Cairo University. Discrete Structures (Msc521).

FGSSR. Lecture 11.

Department of Mathematical Statistics. Prof. Ali. A. A-Rahman.

The Laws of Set Theory

We shall write the laws of set theory. One may easily note the similarity between logic and set laws.[replace negation by complement, "or" by union and " and" by intersection.

For any sets A, B, and C taken from a universe U

(1) $\bar{\bar{A}} = A$, (double complement).
(2) $\overline{A \cup B} = \bar{A} \cap \bar{B}$, (DeMorgan's law).
(3) $\overline{A \cap B} = \bar{A} \cup \bar{B}$, (DeMorgan's law).
(4) $A \cup B = B \cup A$, (commutative law).
(5) $A \cap B = B \cap A$, (commutative law).
(6) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, Distributive law.
(7) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, Distributive law.
(8) $A \cup (B \cup C) = (A \cup B) \cup C$, Associative law.
(9) $A \cap (B \cap C) = (A \cap B) \cap C$, Associative law.
(10) $A \cup A = A$, Idempotent law.
(11) $A \cap A = A$, Idempotent law.
(12) $A \cup \emptyset = A$, Identity law.
(13) $A \cap U = A$, Identity law .
(14) $A \cup \bar{A} = U$,
(15) $A \cap \bar{A} = \emptyset$
(16) $A \cup U = U$, Domination law.
(17) $A \cap \emptyset = \emptyset$ Domination law.

Example 11.1: simplify the following:

$$\overline{\overline{(A \cup B) \cap C} \cup \overline{B}}.$$

Solution:

$$\overline{\overline{(A \cup B) \cap C} \cup \overline{B}} = \overline{\overline{(A \cup B) \cap C}} \cap \overline{\overline{B}}$$

$$= [(A \cup B) \cap C] \cap B = (A \cup B) \cap (C \cap B)$$

$$= (A \cap C \cap B) \cup (B \cap C \cap B)$$

$$= [A \cap (C \cap B)] \cup (C \cap B) = C \cap B.$$

Explanation of solution steps

(1) **At first we use DeMorgan's law.**
(2) **Then we use double complement law.**
(3) **Then distributive law is used.**
(4) **Finally we apply the absorption law.**

Important homework:
Simplify the following expression:

$$\neg[\neg[(\boldsymbol{p} \vee \boldsymbol{q}) \wedge \boldsymbol{r}] \vee \neg \boldsymbol{q}].$$

Answer: $\boldsymbol{q} \wedge \boldsymbol{r}$.

The student is advised to perform this problem to understand the similarity between set theory and mathematical logic.

Example 11.2:

Express $\overline{\boldsymbol{A} - \boldsymbol{B}}$ in terms of the union and complement operations only.

Solution: $\overline{\boldsymbol{A} - \boldsymbol{B}} = \overline{\boldsymbol{A} \cap \overline{\boldsymbol{B}}} = \overline{\boldsymbol{A}} \cup \overline{\overline{\boldsymbol{B}}} = \overline{\boldsymbol{A}} \cup \boldsymbol{B}$.

Example 11.3: Determine the sets A,B, where A-B={1,3,7,11},

B-A={2,6,8}, and $\boldsymbol{A \cap B = \{4, 9\}}$.

Solution: $\boldsymbol{A - B = A \cap \overline{B} = \{1, 3, 7, 11\}}$ .To find the set A, we use the following important relation between A and B

$$\boldsymbol{A = (A \cap B) \cup (A \cap \overline{B}) = \{1, 3, 7, 11, 4, 9\}}.$$

Also, $\boldsymbol{B = (B \cap A) \cup (B \cap \overline{A})}$=$\boldsymbol{\{4, 9, 2, 6, 8\}}$.

Example 11.4: Determine the sets C,D where C-D= $\boldsymbol{\{1, 2, 4\}}$,

D-C= $\boldsymbol{\{7, 8\}}$, and $\boldsymbol{C \cup D = \{1, 2, 4, 5, 7, 8, 9\}}$.

Solution:

$\boldsymbol{C - D = C \cap \overline{D} = \{1, 2, 4\}}$**,**

$\boldsymbol{D - C = D \cap \overline{C} = \{7, 8\}}$ **, and**

$$\boldsymbol{C \oplus D = (C - D) \cup (D - C) \ = (C \cup D) - (C \cap D)}$$

$C \oplus D = \{1,2,4,7,8\}$.

Hence

$C \cap D = \{5,9\}$. Therefore

$C = (C \cap D) \cup (C \cap \bar{D}) = \{1,2,4,5,9\}$, and

$D = (D \cap C) \cup (D \cap \bar{C}) = \{5,9,7,8\}$.