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Application of Genetic Algorithms to the Optimization of Pressure Transient Analysis of Water Injectors using Type Curves

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Abstract

Injection pressure fall-off (PFO) test analysis has proven to be a reliable vehicle for understanding and evaluating well performance. Recently the methodology has been extended to the understanding of injectors. This paper presents an optimization model for analysis and interpretation of PFO tests for fractured water injectors. An elliptical composite flow model mathematically represents the fractured injector well. The optimization scheme couples the mathematical model of the well with a Genetic Algorithm (GA) to reach the final solution. The pressure transients representing the behavior of an injector a closing fracture and the discontinuity in fluid mobility are best developed in elliptical coordinates. The methodology derives the dimension of the induced fractures, formation permeability, fracture conductivity and fracture face skin.

The current paper illustrates the solution methodology by showing the attained match for a West Africa offshore field case. The field case provides reasonable agreement for the fracture dimensions and characteristics as verified by other techniques.

Genetic Algorithms are one of the most common artificial intelligence techniques for optimization. The reported solution is obtained by applying a GA with a non-linear least square error function as an objective function. A special penalty function, mutation, crossover probabilities, and stopping criterion are used to obtain the global minimum of the objective function. The test data analysis is done through type curve matching of the pressure and its derivative by minimizing the objective function to help determine the parameters that provide the best match between the field data and the presented novel fractured injector type curves.

Introduction

Water-injection wells are frequently fractured either intentionally or unintentionally. For instance, in low-permeability formations or when low-quality water is injected, fracturing may be used in an attempt to increase well injectivity. Unintentional fracturing of water-injection wells may result when cold water is injected into a relatively hot reservoir. The cooling of the reservoir rock can reduce the rock stress such that the injection pressure exceeds the reduced formation strength and fracturing occurs. A considerable amount of research in the field of waterflood-induced fracturing has been carried out in recent years, which has resulted in the analytical description of fracture propagation from a single well in an infinite reservoir. Injection fall-Off (IFO) test analysis offers a cheap way to infer the dimensions of induced fractures from well tests.

An important first step towards the analysis of a fall-off test for closing waterflood-induced fractures was made by Hagoort in his thesis [(1)]. Application of a simple rock mechanical model for the fracture enabled him to relate fracture closure to early-time fluid flow. He presented an analytical expression for the theoretical pressure response which is valid as long as formation fluid flow is still linear and perpendicular to the fracture and before the fracture has completely closed.

Koning [(2)] presented an extension of Hagoort's [(1)] model to account for different fracture geometries, transition from early time linear flow to late time pseudo-radial flow, pressure response during and after closure and the effect of an elliptical discontinuity in fluid mobility. He solved the fully transient diffusivity equation in two dimensions, and incorporating the fracture closure-induced flow via convolution techniques.

Van den Hoek [(3)] rebuilt Koning [(2)] model from scratch and subsequently extended it with a number of features that are essential for the interpretation of fall-off test on fractured water injectors. His extension relates to fractures with a finite (possibly changing) conductivity and to fracture face skin.

Much work was presented in fall-off test analysis based on straight line analysis other than type curve matching. In the current paper, we present the development of a practical interpretation methodology dedicated to fall-off test analysis on fractured water injectors based on automatic type curve matching.

Simultaneous regression with Genetic Algorithm (GA) is then carried out to achieve type curve matching. Each population consists of subpopulations, one for each model. The GA operators, crossover and mutation are applied to create new populations. The parameters with a better fit to the data eventually dominate the population while the subpopulations representing parameters with less fit become extinct. Thus at the end of such a run, the reservoir and model parameters that result in the best least squares fit to the test data are obtained. The dependency of the nonlinear regression process to the initial guess is another obstacle to interpretation. Some parameters do not have simple heuristics to help make a reasonable initial guess and noise in the data may complicate visual determination (e.g., straight line analysis) of the initial guess parameters. Conventional regression techniques may fail to converge to the right solution and fall into suboptimal solutions if the initial estimate is not appropriate. In the case of automated analysis, the consequences of such suboptimal convergence can be highly problematic. In the case of conventional manual well-test analysis, precious time can be wasted in the regression procedure. In order to reduce the dependency of the behavior of the nonlinear regression to the initial guess, a GA algorithm can first be applied.

The use of GA needs only to specify the bounds of the parameters rather than a single deterministic initial guess. These bounds can even be the physical bounds of the reservoir model parameters. This approach virtually eliminates the need to spend time assessing consistent parameter values for an initial guess.

Model Formulation

The objective is to construct a practical methodology for fall-off test data interpretation on fractured water injection wells. This can be achieved in several steps. First, to build two mathematical models, one for a fractured well with an infinite conductivity fracture in two mobility zones and the other for a finite conductivity fracture with fracture face skin in two mobility zones. This step would help to predict pressure –time data for a falloff test on fractured water injector. Second, the models are then inverted to use the real pressure-time data for fall-off test to estimate the reservoir parameters and fracture characteristics. The use of an optimization algorithm (Genetic algorithm) in this last step would facilitate the quick estimation of proper characteristics.

Infinite conductivity closing Fracture Model

After a fractured water injector is shut-in, the leak-off into the formation is still approximately equal to the injection rate before shut-in. Subsequently the fracture will gradually close, which by itself will generate a closure-induced flow rate change. The computations for the pressure transient solution around an infinite conductivity fracture are presented in two periods, one before fracture closure and other after the fracture closure.

For time $t > t_{sh}$ the pressure transient solution will becomes [4]:

$$\Delta p_D(t_D) = \int_0^{t_D} d\tau_D \left[1 - C_{fD}\frac{\partial \Delta p_D(\tau_D)}{\partial \tau_D}\right]\frac{\partial \Delta p_D{}^{Cr}(t_D-\tau_D)}{\partial t_D} \quad t_D < t_{D\,closure} \quad \text{.....} \quad (1)$$

Equation (1) can be solved first in Laplace space using mathieu functions and considering the presence of an elliptical discontinuity in fluid mobility:

$$\Delta \bar{p}_D(s) = \frac{\Delta \bar{p}_D{}^{Cr}(s)}{1+s^2 C_{fD}\Delta \bar{p}_D{}^{Cr}(s)} \quad \text{.....} \quad (2)$$

For shut in time exceeding the fracture closure time ($t > t_{closure}$) the pressure transient solution becomes [4]:

$$\Delta p_D(t_D) = \int_0^{t_{Dclosure}} d\tau_D \left[1 - C_{fD}\frac{\partial \Delta p_D(\tau_D)}{\partial \tau_D}\right]\frac{\partial \Delta p_D{}^{Cr}(t_D-\tau_D)}{\partial t_D} + \Delta p_D{}^{Cr}(t_D - t_{Dclosure}) \;\; t_D > t_{D\,closure} \quad \text{.....} \quad (3)$$

The solution in Laplace space is numerically inverted using stehfest algorithm to obtain the wellbore pressure response in the time domain. On other hand Equation (3) has to be computed via numerical integration for its first integral part while the second one is computed in Laplace space to be numerically inverted using stehfest algorithm. It should be noted that the variable $\Delta p_D{}^{Cr}$ is the transient pressure corresponding to the constant terminal rate solution around an infinite conductivity fracture in two mobility zones.

The type curves computed from equation (1) and (3) are presented in **Fig.1a** using different values for dimensionless fracture compliance, and dimensionless fracture closure time. The effect of various mobility front positions on the pressure change after shut-in is shown on the logarithmic derivative of the pressure change plot **Fig.1b**

Based on known reservoir parameters (such as permeability and fracture half length) the pressure change after shut-in could be estimated from the computed dimensionless pressure as:

$$\Delta p = 141.2\ Q\mu\beta\Delta p_D/Kh \qquad (4)$$

The corresponding time function could be estimated from the dimensionless time as:

$$\Delta t = \phi\mu c L^2 \Delta t_D/0.000263K \qquad (5)$$

The Derivative of the pressure change after shut-in could be estimated as:

$$\Delta p' = \Delta t\ \frac{d(\Delta p)}{d(\Delta t)} \qquad (6)$$

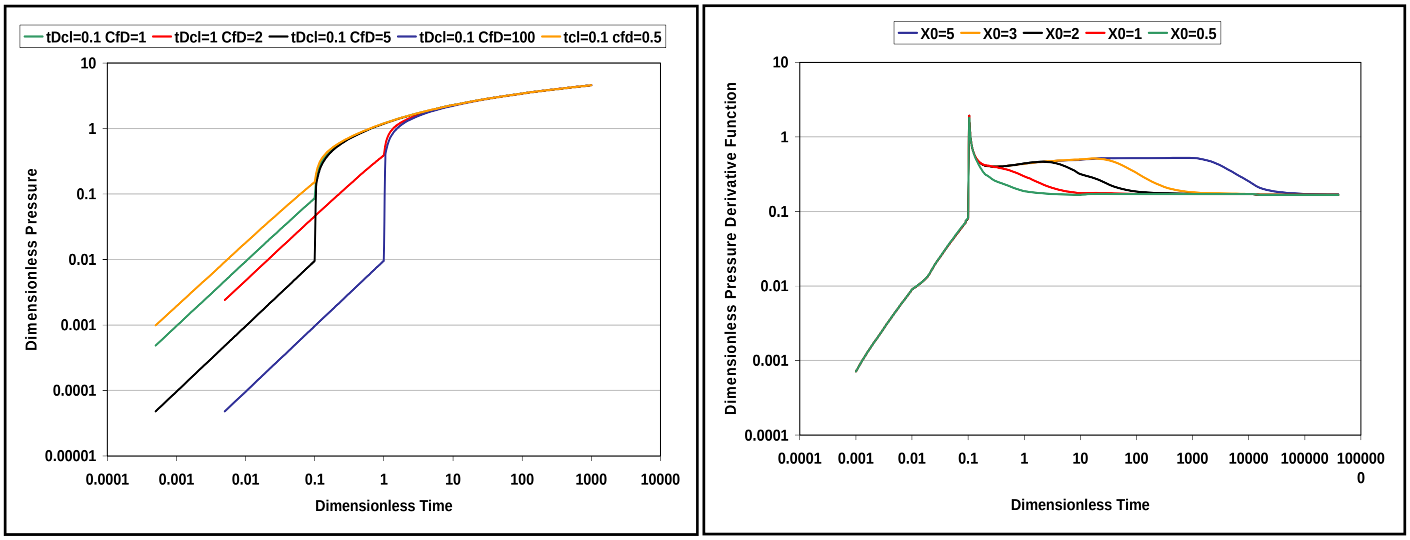


Fig.1 a) Computed pressure change Δp_D after shut-in for various fracture compliance and fracture closure time. Dual mobility Zones; mobility ratio=0.33 b) Computed Logarithmic derivative of the pressure change after shut-in for various mobility interface positions. Dual mobility Zones;$C_{fD} = 1$; $t_{Dcl} = 0.1$; mobility ratio=0.33

Finite conductivity Fracture with fracture face skin

Equation (1) and (3) can form the basis to compute the dimensionless pressure change Δp_D following the shut-in for a closing finite conductivity fracture by solving the transient pressure $\Delta p_D{}^{Cr}$ corresponding to the constant terminal rate solution around a finite conductivity fracture with fracture face skin in two mobility zones. **Fig. 2a** shows the computed dimensionless pressure after shut-in for a finite conductivity fracture as a function of dimensionless time after shut-in and as a function of dimensionless fracture conductivity. **Fig.2b** shows the effect of various mobility front positions on the pressure change after shut-in on the logarithmic derivative of the pressure change plot. **Fig. 3** shows the effect of inclusion of fracture face skin on the pressure change after shut-in.

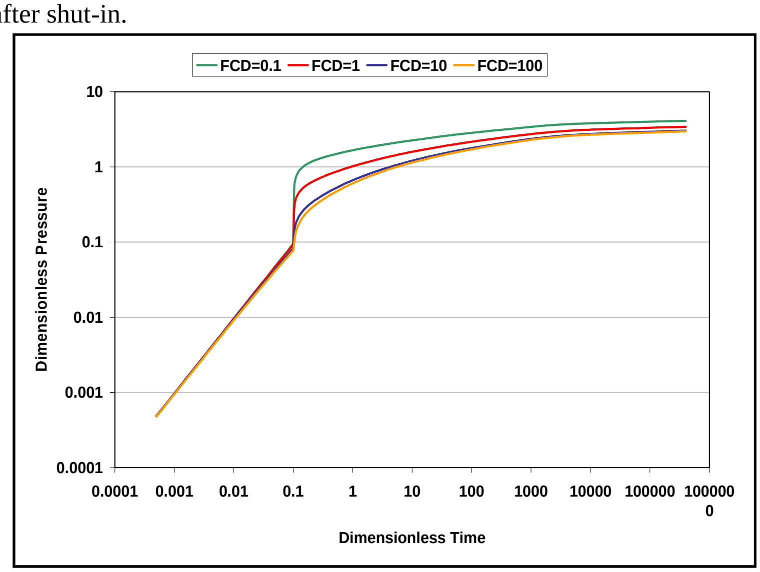
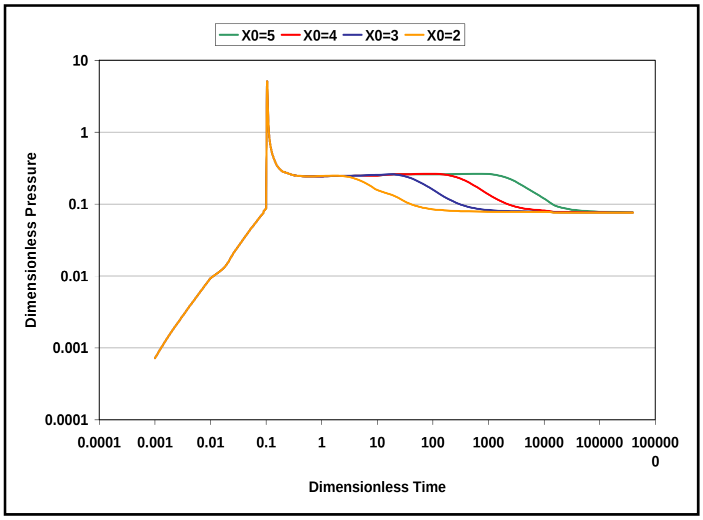


Fig.2 a) Computed pressure change Δp_D after shut-in for various fracture conductivity. Dual mobility Zones; fracture closure time=0.1, mobility ratio=0.33, fracture compliance=1, Skin=0



b) Computed Logarithmic derivative of the pressure change after shut-in for various mobility interface positions. Dual mobility zones; fracture conductivity=1, fracture closure time=0.1, fracture compliance =1, Mobility ratio=0.33, Skin=0

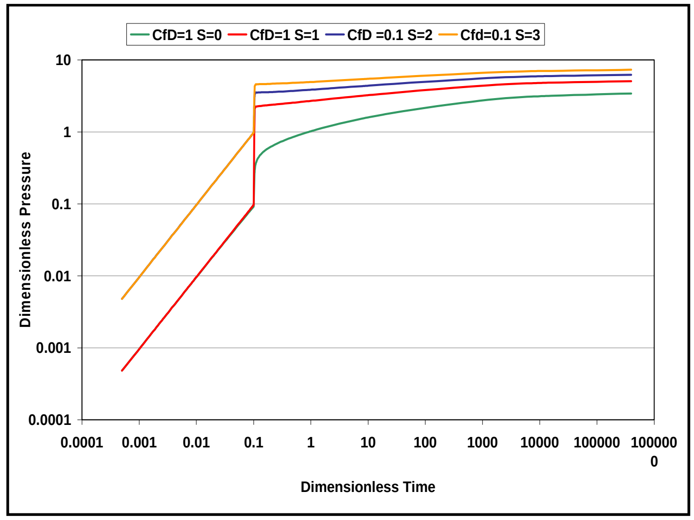


Fig.3 Computed pressure change Δp_D after shut-in for various fracture compliance and fracture skin. Dual mobility Zones; fracture conductivity=1, fracture closure time=0.1, mobility ratio=0.33, fracture compliance=1

Fall-off Test Interpretation

Koning Approach

Koning proposed three independent methods based on straight line analysis to determine fracture characteristics and reservoir parameters from a pressure injection fall-off test.

Method 1: Fracture Storage Dominated Flow

During the very early part of the fracture storage-dominated flow regime p varies linearly with Δt. In this period the leak-off into the formation is still approximately equal to the injection rate before shut-in. The fracture must close to provide the necessary fluid volume. At very early time Equation (2) is converted to the following form:

$$\Delta p_D(t_D) = t_D/C_D \quad (7)$$

It is clear from Equation (7) that the fracture storage dominated flow is characterized by unit slope on log-log plot of Δp_D vs. t_D as shown in type curves in **Fig.1a**

The non-dimensional form of the previous equation yields the following:

$$Q/C_f = (p_{shut-in} - p(t))/(t - t_{shut-in}) \quad (8)$$

Thus, the slope of a linear plot of $p(t)$ vs. t is equal to Q/C_f from which the fracture compliance can be determined. Different models are proposed to compute the fracture dimensions from its compliance. These models are the Christianovic-Geertsma-de Klerk (CGD) model (5), applicable to fractures for which $h_f/2L \gg 1$, the Perkins- Kern- Nordgren (PKN) model (5), for fractures with $h_f/2L \ll 1$, and ellipsoidal fractures (6) for the Intermediate case.

Method 2A: Linear formation flow before fracture closure

This period is characterized by a slope between half slope (slope=1/2) for linear formation flow and unit slope (slope=1) for fracture storage dominated flow as shown in **Fig.1a** on log-log plot of Δp_D vs. t_D. In his work, Van den Hoek (3) compares the analytical expression presented by Hagoort with the exact solution; the results show an excellent agreement between Hagoort equation and the exact solution for $t_D < 0.01$. For values of fracture compliance about 0.1, the Hagoort equation still gives reasonable results for $t_D < 0.1$, but for higher values of t_D, this equation can no longer be used.

Method 2B: Linear formation flow after fracture closure

The linear formation flow equation:

$$\Delta p_D = \sqrt{\pi t_D} \qquad (9)$$

It is clear from Equation (9) that the linear formation flow is characterized by half slope on log-log plot of Δp_D vs. t_D as shown in type curves in **Fig.1a.** With,

$$\Delta p_D = 2\pi Kh(\Delta p)/Q\mu \qquad (10)$$

$$\Delta t_D = Kt/\phi\mu c L^2 \qquad (11)$$

Equation (9) can be written as:

$$\Delta p = Q\sqrt{\mu t}/2hL\sqrt{K\pi\phi c} \qquad (12)$$

Thus a linear plot of Δp vs. $\sqrt{t}$ gives a slope:

$$Q\sqrt{\mu}/2hL\sqrt{K\pi\phi c} \qquad (13)$$

where L is the fracture half length.

Method2C: Bilinear/Linear formation flow before fracture closure

This method consists of a generalization of method 1 "fracture storage dominated flow" to the arbitrary values of time t_D not only the very early times. Van den Hoek (3), in his work, noted that the fracture length and conductivity before fracture closure for $t_D \leq 0.1$ could be determined by matching the modified Cinco expression (7) to the observed pressure change after shut-in through variation of the dimensionless fracture compliance and fracture conductivity.

Method 2D: Bilinear/Linear formation flow after fracture closure

This flow period can be represented by the work of Cinco-Ley (8). He derived for the finite conductivity fracture the dimensionless wellbore pressure for this flow period:

$$\Delta p_D = \frac{2.45}{\sqrt{F_{cD}}} t_D^{1/4} \qquad (14)$$

Bilinear flow period is characterized by a straight line with a slope equal to ¼ on a log-log plot. It refers to the simultaneous linear flow along the fracture and linear formation flow perpendicular to the fracture face. Plots of the pressure change Δp_D as a function of the quadratic root of dimensionless time will yield the fracture conductivity according equation (14) but the fracture length could not be determined using the analysis of this flow period.

Method 3: Transition Flow

This period reflects the transition flow from the inner injected fluid region to the outer reservoir fluid region across the mobility discontinuity front. The mobility contrast front position, if assumed to be elliptical, can be computed from analysis of pressure fall-off test data. **Figures 1b and 2b** depict the logarithmic derivative plot of the pressure change versus the injection time. The plots show that the larger values for the mobility front position correspond to larger injection times. The late drop in the logarithmic pressure derivative occurs after shut in. Type curve matching of the field data is the only proposed solution for determining the mobility front position. However, the disadvantage of this method is the need for long fall-off test data to detect the boundary of mobility contrast, especially for long injection periods.

Koning's approach has been used over the past years in IFO test analysis. However, type curve matching would be required in lieu of Koning's approach in the following situations:

- For infinite conductivity fractures, the half slope that characterized the linear formation flow after fracture closure is not clearly indentified for longer fracture closure time $t_D > 0.01$. Therefore, type curve matching provides more confidence to produce meaningful results than the Cartesian plot analysis
- For infinite conductivity fractures, the Cartesian or semi-log plot analysis is rarely proposed in the literature for analysis of the linear formation flow period before fracture closure.
- Type curve matching field data is the only proposed analysis approach for transition flow period to estimate the mobility interface location.

- The quarter slope that characterized the bilinear/linear formation flow after fracture closure is not clearly identifiable when fracture conductivity is low and for longer fracture closure time $t_D > 0.1$.
- Even for low fracture closure times the quarter slope is not easily detected for medium to high fracture conductivities. Hence, the need for type curve matching is necessary to estimate the fracture characteristics and fall-off test analysis.
- The presence of fracture face skin has an impact on the computed dimensionless pressure after shut-in that makes Koning approach not applicable.

Automatic Type Curve Matching Approach

Automatic type curve analysis requires matching the derivative of the pressure-time field data with the computed derivative of pressure change from the typical mathematical models. This is reached by adjusting the parameters that control the computed values from the models to reach the best overlap.

In the case of the infinite conductivity fracture model, the controlling parameters are the permeability; fracture half Length, dimensionless fracture closure time, diffusivity ratio, mobility front position, dimensionless fracture storage constant, and mobility ratio. Dimensionless fracture conductivity, and fracture face skin are two additional controlling parameters for the case of finite conductivity fracture model.

In order to reduce the selection parameters, for both models, the dimensionless fracture closure time could be estimated from:

$$t_D = 0.000263 \frac{\Delta t \ K}{\mu \ \phi \ C \ L^2} \quad \text{.......... (15)}$$

The mobility front position can be calculated as a function of fracture half length [9] as:

$$V_I = \frac{\pi}{2} \ . L \ . Sinh(2\zeta_0) . h_f . \phi \ . (1 - S_{wi} - S_{ro}) \quad \text{.......... (16)}$$

Matching the observed pressure and its derivative test data is achieved by searching for a combination of the controlling parameters that minimize the objective function defined in the genetic algorithm.

Genetic Algorithm

A genetic algorithm (GA) is a search technique used in computations to find exact or approximate solutions to optimization and search problems. Genetic algorithms are categorized as global search heuristics that represent a particular class of evolutionary algorithms (also known as evolutionary computation). These algorithms use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover. The genetic algorithm method, first introduced by Holland [10] in 1975, has earned many interests in petroleum engineering aspects. GA allows precise modeling of the optimization system, although not usually providing mathematically optimal solutions. Another advantage of using GA techniques is the absence of a need for an explicit objective function. Moreover, when the objective function is available, it does not have to be differentiable. The GA has four advantageous features:

1. GA begins the search with a population of parameter realizations, rather than a single realization as most of the conventional optimization methods might. In this way, the search domain is covered in a random distribution.
2. The realizations are perturbed by probabilistic rules rather than deterministic ones.
3. The parameter itself is not manipulated directly by the GA operators. GA would alter the chromosome (or individual or string) that is a pattern of zeros and ones representing the whole set of parameters put all together in one binary entity. For binary alphabets, the smaller piece of a chromosome is called a bit.
4. Only function evaluations are used rather than derivatives or other secondary descriptors

Genetic Algorithms are a subset of reproductive population algorithms. This type of algorithm starts from a population of parameter realizations and repeatedly performs the following cycle of operations (see Apendix I for more detailed definition of the operations) until the stopping criteria are reached [11]:

1. **Evaluation**: evaluate the fitness of each chromosome.
2. **Selection**: based on each chromosome’s fitness, select the ones to build the reproducing set in the population based on determined selection criteria.
3. **Reproduction**: from each reproducing set, apply reproductive strategies to generate a number of new chromosomes.
4. **Replacement**: replace some or all of the original population with new chromosomes.

In this way, a single vector of the parameters for an optimization problem would be analogous to an individual chromosome. We assign a specific length for each of the parameters compatible with their original domain and the desired resolution. The parameters are then mapped into the binary domain to compose the chromosome. After the three operations of reproduction, crossover and mutation have taken place; the new individuals are mapped back to their domain (real or integer) and then used as input to the objective function computation. The measure of evaluation for the population will be the highest value of the objective function obtained in this generation. The GA will continue to cycle through the three procedures of reproduction, crossover and mutation in its search for the best patterns, until some convergence or stopping criterion is reached. Other variations of the technique have been suggested. We can establish a threshold heuristic so that the best individual is always saved from generation to generation *(elitism)* as is used in *Genitor model* [15] .This approach assures that evaluation values will never decrease from one generation to the next and assures that crossover and mutation do not lead to degradation.

Objective Function for the Pressure Transient Problem: The fitness of each individual in the GA population is based on the least squares fit of the represented model/parameter combination to the measured data from the well test. The objective is to minimize the function:

$$F = \frac{1}{N}\sqrt{\sum_{i=1}^{i=N} \frac{\Delta p'(mesaured) - \Delta p'(calculated)}{\Delta p'(mesaured)}} \qquad (18)$$

The derivative of the pressure was used instead of the pressure values themselves since the characteristics of the reservoir that are of interest here are more clearly defined by the derivatives. Individuals attributes that produce values that result in a better least squares fit to the measured data have higher fitness. Accordingly, the model parameters that represent the physics of flow in the reservoir and the well will eventually dominate the population, since they will be more successful in capturing the characteristics of the measured pressure response.

Termination Criteria: The optimization is stopped when one of the following criteria is satisfied:

- The maximum number of generations have been reached
- The fitness function has not improved over a specified number of generations.

Field Case Offshore West Africa

A fall-off test was carried out in the injector well after a seawater volume of 96075 bbls has been injected into soft formation that induces fracture propagation. **Fig.4** shows the pressure change after shut-in and its logarithmic derivative on log-log plot. The characteristics of the flow dominated by fracture storage is followed by a gradual change in pressure as a result of fracture closure as also shown in **Fig. 4** The pressure keeps increasing long after fracture closure. This behavior indicates the existence of no-flow reservoir boundary. Automatic type curve matching technique will be applied on this field case through the use of Genetic algorithm to test the robustness of our current approach.

Genetic algorithm characteristics that were adopted in this study are: initial population generated randomly, single-point crossover, crossover probability of 0.6, mutation probability of 0.05, elitism, memory used for sets of parameters with exact match, and polytope activated every other generation with one movement allowed.

Two Genetic algorithm runs was carried out on the data set in **Fig.4** .Run 1 used the analytical model corresponding to the infinite conductivity fracture while Run 2 used the finite conductivity fracture model. All runs were carried out with Young's modulus 725200 psi, Poisson's ratio 0.25, and Total compressibility of 0.0000455 psi^{-1} .

In Run 1, genetic algorithm develops 21 generation to get the parameters that best match the field data with the computed pressure change and its derivative from the analytical model for the infinite conductivity fracture. The variables used in the optimization process were presented in **Table 1,** varying in the range shown in the table. The best fit parameters for the infinite conductivity fracture model are: permeability 21 mD, fracture half-length 22 ft, dimensionless fracture storage constant 100, and the diffusivity and mobility ratios are 0.15 and 0.15 respectively. The dimensionless fracture closure time was calculated through Eq. (15) to have a value of 0.18, while 4 is the value for the mobility front position estimated from Eq. (16).**Fig.5** shows the very well match between the field data and the computed pressure change and its derivative. Fitness function of the total population for run 1 on the field data set is given in **Fig.6**

In Run 2, **Fig.7** shows the match between the field data and the computed pressure change and its derivative from the analytical model for the finite conductivity fracture through the use of the genetic algorithm. The variables used in the optimization process are presented in **Table 2. They** vary in the range shown in the table. In this run the algorithm requires about 25 generation to get the following best fit parameters: Dimensionless fracture conductivity 98, fracture face skin 0.8, permeability 25 md, fracture half length 15 ft, dimensionless fracture storage constant 200, the diffusivity and mobility ratio are 0.15 and 0.15 respectively. The dimensionless fracture closure time was calculated through Eq. (15) to be 0.18, while 4 is the value for the mobility front position estimated from Eq. (16). **Fig.8** shows the fitness function along the total population for run 2 on the field data set.

Table 1 Optimization Variables for Infinite Conductivity Model

Data	Lower Bound	Upper Bound
Dimensionless fracture storage constant	*10*	*500*
Diffusivity Ratio	*0.1*	*1*
Mobility Ratio	*0.1*	*1*
Permeability, mD	*1*	*50*
Fracture half Length, ft	*5*	*50*

Table 2 Optimization Variables for Finite Conductivity Model

Data	Lower Bound	Upper Bound
Dimensionless fracture storage constant	*10*	*500*
Dimensionless fracture conductivity	*10*	*500*
Diffusivity Ratio	*0.1*	*1*
Mobility Ratio	*0.1*	*1*
Permeability, mD	*1*	*50*
Fracture half Length, ft	*5*	*50*
Fracture face skin	*0.1*	*10*

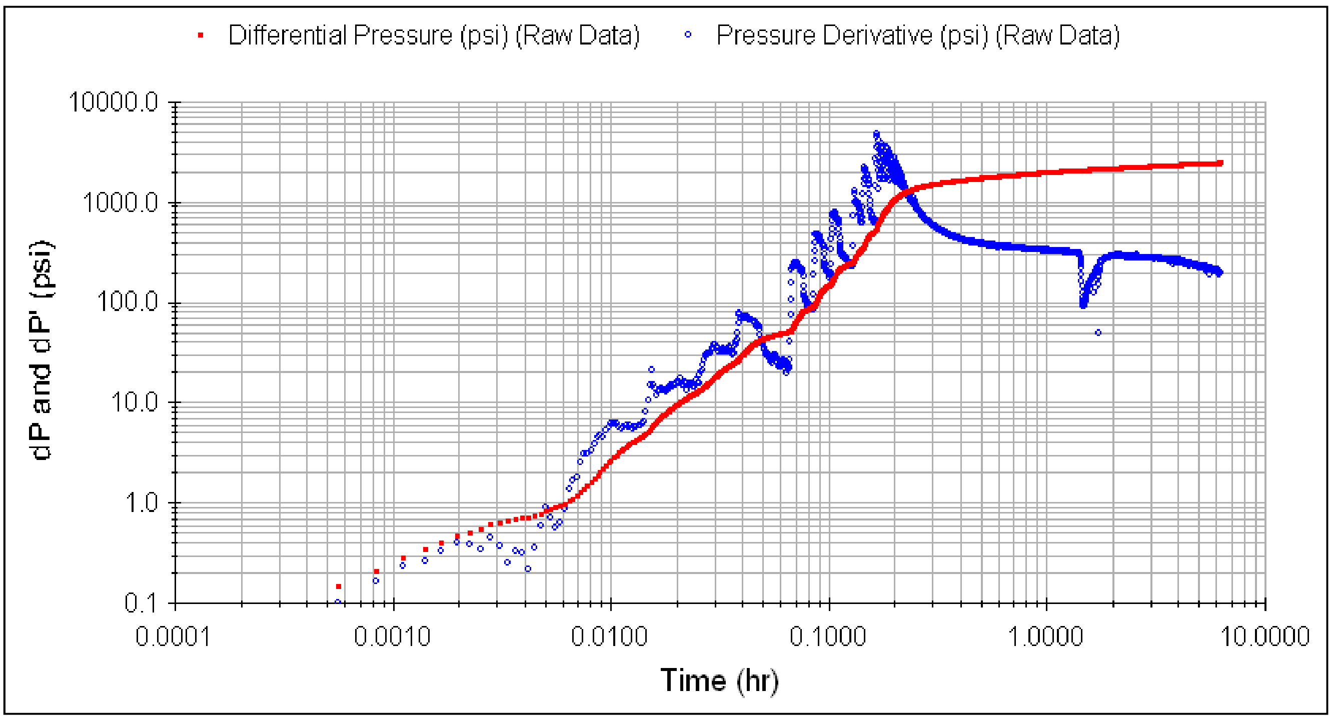


Fig. 4 Log-Log plot of observed pressure change and it's logarithmic during injection fall-off test in West Coast Africa Field case

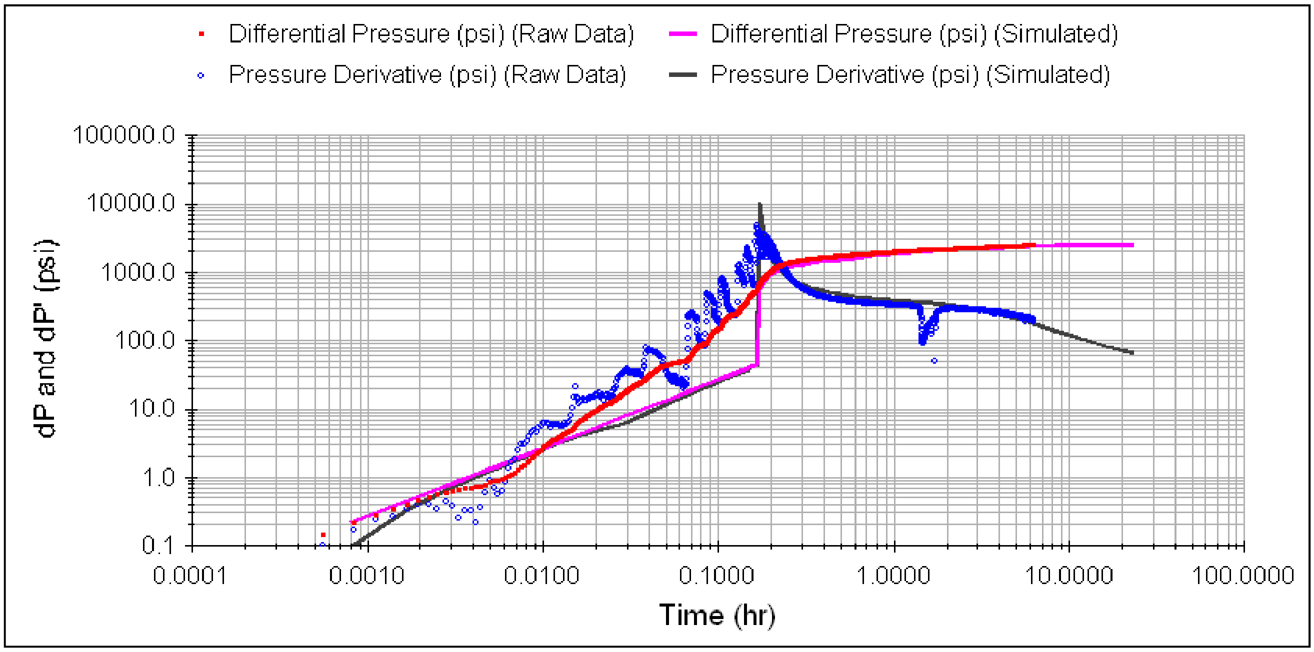


Fig. 5 Log-Log plot of observed pressure change and it's logarithmic during injection fall-off test in West Coast Africa Field Case (A match with the infinite conductivity model)

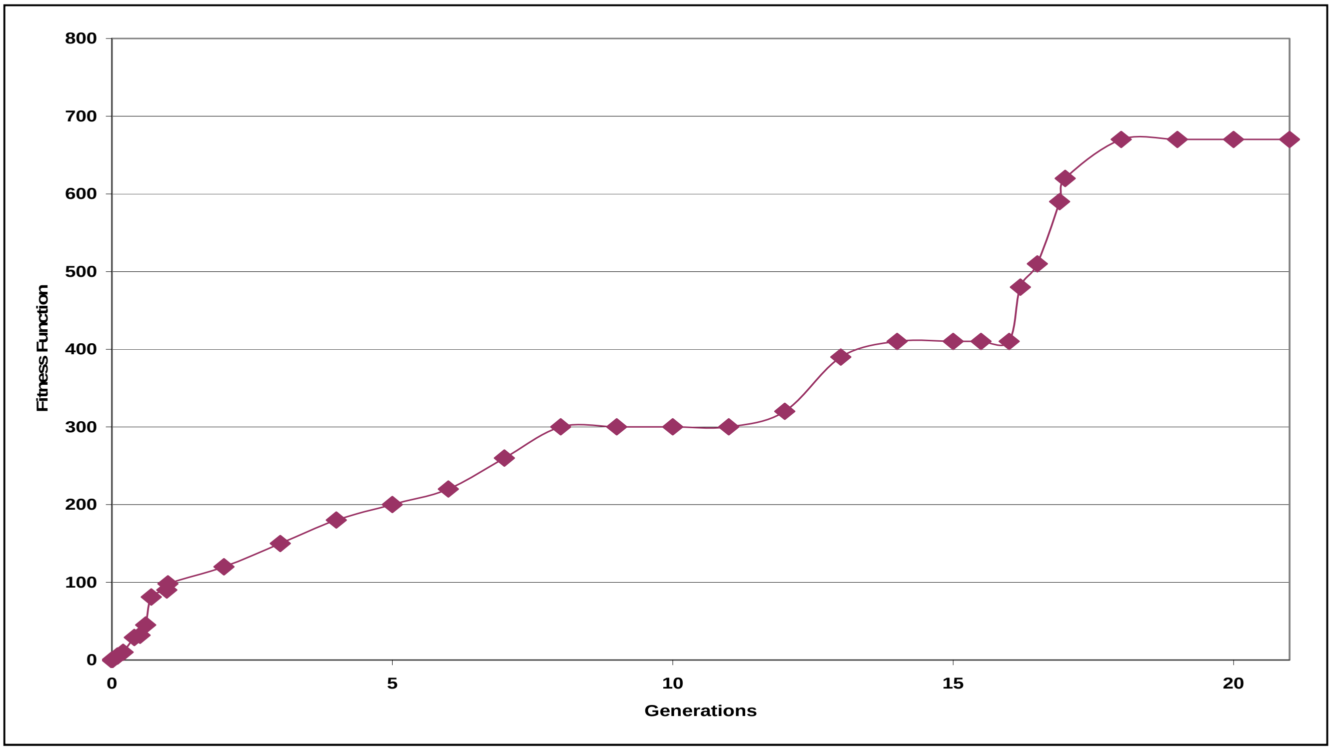


Fig. 6 Fitness function of total populations for Run 1 on field data set for West Coast Africa Field Case

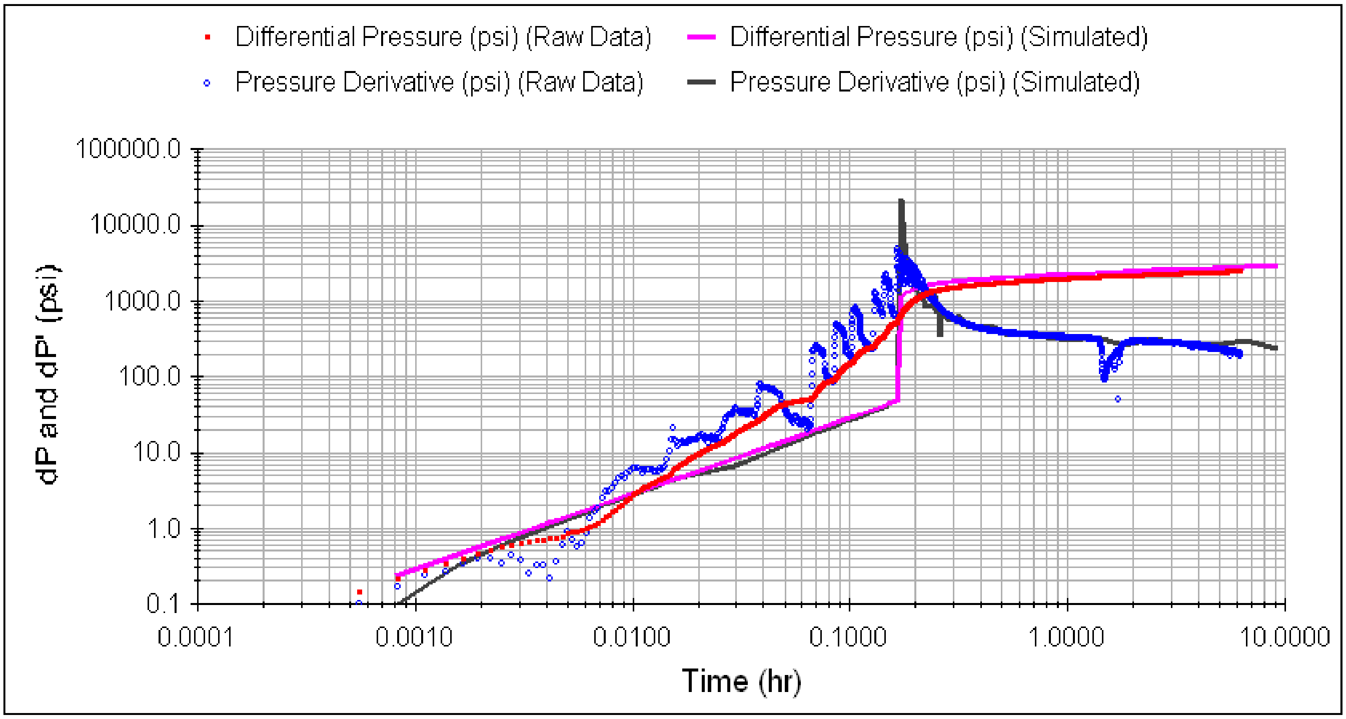


Fig. 7 Log-Log plot of observed pressure change and it's logarithmic during injection fall-off test in West Coast Africa Field Case (A match with the finite conductivity model with dual mobility zones and fracture face skin)

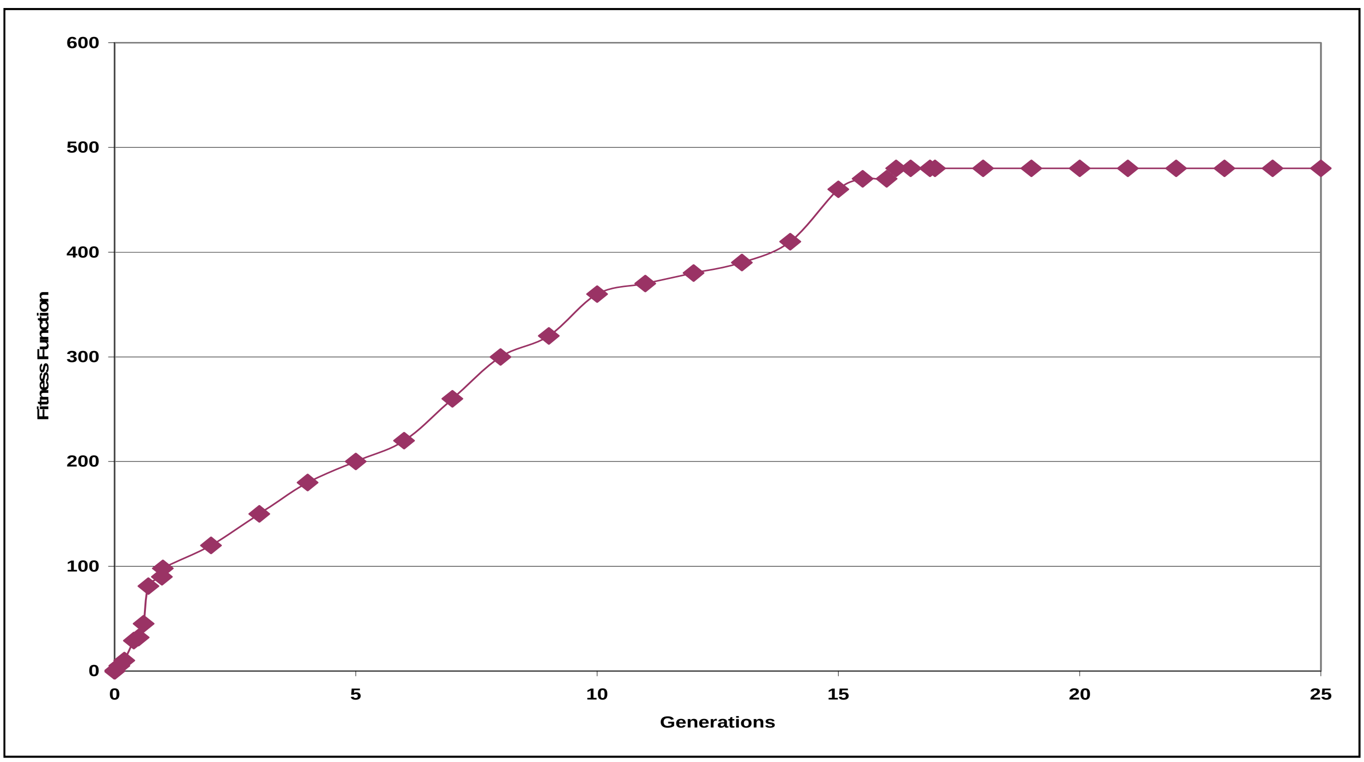


Fig. 8 Fitness function of total populations for Run 2 on field data set for West Coast Africa Field Case

Conclusion

This paper presents a practical pressure transient interpretation methodology using GA technique. The method is dedicated to fall-off test analysis on fractured water injectors based on automatic type curve matching. Two mathematical models are built, one for infinite conductivity fracture in two mobility zones and other for finite conductivity fracture with fracture face skin in two mobility zones, these models are coupled with the Genetic Algorithm for fall-off test analysis on a water injection fractured well to get the reservoir parameters and fracture characteristics that provides the best match with fall-off test data. The proposed method of simultaneous regression with Genetic Algorithm was able to automate the entire well test analysis process, eliminating the need to make an initial guess and replaced it by a set of bounds assigned to the parameters. This application was investigated through the presented field case and proved to be robust.

Nomenclature

C	*Total compressibility of rock-fluid system*
C_{fD}	*Dimensionless fracture compliance*
F_{cD}	*Dimensionless fracture conductivity*
h	*Formation thickness*
h_f	*Fracture height*
K	*Formation permeability*
L	*Fracture half length*
Δp_D	*Dimensionless pressure change after shut-in*
$\Delta p_D{}^{cr}$	*Constant terminal rate solution of dimensionless pressure change after shut-in*
Δp	*Pressure change after shut-in*
$\Delta p'$	*Derivative pressure change after shut-in*
Q	*Injection rate*
s	*Laplace parameter*
S	*Fracture face Skin*
S_{wi}	*Initial water saturation*
S_{or}	*Residual oil saturation*
$t, t_D, t_D{}^{closure}$	*Time, Dimensionless Time, Dimensionless fracture closure time*
V_I	*Total injected volume*
ζ_0	*Mobility front position*
φ	*Porosity*
μ	*Viscosity*
β	*Formation volume factor*

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Appendix A
Detailed Description of GA Operations

Selection can use one of the four common selection schemes [12]: Proportional selection, Ranking selection, Tournament selection, Genitor (or "steady state") selection. We present here a subset of the first selection scheme (Proportional selection) which was the approach used in the GA method [13] that formed the basis for this work. We adopted the model where all strings are able to mate with one another (*random mating*). The strategy of selecting strings for reproduction is based on their fitness values and, hence, different methods can be used.

Reproduction is an operator by which parameter strings are selected for possible propagation into the next generation. An intermediate population or reproducing set, which is submitted to the other genetic operations, is filled with copies of the selected strings. The value of the objective function evaluated for each particular parameter realization is used in deciding whether the information will survive. One of the easiest ways to apply the reproduction operator is using the roulette wheel selection. In this procedure the evaluations of a population are summed and then the contribution of each string creature is calculated. This fitness of a specific string creature will determine how *many* times the creature gets into the mating pool or reproducing set, an intermediate step to the next generation. The roulette wheel selection [14] method uses the ratio of the fitness of each string to the total fitness of all strings to define its probability of selection:

$$P_{selection}(i) = F_i(x) / \sum_{i=1}^{all} F_i(x) \qquad \text{(A-1)}$$

Crossover is a process in which pairs of chromosomes are selected from the intermediate population and their strings are cut at a random location(s) and joined to the corresponding piece(s) of the partner string. There are several ways to implement such an operator. The *n-point crossover* determines random crossover sites on the entire length of the chromosome. The crossover sites become the boundary points at which all the information on the first string is exchanged with all the information on the second string within the same boundary. This exchange occurs alternately from boundary to boundary until all crossover sites are exhausted. The *uniform crossover* generates one bit at a time. Each bit is inherited from one of the parents according to the crossover probability. Each one of these crossover operators has its advantages and disadvantages. *Disruption* is one of the effects caused by crossover. It causes the loss of a specific pattern of bits, called *schemata*, that can degrade the search when the bit value of the fixed position is changed (*allele loss*) causing the disruption of the schema. De Jong [14] showed that two-point crossover is less disruptive than single-point crossover. However operators that use higher order *n-point* crossover are much more disruptive and should be avoided. Uniform crossover is highly disruptive because alleles not found in the parents can be produced.

Mutation is surrounded by some controversy in genetics. Mutation is designed to avoid the loss of valuable genetic material, which theoretically may result from reproduction and crossover. Although mutation is one of the most familiar terms in genetics, its role in the GA is sometimes misconceived. Its function is not to generate new structures or patterns; the crossover operator does that in a very efficient way. Mutation's primary and unique role is to create a mechanism by which information or small segments of parameter strings can be reintroduced into a population. In the simple GA, mutation is a random alteration of a variable in a string. However, one might randomly choose a mutation site bit once every second or third generation. Because of its nature, mutation is highly disruptive and the assumption of very low mutation probability is sometimes recommended.