Cairo University. **Discrete Structures (Msc521).**

FGSSR. **Lecture 9.**

Department of Mathematical Statistics. **Prof. Ali. A. A-Rahman.**

Sets (Continued)

Definition 9.1: For a given universe U, the sets C and D(taken from U) are said to be equal, and we write $C = D$, when $C \subseteq D$ and $D \subseteq C$.

Example 9.1: Let $U = \{1,2,3,4,5\}$, consider the set $A = \{1,2\}$.

If $B = \{x : x^2 \in U\}$. Then $A \subseteq B$ and $B \subseteq A$. Hence $A = B$.

Example 9.2: Let $U = \{1,2,3,4,5,6, x, y, \{1,2\}, \{1,2,3\}, \{1,2,3,4\}\}$

(Where x, y are the 24th ,25th lowercase letters of the alphabet and do not represent anything else, such as 3,5,{1,2}. Then $|U| = 11$.

(a) If $A = \{1,2,3,4\}$, then $|A| = 4$ and we have:
(i) $A \subseteq U$, (ii) $A \subset U$, (iii) $A \in U$,

(iv) $\{A\} \subseteq U$, (v) $\{A\} \subset U$, (vi) $\{A\} \notin U$.

(b) Let $B = \{5,6, x, y, A\} = \{5,6, x, y, \{1,2,3,4\}\}$
Then $|B| = 5$ and
(i) $A \in B$ (ii) $\{A\} \subseteq B$ (iii) $\{A\} \subset B$.

Theorem 9.1: Let $A, B, C \subseteq U$.

(a) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
(b) If $A \subset B$ and$B \subseteq C$, then$A \subset C$.

(c) if $A \subseteq B$ and $B \subset C$, then $A \subset C$.

(d) if $A \subset B$ and $B \subset C$, then $A \subset C$.

Definition 9.2: The null set, the empty set, is the (unique) set containing no elements. It is denoted by ∅ or { }.

Remarks 9.1:

(1) $|\emptyset| = 0$.
(2) $\{0\} \neq \emptyset$.
(3) $\emptyset \neq \{\emptyset\}$.

Theorem 9.2: For any universe U, let $A \subseteq U$. Then $\emptyset \subseteq A$, and if $A \neq \emptyset$, then $\emptyset \subset A$.

Definition 9.3: if A is a set from the universe U, the power set of A, dented by $P(A)$, is the collection (or set) of all subsets of A.

Example 9.3: Let $U = \{1,2,3,4,5\}$. Determine the number of subsets of the set $C = \{1,2,3,4\}$, i.e., $|P(C)|$.

Solution:

The number of subsets of C of size zero $C(4,0) = 1$.

Number of subsets of size one $= C(4,1) = 4$.

Number of subsets of size two $= C(4,2) = 6$.

Number of subsets of size three $= C(4,3) = 4$.

Number of subsets of size four $= C(4,4) = 1$.

$|P(C)| = 1 + 4 + 6 + 4 + 1 = 16$.

Now $P(A)$ can be written as follows:

$$P(A) = \left\{ \begin{matrix} \emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \\ \{1,2,3\}, \{1,3,4\}, \{1,2,4\}, \{2,3,4\}, \{\{1,2,3,4\}\} \end{matrix} \right\}$$

Theorem 9.3: Let A be a finite set with size n. Then

$|P(A)| = 2^n$.

Proof : $\boldsymbol{|P(A)| = \sum_{i=0}^{n} C(n,i) = 1 + C(n,1) + C(n,2) + \cdots C(n,n) = 2^n.}$

Example 9.4: let $A = \{1,2,3,4,5,6,7\}$, determine the number of :

(1) Subsets of A. (2) nonempty subsets of A.

(3) proper subsets of A. (4) nonempty proper subsets of A.

(5) subsets of A containing five elements including 1,2.

(6) proper subsets of A containing 1,2.

(7) subsets of A with an even number of elements.

(8) subsets of A with an odd number of elements.

(9) subsets of A with an odd number of elements including 3.

Solution: (1) $|P(A)| = 2^7$.

(2) $2^7 - 1$.

(3) $2^7 - 1$.

(4) $2^7 - 2$.

(5) $C(5,3) = 10$.

(6) $C(5.0) + C(5,1) + C(5,3) + C(5,4) = 1 + 5 + 10 + 5 = 21.$

(7) $C(7,0) + C(7,2) + C(7,4) + C(7,6)$
$= 1 + 21 + 35 + 1 = 64.$

(8) $C(7,1) + C(7,3) + C(7,5) + C\ (7,7)$ =7+35+21+1=64.

(9) $C(6,0) + C(6,2) + C(6,4) + C(6,6) =$ 1+15+15+1=32