Cairo University | Msc 521
FGSSR | Lecture 2
Department of Mathematical Statistics | Discrete Structures
Prof. Ali A.A-Rahman

Permutations

Definition2.1 : Let $A = \{a_1, a_2, a_3, \cdots, a_n\}$. Assume that r, n be two non-negative integers such that $r \leq n$. An ordered arrangement of r elements of A is called an r- permutation.

Example 2.1 : a_1a_2, a_3a_1, a_5a_1 are examples for a 2-permutation.

Remark 2. 1 : There are $n(n-1)$ 2-permutations can be formed using the elements of A. (why?).

Example 2.2: Let $B = \{1,4,6,5\}$. How many 1-permutation ,2-permutation,

3- permutation and 4-permutation can be formed from B. Write 3 permutations from every type.

Solution: We have four 1-permutation namely 1,4,6,5.

To form 2-permutation we have 4 ways to choose the first element, and 3 ways to choose the second element , therefore we have 4x3=12 (2 –permutation), e.g., 14,41,56.

To form a 3-permutation we note that we have to choose 3 elements from B. The first element can be chosen with 4 ways ,the second can be chosen with 3ways while the third can be chosen by 2 ways therefore we have 4x3x2=24 permutations of length 3 (3-permutation). Some of them are ,e.g., 145, 146, 541.

In the same manner, one can see that there are 24 permutations of length 4 (4-permutation). Some examples are ,1456,6541,5614**.**

The number of r −permutations of a set with n elements is denoted by $P(n,r)$.

Using the product rule for counting we have the following theorem.

Theorem 2.1: The number of r-permutation from a of n distinct elements is

$$P(n,r) = n(n-1)(n-2)\cdots(n-r+1)\,, \quad 1 \le r \le n.$$

Remarks 2.2

(1) if we set $r = n$ in the last equation we have:

$$P(n,n) = n(n-1)(n-2)(n-3)\cdots 3 \times 2 \times 1 = n!$$

The last symbol is read as " factorial"

(2) if we set $r = 1$in theorem 2.1, we get
$P(n,1) = n.$

(3) The formula in Theorem 2.1 can be written as follows:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Set $r = 0$.one gets: $P(n,0) = 1.$

Example 2.3 : $4! = 4 \times 3 \times 2 \times 1 = 24.$

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720.$

Example 2.4: How many different ways are there to select 4 different players from 10 players on a team to play 4-tennis matches where the matches are ordered .

solution : we have $P(10,4) = 10 \times 9 \times 8 \times 7$=5040.

Example 2.5 : Suppose that there are 8 runners in a race. The winner receives a gold medal, the second –place finisher receives a silver medal, and the third place finisher receive a bronze medal. how many different ways are there to award these medals, if all possible outcomes of the race can occur ?

Solution : from the text we understand that no one can win more than one prize also the order is important so we have

$$P(8,3) = 8 \times 7 \times 6 = 336.$$

Example 2.6 : A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

Solution : We have two possibilities

(1) A man is seated in the first place with n ways, the second place must be filled by a woman with n ways then the third place must be filled by a man with (n-1) ways, then a woman must be seated the fourth place with (n-1), continuing in this manner we find that the number of ways =

nxnx(n-1)x(n-1)x(n-2)x(n-2)…..x3x3x2x2x1x1=$(n!)^2$.

(2) The first place is filled by a woman with n ways ,the second place must be filled by a man with n ways, the third place is occupied by a woman with (n-1) ways, then the fourth place will be filled by woman with(n-1) and so on. Therefore in this case we have the number of ways will be $(n!)^2$.

Combining the above two possibility (using the sum rule),it follows that the number of ways= $2(n!)^2$