

## Important questions 6

Question 1: Let  $A = \{a, b, c, d\}$ ,  $B = \{1, 2, 3\}$ .

- (1) How many relations are there on  $A$ ?  
[a] 256. [b] 65 536. [c] 4096. [d] 16384.
- (2) How many relations on  $A$  of size one?  
[a] 16. [b] 4. [c] 8. [d] 12.
- (3) How many relations on  $B$  are there ?  
[a] 256. [b] 81. [c] 512. [d] 320.
- (4) How many relations on  $B$  of size 3 are there?  
[a] 20. [b] 84. [c] 30. [d] 220.
- (5) how many relations from  $A$  to  $B$  are there ?  
[a] 64. [b] 128. [c] 256. [d] 4096.
- (6) how many relations from  $A$  to  $B$  of size 4 are there ?  
[a] 495. [b] 11880. [c] 2970. [d] 5940.
- (7)  $\{(1, a), (2, d), (1, c)\}$  is a relation :  
[a] on  $A$ . [b] from  $A$  to  $B$ . [c] on  $B$ . [d] from  $B$  to  $A$ .
- (8)  $\{(a, a), (a, c), (c, c), (b, b), (d, d), (c, a)\}$  is a relation on  $A$  such that :  
[a] Reflexive. [b] Symmetric. [c] Transitive. [d] reflexive, symmetric and transitive.
- (9)  $\{(a, b), (b, a), (a, c), (a, a), (b, c), (b, b)\}$  is a relation on  $A$  such that :  
[a] antisymmetric. [b] symmetric. [c] transitive. [d] reflexive.
- (10)  $\{(1, 1), (1, 2), (2, 2), (3, 3)\}$  is a relation on  $B$  such that :  
[a] Reflexive . [b] Antisymmetric .  
[c] Symmetric. [d] Reflexive, Antisymmetric and Transitive.
- (11)  $\{(a, a), (b, b), (c, c), (d, d)\}$  is a relation on  $A$  such that:  
[a] reflexive . [b] transitive.  
[c] antisymmetric and symmetric. [d] satisfies all properties stated in [a] , [b] , and [c].

**Question 2:** Let  $\mathfrak{R} \subseteq \mathbb{Z} \times \mathbb{Z}$ , where  $\mathbb{Z}$  is the set of integers.

If  $\mathfrak{R} = \{(a, b) : a < b, a, b \in \mathbb{Z}\}$ . Then  $\mathfrak{R}$  is :

- [a] Reflexive and Transitive.      [b] Symmetric and Transitive.  
[c] Antisymmetric and Reflexive.      [d] Antisymmetric and Transitive.

**Question 3:** Let  $\mathcal{R} = \{(a, b) : a \leq b, a, b \in \mathbb{Z}\}$ . Then  $\mathcal{R}$  is :

- [a] Reflexive, Transitive and Symmetric.      [b] Reflexive, Antisymmetric and transitive.  
[c] Antisymmetric, Reflexive, and Symmetric.      [d] Transitive, Reflexive, and Symmetric.

**Question 3:** Let  $\mathcal{R} = \{(a, b) : a = b, a, b \in \mathbb{Z}\}$ . Then  $\mathcal{R}$  :

- [a] Is reflexive and antisymmetric.      [b] Is symmetric.  
[c] Is transitive.      [d] Has all properties stated in [a], [b], and [c].

**Question 4:** Negate the following:

- [a]  $p \vee q \vee (\neg p \wedge \neg q \wedge r)$ .  
[b]  $p \vee q \vee [(\neg p \wedge \neg q) \vee r]$ .  
[c]  $p \vee q \vee [\neg p \wedge (\neg q \vee r)]$ .

**Solution:**

$$\begin{aligned} \text{[a]} \quad & \neg[p \vee q \vee (\neg p \wedge \neg q \wedge r)] \Leftrightarrow \\ & \neg[(p \vee q) \vee (\neg p \wedge \neg q)] \wedge \neg[(p \vee q) \vee r] \Leftrightarrow \\ & \neg[(p \vee q) \vee \neg(p \vee q)] \wedge \neg[p \vee q \vee r] \Leftrightarrow \\ & \neg[T \wedge (p \vee q \vee r)] \Leftrightarrow \neg[p \vee q \vee r] \Leftrightarrow \neg p \wedge \neg q \wedge \neg r. \\ \text{[b]} \quad & \neg[p \vee q \vee [(\neg p \wedge \neg q) \vee r]] \Leftrightarrow \\ & \neg[(p \vee q) \vee (\neg p \wedge \neg q) \vee r] \Leftrightarrow \\ & \neg[((p \vee q) \vee \neg(p \vee q)) \vee r] \Leftrightarrow \neg[T \vee r] \Leftrightarrow \neg T \Leftrightarrow F. \end{aligned}$$

$$\begin{aligned}
[c] \neg[p \vee q \vee [\neg p \wedge (\neg q \vee r)]] &\Leftrightarrow \\
\neg[(p \vee q) \vee \neg p] \wedge [(p \vee q) \vee (\neg q \vee r)] &\Leftrightarrow \\
\neg[(q \vee p) \vee \neg p] \wedge (p \vee (q \vee \neg q) \vee r) &\Leftrightarrow \\
\neg[q \vee (p \vee \neg p)] \wedge [p \vee T \vee r] &\Leftrightarrow \neg[T \wedge T] \Leftrightarrow \neg T \Leftrightarrow F.
\end{aligned}$$