Cairo University. Discrete Structures (Msc521).

FGSSR. Lecture 8.

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Sets

A set is well-defined collection of objects. These objects are called elements and said to be members of the set. The adjective well-defined implies that for element we care to consider we are able to determine whether it is in the set under consideration.

A set can be designated by listing its elements within set braces. For example, if A is a set consisting of the first five positive integers, then we write $A = \{1,2,3,4,5\}$.

Another standard notation for this set provides us with:

$$A = \{x: x \ is \ integer \ and \ 1 \leq x \leq 5\}.$$

The symbol: $\{x: \cdots\}$ is read " the set of all x such that $\cdots$ ". The properties following " :" help us determine the elements of the set that is being described.

Remark 8.1: Be aware the notation " $\{x : 1 \leq x \leq 5\}$ is not an adequate description of the set A unless we have agreed in advance that the elements we are considering are integers. When such an agreement is adopted, we say that we are specifying a universe, or universe of discourse, which is usually denoted by U. We then select only elements from U to form our sets.

Example 8.1: Let $U = \{1,2,3, \cdots\}$, the set of positive integers, let

a) $A = \{1,4,9, \cdots ,64,81\} = \{x^2: x \in U, x^2 < 100\}$
$= \{x^2: x \in U \wedge x^2 < 100\}$.

b) $B = \{1,4,9,16\} = \{y^2 : y \in U, y^2 < 20\}$
$\{y^2 : y \in U, y^2 < 23\} = \{y^2 : y \in U \wedge y^2 < 17\}$
$= \{y^2 \in U : y^2 \leq 16\}$.

c) $C = \{2,4,6,8, \cdots\} = \{2k : k \in U\} =$ the set of positive even integers.

d) $D = \{2k - 1 : k \in U\} =$ the set of positive odd integers.

Remarks 8.2:

(**1**) Sets A and B are examples of finite sets., whereas C and D are ofexamples infinite set.

(2) When dealing with sets like A or C, we can either describe the sets in terms of properties the elements must satisfy or list enough elements to indicate what is, we hope an obvious pattern.

(3) For any finite set A, $|A|$ denotes the number of elements in A and is referred to as the cardinality, or size of A. In this example $A = 9$ and $|B| = 4$.

(4) every element of B is also an element of A.

Definition 8.1: If C, D are sets from the universe U, we say that C is a subset of D and write $C \subseteq D$ or $D \supseteq C$, if every element of C is an element of D. If in addition, D contains at least one element that is not in C, then C is called a proper subset of D, and this is denoted by $C \subset D$ or $D \supset C$.

Remarks 8.2:

(1) This fact can be written in mathematical form as follows:

If $C \subseteq D$, then $\forall \boldsymbol{x}[\boldsymbol{x} \in \boldsymbol{C} \Rightarrow \boldsymbol{x \epsilon D}]$

And if $\forall x[x \in C \Rightarrow x \in D]$, then $C \subseteq D$.

The symbol $\forall x$ means " for all x" or " for every x" or "for any x" .

(2) If $C \subseteq D$ then $|C| \leq |D|$.
(3) If $C \subset D$, then $\forall x\ [x \in C \Longrightarrow x \in D]$ and $[\exists y \in D \wedge y \notin C]$.
(4) If $C \subset D$, then $|C| < |D|$.

.**Example 8.2:** A variable name (in some institute) in FORTRAN consists of a single letter followed by at most 5 characters (letters or digits**). If** U is the set of all variable names. Find $|U|$.

Solution : number **of** varibles with one letter =26.

Number of variables with two places = 26x36.

Number of variables with 3 places=26x$(36)^2$.

Number of variables with 4 places $= \mathbf{26x(36)^3}$**.**

Number of variables with 5 places $= \mathbf{26x(36)^4}$**.**

Number of variables with 6 places $= \mathbf{26x(36)^5}$**.**

Number of variables$\sum_0^5 \mathbf{26x(36)^i}$ **.**