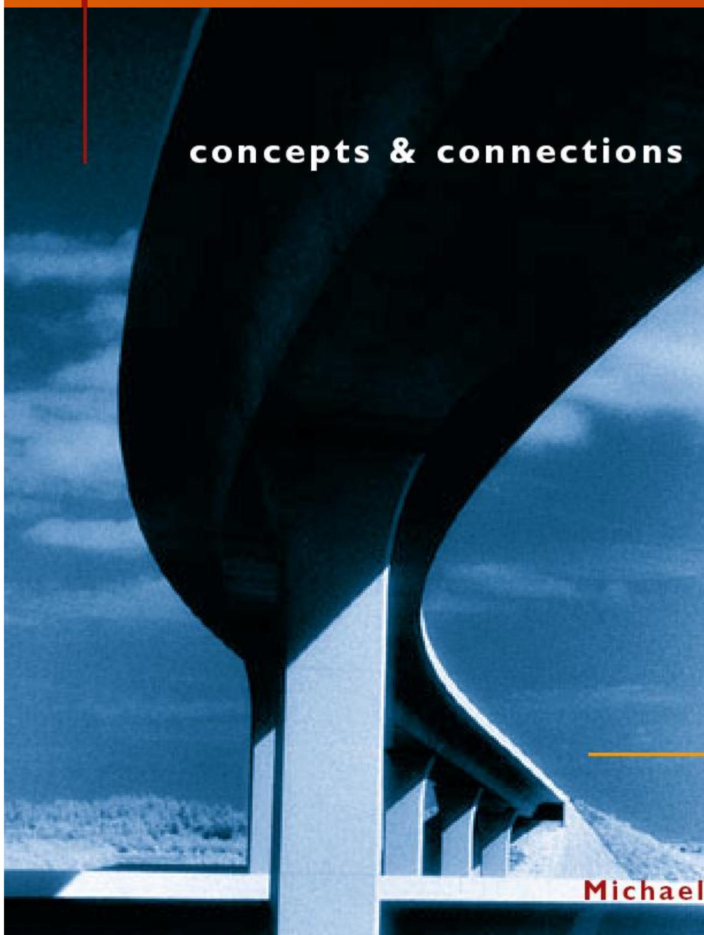
MICROECONOMIC THEORY



Utility Maximization

L.5

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Introduction

- Basic hypothesis
  - Rational household will always choose a most preferred bundle from set of feasible alternatives
- Belief in utility maximization belongs to Austrian school of thought
  - Maximization hypothesis is the fundamental axiom of human action that is known to be true *a priori*
- Form of hedonism doctrine
  - Doctrine that pleasure is the chief good in life
- Competitive market model is often used as an argument for profit maximization
  - Firms that do not maximize profits are driven out of the market by competitive forces

Introduction

- Households are rewarded for their usefulness to society
  - Through compensation
- Use income to purchase commodities to satisfy some of their wants
- Chapter objective
  - Show how household choices among alternative commodity bundles are determined for satisfying these wants
- Household is constrained in its ability to consume commodity bundles by
  - Market prices
  - Fixed level of income
    - Called a budget constraint

Introduction

- Given this budget constraint and a utility function representing preferences
  - Can determine utility-maximizing commodity bundle
- Decentralized determination of utility-maximizing bundle
  - Key element in efficient allocation of society's limited resources
- Economists do not directly estimate the utility-maximizing set of commodities for each household
  - Not possible to estimate individual utility functions and then determine optimal consumption bundle for each consumer
- Economists are interested in efficient resource allocation for society as a whole
  - Aggregate (market) response of households to prices and income choices
- However, utility-maximizing hypothesis for individual households does yield some interesting conclusions
  - Associated with government rationing of commodities, taxes, and subsidies

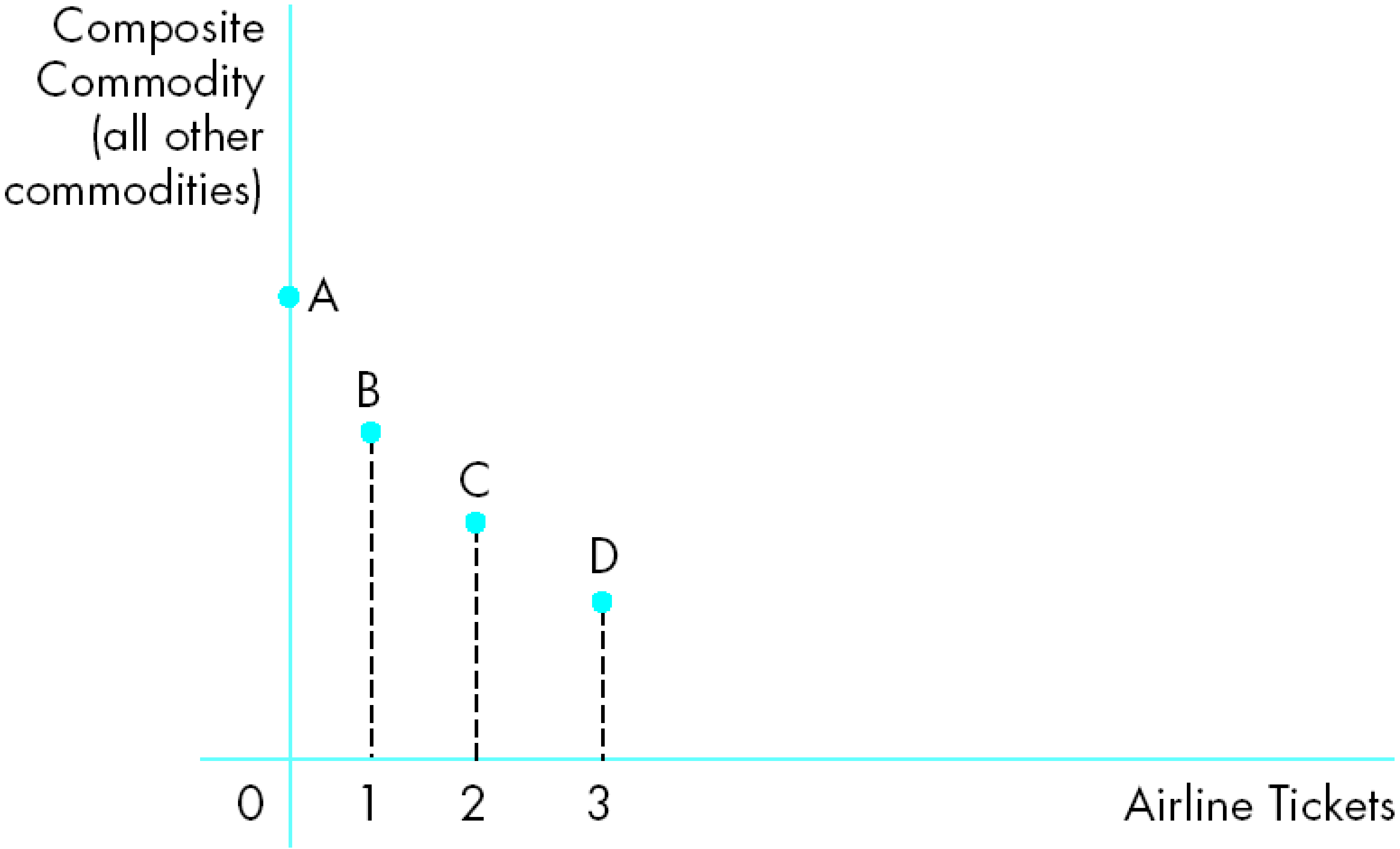
Budget Constraint

- Major determinant of consumer behavior, or utility maximization
  - Utility or satisfaction received from commodities
- Commodity prices and budget constraint are also important
  - In choosing a commodity bundle, household must
    - Reconcile its wants with its preference relation (or utility function) and budget constraint
- Certain physical constraints may be embodied in consumption set
  - May further limit choice of commodity bundles

Discrete Commodity

- Physical constraint example—commodity that must be consumed in discrete increments
  - For example, a household can either purchase an airline ticket and fly to a given destination or not
  - Cannot purchase a fraction or continuous amount of a ticket
- Figure 3.1 illustrates the discrete choice and continuous choices
  - Each point or commodity bundle yields same level of utility
  - As ability to consume more units of discrete commodity within a budget constraint increases
    - Discrete bundles blend together to form an indifference curve
- Generally in econometrics five or more discrete choices can be analyzed as a continuous choice problem
- Will assume that households have a relatively large number of choices in terms of number of units or volume of a commodity
  - Can investigate household behavior as a continuous choice problem

Figure 3.1 Airline tickets representing a discrete commodity



Continuous Choice

- Principle of completeness or universality of markets
  - Prices are based on assumption that commodities are traded in a market and are publicly quoted
- Price for a commodity could be negative
  - Household is paid to consume commodity
    - Example: pollution
- We will assume
  - All prices are positive
  - All prices are constant
    - Implies that households are **price takers**
      - Take market price for a commodity as given

Continuous Choice

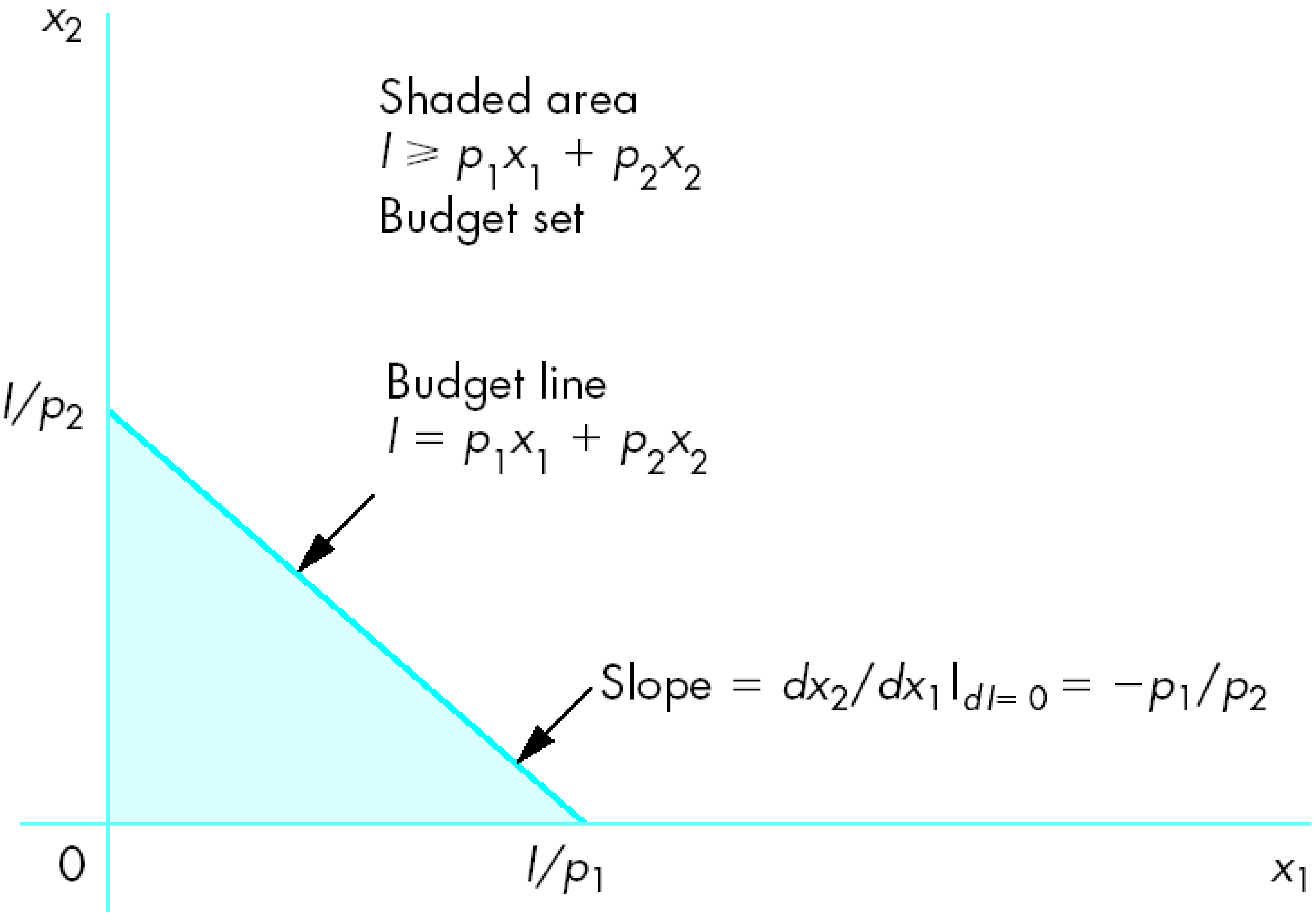
- A household is constrained by a budget set (also called a feasible set)
  - $p_1x_1 + p_2x_2 \leq I$
    - p_1x_1 (p_2x_2) represents expenditure for x_1(x_2) per-unit price times quantity
      - For example, if x_1 is your level of candy consumption at school, then p_1x_1 is your total expenditure on candy
- Total expenditures on x_1 and x_2 cannot exceed this level of income
- Budget set contains all possible consumption bundles that household can purchase
- Boundary associated with this budget set is **budget line** (also called a **budget constraint**)
  - $p_1x_1 + p_2x_2 = I$

Continuous Choice

- Bundles that cost exactly I are represented by budget line in Figure 3.2
  - Bundles below this line are those that cost less than I
- Budget line represents all the possible combinations of the two commodities a household can purchase at a particular time
- Consumption bundles on budget line represent all the bundles where household spends all of its income purchasing the two commodities
- Bundles within shaded area but not on budget line represent bundles household can purchase and have some remaining income
- Rearranging budget line by subtracting p_1x_1 from both sides and dividing by p_2 results in

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2} x_1$$

Figure 3.2 Budget set



Continuous Choice

- Previous equation indicates how much of commodity x_2 can be consumed with a given level of income I and x_1 units of commodity 1
- If a household only consumes x_2 ($x_1 = 0$)
  - Amount of x_2 consumed will be household's income divided by price per unit of x_2, I/p_2
  - In Figure 3.2, this corresponds to budget line's intercept on vertical axis
- If only x_1 is purchased ($x_2 = 0$), then amount of x_1 purchased is I/p_1, corresponding to horizontal intercept
- Slope of budget line measures rate at which market is willing to substitute x_2 for x_1 or

$$\left.\frac{dx_2}{dx_1}\right|_{dI=0} = -\frac{p_1}{p_2}$$

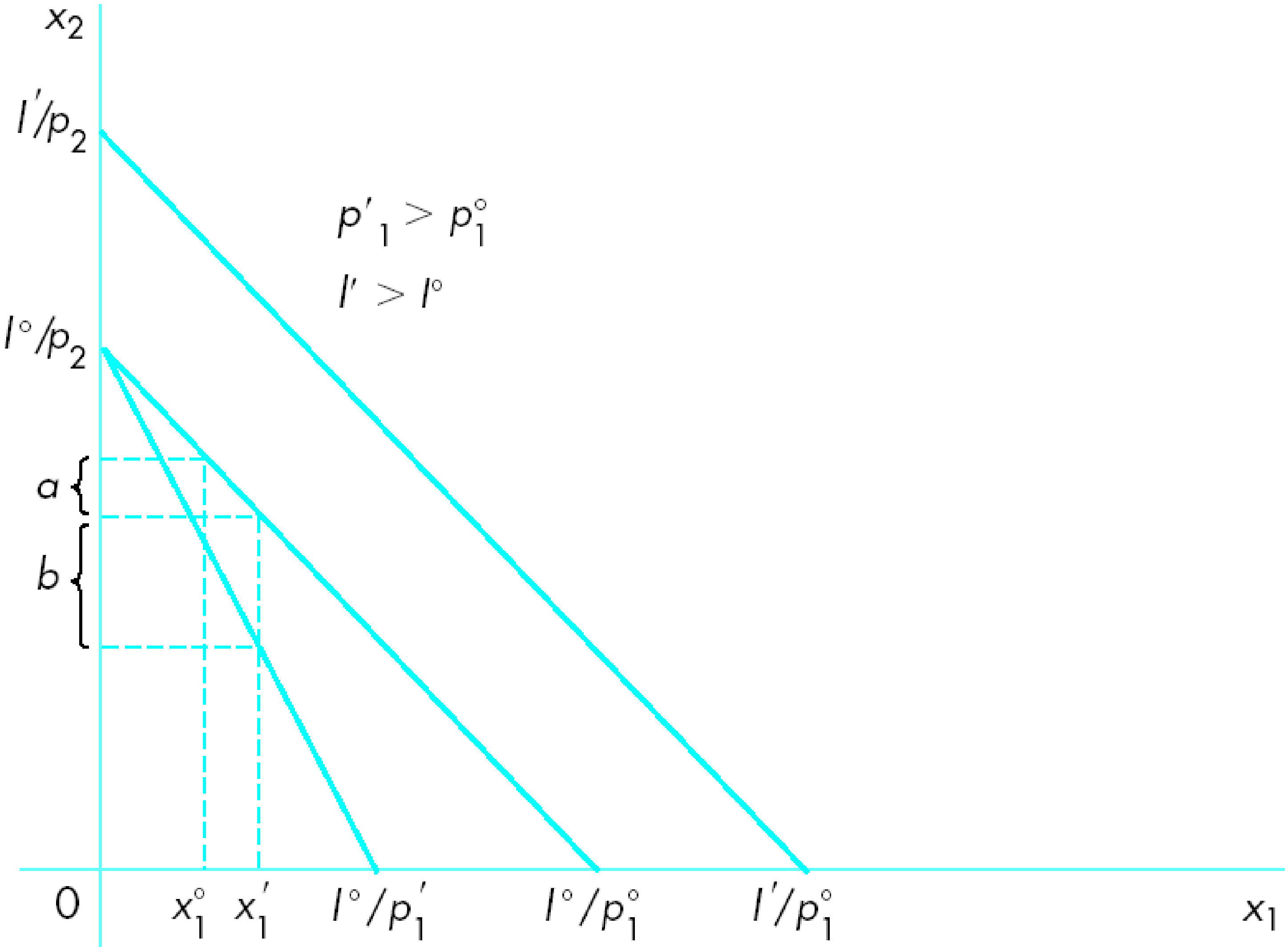
Continuous Choice

- dI = 0 indicates that income is remaining constant
  - So a change in income is zero
- If a household consumes more of x_1
  - It will have to consume less of x_2 to satisfy the budget constraint
  - Called **opportunity cost** of consuming x_1
  - Slope of budget line measures this opportunity cost
    - As rate a household is able to substitute commodity x_2 for x_1

Continuous Choice

- A change in price will alter slope of budget line and opportunity cost
- For example, as illustrated in Figure 3.3, an increase in the price of p_1 will tilt budget line inward
  - Does not change vertical intercept
- Price of x_2 has not changed
  - If household does not consume any of x_1, it can still consume same level of x_2
- Opportunity cost does change
  - Rate a household can substitute x_2 for x_1 has increased
- Increasing consumption of x_1 results in a decrease in x_2
- A change in income does not affect this opportunity cost
  - Income is only an intercept shifter
    - Does not affect slope of budget line

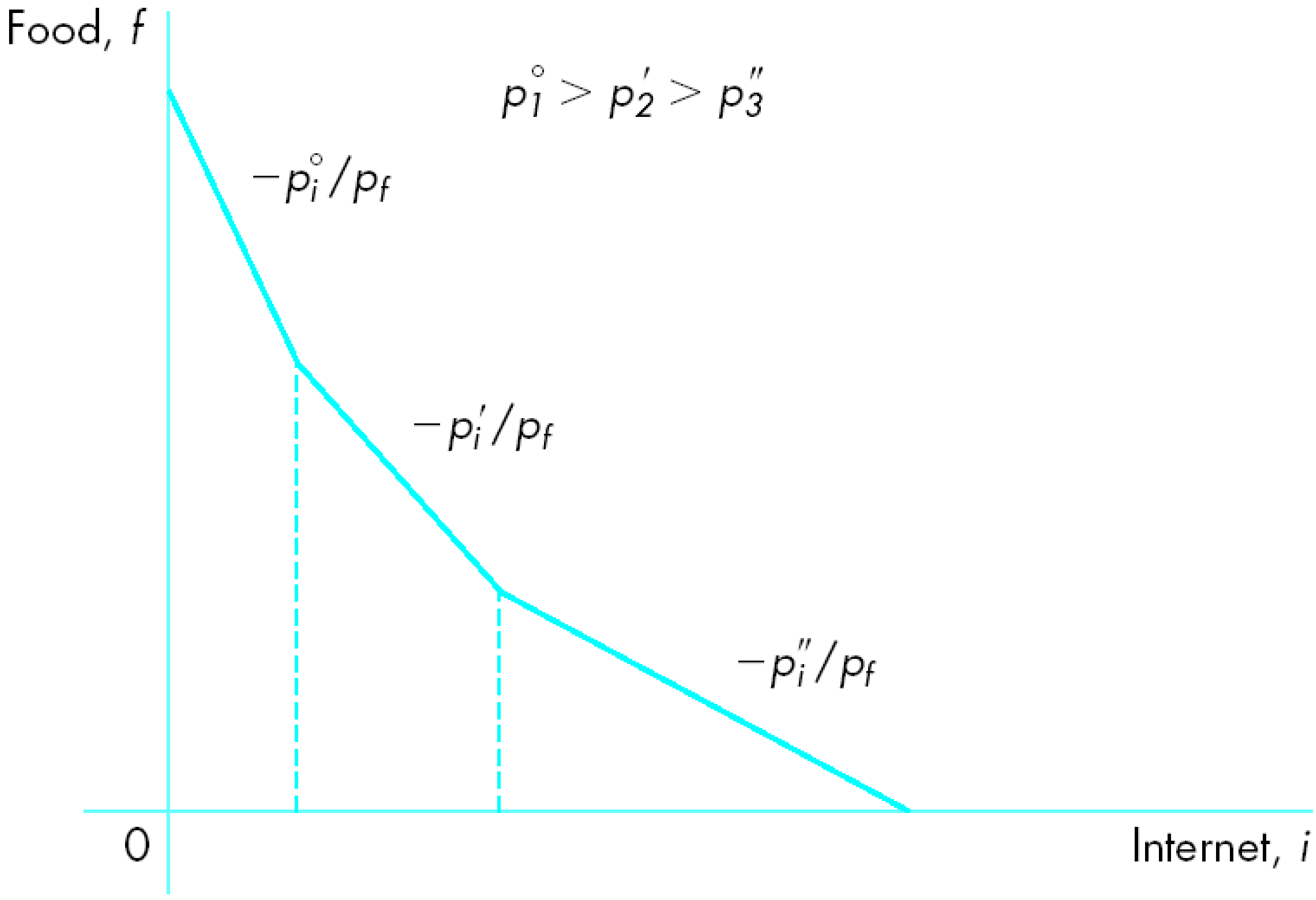
Figure 3.3 Increased opportunity cost …



Nonlinear Budget Constraint

- A budget constraint is linear
  - Only if per-unit commodity prices are constant over all possible consumption levels
- In some markets, prices will vary depending on quantity of commodity purchased
- Firms may offer a lower per-unit price if a household is willing to purchase a larger quantity of commodity
  - For example, per-unit price of breakfast cereals is lower when purchased in bulk
- Quantity discounts result in a nonlinear (convex) budget constraint
  - Price ratio varies as quantity of a commodity changes for a given income level
- Figure 3.4 shows a convex budget constraint with quantity discounts for Internet access and an assumed constant price for food, p_f
  - Price per minute for limited Internet usage is higher than price for moderate usage
    - Lowest price per minute is reserved for Internet addict

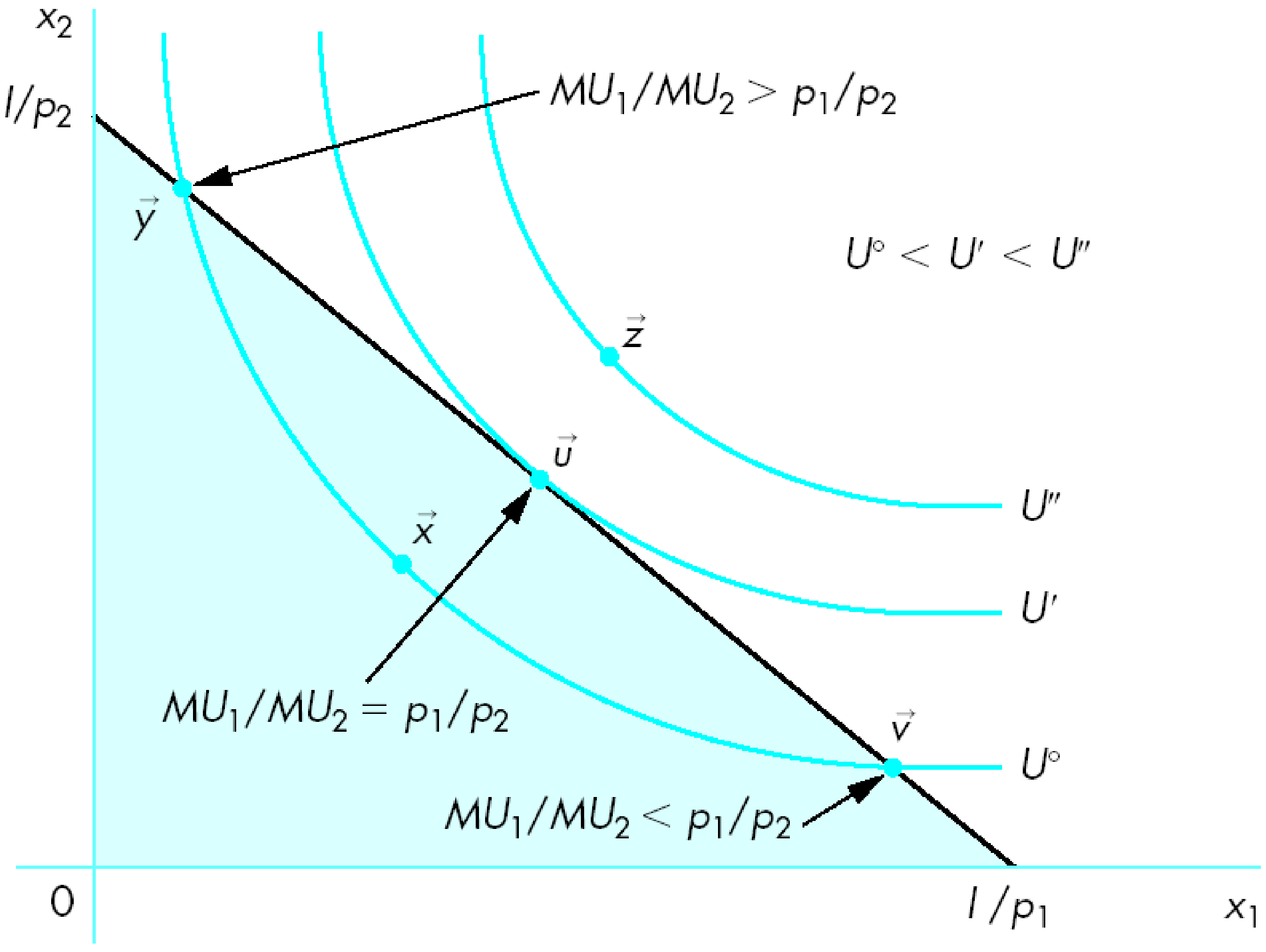
Figure 3.4 Quantity discount resulting in a convex budget constraint



Household's Maximum-Utility Commodity Bundle

- Budget constraint, along with a household's preferences, provides information needed to determine consumption bundle that maximizes a household's utility
- Indifference curves contain information on a household's preferences
- Superimposing budget constraint on household's indifference space (map) results in Figure 3.5
  - Budget line indicates possible combinations of x_1 and x_2 that can be purchased at given prices of x_1 and x_2 and income
- Moving along budget line, possible combinations of x_1 and x_2 change
  - But household's income and market prices remain constant

Figure 3.5 Budget set superimposed on the indifference map



Household's Maximum-Utility Commodity Bundle

- Indifference curves U°, U', and U'' indicate various combinations of x_1 and x_2 on a particular indifference curve that offer same level of utility
  - For example, moving along indifference curve U°
    - Total utility does not change but consumption bundles containing x_1 and x_2 do
- However, shifting from indifference curve U° to U' does increase total utility

Household's Maximum-Utility Commodity Bundle

- According to Nonsatiation Axiom
  - Household will consume more of the commodities if possible
- However, a household cannot consume a bundle beyond its budget constraint
- Within shaded area of budget set
  - Household has income to purchase more of both commodities and increase utility
- For utility maximization subject to a given level of income (represented by budget line)
  - Household will pick a commodity bundle on budget line
- Moving down budget line results in household's total expenditures remaining constant
  - But level of utility increases

Household's Maximum-Utility Commodity Bundle

- As household moves down budget line
  - Combinations of x_1 and x_2 purchased are changing
    - Commodity x_1 is being substituted for x_2
  - Household is shifting to higher indifference curves
    - Total utility is increasing
- At all other bundles except for u
  - Budget constraint cuts through an indifference curve, so utility can be increased

Household's Maximum-Utility Commodity Bundle

- However, at u budget constraint is tangent to an indifference curve
  - Indicates that there is no possibility of increasing utility by moving in either direction along the budget constraint
- In reality, complications may prevent a consumer from reaching this theoretical maximum level of utility
  - Consumer tastes change over time due to new products, advertising
  - Consumers grow tired of some commodities
  - Commodity prices change over time
    - Households are constantly adjusting their purchases to reflect these changes

Tangency Condition

- Geometrically, tangency at commodity bundle u is where slope of budget constraint exactly equals slope of indifference curve at point u

$$\left.\frac{dx_2}{dx_1}\right|_{dU=0} = -MRS(x_2 \text{ for } x_1)$$

  - dU = 0 indicates utility remains constant along indifference curve
- Only at this tangency point are slopes of budget and indifference curves equal along budget line
  - Shown in Figure 3.5
- For utility maximization MRS should equal ratio of prices
  - MRS (x_2 for x_1) = $p_1 \div p_2$
  - Called **optimal choice** for a household
  - Price ratio is called economic rate of substitution
    - At utility-maximizing point of tangency economic rate of substitution equals marginal rate of substitution
- Per-unit opportunity cost is equal to how much household is willing to substitute one commodity for another

Marginal Utility Per Dollar Condition

- When deciding what commodities to spend its income on
  - Household attempts to equate marginal utility per dollar for commodities it purchases
  - Basic condition for utility maximization
- For k commodities, purchase condition is
  - $MU_1 \div p_1 = MU_2 \div p_2 = \ldots = MU_k \div p_k$
  - Expresses a household's equilibrium
- Equilibrium is a condition in which household has allocated its income among commodities at market prices in such a way as to maximize total utility

Marginal Utility Per Dollar Condition

- For a household to be in equilibrium
  - Last dollar spent on commodity 1 must yield same marginal utility as last dollar spent on commodity 2
    - As well as all other commodities
- If this does not hold, a household would be better off reallocating expenditures
- Marginal utility per dollar indicates addition in total utility from spending an additional dollar on a commodity
- If marginal utility per dollar for one commodity is higher than that for another commodity
  - Household can increase overall utility by
    - Spending one less dollar on commodity with lower marginal utility per dollar and
    - One more dollar on commodity with higher marginal utility per dollar

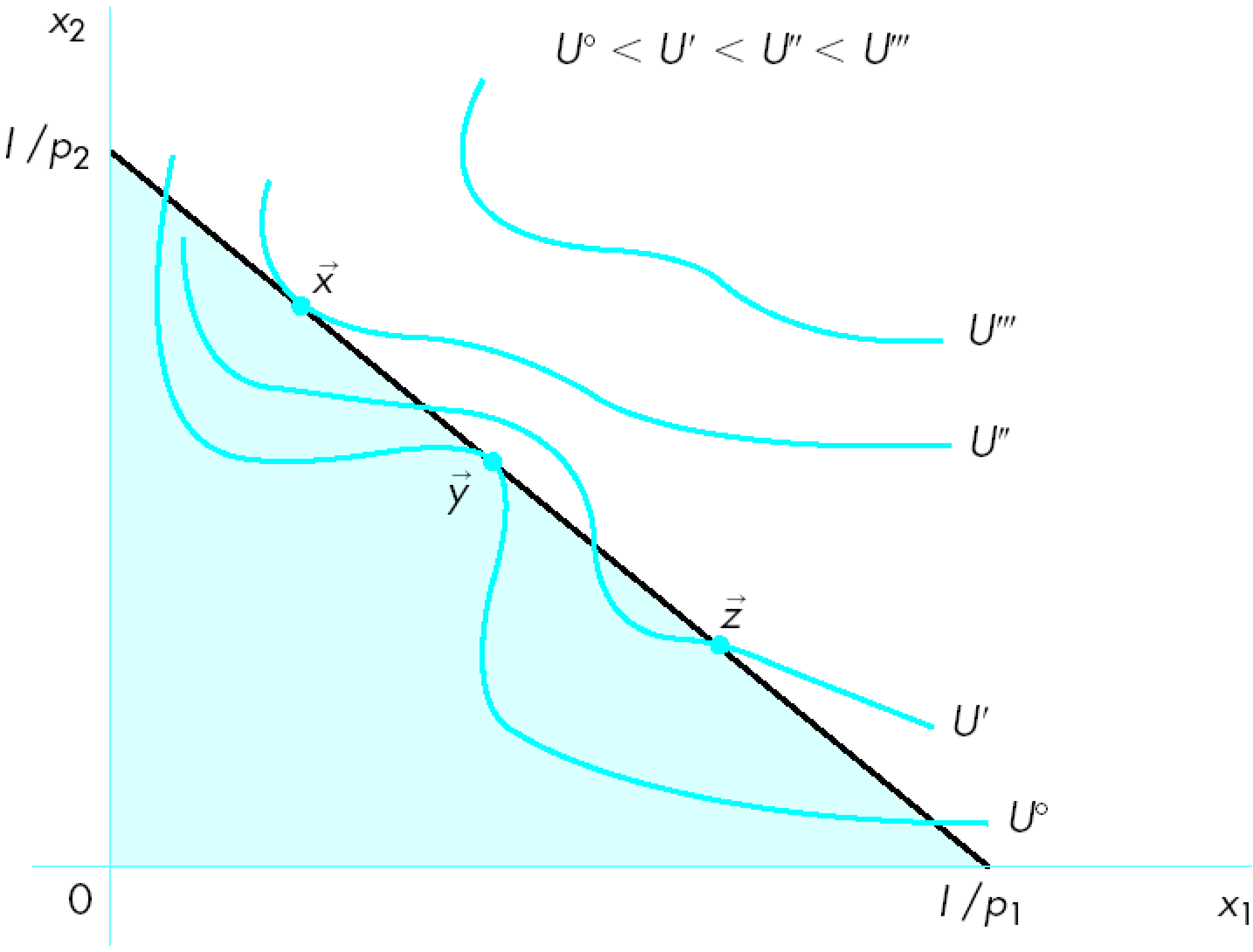
Marginal Utility Per Dollar Condition

- Suppose $MU_1 \div p_1 > MU_2 \div p_2$
  - Represented by bundle y in Figure 3.5
- Household is not in equilibrium
  - Not maximizing total utility given its limited income
- Household can increase total utility by increasing consumption of x_1 and decreasing consumption of x_2
- More additional total utility per dollar is received from x_1 than from x_2
- If one more dollar were used to purchase x_1 and one less dollar to purchase x_2,
  - Total expenditures would remain constant
    - But total utility would increase
- Household can continue to increase total utility by rearranging purchases until $MU_1 \div p_1 = MU_2 \div p_2$
  - Results in maximum level of utility for a given level of income
  - Corresponds to bundle u in Figure 3.5

Nonconvexity

- Optimal choice illustrated in Figure 3.5 involves consuming some of both commodities
  - Called an **interior optimum**
- Tangency condition associated with interior optimum is only a necessary condition for a maximum
- Tangency point y is inferior to a point of nontangency z
  - Shown in Figure 3.6
  - True maximum is point x
- If optimal choice involves consuming some of both commodities
  - Diminishing MRS Axiom (Strict Convexity) and tangency condition are a necessary and sufficient condition for a maximum

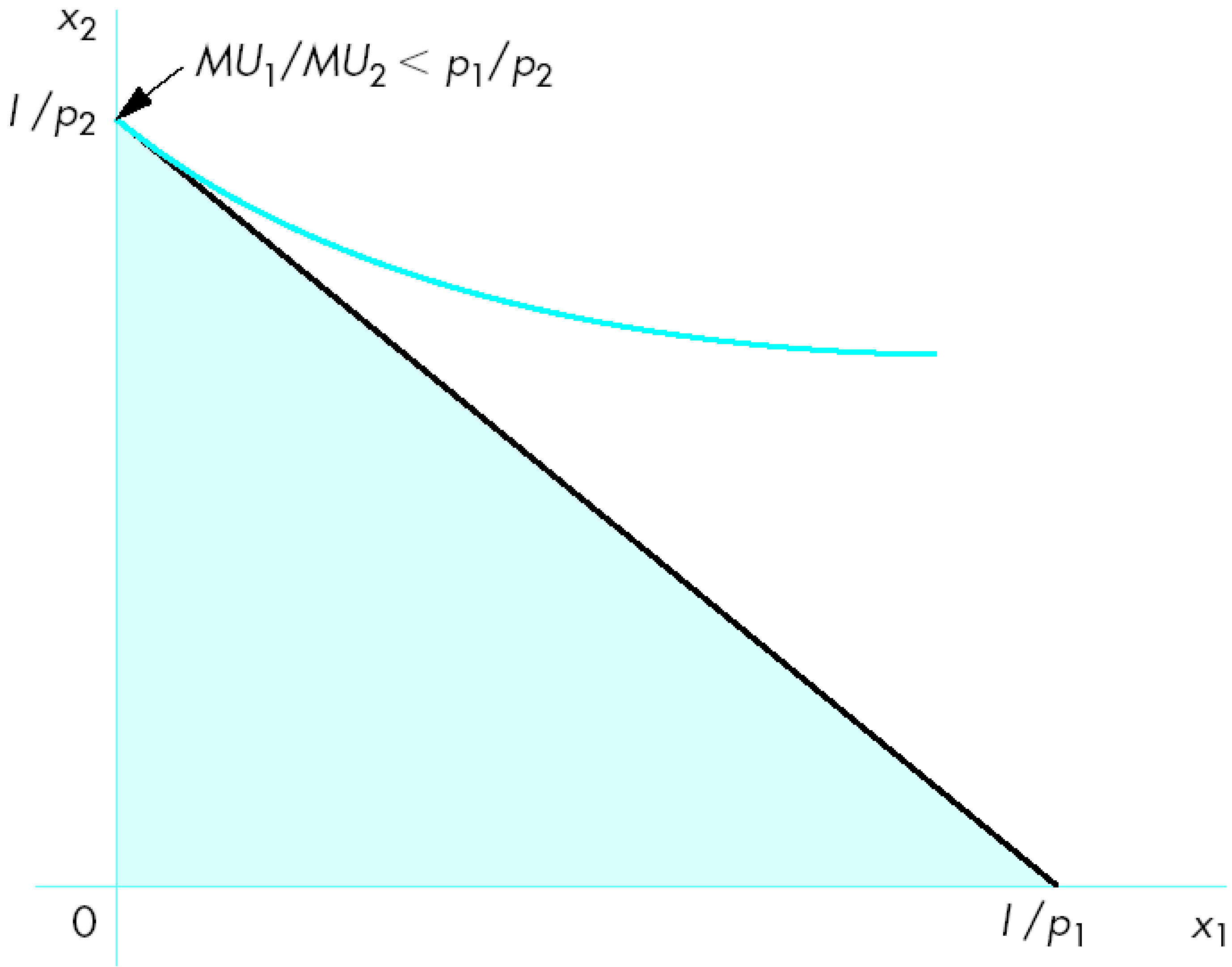
Figure 3.6 Tangency rule is only a necessary condition for a maximum



Corner Solution

- A couple may intend to purchase a combination of chicken and beef for this week's dinners
  - However, there is a weekly special on chicken, so couple decides to purchase only chicken
- This decision to not purchase a combination of the commodities is called a **corner solution** (boundary optimal)
  - Shown in Figure 3.7
- Utility-maximizing bundle
  - Consume only x_2 (chicken) and none of x_1 (beef)
- At this boundary optimal, tangency condition does not necessary hold
- Specifically, boundary-optimal condition in Figure 3.7 is
  - $MU_1 \div p_1 \leq MU_2 \div p_2$

Figure 3.7 Boundary-optimal solution with only commodity x_2 being consumed



Corner Solution

- Possible for marginal utility per dollar to be larger for commodity x_2
  - Household would prefer to continue substituting x_2 for x_1
- However, further substitution at the boundary is impossible
  - Where $x_1 = 0$
- Corner solution will always occur when Diminishing MRS Axiom is violated over whole range of possible commodity bundles

Corner Solution

- If a household has preferences, it may consume only x_2 and purchases I/p_2 units
  - Shown in Figure 3.8
- Similarly, for a household with strictly concave preferences (increasing MRS)
  - Extremes are preferred so optimal allocation will be at an extreme boundary
- Occurs where marginal utility per dollar for one of commodities is maximized
  - Shown in Figure 3.9
    - Results in household consuming only x_2 and purchasing I/p_2 units
- Point A in Figure 3.9
  - Tangency condition results in minimizing rather than maximizing utility for a given level of income

Figure 3.8 Corner solution for perfect substitutes

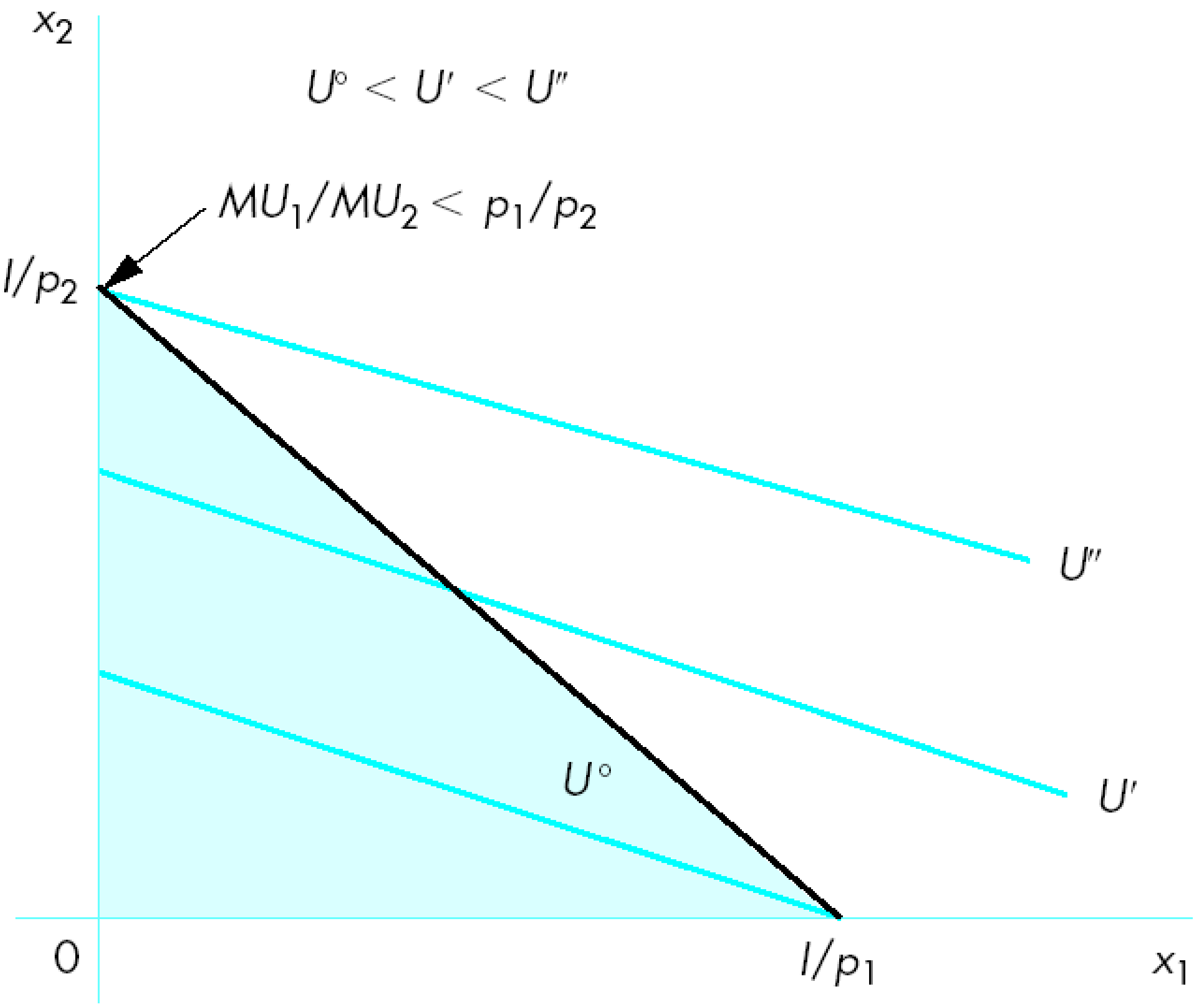
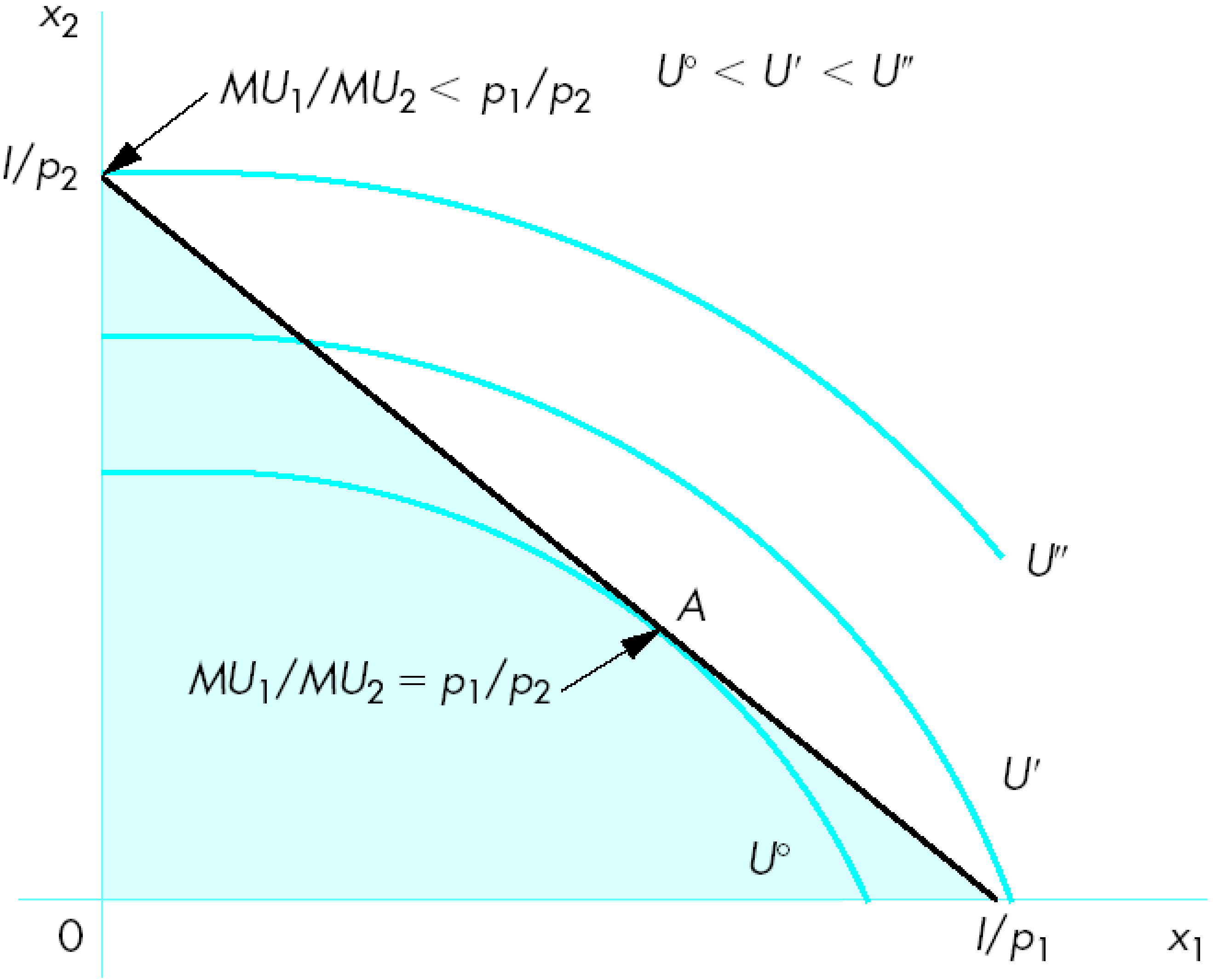


Figure 3.9 Corner solution with strictly concave preferences



Lagrangian

- Mathematically, maximum level of utility is determined by
  - Max U = max $U(x_1, x_2)$, s.t. $I = p_1x_1 + p_2x_2$
  - (x_1, x_2),
    - $U(x_1, x_2)$ is utility function representing household's preferences
- Budget constraint is written as an equality
  - Given Nonsatiation Axiom household will spend all available income rather than throw it away
- Lagrangian is

$$\mathcal{L}(x_1, x_2, \lambda) = U(x_1, x_2) + \lambda(I - p_1x_1 - p_2x_2)$$

Lagrangian

- F.O.C.s are

$$\frac{\partial \mathscr{L}}{\partial x_1} = \frac{\partial U}{\partial x_1} - \lambda^* p_1 = 0,$$

$$\frac{\partial \mathscr{L}}{\partial x_2} = \frac{\partial U}{\partial x_2} - \lambda^* p_2 = 0,$$

$$\frac{\partial \mathscr{L}}{\partial \lambda} = I - p_1 x_1^* - p_2 x_2^* = 0$$

- These F.O.C.s along with the three variables can be solved simultaneously for optimal levels of x_1*, x_2*, and λ*
- In terms of second-order-condition (S.O.C.), Diminishing MRS Axiom is sufficient to ensure a maximum
- Figure 3.6 illustrates possibility of not maximizing utility when this axiom is violated

Implications of the F.O.C.s Tangency Condition

- Rearranging first two F.O.Cs by adding l* times the price to both sides yields
  - $\partial U \div \partial x_1 = \lambda^* p_1$
  - $\partial U \div \partial x_2 = \lambda^* p_2$
- Taking the ratio gives

$$\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = \frac{MU_1}{MU_2} = MRS(x_2 \text{ for } x_1) = \frac{p_1}{p_2}$$

- For maximizing utility, how much a household is willing to substitute x_2 for x_1 is set equal to economic rate of substitution, p_1/p_2
- Result is identical to tangency condition for utility maximization between budget line and indifference curve

Marginal Utility per Dollar Condition

- "Dig where the gold is, unless you just need some exercise" (John M. Capozzi)
  - Gold is marginal utility per dollar for a commodity
    - So dig (consume) where marginal utility per dollar is highest
- Results in equating marginal utility per dollar for all commodities consumed
- From F.O.C.s, this marginal utility per dollar condition is derived by solving for λ* in first two equations

$$\lambda^* = \frac{\partial U/\partial x_1}{p_1} = \frac{\partial U/\partial x_2}{p_2}$$

$$\lambda^* = MU_1/p_1 = MU_2/p_2$$

- At utility-maximizing point, each commodity should yield same marginal utility per dollar
  - Each commodity has an identical marginal-benefit to marginal-cost ratio

Marginal Utility per Dollar Condition

- Extra dollar should yield same additional utility no matter which commodity it is spent on
  - Common value for this extra utility is given by Lagrangian multiplier
    - λ^* of income I
- Multiplier can be regarded as marginal utility of an extra dollar of consumption expenditure
  - Called **marginal utility of income (MUI)**
    - $\lambda^* = \partial U/\partial I = MU_I$
- Solving each of first two conditions for price yields
  - $p_j = MU_j \div \lambda^*$ for every commodity j
    - Price of commodity represents a household's evaluation of the utility associated with last unit consumed
- Price for every commodity j represents how much a household is willing to pay for this last unit of the commodity
  - If $MU_j/\lambda^* < p_j$ a household will not purchase any more units of commodity j

Figure 3.10 Governmental rationing

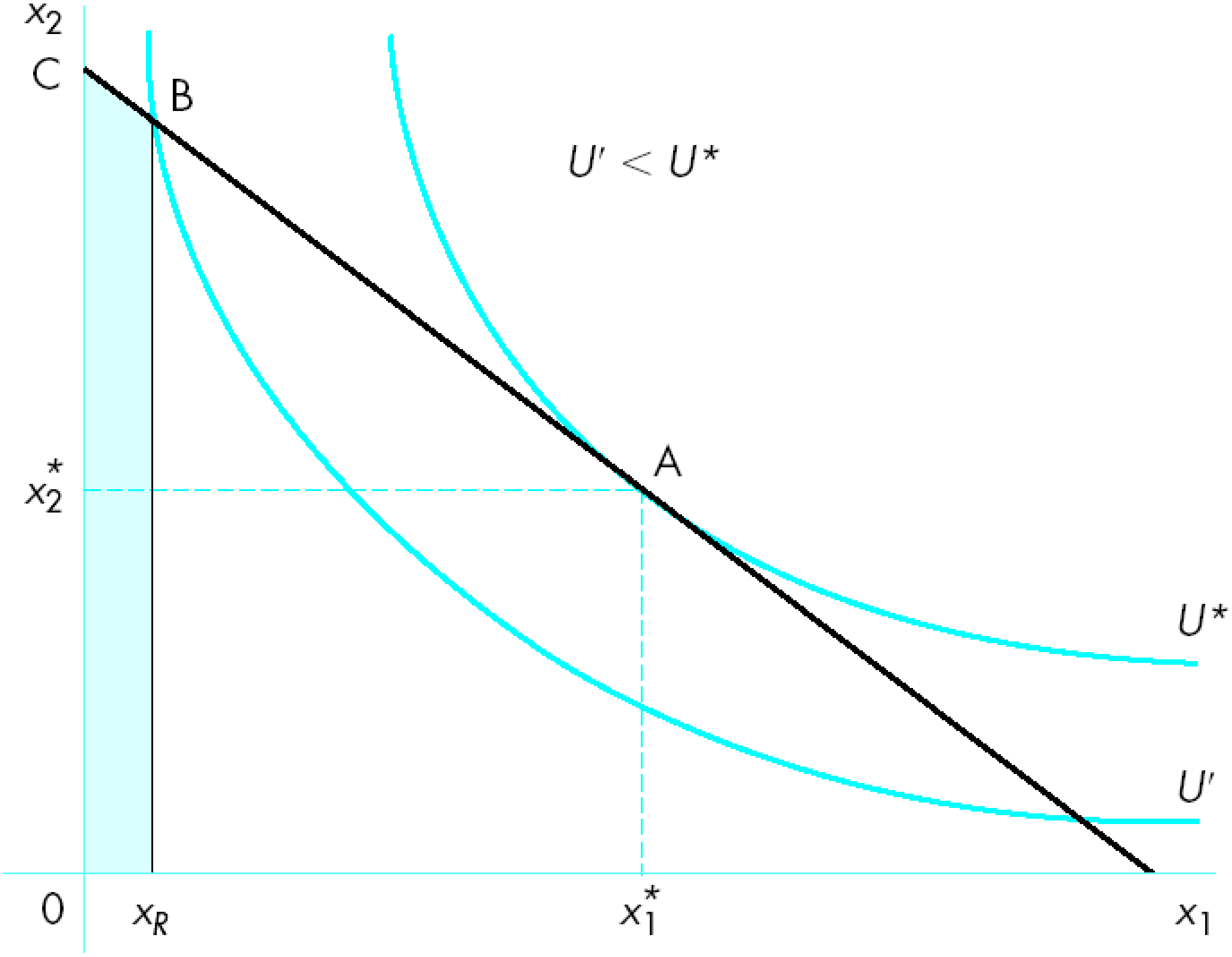


Figure 3.11 Taxation and the lump-sum principle

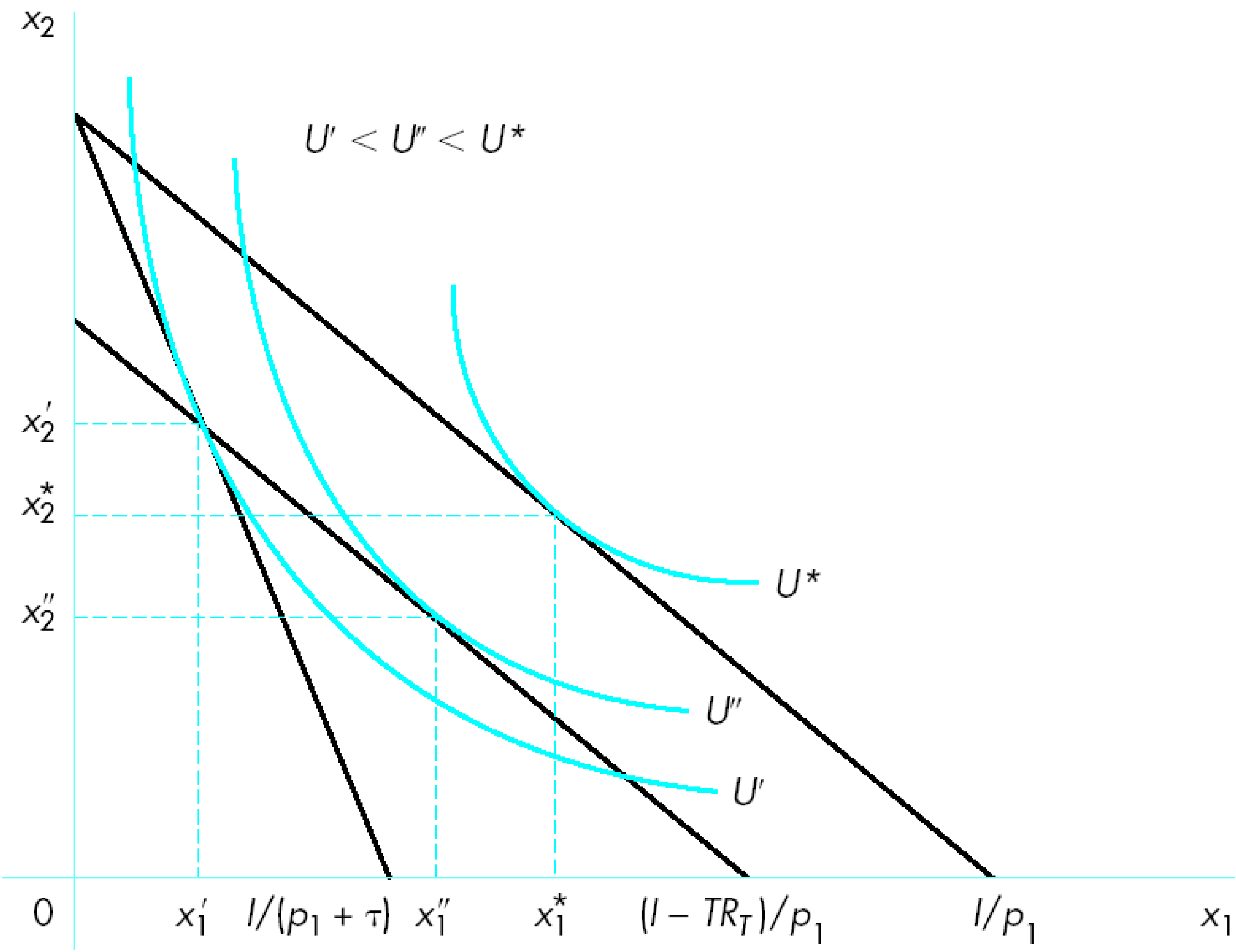


Figure 3.12 Subsidy