Cairo University **Discrete Structures(Msc521)**

FGSSR. **Lecture 10.**

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Set Operations

Definition 10.1: For $A, B \subseteq$ U we define the following:

a) $\bar{A}$ (the complement of A) $= \{x: x \notin A\}$.
b) $A \cup B$ (the union of A and B) $= \{x: x \in A \vee x \in B\}$.
c) A ∩ B **(**the intersection of A and B**)**= $\{\boldsymbol{x}: \boldsymbol{x} \in \boldsymbol{A} \wedge \boldsymbol{x} \in \boldsymbol{B}\}$**.**
d) A−B (the difference between A and B)= $\{x: x \in A \wedge x \notin B\}$.
e) A⊕B (the symmetric difference) $= \{x: (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\}$.

Example 10.1: Let U= $\{1,2,3, \cdots 10\}$, $A = \{1,2,3,4,5\}$,$B = \{3,4,5,6,7\}$, and $C = \{7,8,9\}$. We have:

(1) $\bar{A} = \{6,7,8,9,10\}$. (2) A$\cap B = \{3,4,5\}$.
(3) $A \cup C = \{1,2,3,4,5,7,8,9\}$
(4) $A \oplus B = \{1,2,6,7\}$. (5) $A \cap C = \emptyset$.
(6) $B \cap C = \{7\}$. (7) $B \oplus C = \{3,4,5,6,8,9\}$.

Definition 10.2: Let$A, B \subseteq U$. The sets A and B are called disjoint if $A \cap B = \emptyset$.

Example 10.2: The sets A and C in example 10.1 are disjoint.
Theorem 10.1: For any universe U and any sets $A, B \subseteq U$, the following statements are equivalent:

a) $A \subseteq B$. b) $A \cup B = B$.
c) A $\cap B = A$. C) $\bar{B} \subseteq \bar{A}$.

Proof: $(a) \Longrightarrow (b)$

Let $x \epsilon\ A \cup B$ then either $x \in A$ (1) or

$x \in B$ (2)

Since by assumption $A \subseteq B$, it follows that $x \in B$.

Therefore, $A \cup B \subseteq B$ (3).

It is clear that $B \subseteq A \cup B$ (4).

From (3) and (4), we get $A \cup B = B$.

(b)$\Longrightarrow (c)$

Let $x \in A \cap B$ then $x \in A$ and $x \in B$, hence

$A \cap B \subseteq A$ (5).

Let $x \in A \xRightarrow{A \subseteq A \cup B} x \in A \cup B \xRightarrow{from\ (b)} x \in B$

$\Longrightarrow \boldsymbol{x \in A \cap B}$ **(6).**

From (5) and (6), we get $A \cap B = A$.

(c) $\Longrightarrow (d)$

Let $x \in \bar{B} \xRightarrow{by\ definition} \boldsymbol{x \notin B \Rightarrow x \notin A \cap B} \xRightarrow{\boldsymbol{by\ (C)}} \boldsymbol{x \notin A} \xRightarrow{\boldsymbol{by\ definition}} \boldsymbol{x \in \bar{A}}$**.**

Hence $\boldsymbol{\bar{B} \subseteq \bar{A}}$**.**

$(\boldsymbol{d}) \Rightarrow (\boldsymbol{a})$

Let $x \in A \xRightarrow{by\ definition} x \notin \bar{A} \xRightarrow{by\ (d)} x \notin \bar{B} \xRightarrow{by\ definition} x \in B \Longrightarrow A \subseteq B$.

Cartesian products:

Let A and B be sets. The Cartesian product of A and B, denoted by $A \times B$ is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. Hence,

$$\mathrm{A \times B = \{(a, b): a \in A \wedge b \in B\}}.$$

Example10.3:

Let $A = \{1,2\}$ and $B = \{a, b, c\}$.

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

$B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$.

$|A \times B| = |A| \times |B| = 6 = |BxA|$.

Remarks 10.2:

(1) $A \times B \neq B \times A$
(2)$A \times \emptyset = \emptyset \times A = \emptyset$
(3) $A \times B = \emptyset$ iff $A = \emptyset \; or \; B = \emptyset$
(4) $|A \times B| = |B \times A| = |A| \times |B\,|$.

Example 10.4:

Let $A = \{\emptyset, 1\}$ and $B = \big\{\{1\}, 2, \{\emptyset\}\big\}$.

$A \times A = \{(\emptyset, \emptyset), (\emptyset, 1), (1, \emptyset), (1,1)\}$.

$$B \times B = \left\{\begin{matrix} (\{1\}, \{1\}), (\{1\}, 2), (\{1\}, \{\emptyset\}), \\ (2, \{1\}), (2,2), (2, \{\emptyset\}), (\{\emptyset\}, \{1\}), \\ (\{\emptyset\}, 2), (\{\emptyset\}, \{\emptyset\}) \end{matrix}\right\}$$

$A \times \emptyset = \emptyset$.
$A \times \{\emptyset\} = \{(\emptyset, \emptyset), (1, \emptyset)\}$.

$\mathrm{A} \times \mathrm{B} = \{(\emptyset, \{1\}), (\emptyset, 2), (\emptyset, \{\emptyset\}), (1, \{1\}), (1,2), (1, \{\emptyset\})\}$.