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MODELING for OPTIMIZATION (MO-OP): TOOLS for MANUFACTURING and DESIGN ENGINEERING PROBLEMS

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ABSTRACT

In this paper, a new statistical optimization technique is proposed. The technique employs new variance reduction schemes (VRTs). The performance of three standard designs: L27/ L27 OA, L54/ L27 OA and L243 / L27 OA are studied. These designs, although both orthogonal and balanced, exhibit high variance reduction properties with questionable convergence in very short number of iterations. Four new composite designs are developed, implemented and compared with the standard ones. These designs are known as: 5-, 7-, 9- and 11-point composite L27 OA. The problem of tolerance allocation with optimal process selection is revisited as a case study for simulation. Results indicate the efficiency of these new designs to reduce variances to lower levels than standard designs and better convergence in fraction of experiments. These designs are then integrated in an optimization algorithm previously developed (Gadallah, M.H., 2000). The algorithm is then modified to deal with the least sensitive optimal solutions for standard and composite designs. Particularly, the parameters that affect the algorithm are varied and their effects on performance of algorithm are studied. A standard manufacturing case study is used for analysis and simulation results for the composite designs are also given.

KEYWORDS: Simulation, Statistical Optimization, Variance Reduction Techniques, and Least Sensitivity.

BACKGROUND

Statistical optimization techniques are developed to deal with non-deterministic engineering and non-engineering models. Although most systems utilize statistical models for detailed analysis, the use of these models in optimization is still limited. In this paper, foundations for statistical optimization algorithm are given, particularly the tools used to build the search.

Houser and Ishii formulated a statistical optimization based on design of experiments (DOE) and sensitivity index. This index is a combined function of the original function and the values at the target points in space. The study also considered the correlation existing between some design and manufacturing variables.

Statistical optimization formulations were presented analytically using Taguchi Loss function (Rao, S. S. 1991). This formulation is derived for analytical objective and constraint equality and inequality functions. This formulation assumes the value of variances for any system and also the target value for the system performance. This information might not be available for any system. Besides, the method assumes that the analytical form of the objective function and/ or constraints is known. Very often, designers formulate objectives and/ or constraints from simulation experimental data. The same approach was followed by Webb and Parkinson, 1995. Their work, however, was limited to linear variance functions.

Ragsdell et al. attempted to formulate statistical optimization based on surface modeling and space decomposition techniques. Several matrix decomposition approaches are used and their results are compared.

Gould, S. (1989) emphasized that large-scale simulation models are very complex and require long runs. Accuracy levels become unachievable without further complication except through the use of VRT. The application of some VRTs to large scale models was investigated and a modification to the antithetic variates to suit larger models was given. Results indicated that antithetic variates are not really suited for large complex models.

Kleijnen (1996) reviewed 5 related analysis types namely sensitivity analysis, uncertainty (risk analysis), screening, validation and optimization. These analysis types are evaluated with respect to simulation models. The study points to the fact that there are no standard simulation definitions. Insua et al. presented a similar study on the use of sensitivity analysis in statistical decision theory.

Vignaux, G. (1999) mentioned that the value and efficiency of VRT depends very much on the characteristics of the model. Several sampling methods, especially Importance Sampling, are reviewed. These methods include Monte Carlo method, Common Random Numbers, Antithetic Methods, Control Variates, Stratified Sampling and Importance Sampling.

Mc Geoch gave a tutorial discussion on the use of Variance Reduction Techniques and simulation in algorithm studies. The study reviews other VRTs such as conditional expectations, simulation shortcuts, splitting and stratification. The author continued to stress the fact that it is hard to determine which test to apply to any specific algorithm. Others interested in the subject of variance reduction include Trich, M, 1995, Calvin and Nakayama, and others.

Kock et al. (1999) elaborated on the problem of size and dimensionality with respect to multi-disciplinary design optimization. Two techniques are named, these are the experimental design and kriging techniques. System decomposition and multi-level optimization are used to tackle the dimensionality problem.

The Response Surface Models (RSM) are exercised to explore the design space efficiently. The incorporation of RSM and robust design principles within the compromise decision support problem presented a new approach to multi-disciplinary analysis. The CDSP is a multi-objective mathematical construct that is a hybrid formulation based on mathematical and goal programming. The CDSP is used to determine the values of the design variables that satisfy a set of constraints and achieve as closely as possible a set of conflicting goals as is often the case in robust and complex system designs.

Lately, Rajadas and Jury used the Kreisselmeir - Steinhauser (K-S) function approach as it combines different disciplines of the optimization problem. The objective functions and the constraints are combined to form a single unconstrained composite function. The resulting function is solved via any unconstrained solver. In case more than one objective is used, appropriate weights are employed to stress the relative importance of individual objectives.

Webb and Parkinson discussed the properties and optimization of the linear variance function. The interest is to devise a means how variations can be transmitted to variables and functions. Another objective, these functions represent engineering systems. The minimization of these functions would result in robust systems to variations. In this study, we are interested in the variance properties of certain arrays prior to use as search schemes.

Nigam and Turner reviewed various statistical approaches to tolerance analysis problem. The study considered the extension to solid modeling systems and to geometric tolerancing standards. Parkinson presented a similar method to deal with uncertainty of design parameters.

We can conclude that statistical optimization techniques are very limited in applications for several reasons:

a. The engineering community does not own sound, consistent and rigorous modeling tools.
b. The engineering community, though believes in optimization theories and algorithms as valid and useful tools, the integration with the engineering science is still far away from being accepted.

This paper offers developments in several important areas:
First, in the area of modeling, it models two search domains using several standard and non-standard arrays. The arrays developed so far (often called) standard limit the capacity of optimization algorithms to deal with large realistic problems. In this study, we offer means to construct larger size arrays.
Second, in the area of simulation, it simulates the problem of tolerance allocation with process selection, particularly the statistical part of the problem.
Third, in the area of statistical optimization, it optimizes the problem of tolerance allocation with a study on the variances of several given arrays.

THE LEAST SENSITIVE FORMULATION

The optimization model given in Gadallah, M.H., 2000 is modified to adapt the least sensitive optimization problem. The cost models for each variable are differentiated with respect to each variable. The process models are varied in combinatorial ways using the proposed arrays. The first derivatives resulted in negative function, we decided to take the absolute addition of process functions for minimization. Now, the analyst will have the chance to either obtain the optimum solution, the least sensitive solution or both solutions.

PROBLEM STATEMENT

This paper offers a study experience on modeling of search domains using standard orthogonal arrays. Three standard orthogonal arrays are used to model the tolerance/ process cost domains. These are L27/ L27 OA, L243/ L27 OA and L54/ L27 OA respectively. As the standard arrays become unpractical for large optimization problems, we tend to approximate the search space using composite arrays. In this study, L27 OA (27 experiments, 3 levels and 5 variables) is used to investigate the approximation using 5-point, 7-point, 9-point and 11-point composite arrays. Composite array is an array approximated to model more levels for decision variables than the existing standard arrays. For instance, the standard L27OA has factors at –1, 0, and +1 coded levels. The 5-point array will have factors at –1,-0.5, 0, +0.5, and +1 coded levels. The 7-point array will have factors at –1, -0.66, -0.33, 0, +0.33, +0.66 and +1.0 coded levels. The 9-point array will have factors at –1, -0.75, -0.5, -0.25, 0, +0.25, +0.5, +0.75 and +1 coded levels. Finally, the 11-point array will have factors at –1, -0.8, -0.6, -0.4, -0.2, 0, +0.2, +0.4, +0.6, +0.8, +1 coded levels. Full corresponding arrays would need 5^5, 7^5, 9^5 and 11^5 trials for the 5, 7, 9 and 11 point composite arrays respectively. Clearly, the computational effort behind these large numbers of experiments is huge and the use of composite L27OA with point approximation represents a tremendous computational approximation for optimization modeling. Table 1 shows the standard L27OA, 4 composite L27 OA arrays and corresponding scaling factors.

The use of L27OA to host 5-, 7-, 9- and 11-point approximation can be extended simply to any existing orthogonal arrays. We preferred to experiment with the least expensive and time consuming 3-level array. Once the engineering community is convinced with the validity and usefulness of the technique, other more detailed larger size arrays can be constructed. The idea of composite arrays can be extended simply to higher number of levels. Several illustrations are given next.

The Standard L27 OA

The standard L27 OA array is made up of 27 experiments, 7 variables in 3 levels. These variables can be assigned to columns 1, 2, 5, 9, 10, 12 and 13 respectively. The 3 level array searches the space between coded levels (-1.0, 0.0, +1.0). For a 5-variable problem, the L27OA is equivalent to 1/9 Full Factorial Experiments (1/9 FFE = 27/ 243). In other words, 1/9 of the search space is checked at every iteration.

5-Point and Higher Composite L27 OA

The 5-point array attempts to vary the same number of variables using 5 points with different weights. The 5-point coded levels are (-1.0,-0.5,0.0,+0.5,+1.0). The 5-point approximation is equivalent to an array with 5^5 = 3125 experiments. Since we use 27 experiments, this is equivalent to 27/3125 = 1/116. In other words, 1/116 of the search space is searched at every iteration. Similarly, the use of 7-,9- and 11-point is equivalent to 7^5, 9^5 and 11^5 experiments respectively. Similarly, 27/16,807, 27/59,049 and 27/161,051 of the search space is checked respectively. The full composite arrays are given in appendix 1.

DISCUSSION

Table 2 gives the cost tolerance function coefficients for the example used for simulation (Zhang and Wang, 1993). The same work can be extended to non-linear cost functions and constraints. Figure 1 shows the system variance level for 7 arrays (3 standard arrays, these are L54/L27 OA, L27/L27 OA and L243/L27 OA) and (4 composite arrays, these are 5-point, 7-point, 9-point and 11-point composite L27OA as mentioned earlier). The expected process cost is determined for the 7 arrays as shown in Figure 2. Based on simulation results, the following conclusions can be stated:

1. The least expected cost is $21.167, achieved using L243/L27 OA followed by 9-point and 7-point composite L27OA respectively.
2. The highest expected cost is $23.64, achieved by the standard L27OA followed by L54OA.
3. The 4-composite L27OA arrays returned the least cost compared with the standard L27OA but in larger number of iterations. This is in line with high order approximations.
4. The least variance is returned by the 7-point and 9-point composite L27OA; although the number of experiments for 5-point L27 OA is 50% that of L54OA.
5. The variance of the 5-point L27OA is comparable to that of L54OA and standard L27OA respectively.
6. The variance of the 11-point composite array is comparable with the L243OA.

Figure 3 shows the cost tolerance functions for dimension 1 using process 1, 2 and 3 respectively (here denoted by P1, P2 and P3). The sensitivity function is also plotted versus the tolerance values for different processes. Figure 4 gives the sensitivity function versus iteration number for 5 arrays (here, the L27OA standard array is included with the 4 composite arrays for comparison purposes). The figure shows 2 regions: the high sensitivity region and low sensitivity region. In the first region, any change in tolerance values causes the sensitivity function to fluctuate. As the number of iterations increase, the function converges towards zero. The least sensitive function is also accompanied by lower variances. Out of the 4 composite arrays, the 11 point composite L27OA has the lowest starting sensitivity value and converged to a solution in 6 iterations (6 x 27 experiments). The lowest sensitivity value is achieved by the standard L27OA in 7 iterations (7 x 27 experiments). Table 3 gives the least sensitivity solutions for different arrays. It should be noted that this is not the optimum least cost solution but rather the optimum least sensitive solutions. The least sensitive cost is $22.756 using 7-point L27OA, achieved in 3 iterations compared to $22.831 using the standard L27OA and 8 iterations.

The idea of standard and composite arrays is utilized to solve the least sensitive problem. The least sensitive solution ranges from $22.756 - $23.026. The minimum sensitivity solution ranges from 1438 - 1591 using the 5-point L27 OA. The maximum sensitivity solution ranged from 11,698 for 11-point L27OA to 24,996 using the 7-point L27 OA. The number of iterations is generally low.

The least sensitive solution presented here is new for several reasons: a) the objective function is discrete (in a certain domain) due to the existence of several cost functions per dimension; b) the objective function is a multi-objective for the resulting mechanical assembly. The number of combinations to evaluate by other algorithms such as Branch& Bound, Exhaustive Search, Univariate Search, Sequential Programming and lately Simulated Annealing and Genetic Algorithms is simply huge. That's why the least sensitivity approach for this multi-objective combinatorial problem is considered a new advancement to previous formulations. The effect of algorithm parameters is investigated next.

EFFECT OF ALGORITHM PARAMETERS

1. Effect of Space Reduction Factor

Figure 5 shows the effect of the space reduction factor on the system variance level. This factor usually varies from 0.0-1.0. We studied that factor in the range 0.90-0.995. The performance of the algorithm shows that the maximum variance occurs near R > 0.9, although it converges in a non-smooth way. The least variance is achieved at R=0.95.

2. Effect of Starting Search Points

Figure 6 gives the system variance level versus the system level iterations for 4 starting search points and R=0.98. When a point (.0005,.0045, 0.0085) is used, no feasible solution was obtained with the maximum variance during the initial run.

3. Effect of Search Domains

When the search is forced to operate in a certain region, different mean to variance ratios are obtained. This study is carried versus (-1.0,0.0,+1.0) coded level search as a reference for comparison. Three different coded search are used, these are (-0.25,+0.25), (-0.35,+0.35) and (0.45,+0.45). The three designs exhibited a constant mean to variance ratio later during the search. At iteration 10, the mean to variance ratio is higher for (-0.25,+0.25) than for (-0.45,+0.45), although convergence to a better variance value was not possible. This comparison was done using a Delta = 0.00075, R=0.98 and the variables assigned to column number 1-2-5-9-10 respectively. Figure 7 shows the mean to variance ratio for non-equi-spaced search domain.

4. The Hatch-Spaced Search

When a search is done with the coded level (-1.0,0.0,+1.0), this is called a complete equi-spaced search (CESS). We tried to investigate the possibility of trimming the space with the attempt to guide the search in certain regions. The resulting space is called Hatch-Space Search (HSS). When a coded level (-1.0,0.0,+1.0) is used, the program takes 11 iterations to converge to higher mean to variance ratio levels than (-0.5,0,+1.0), (-0.25,0,+1.0) and (-0.75,0,+1.0) hatch spaces respectively. The highest mean to variance ratio was achieved for (-0.25,0.0,+1.0).

When the hatching takes place from the positive side,.(-1.0, 0.0, +0.5), we call this positive hatch space. Similarly, when hatching takes place from the negative side, (-0.5, 0.0, +1.0), we call this negative hatch space. This same idea was explored for three hatch spaces, namely (-1.0, 0.0, +0.5),(-1.0, 0.0, +0.25) and (-1.0, 0.0, +0.75) respectively. The system mean to variance ratio exhibited a flat pattern as number of iterations increase. The highest variance occurred at (-1.0,0.0,+0.75). When (-1.0, 0.0, +0.25) was used, the system did not converge. Figures 8 and 9 show the system mean to variance ratio versus system iteration for positive and negative hatch space. In both cases, the {-1.0,0.0,+1.0} was used as a reference.

5. Effect of Delta

The system mean to variance ratios are studied for 6 Delta values in the range 0.0004 - 0.0010. It is found that as Delta reaches 0.001, the system could not converge. This is true for Delta = 0.0009 and Delta = 0.001. The lowest variance was achieved for Delta = 0.0006. When this value is reduced to 0.0005 or to 0.0004, the system could not converge and the lowest variance value was taken as intermediate solution. These conclusions were based on a study using 9-point composite L27 OA. Figure 10 gives the system mean to variance ratios versus system level iteration for different Delta values.

CONCLUSION

In this paper, a new statistical optimization algorithm given in a previous paper (Gadallah, M.H. 2001) is herein extended and studied in details. As it stands, although positive promising results are obtained, the algorithm needs further validation and experimentation versus more complex problems. Few conclusions can be stated:

1. The composite arrays, though highly fractional with respect to the number of design levels are more efficient than standard designs. This is true for the 3 standard designs studied.
2. The composite arrays, given in this study show consistent high and low sensitivity regions. Most studies attempted to analytically derive and minimize the first derivative of the objective functions.
3. The use of the tolerance allocation with process selection as an example for simulation does not limit the algorithm as a general statistical based optimization technique.
4. The extension of the algorithm to more search domains is thought to affect the variance limits, regardless of the design used. This claim needs further experimentation.

5. It is true that the algorithm developed is highly data intensive; however, the computational and analysis savings are tremendous.
6. The least sensitive formulation has not been dealt with in the local/ global optimization context. In this paper, various solutions are given for different structures.

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Appendix 1

Sample 5-Point Approximation Using L27 OA

	X1	X2	X3	X4	X5
1	-1	-1	-1	-1	+1
2			-0.5	0	-1
3			0	+1	-1
4		0.0	+0.5	+0.5	+0.5
5			+1.0	-0.5	0
6	-0.5		-1	0	-0.5
7		+1	-0.5	0	-0.5
8			0	+0.5	0
9			+0.5	-0.5	+0.5
10		-0.5	+1.0	0	-1
11	0.0		-1	+0.5	0
12			-0.5	-0.5	+1
13		0	0	-0.5	0
14			+0.5	0	+1
15			+1.0	+0.5	-1
16		+0.5	-1	+0.5	0
17			-0.5	-0.5	-0.5
18	+0.50		0	0	+0.5
19		-1.0	+0.5	+1.0	+0.5
20			+1.0	-1.0	-0.5
21		-0.5	-1	0	-1
22			-0.5	+1.0	0
23	+1.0	0.0	0	+1.0	-1
24		+0.5	+0.5	-1.0	+0.5
25			+1.0	-1.0	0
26		+1.0	0.0	-1.0	-1
27			0.0	+1.0	+1

Sample 7-Point Approximation Using L27 OA

	X1	X2	X3	X4	X5
1	-1	-1	-1	-1	-1
2		-0.66	0	0	0
3		-0.33	+1	+1	+1
4		0.0	-0.66	+0.66	+0.66
5	-0.66	+0.33	0	-0.66	-0.66
6		+0.66	+0.66	0	0
7		+1	-0.33	0	0
8		0	0	+0.33	+0.33
9	-0.330	0	+0.33	-0.33	-0.33
10		-1	-1	0	+1
11		-0.66	0	+1	-1
12		-0.33	+1	-1	0
13	0.0	0	-0.66	-0.66	0
14		+0.33	0	0	+0.66
15		+0.66	+0.66	+0.66	-0.66
16	+0.33	+1	-0.33	+0.33	-0.33
17		0	0	-0.33	0
18		0	+0.33	0	+0.33
19		-1	-1	+1	0
20	+0.66	-0.66	0	-1	+1
21		-0.33	+1	0	-1
22		0	-0.66	0	-0.66
23		+0.33	0	+0.66	0
24	+1.0	+0.66	+0.66	-0.66	+0.66
25		+1	-0.33	-0.33	+0.33
26		0	0	0	-0.33
27		0	+0.33	+0.33	0

Sample 9-Point Approximation Using L27 OA

	X1	X2	X3	X4	X5
1	-1	-1	-1	-1	-1
2			-0.75	+0.75	0
3			-0.5	+1	+1
4	+0.75	+0.75	-0.25	+0.75	+0.75
5			0	-0.75	-0.75
6			+0.25	+0.75	-0.25
7	+0.50	+0.50	+0.5	0	+0.25
8			+0.75	+0.5	+0.5
9			+1	-0.5	-0.5
10	-0.25	-0.25	-1	-0.75	+0.25
11			-0.75	+1	-0.25
12			-0.50	-1	0
13	0.0	0.0	-0.25	-0.5	-1
14			0	-0.25	+0.25
15			+0.25	+0.5	-0.25
16	+0.25	-0.25	+0.50	+0.25	-0.50
17			+0.75	-0.25	+1
18			+1	0	+0.5
19	+0.50	+0.50	-1	+0.5	+1
20			-0.75	-0.5	+0.75
21			-0.50	+0.25	-0.75
22	+0.75	+0.75	-0.25	0	-0.5
23			0	+0.25	0
24			+0.25	-0.25	+0.5
25	+1.0	+1	+0.50	-1	+0.75
26			+0.75	-0.75	-0.75
27			+1	+1	-1

Sample 11-Point Approximation Using L27 OA

	X1	X2	X3	X4	X5
1	-1	-1	-0.8	-1	-1
2	-0.8	-0.8	-0.6	+0.8	0
3		-0.6	-0.4	+0.6	+1
4		+0.6	-0.4	+0.8	+0.8
5	-0.6	+0.4	-0.2	-0.6	-0.8
6		+0.2	+0.4	+0.8	-0.2
7		+0.6	+0.4	0	+0.2
8	-0.4	+0.4	+0.6	+0.4	+0.4
9		+0.2	+0.8	-0.4	-0.4
10		-0.2	-0.4	-0.8	+0.2
11	-0.2	-0.4	-0.6	+0.4	-0.2
12		-0.6	-0.8	-0.4	-0.4
13		0	-0.2	-0.6	-0.8
14	0.0	-0.6	0	-0.2	+0.2
15	+0.2	-0.8	+0.2	+0.6	-0.2
16		+0.2	+0.6	+0.2	-0.6
17		+0.4	+0.8	-0.2	+0.6
18	+0.4	+0.6	+1	-0.8	+0.4
19		-0.2	-1	+0.4	+0.4
20		-0.4	-0.8	-0.4	+0.8
21	+0.6	-0.6	-0.6	+0.2	-0.8
22		-0.2	-0.2	-0.8	-0.6
23		-0.4	+0.2	+0.2	+0.6
24	+0.8	+0.8	+0.2	-0.2	+0.6
25		+0.8	+0.4	-0.6	+0.8
26		+0.8	+0.6	-0.6	-0.6
27	+1	+1	+0.8	+1	-0.4

	0	+0.2	+0.25	+0.33	+0.40	+0.50	+0.60	+0.66	+0.75	+0.80	+1.0
Standard L27OA	v										v
Composite 5-point L27OA	v					v					v
Composite 7-point L27OA	v			v				v			v
Composite 9-point L27OA	v		v			v			v		v
Composite 11-point L27OA	v	v			v		v			v	v

	- 1	-0.8	-0.75	-0.66	-0.6	-0.5	-0.40	-0.33	-0.25	-0.2
Standard L27OA	v									
Composite 5-point L27OA	v					v				
Composite 7-point L27OA	v			v				v		
Composite 9-point L27OA	v		v			v			v	
Composite 11-point L27OA	v	v			v		v			v

Table 1: Standard L27OA and Five Composite L27OA and Corresponding Scaling Factors

Variable	Process 1		Process 2		Process 3	
	A	B	A	B	A	B
1	3.0	0.012	2.0	0.016	-1.20	0.029
2	-0.33	9.3E-3	-8.0	0.042	-2.0	0.012
3	3.0	0.003	2.0	0.008	-	-
4	4.0	0.008	3.0	0.012	-	-
5	6.0	0.004	5.0	0.010	-4.70	0.047

Table 2: Cost – Tolerance Function Coefficients (ZHANG, C. and WANG, H.P.).

Index	**O-T**	L-S-O-F	**C-V**	**O- P- C**	**Comment**	**Maximum Solution**	**Minimum Solution**	**Number of Iterations**
1	0.0055, 0.0061, 0.0049, 0.0049, 0.0033	$ 22.831	0.0139 0.0249	{1, 1, 1, 1, 1}	Standard L27 OA	14, 828	1438	7
2	0.0058,0.0057, 0.0025, 0.0047, 0.0053	$ 23.026	0.0137 0.0242	{1, 1, 1, 1, 1}	5-Point L27 OA	14,184	1591	8
3	0.0065, 0.0053, 0.0043, 0.0055, 0.0030	$ 22.756	0.0139 0.0248	{1, 1, 1, 1, 1}	7-Point L27 OA	24, 996	1466	3
4	0.0057, 0.0051, 0.0045, 0.0049, 0.0036	$ 23.007	0.0139 0.0241	{1, 1, 1, 1, 1}	9-Point L27 OA	12, 038	1477	8
5	0.0057, 0.0067, 0.0037, 0.0044, 0.0035	$ 22.934	0.0130 0.0242	{1, 1, 1, 1, 1}	11-Point L27 OA	11, 698	1504	6

N-o-I = Number of Iterations, C-V = Constraint Value, L-S-O-F = Least Sensitive Objective Function, O-T = Optimum Tolerance, O-P-C = Optimum Process Combination

Table 3: Sensitivity Solutions for Example Problem

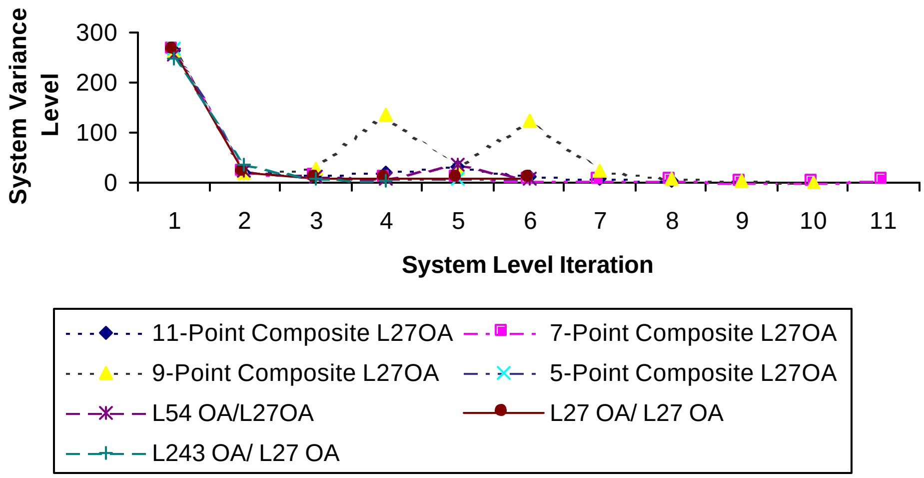

Figure 1: System Variance Level for Different Standard & Composite Designs

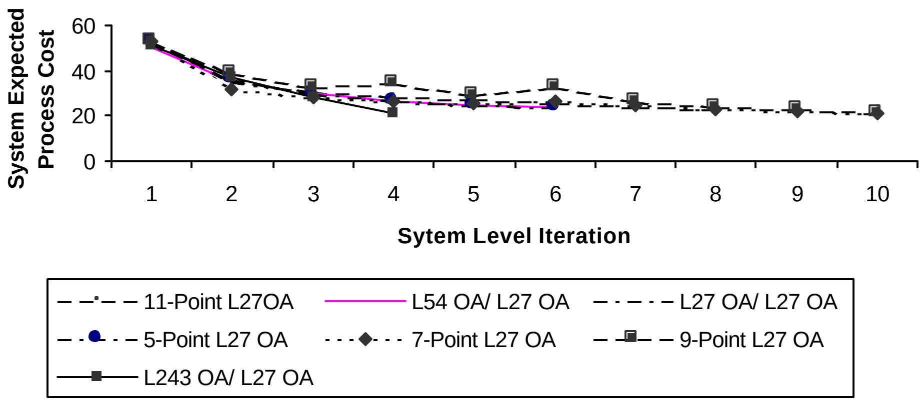

Figure 2: System Expected Process Cost for Different Standard & Composite Designs

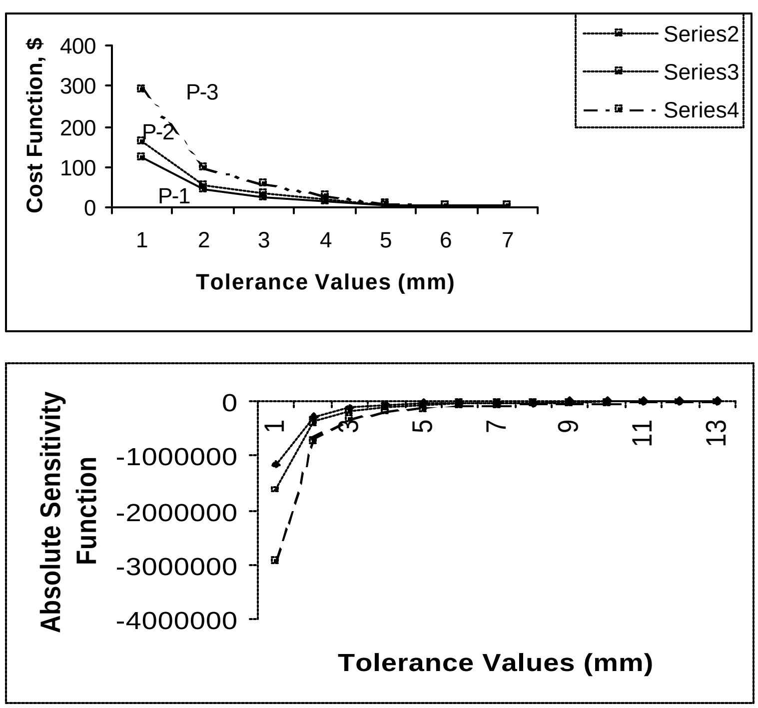

Figure 3: Cost Tolerance Functions and Corresponding Sensitivity Function

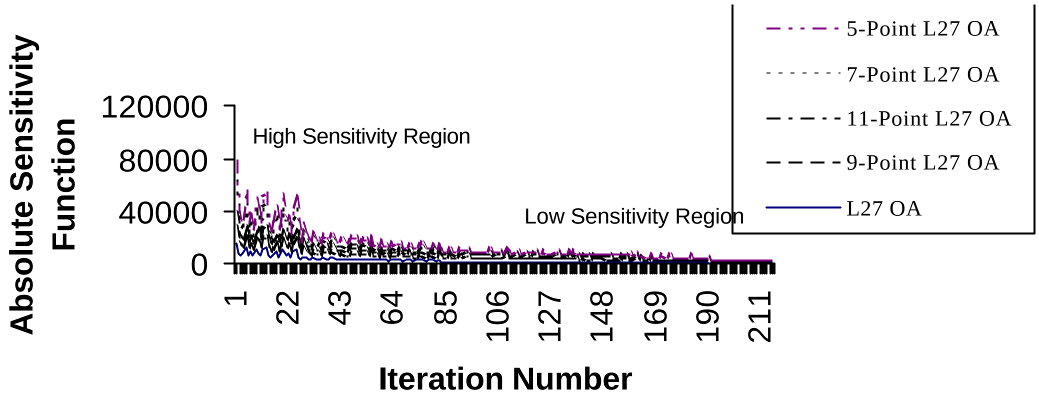

Figure 4: The sensitivity Diagram for Standard & Composite Designs

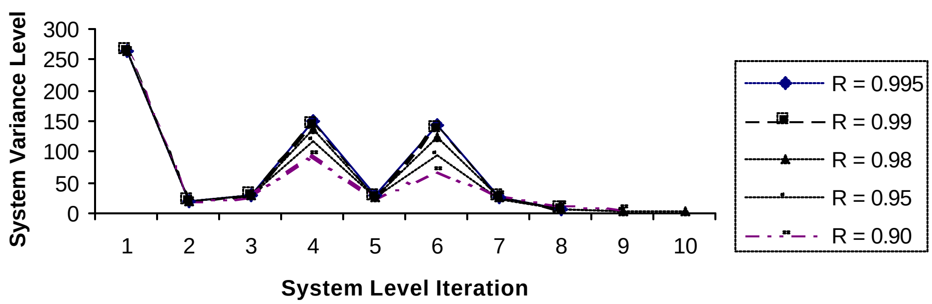

Figure 5: Variance Level versus Reduction Factors

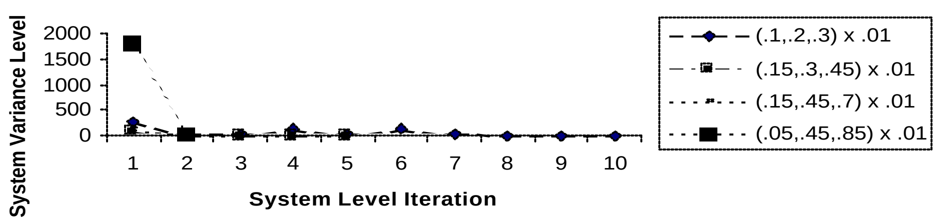

Figure 6: Variance Level for Different Starting Search Points

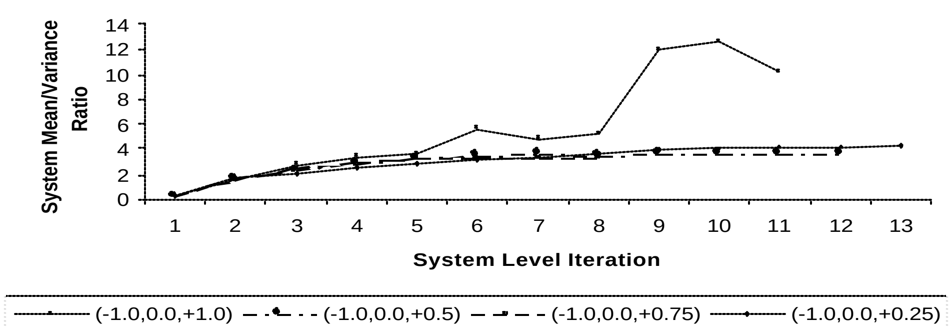

Figure 7: Mean/ Variance ratio for Equi-Spaced Space

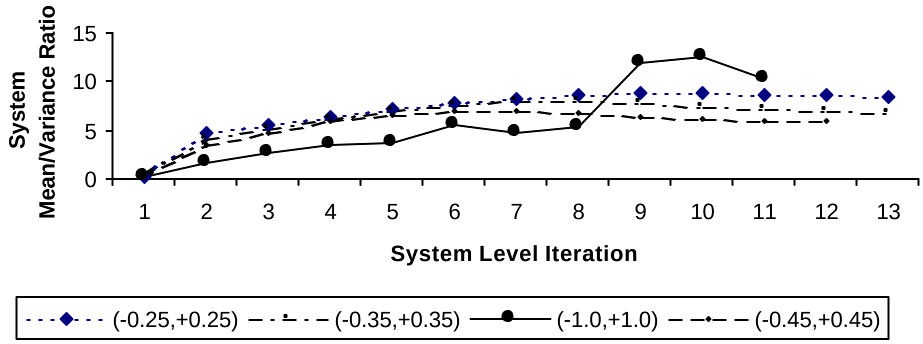

Figure 8: Mean/ Variance ratio for Non-Equi-Spaced Negative Hatch Space

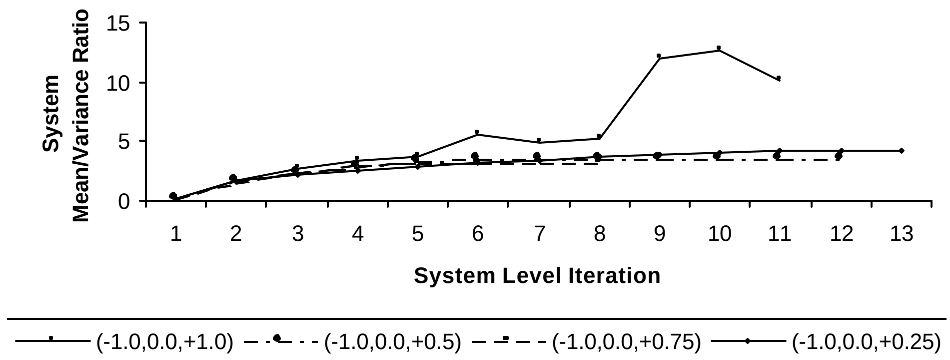

Figure 9: Mean/ Variance ratio for Non-Equi-Spaced Positive Hatch Space

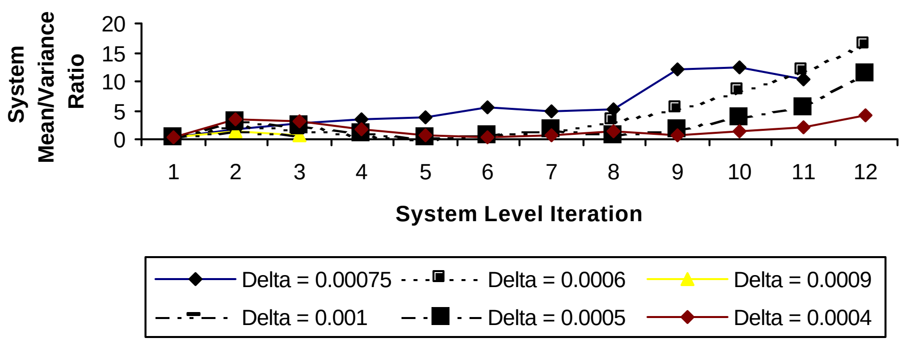

Figure 10: Mean/ Variance ratio for Different Delta Values