit with $\Omega = \frac{\pi}{4}$, $h_{\times} = 0$ and $\alpha_1 = 0^{\circ}$.

Figure 5 displays the variation of the eccentricity e for equatorial orbit with the case $i = h_{\times} = 0$ and $\alpha_1 = \pm\frac{\pi}{2}$.

Figure 6 displays the variation of the angular distance ω in radian for equatorial orbit with the case $i = h_{\times} = 0$ and $\alpha_1 = 0^{\circ}$.

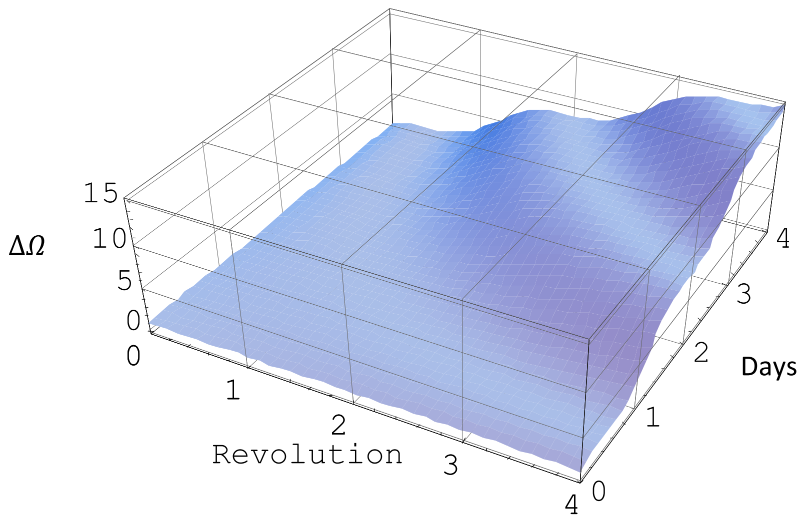

Figure 2. The variations of the orbital plane by angle Ω for polar orbit

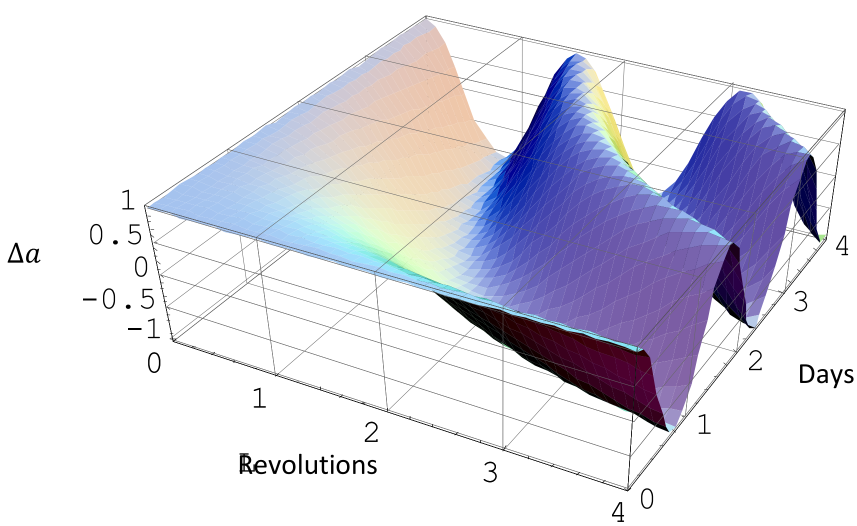

Figure 3. The variations of the semi-major a for polar orbit

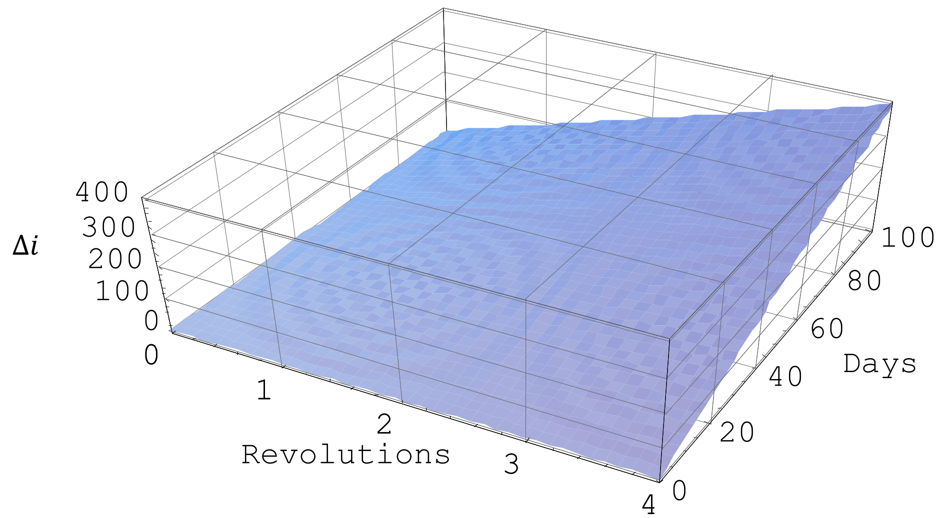

Figure 4. The variations of the inclination i for polar orbit

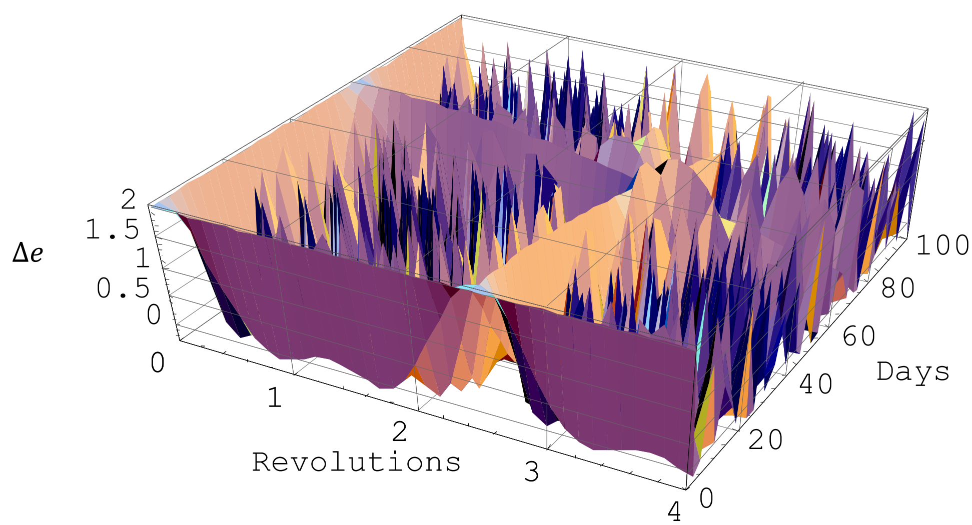

Figure 5. The variations of the eccentricity e for equatorial orbit

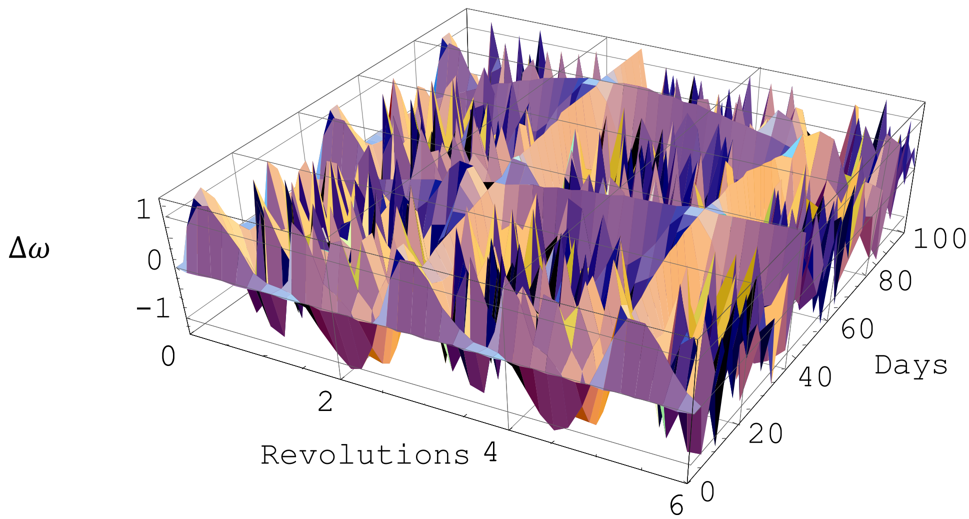

Figure 6. The variations of the angular distance ω for equatorial orbit

5- Discussion and Conclusions

Summarizing the results we conclude that there is secular effect due to geomagnetic and gravitational waves on the orbital plane of polar orbits (the longitude of node Ω and inclination i) and on the shape (the semi-major axis a) according to phase and phase difference of gravitational waves. For equatorial orbits the secular effect will on the size on the orbit (the eccentricity e and the argument of perigee ω). The amount of the variation of the orbital elements depend on the direction of propagation of gravitational waves and on the frequency and its phase difference which changes the amount of charges on the satellite and consequently the perturbations on the orbital elements. This effect is small but it is important for studying the effect of GW with the magnetic field of the Earth during interval of time and their effects on artificial satellites.

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