A NEW ALGORITHM FOR FORM TOLERANCE EVALUATION

M. H. Gadallah & H.A. El Maraghy

Mechanical Engineering Department
McMaster University, Hamilton, Ontario, Canada, L8S 4L7

Abstract

A new algorithm for form tolerance evaluation has been developed. Evaluation of the minimum tolerance zone is formulated as an optimization problem following the definitions of geometric tolerances in the current ANSI standards. The algorithm utilizes the experimental optimization techniques and the combinatorial nature of orthogonal arrays to plan the experimentation and evaluate the minimum tolerance zone. The approach is applied to 2–dimensional features tolerances such as straightness and circularity (roundness) and 3–dimensional features such as flatness. The obtained results are compared with other approaches using the Least Square method, the constrained optimization techniques and the Convex Hull approach.

Keywords

Form errors, Orthogonal arrays, experimental optimization

1. Introduction

In any manufacturing process, features deviate systematically or randomly from their ideal form, size or position. In order to ensure acceptability of parts, features must be manufactured within predetermined design limits defined by dimensions and tolerances. Geometric tolerances are used, in this paper, as defined by the ANSI Geometric Dimensioning and Tolerancing Standard, Y 14.5M [1, 7, 8, 10]. The minimum tolerance zone is one measure of conformance of manufactured features with the ideal ones. A set of measurements is taken using Coordinate Measuring Machines. The tolerance zone deviations determine the parts acceptance or rejection. In this article, a new algorithm is given to evaluate the minimum zone deviations based on a set of discrete measurments using CMM.

Three techniques were investigated for evaluating the minimum tolerance zone, namely the Monte Carlo technique, the Simplex search technique and the Spiral search technique. In the Monte Carlo technique, the minimum zone mean surface is assumed to lie close to the Least squares mean surface and within the deviation zone obtained by the Least squares method. The value of variables defining the surface were selected randomly to determine the actual minimum zone. The minimum deviation is calculated for each variable. One disadvantage, however, is the possibility of missing the actual minimum because of this random selection of variables. When the number of variables increase, a more suitable method such as the Simplex Search method would be needed.

The Simplex Search is a sequential gradient method designed to climb mathematical hills and valleys. Three points are initially chosen (for 2–D cases) such that they are equidistant, forming an equilateral triangle. The value of the objective function is evaluated at the three vertices. The algorithm rejects the point with the lowest value of objective function and replaces it by another point. The process is repeated until the region containing the desired optimum is attained. When the number of variables is small, the Spiral Search technique is used to scan the design space for the absolute minimum. The search commences in a spiral manner around the Least squares solution. This technique has been used for circles and planes and also in conjunction with the Simplex Search technique [5, 6].

0-8186-6510-6/94 $4.00

Four commonly used algorithms were analyzed with particular emphasis on the effects of errors in measuring the data points on the resulting plane. These are: the exact fit algorithm, the best of exact fits algorithm, the average of exact fit algorithm and the least RMS plane [4].

Traband et al. summarized the problems in using the Coordinate Measuring Machines (CMMs) [9]. The majority of CMMs depend on point sampling to evaluate the specified dimensions and tolerances. The basic requirements of CMM software and their validation using virtual volumetric standards were discussed.

A methodology was proposed to evaluate two of the ANSI – geometric form tolerances: straightness and flatness using the concept of Convex Hull algorithm [9]. ElMaraghy et al. focused on the development of procedures and algorithms for the systematic comparison of geometric variations of measured features [2].

A general algorithm was proposed by Dhanish and Shunmugam to approximate the real function using linear Chebychev approximation. The algorithm was coupled with the exchange algorithm to exclude a point during the search [1].

This paper deals with different techniques for assessing and evaluating geometric errors. It also includes a new approach which makes use of the combinatorial nature of orthogonal arrays and experimental optimization techniques to plan the search for the minimum tolerance zone. This article is organized in five sections; section 1 reviews the related work; section 2 introduces the proposed methodology in detail; and section 3 presents some applications for 2 and 3–dimensional features. The algorithm and its adaptation as a special case are also presented in section 4. Results obtained using the Least Squares method, the Convex Hull approach, constrained optimization techniques as well as CMM measurments results are also included for comparison. Discussions and conclusions are finally included in section 5.

2. Methodology

The algorithm developed, in this article, uses orthogonal arrays and experimental optimization techniques to approximate the actual features from a set of discrete measurments using automated measuring machines. An evaluation criterion is needed to determine the minimum zone deviation of 2–and 3–dimensional features. Let ψ_i be a function representing the ideal forms of engineering features such as straightness and circularity (2–D) and flatness, cylindricity and spherecity (3–D). Assuming that the actual function is $\hat{\psi}_i$, the problem can now be stated in an optimization form as:

$$\text{Minimize } \sum e_i = \psi_i - \hat{\psi}_i \qquad (1)$$

The form error can be computed as:

$$\text{Form Errors} = | e_{max} | + | e_{min} | \qquad (2)$$

Where $| e_{max} |$ and $| e_{min} |$ refer to the maximum and minimum errors of the ideal and measured features.

Orthogonal arrays allow design variables to be changed with other design parameter settings an equal number of times. We have to distinguish between design parameters and their settings. The designer can determine the value of error function for each combination of design parameter settings. This can be explained using an example. Fig. 1.a shows an L9 OA with X_1 & X_2 as two design parameters. Each design parameter has three design levels or design parameter settings. In this case, we can write X_{11}, X_{12} and X_{13} to represent the design parameter settings of parameter 1. Similarly, X2 can be written as X_{21}, X_{22} and X_{23} to represent the first, second and third design parameter settings of design parameter 2. Accordingly, nine design parameter setting combinations can be listed. These are: X_{11} and X_{21}, X_{11} and X_{22}, X_{11} and X_{23}, X_{12} and X_{21}, X_{12} and X_{22}, X_{12} and X_{23}, X_{13} and X_{21}, X_{13} and X_{22} and finally X_{13} and X_{23}. This is exactly the case if an L9 OA (2–3 two–three level design parameters and 9 experiments) was used.

The algorithm picks the design level and design settings which minimize the combined form error. New design levels and settings are generated by shifting the design setting by $\pm\Delta$. The objective function is evaluated at these new design levels and design settings. The algorithm picks the design levels and settings which yield the lowest combined errors. The search proceeds until either the Δ used for design level generation is very small ($\approx 1.0E-9$) or the objective function does not decrease after a sufficient number of iterations. The minimum tolerance zone is evaluated in this last iteration. Fig. 1. b shows a representation of the design search problem with one minimum at node 9.

When there are two points with equal minimum function value, they will have different search directions. For instance, let point 2 and 8 have the same minimum function value, using an L9 OA. Then point 2 has a search direction of X_{11} and X_{22} and point 8 has a search direction of X_{13} and X_{22}. If (X_{11} , X_{22}) is used as a starting point, the design level construction will result in nine new search points. These are $(X_{11}-\Delta, X_{22}-\Delta)$, $(X_{11}-\Delta, X_{22})$, $(X_{11}-\Delta, X_{22}+\Delta)$, $(X_{11}, X_{22}-\Delta)$,

(X_{11}, X_{22}), $(X_{11}, X_{22} + \Delta)$, $(X_{11} + \Delta, X_{22} - \Delta)$, $(X_{11} + \Delta, X_{22})$ and $(X_{11} + \Delta, X_{22} + \Delta)$ for points 1, 2,3, 4, 5, 6, 7, 8 and 9 respectively. If point (X_{13} , X_{22}) is used as a starting point, design level construction will result in nine new search points. These are $(X_{13} - \Delta, X_{22} - \Delta)$, $(X_{13} - \Delta, X_{22})$, $(X_{13} - \Delta, X_{22} + \Delta)$, $(X_{13}, X_{22} - \Delta)$, (X_{11}, X_{22}), $(X_{13}, X_{22} + \Delta)$, $(X_{13} + \Delta, X_{22} - \Delta)$, $(X_{13} + \Delta, X_{22})$ and $(X_{13} + \Delta, X_{22} + \Delta)$ for points 1, 2, 3, 4, 5, 6, 7, 8 and 9 respectively. Fig. 1.c shows a representation of the design search problem with two equal minima at nodes 5 and 8. Clearly, the two searches using point 2 or 8 are different and could lead to different minimum zone evaluation. A modified algorithm (number 2) is used to deal with cases when there are two or more design points with equal minimum. Each point is searched individually and the minima are compared. This is equivalent to the usual exhaustive search techniques in numerical optimization.

Trial #	X_1	X_2	$X_1 . X_2$	$X_1 . X_2$
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

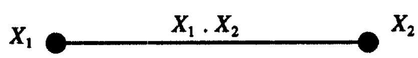

Fig. 1.a Layout Assignment Using L9 OA

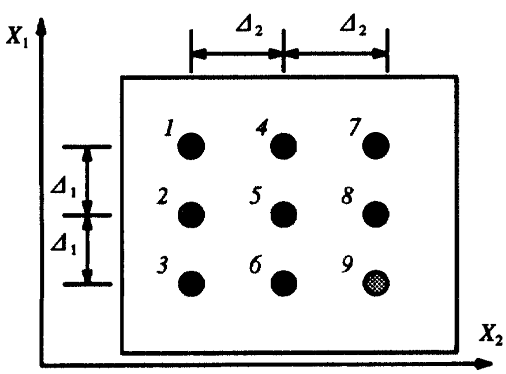

Fig. 1.b Representation Of The Design Search Problem With One Minimum At Node # 9

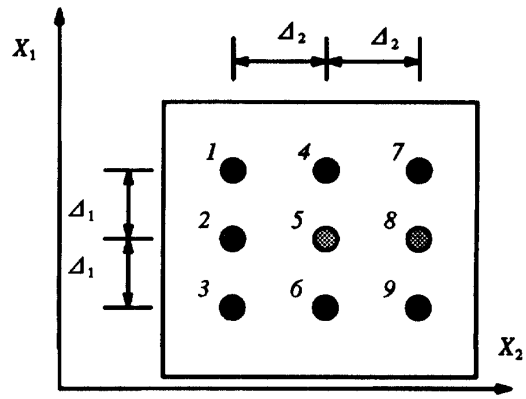

Fig. 1.c Representation Of The Design Search Problem With Two Equal Minima At Nodes # 5 and 8

3. Straightness

Fig. 2 shows the measurments of straightness errors from a reference line. The data representing the straightness measurments are given by (X_i, Y_i). The reference line is given by:

$$Y_i = a\ X_i + c \tag{3}$$

The error, e_i, is expressed as:

$$e_i = Y_i - (a\ X_i + c) \tag{4}$$

for a feature aligned with the X–axis.

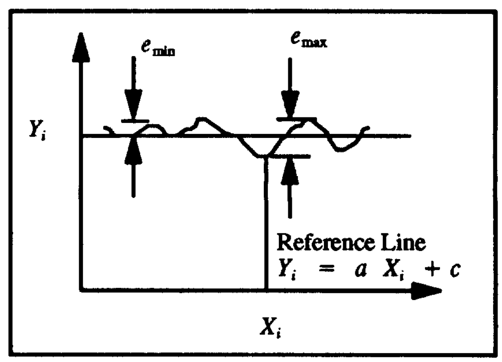

Fig. 2 Evaluation Of Straightness Errors

Now, given the measured surface points, it is required to estimate a and c such that $\sum e_i$ is minimum.

Results from the Orthogonal–based–algorithm are compared with others using the Least Squares and the Convex Hull algorithm. As can be seen from Table 1, the algorithm developed in this paper yielded results which differed only by 0.0011 % , 4.3522 % , 0.418459 % and 1.017809 % for examples 1, 3, 4 and 5 respectively from those obtained using the Convex Hull algorithm. In example 2, the minimum zone calculated using the orthogonal–based–algorithm is 30.0 % higher than those obtained from the Least Squares and Convex Hull approach. This is due to the fact that only 10 points are used for evaluation. The coordinate measurments data range from 2.428 to 4.303. In examples 3 and 4, results from the orthogonal–based–algorithm coincide with the CMM measured zones [9]. The efficiency of the Orthogonal–based–algorithm is measured in terms of CPU and the number of iterations to reach the minimum zone.

4. Flatness

Fig. 3 shows the measurments of flatness from a reference plane. The reference plane is given by:

$$\psi_i = z_i + aX_i + cY_i \qquad (5)$$

Where a and c are the slopes on the X and Y axis respectively and Z_o is the intercept on the Z–axis. The 3–dimensional measurments are given by (X_i, Y_i, Z_i).

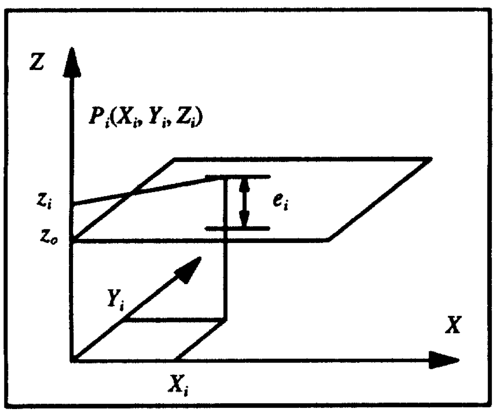

Fig. 3 Evaluation Of Flatness Errors

An L27 OA is used to plan experimentation for the minimum zone evaluation with a, c and Z_o assigned to columns 1,2 and 5 respectively. Points (–1.0, 0.0, –1.0) are used as a starting point for search.

Tables 2 includes the results for the calculated flatness tolerance zones using the Least Squares, the Convex Hull and Orthogonal–Based–algorithm. Five examples are used with data point measurments as given in reference [9]. In examples 1, 2 and 5, the tolerance zones calculated according to the Orthogonal–based– algorithm are smaller (4.72%, 37.72% and 9.25%) than those calculated using the Least Squares method. In examples 3 and 4, the differences between our results, and those calculated by the Least Squares and the Convex Hull method are 1.023%, 6.776 % and 11.780 %, 17.0155% respectively (for example $\frac{0.1875 - 0.1856}{0.1875} X100 = 1.023\%$). The CPU time (second) and the number of iterations for the five example problems are also given.

5. Circularity

The circularity measurments are given by (r_i, θ_i), where r_i is the radial deviation from the measurment reference circle at an angle θ_i. The reference circle is given by

$$\psi_i = r_o + X_o \cos\theta_i + Y_o \sin\theta_i \qquad (6)$$

Where r_o is the radius and (X_o, Y_o) is the centre. Fig. 4 shows the coordinate system for circularity error evaluation. An L27 OA is used to plan experimentation for r_o, X_o, Y_o calculation with X_o, Y_o and r_o assigned to columns 1, 2 and 5 respectively. Point (–1.0, 0.0, 1.0) is used as a starting point for search. It is realized that r_o cannot take a negative value. In this case, another point (1.0, 2.0, 3.0) is used to search for the optimum value of r_o. Table 3 lists the results for two example problems used to compare the orthogonal–based algorithm with the Least Squares and the constrained optimization methods. The orthogonal–based algorithm gave results very close to those calculated by the Least Squares method and the Simplex search techniques (15.22% and 26.21% for the first example, 0.3185 % and 4.467% for the second example respectively).

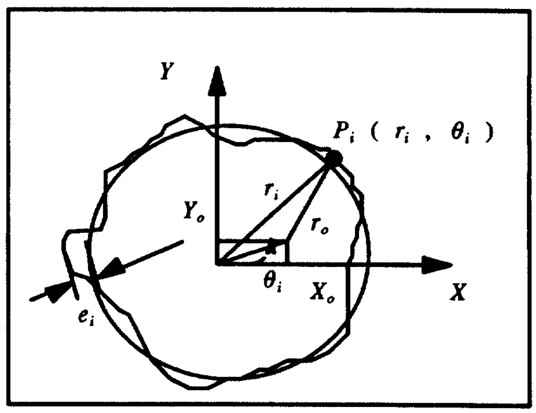

Fig. 4 Evaluation Of Circularity Errors

6. Algorithms

6.1 Algorithm 1

1. Assign the number of constants (a and c in case of straightness) to a suitable orthogonal array (preferably L9 OA).

2. Pick a starting point for the algorithm to start with. Here, (a, c) = (0.0 , 0.0) is a good choice.

3. Take a design level of 1.0. Therefore, a and c will have 9 design level combinations.

4. For each design level i (i = 1 , ... 9), determine the difference between the perfect feature form and the individual measured points.

5. Calculate $\sum e_i = \phi_i - f_i(y_i, x_i, a_i, c_i)$.

6. Pick the point (i = 1, ... 9) with the smallest sum of errors. This point is used as the basis for the next search point. It is possible to have two or more points with the same sum of errors. Each point refers to a different search direction and different zone evaluation. The smallest zone is the minimum zone. This is described in algorithm 2.

6.2 Algorithm 2

1. Follow steps 1. to 5. in Algorithm 1.

2. Pick point (i = 1) and start the search. This point will serve as one design level. The other two design levels are i + 1 and i–1 respectively. Pick the design point that gives the smallest combined maximum error. This point will be the next search point. The smallest zone at the end is the minimum zone. This procedure is repeated for the other 9 points. Select the minimum of the resulting 9 zones.

Orthogonal arrays are used to plan the minimization search using experimental optimization. An L9 OA (2 design parameters and 9 experiments) is used to evaluate a and c in the case of straightness. For instance, an L9 OA is used to evaluate the minimum straightness zone with a and c assigned to columns 1 and 2 respectively. An L27 OA is used to plan experimentation in case of flatness, circularity, cylindricity and sphericity with (a, c, z_o), (X_o, Y_o, r_o), (X_o, Y_o, a, c, r_o) and (X_o, Y_o, z_o, r_o) assigned to columns (1, 2, 5) , (1, 2, 5), (1, 2, 5, 8, 11) and (1, 2, 5, 8) respectively. Results related only to straightness, flatness and circularity are included here due to space limitations.

7. Conclusion

A new algorithm for form tolerance evaluation using orthogonal arrays and experimental optimization techniques has been developed. The new algorithm is applied to the problem of minimum tolerance zone evaluation. It uses the combinatorial nature of orthogonal arrays to detect the design level settings that minimize the deviations of a fitted surface based on a set of measurments from the ideal one. The algorithm is applied to two–dimensional features such as straightness and circularity tolerances as well as three–dimensional features such as flatness tolerances.

In most cases studied, the developed Orthogonal–based algorithm yielded results that are either smaller or very close to the minimum tolerance zones calculated using the Least Squares method. The new algorithm is validated by comparing its results with others using the Least Squares method, the Convex Hull algorithm, the constrained optimization techniques and the Simplex search techniques. As such, the orthogonal–based–algorithm can be used as an additional design tool for evaluation of the minimum tolerance zones.

References

[1]Dhanish , P.B. and Shunmugam , M.S. (1991) " An Algorithm For Form Error Evaluation – Using The Theory Of Discrete and Linear Chebychev Approximation ", Computer Methods In Applied Mechanics and Engineering , pp. 309 – 324.

[2]ElMaraghy , W.H. , ElMaraghy H.A. and Wu Z. (1990) " Determination Of Actual Geometric Deviations Using Coordinate Measuring Machine Data", Manufacturing Review Vol. 3 , no. 1 , pp. 32–39.

[3]Gadallah , M.H. and ElMaraghy H.A. (1994) "The Tolerance Optimization Problem Using A System Of Experimental Design ", submitted to the sixth International conference on Design Theory and Methodology.

[4]Hurt , J. J. (1980) " A Comparison Of Several Plane Fit Algorithms , Annals Of The CIRP 29 (1) , 381–384.

[5]Murthy T.S.R. and Abdin , S.Z. (1980) "Minimum Zone Evaluation Of Surfaces", International Journal Of Machine Tool Design & Research , Vol. 20 , pp. 123 – 136.

[6]Murthy T.S.R. (1982) " A Comparison Of Different Algorithms For Cylindricity Evaluation ", International Journal Of Machine Tool Design & Research , Vol. 22, no. 4 , pp. 283 – 292.

[7]Shunmugam , M.S. (1991) " Criteria For Computer Aided Form Evaluation ", Journal Of Engineering For Industry , Vol. 113, pp. 223 –238.

[8]Shunmugam , M.S. (1987) " New Approach For Evaluating Form Errors Of Engineering Surfaces ", Computer Aided Design , Vol. 19, no. 7 , pp. 368–374.

[9]Traband , T. M. , Josh S. , Wysk , R.A. and Cavalier, T.M. (1989) "Evaluation Of Straightness and Flatness Tolerances Using The Minimum Zone ", Manufacturing Review Vol. 2, no. 3 , pp. 189 – 195.

[10]Wang Y. (1992) "Minimum Zone Evaluation Of Form Tolerances ", Manufacturing Review Vol. 5 , no. 3 , pp. 213– 220.

Example Number	Least Squares Tolerance Zone [9 , 10]	Convex Hull Zone [9]	CMM Zone	Orthogonal Based Algorithm: a	c	Tolerance Zone	CPU (sec)	Number of iterations
1	2.4010	2.1213		–0.9999	2.000	2.1213	–	19
2	0.8877	0.8479		0.2951	2.3006	1.1587	20.7	508
3	5.377 E–2	5.186E–3	7.30E–3	2.1068 E–3	–4.6172 E–3	5.4117 E–3	5.60	138
4	1.463 E–3	1.311E–3	1.50E–3	0.1251 E–3	–0.3121 E–2	1.3055 E–3	1.70	31
5	0.1706	0.1646		0.0279	–0.0283	0.1662	3.0	65

Table 1 Evaluation Of Straightness Tolerance Zones

Example Number	Least Squares Tolerance Zone [9,10]	Convex Hull Tolerance Zone [9]	CMM Zone	Orthogonal Based Algorithm: a	b	z	Tolerance Zone	CPU (sec)	Number of iterations
1	2.800	2.000		–0.6666	0.6666	2.6666	2.6678	18.10	156
2	9.1797	6.2343		0.0105	0.0426	4.7372	5.7168	6.80	49
3	0.1856	0.1756		0.1120	0.0448	–0.0920	0.1875	2.70	17
4	4.381E–2	4.185 E–2	4.47E–2	–0.3629 E–2	–0.0156	–0.4882 E–1	4.8971 E–2	3.80	30
5	3.033E–3	2.817 E–3	6.60E–3	–0.1966 E–3	0.1272 E–3	0.8434 E–3	2.7524 E–3	10.0	73

Table 2 Evaluation Of Flatness Tolerance Zones

Least Squares [7, 8]
Tolerance Zone=2.457

Constrained Optimization [10]
Tolerance Zone=2.243

Orthogonal Based Algorithm

r_o	X_o	Y_o	Zone	CPU (sec)	Number of iterations
2.5855	0.5837	1.4162	2.8311	40.60	407

Least Squares [7, 8]
Tolerance Zone= 1.0008

Simplex Search Technique [5]
Tolerance Zone=0.9550

Orthogonal Based Algorithm

r_o	X_o	Y_o	Tolerance Zone	CPU (sec)	Number of iterations
16.0197	–2.2625	–0.5961	0.9976	133.4	818

Table 3 Evaluation Of Roundness (Circularity) Tolerance Zones