ARTIFICIAL NEURAL NETWORK MODELS USING STATISTICAL BASED TECHNIQUES

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ABSTRACT

The initial and crucial step in developing an accurate end milling model using artificial neural network (ANN) requires an enormous number of experiments over several ranges of feasible input parameters. However, approximately, similar modeling results can be achieved with lower number of experiments and consequently lower experimental cost, when the selection of experiments is properly planned. This paper provides a powerful methodology for selecting efficient experiments using orthogonal arrays (OAs) and design of experiments (DOE). Process variables include depth of cut (a), spindle speed (n), feed rate (f), and tool diameter (d). The interest is to measure the resulting dynamic cutting forces in the time domain. An ANOVA study for an initial experimental model based on 3-levels OA is presented and discussed. Consequently, several experimental models are selected including 2-levels, 3-levels, 4-levels, and 5-levels OAs aiming to provide larger experimental model that host more variations for significant parameters. A number of preliminary experiments is conducted to validate the used experimental setup. Previous work in the field of metal cutting using analytical and numerical modeling as well as

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artificial intelligence techniques are reviewed and discussed. Results indicate the validity of neural based models for end milling process modeling.

KEYWORDS: Artificial Neural Networks, DOE, Orthogonal Arrays and Modeling.

1. INTRODUCTION

Developing a precise model that can predict the performance of metal cutting system and consequently, lessen the requirements of carrying out experiments is one of the most important topics that received a tremendous attention from both academic researchers and industry practitioners. Satisfactorily achieving that task is very difficult because the features of metal cutting system are complex and contain many interacting factors. In particular, modeling the process of milling is more complicated because the dynamics are more difficult and the resulting cutting forces are periodic [19]. Therefore, recently, there has been a strong need for utilizing powerful modeling tools. One of these modeling tools is artificial neural network (ANN). It can accurately model complex systems like milling system after using some experiments for training phase and other experiments for validation purposes. Nevertheless, neural modeling is an expensive modeling technique that needs several hundred milling experiments over several ranges of feasible input parameters. The purpose of this article is to provide a powerful methodology using the orthogonal arrays and design of experiments for selecting efficient end milling experiments that can be used for constructing a neural model with low experimental cost for end milling process.

2. BACKGROUND

2.1 Milling Process

In the milling operation, tools with multiple cutting edges are rotated and fed towards the machined part to form the desired shape by removing the unwanted material. Milling is a very flexible process capable of producing simple two-dimensional flat shapes to complex three-dimensional surface. Milling has several geometries which can be categorized into two main groups: face milling (or end milling) and peripheral milling (or slab milling).

Milling can be viewed as an intricate system with several input and output parameters. Such system can be divided into four elements. They are: input parameters, a milling machine, internal parameters, and output parameters. The system has several input machining parameters to be considered and planned before machining to get desirable output parameters. Tool material and geometry, depth of cut, spindle speed, feed rate, and cooling fluids are some significant examples of these decision variables. The optimum value of one input variable for one cutting situation can be undesirable in other situations. For instance, cooling fluids, in many milling applications, have vital benefits such as reducing the cutting temperature and lubricating the tool and the work-piece. In other applications, however, using cooling fluids will not be necessary, and in some cases will not be recommended to avoid the thermal shock that can result from sudden cooling. Besides, searching for the most suitable values of input parameters has to consider the values of other parameters. For example choosing a cutting tool for a particular milling operation will depend on many other factors such as the feature to be machined and the work-piece material to be cut. Moreover, input parameters can rely on the type and condition of the machine that will be used. The type of machine will place a limit on the input parameters and their assigned values. However, knowing the actual performance of the machine condition usually requires deep experience along some estimation of the planners for determining the most suitable input process values.

Cutting forces, machine vibrations and quality of parts are important issues that should be studied. These internal parameters have strong relation with the milling system's output parameters such as quality of parts. As the stability of internal parameters is obtained, the throughput parameters can be improved. Moreover, instability of these internal parameters can badly affect the machine condition, and lead to additional loss. Therefore, measuring these parameters helps in process monitoring and enhances quality of machined parts.

Contact between cutting tool and machining parts generates significant and irregular forces during the cutting pass. Industry practitioners and researchers tend to control these forces to have constant average force as possible. Excessive forces can lead to some unwanted machining performance such as tool failure, tool deflections, geometric work-piece errors, poor surface finish, and machine structure deflections. Cutting forces have strong influence on tool breakage, tool wear, and work-piece deflection.

2.2 Design of Experiments

An essential objective of a designer of a product or a process is to get a reasonable conclusion about the effect of design parameters on response variables under different conditions. DOE allows for a systematic approach to quantify the effects of these parameters using a technique called the analysis of variance (ANOVA).

2.3 Artificial Neural Network

ANN is an information handling means that is inspired by the way biological nervous systems deal with information. ANN is basically composed of processing elements connected in parallel called neurons [7, 21]. Every connection contains an adjustable parameter called weight. The output of ANN comes from combination of each single neuron's output by these connections. ANN has several types and applications such as self-organizing (unsupervised) ANN that can be used for classification problems, and the supervised ANN that can be used for nonlinear multivariate function mapping. Since this work uses the supervised ANN for process modeling, a description will be presented. The supervised ANN can be trained to acquire knowledge by presenting some different input values of sophisticated nonlinear function. For training pattern, ANN adjusts its weight parameters based on the magnitude of the error between the true output and the ANN output. The role of the ANN algorithms is to minimize the error function with every new training pattern until the error is gradually reduced to become acceptably small. At this time, the ANN can be used to predict new output values for any input values as shown in Figure 1.

3. LITERATURE REVIEW

The discussion of several modeling processes of metal cutting is presented in two categories: analytical and numerical modeling and modeling using artificial intelligence.

3.1 Analytical and Numerical Modeling

By definition, analytical modeling is a set of equations based on physical significance describing the performance of the metal cutting system. These equations are usually easy to implement and can give excellent understanding of metal cutting physics. However, such models have some assumptions. Cutting process is assumed to be orthogonal and the material is removed by a cutting edge that is perpendicular to the direction of relative tool-part motion. Using such simplification, the cutting force is uniform along the cutting edge, the resulting chip is uniform flat, and the resulting stress and shear distributions are uniform. Furthermore, dynamic features such as machine vibrations, the spindle run-out and the thermal effect are neglected.

Since the developed equations contain a number of coefficients, it is not applicable to use the analytical model alone. It is more sensible to carry out few experiments used to identify these coefficients. Then, these coefficients can be used to model the processes within the range of conducted experiments. In this case, the model will be called mechanistic model. For example, a handbook containing several cutting coefficients for many alloys was presented by Cincinnati [5]. This approach was used to predict the cutting force, torque and power in a simple and fast way for a set of process variables such as depth of cut, spindle speed, tool geometry, feed rate and work-piece material. However, the mechanistic model has some limitations. First, the model can't predict out of range responses which leads to more needed experiments. Second, it is time consuming to search for appropriate tables to choose suitable coefficients. Third, the ignorance of many dynamic features of the cutting process and the simplification applied to the cutting configuration and the developed models are still remote from being considerably complete. Accordingly, there has been a great effort to lessen the disadvantages of this approach and to enhance the cutting analysis.

Among those efforts, a simulation based model for predicting cutting forces of end-milling was presented by Milfelner and Balic in [13]. Abrari and Elbastawi [1] presented a set of force functions that implement an analytical integration of cutting forces along the cutter edge rather than using numerical integration. The developed functions are based on the projection ship of the load area in any tool pass onto the reference coordinate planes. These equations were applied separately to some milling operations of flat and ball end mills and the obtained results were compared to the experimental data. Results showed good agreement between model and experimentation.

A dynamic model of turning process was presented by Acosta, Switek, and Garcia [3]. The model was implemented by developing software to output the machining parameters. Some experiments were performed and showed that the model is capable of illustrating many machining parameters within certain ranges and with some deviations due to ignorance of some dynamic features such as vibration.

Li et al [12] presented a theoretical model for face milling based on predictive machining theory and the mechanics of milling. A windows based simulation system was presented with friendly user-interface. The model outputs milling force variation against cutter rotation in either numerical or graphical form. Oxley's predictive machining theory [16] is used as a foundation in which the cutting forces are calculated from input data of work-piece properties, tool geometry, cutting conditions, and the type of milling. A number of experiments was performed to validate and test the model. Results confirmed good prediction accuracy with error range of 1% ~ 12%.

Orthogonal cutting considering the dynamics of cutting forces was studied by Abrari et al.[2]. This method considers the tool as very thin slices and the cutting force as the summation of cutting forces applied to each slice. The model considers the effects of surface undulations, instantaneous deflection and the interface of flank face with the finished surface. The paper also considers the semi-finishing operation of die cavities.

Another way for reducing the limitation of analytical models is by simulating the process using the finite element methods (FEM). The experiments conducted on a horizontal high speed milling center were used to compare the resulting values with the predicted ones. The comparison demonstrated the effectiveness of FEM simulations in predicting process variables in simple flat end milling.

3.2 Modeling Using Artificial Intelligence

Artificial Intelligence Techniques (AIT) have been used to efficiently predict many dynamic features both on and offline. Surface roughness and tool flank wear have been predicted using back-propagation neural network models (Ozel, and Karpat [15]). A regression model was developed and compared with the neural network model. Superior results were noticed by the model. A similar approach was taken by Chien and Tsai [4]. Optimization model was then formulated to maximize metal rate.

A new approach using neural network for modeling flat end milling operation was presented by El-Mounayri et al. [6]. Feed rate, spindle speed, and radial depth of cut were used as input parameters to give a representation of the cutting forces. The full factorial design techniques were used to plan the experiments needed for training the neural network. This includes using four OAs specifically, L9 OA, L27 OA, L27 OA (extended range), and L36 OA. Comparison between these arrays for training the neural model was specified. Results indicated that L36 OA model results in better predictive model.

Online predictive model for surface roughness in turning operations was presented by Ho Shinn-Ying et al [8] using an adaptive NEURO-FUZZY inference system (ANFIS) and computer vision. The model aims to precisely predict the features of surface roughness for certain cutting parameters. Experimental results demonstrated better modeling and prediction accuracy than previous models.

Literature indicated efforts directed to advance the mathematical models based on the physics and the geometry of metal cutting. This included the effect of some dynamic features and more comprehensive analysis for metal cutting process. Implementation of these models have received attention including faster techniques for equation solving computer software that enhances model manipulation. This approach has relatively added more accuracy and has made the mathematical model easier to use. However, it is relatively complex to obtain accurate models that totally consider the dynamic nature of metal cutting.

4. METHODOLOGY

End milling is selected as the cutting process specifically, slot end milling process with one path. Depth of cut (a), spindle speed (n), feed rate (f), and tool diameter (d) are chosen as the process variables. These four variables are assumed to be independent variables. Other process variables are chosen at fixed levels for all experiments. The resulting dynamic 3-D cutting forces will be measured as the output response. The three resulting force components F_X, F_Y, and F_Z are combined to one resultant force using the following equation.

$$\mathbf{F} = \sqrt{[(F_X)^2 + (F_Y)^2 + (F_Z)^2]} \qquad \mathbf{1}$$

Figure 2 illustrates the resultant force of three cutting revolutions as a sample. In this illustration, it is clear that the resultant cutting force is periodic. However, the cutting force behavior of each revolution is not identical. For example, the maximum force and minimum force are different for each revolution. The reasons for this behavior are due to the noise in the measurement, machine vibrations, and the irregularities of work-piece material. It is also shown that within a revolution the two maximum peaks and two minimum peaks are not similar because of spindle run-out. Therefore, using the resultant force, the outputs are presented in the form of six values. The first four values are: the maximum, the minimum, the mean, and the standard deviation of resultant force: F_{Max}, F_{Min}, F_{Mean}, and F_{Stdev}. $F_{M\text{-}Max}$ and $F_{M\text{-}Min}$ are the symbols of the mean of maximum and minimum force respectively. The resultant cutting force values are determined only at the cutting time while the force values before and after cutting time are not considered in the calculation. The uncut force variations represent the machine vibration and other noise variations. Trying to reduce the effect of noise variations, the mean of uncut force variations was calculated and subtracted from the cut force variations as illustrated in Figure 3.

Using fractional factorial designs (FFD), a number of Orthogonal Arrays (OAs) are presented considering 2 ~ 5 levels to cover the chosen space of the given input variables. At the beginning, an experimental model of 3-levels called UL27-1 is carried out and an ANOVA study is presented for this model. Other 4 experimental models called UL8-1, UL9, UL32, and UL25 considering 2, 4, and 5 levels for each variable parameter. Additional models are presented called UL8-2, and UL27-2 using two and three variable levels. Besides, a set of 21 experiments covering different input values was performed. This model has some input values within the selected range and other points outside the range. The role is to provide validation within and outside the range.

All experimental models are conducted using the CU machine while another two models called NL8-1 and NL8-2 utilizing two variable values, are conducted using the CNC machine. Another set of 17 experiments are chosen to have different input positions in the space are conducted on the CNC machine called N-ad and used for the validation process later. None of the ANOVA results are given in table form due to limitations of space.

5. EXPERIMENTAL SETUP & VALIDATION

The employed equipment and setup consist basically of six main elements: two milling machines, work-pieces, tools, a dynamometer, a data acquisition system, and a pc. This section will identify and describe these elements briefly.

All experiments are conducted using two milling machines. They are CNC milling machine and conventional universal milling machine (CU). The CNC machine can run at spindle speeds from 100 rpm to 2500 rpm. Its axis travels 290 mm in X-direction, 170 mm in Y-direction, and 235 mm in Z-direction. While the conventional machine has some limitations regarding the alternative values of both spindle speed and feed rate.

Cast aluminum work-pieces used are purchased from the local market. Their dimensions are chosen to be 80 x 65 x 60 mm to suit the fixing area of the dynamometer. Surfaces of work-pieces are prepared before milling to avoid any surface irregularities with two holes for clamping purposes.

Six IZAR® HSS 2-flute end-mill tools [11] with different diameter sizes were used. They are 6 mm, 7 mm, 8 mm, 10 mm, 11 mm, and 12 mm. All tools have the same cutting angles and material.

Three Cartesian force components are measured by a KISTLER 9257B dynamometer. A work-piece can be fixed in its top late area of 170 x 100 mm. The dynamometer starts the process of measurements the cutting forces by sending analog signals in the form of voltages proportional to the actual cutting forces occurring to three amplifiers by an integrated cable.

A HUMSOFT MF 614 data acquisition card is used with a pc to interpret these signals. The used card has some important features. It contains a converter unit that receives analog signals and converts them to digital signals. Therefore, it used to receive the analog signals from the amplifiers and output digital signals. Besides, this card can work with a Real Time Toolbox for MATLAB that contains a library of real-time blocks. This enables to create a simulation diagram using SIMULINK and consequently, benefits from SIMULINK capabilities. Therefore, SIMULINK simulation diagram is designed using some real-time blocks provided in the Real Time Toolbox [3] and MATLAB to understand the coming digital signals and give online diagrams, immediately processing the data and store the data in mat-file format. It also gives the ability to specify different sampling periods for each output. The cutting forces were sampled at 500 HZ for 1 second. Figure 4 and 5 show the simulation model used for force measurements and the experimental setup.

A number of initial experiments was performed and repeated on the two milling machines. The repeatability errors were calculated for the six output values: F_{Max}, F_{Min}, F_{Mean}, F_{Stdev}, $F_{M\text{-}Max}$, and $F_{M\text{-}Min,}$ using equation 2.

$$RE = \frac{abs(V_1 - V_2)}{V_1} \qquad \mathbf{2}$$

Where, RE is the repeatability error. V_1 is the measured force value of the first experimental run and V_2 is the measured force value of the repeated experimental run. The repeatability error measures the possibility of each machine to respond the identical performance when using the same input machining parameters. Table 1 gives the control variables used for all experimental models. Table 2 and 3 show that the mean repeatability error is 5.95% for CNC machine and is 8.26% for CU. Resultant errors from F_{Min} and $F_{M\text{-}Min}$ are too high and once excluded the mean repeatability error for CNC and CU become 2.29% and 3.51% respectively. Accordingly, the two machines can be acceptable for the F_{Max}, F_{Mean}, F_{Stdev}, and $F_{M\text{-}Max}$ values, while they are not trustworthy for F_{Min} and $F_{M\text{-}Min}$ values. One possible reason for the increase of repeatability error for F_{Min} and $F_{M\text{-}Min}$ is that their values are very small and consequently will be strongly affected by the noise variations resulting from machine noise, vibrations and force measuring equipment.

6. RESULTS & CONCLUSIONS

An ANOVA study (at a confidence level of 90%) using UL7-1 to determine the significance of input process variables was conducted. F_{max} and $F_{M\text{-}Max}$, F_{Stdev}, F_{Min} and $_{FM\text{-}Min,}$ F_{Mean} are taken as output responses. Results indicate that the four input parameters are significant at 90% confidence. This also strengthens our initial choice to model end milling process. Furthermore, the modeling capability of UL27-1 can be enhanced by adding experiments that have more [a] and [f] levels. Consequently, other experimental models are selected to contain more [a] and [f] levels including 2-levels, 3-levels, 4-levels, and 5-levels OAs. Combining these experimental models into a larger model is expected to provide better modeling capabilities. Besides, the modeling capability of each model when used as training patterns for the neural model can be studied and compared. A sample force measurement was given in Table 4 using L8OA and a sample space representation of CU and different tool diameters is given in Figure 6. A neural network model was developed for flat end milling process. DOE and ANOVA were used to fractionate the huge number of experiments needed for model development. In a sequel paper, we intend to present our final Neural network Model of flat end milling process.

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Table 1. Control Variables Used for All Experimental Models.

a mm		n rpm		f mm/min		d mm	
CU	CNN	CU	CNN	CU	CNN	CU	CNN
0.5	0.5	400	500	35.5	50	6	6
0.75	1	560	750	50	100	7	8
0.8	1.5	800	1000	71	150	8	10
1	2.5	1120	1500	100	200	10	12
1.25		1600		140		11	
1.5				200		12	
1.8				280			

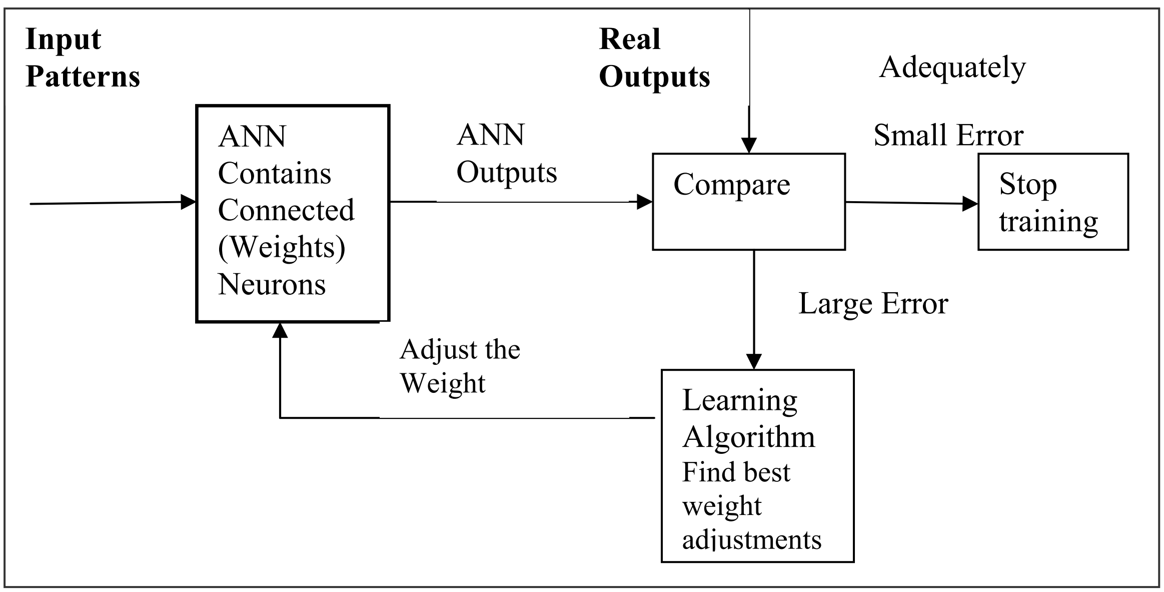

Figure 1. A Block diagram representation of training process [20].

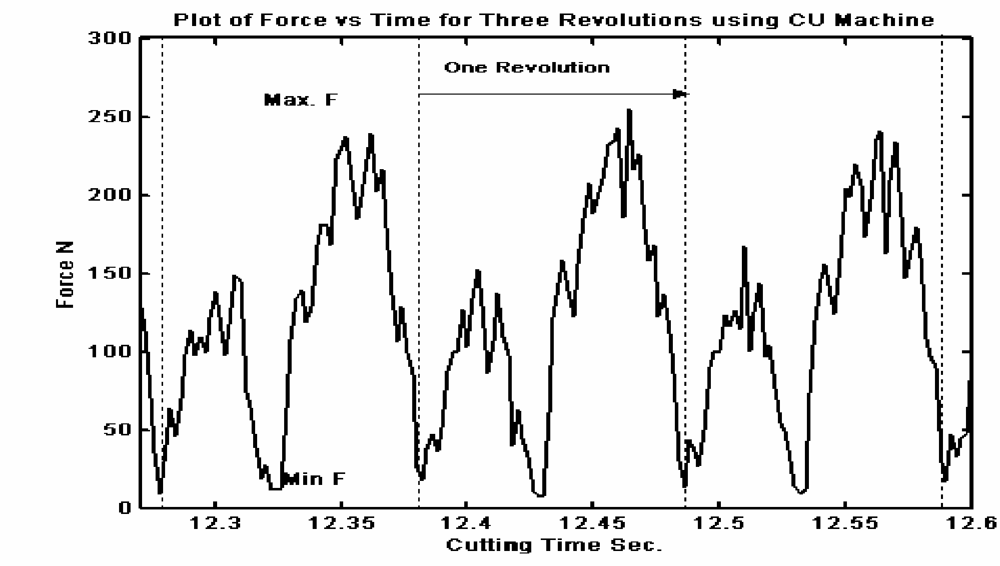

Figure 2. Force vs. Time for Three Revolutions.

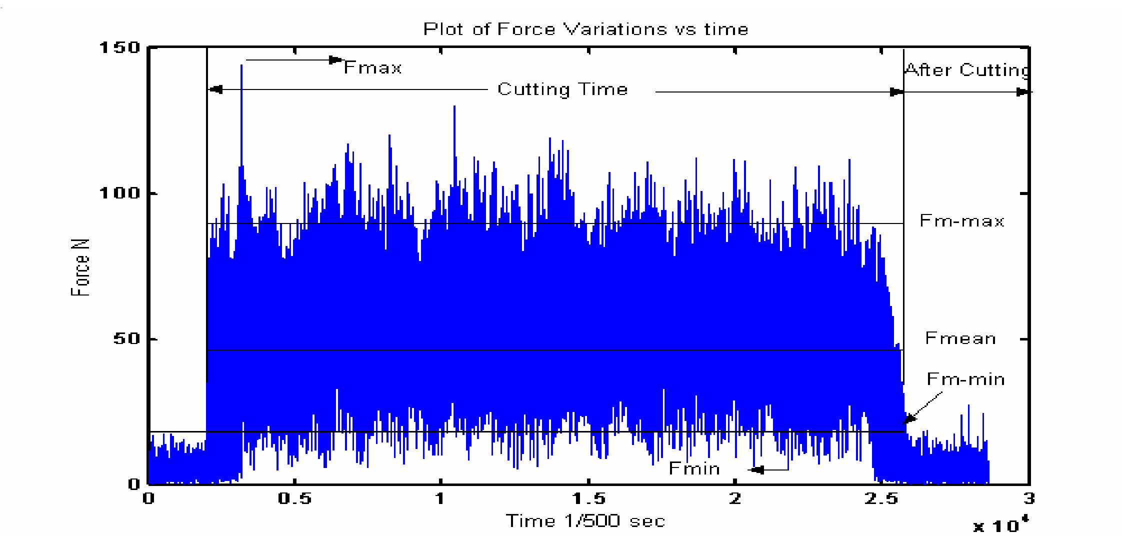

Figure 3. Force Variations vs. Time.

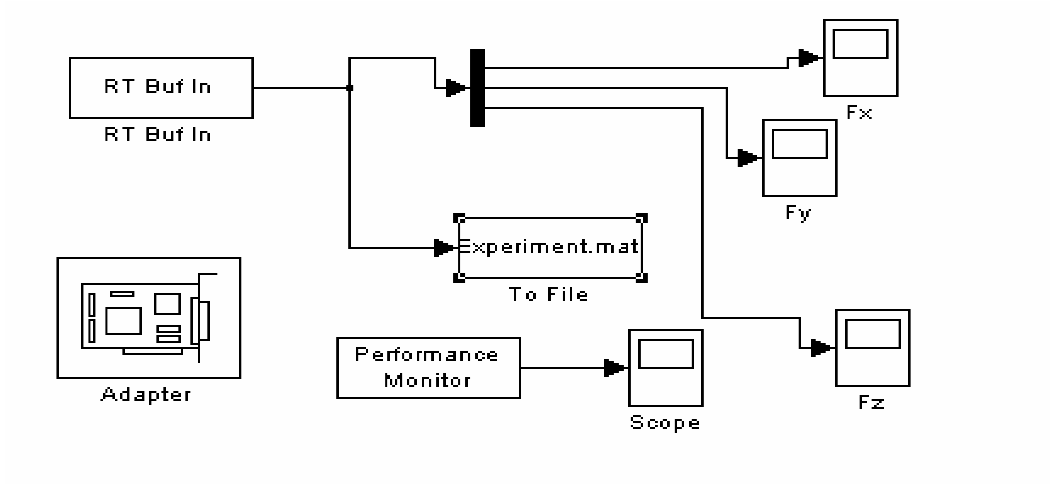

Figure 4. SIMULINK Simulation Model Used to Measure the Cutting Forces.

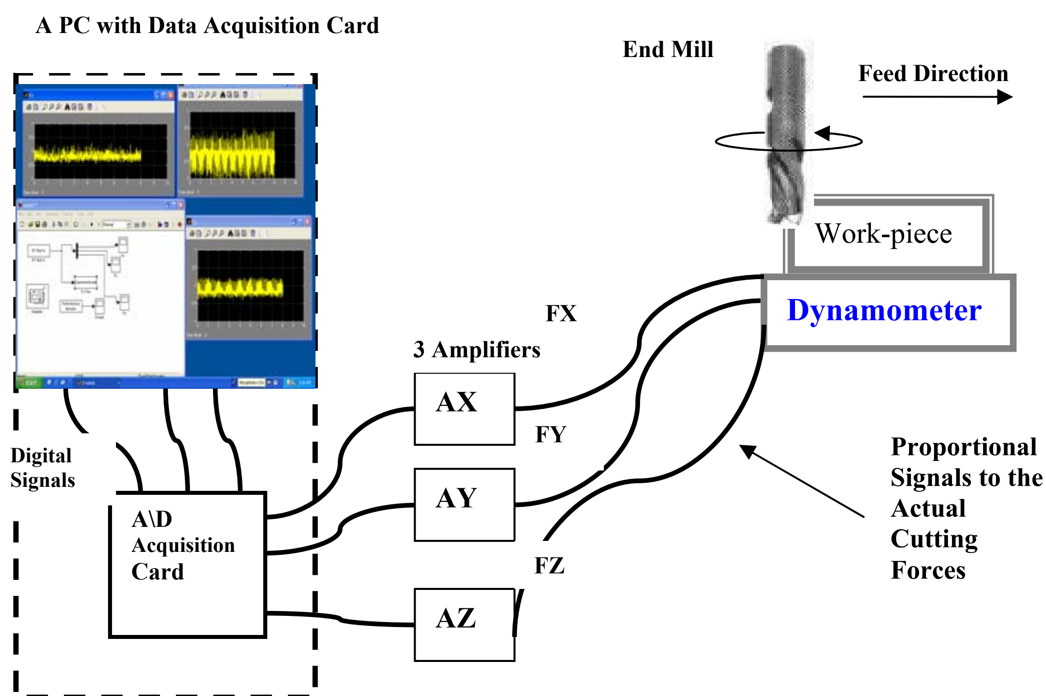

Figure 5. The Experimental Setup.

Table 2. Repeatability Testing Experiments for the CNC machine.

a mm	n rpm	f mm/min	d mm	F_{Max}	F_{Min}	F_{Mean}	F_{Stdev}	$F_{M\text{-}Max}$	$F_{M\text{-}Min}$
1.5	1000	50	10	126.328	0.211	35.888	31.478	95.496	1.607
				125.893	0.248	36.539	31.579	95.562	1.694
The Repeatability Error %				0.34%	17.42%	1.81%	0.32%	0.07%	5.39%
1	750	100	8	124.808	0.247	43.494	34.223	104.865	2.592
				129.444	0.297	45.604	33.207	106.357	3.355
The Repeatability Error %				3.71%	20.31%	4.85%	2.97%	1.42%	29.43%
0.5	500	200	12	170.909	0.433	53.085	38.369	122.287	5.046
				173.491	0.415	54.724	39.796	126.653	5.193
The Repeatability Error %				1.51%	4.31%	3.09%	3.72%	3.57%	2.93%
The Mean Errors = 5.95%				1.86%	14.01%	3.25%	2.34%	1.69%	12.58%

Table 3. Repeatability Testing Experiments for the CU machine.

a mm	n rpm	f mm/min	d mm	F_{Max}	F_{Min}	F_{Mean}	F_{Stdev}	$F_{M\text{-}Max}$	$F_{M\text{-}Min}$
1	1600	230	12	193.296	0.189	51.349	51.647	156.462	3.591
				199.995	0.174	52.057	51.948	158.120	3.449
The Repeatability Error %				3.47%	8.04%	1.38%	0.58%	1.06%	3.93%
2.5	800	230	8	415.081	0.399	222.959	92.558	346.261	23.316
				442.991	0.469	228.989	96.286	353.982	21.453
The Repeatability Error %				6.72%	17.60%	2.70%	4.03%	2.23%	7.99%
0.5	560	100	10	130.995	0.170	31.734	30.935	89.473	2.256
				124.052	0.264	32.986	32.166	92.486	2.201
The Repeatability Error %				5.30%	55.48%	3.95%	3.98%	3.37%	2.44%
1.5	560	100	10	264.828	0.205	70.154	76.550	198.989	1.590
				255.130	0.210	69.240	74.596	194.386	1.922
The Repeatability Error %				3.66%	2.14%	1.30%	2.55%	2.31%	20.86%
The Mean Errors = 8.26%				5.23%	25.07%	2.65%	3.52%	2.64%	10.43%

Table 4. A Sample Model - Force Components for Experimental Models.

Model Name	Value Definition	F_{Max}	F_{Min}	F_{Mean}	F_{Stdev}	$F_{M\text{-}Max}$	$F_{M\text{-}Min}$
UL8-1	Max.	367.49	0.56	138.58	108.10	292.81	4.30
	Mean	253.18	0.34	75.44	62.97	190.97	2.90
	Min.	106.04	0.11	32.37	20.77	69.18	1.79
	Stdev.	105.11	0.15	33.22	29.58	84.39	0.87

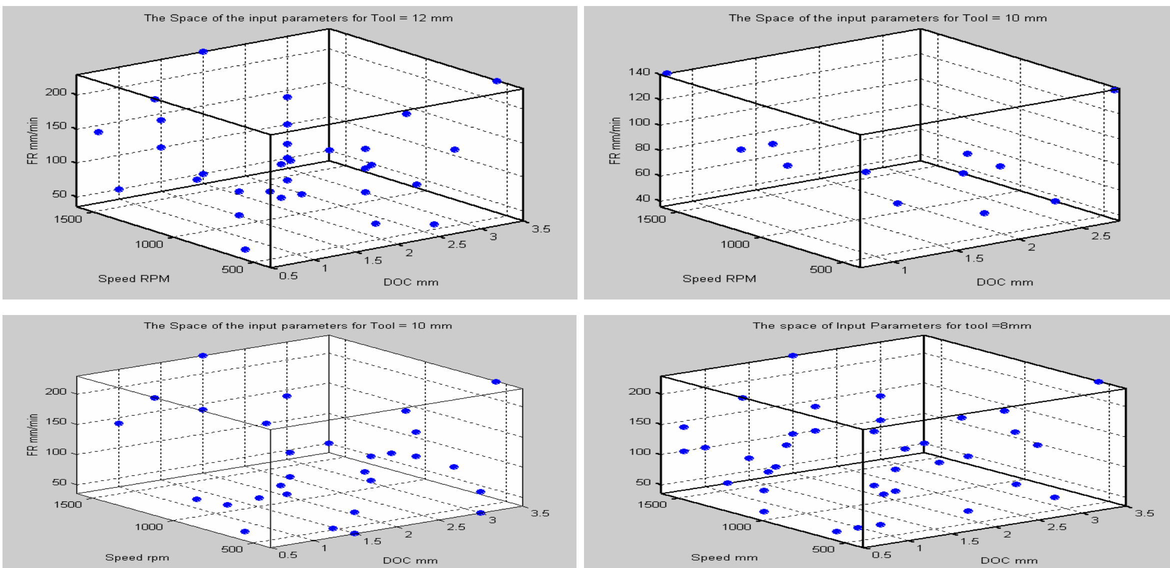

Figure 6. Sample space Representation of CU and Different Tool diameters (TD).