FLUID MECHANICS

SOLUTIONS OF SHEETS

SHEET 1

4. $\rho = \frac{pM}{10^3 RT} = \frac{1.0132\times10^5\times29}{1000\times8.314\times(273+25)} = 1.187$ kg.m^{-3}

5. $\mu = Ae^{\frac{E}{RT}} \rightarrow 1.48 = Ae^{\frac{E}{8.314\times293}}$ (i)

$0.102 = Ae^{\frac{E}{8.314\times333}}$ (ii)

Dividing we get: $14.51 = e^{\left(\frac{1}{8.314\times293}-\frac{1}{8.314\times333}\right)E}$

Hence $\boldsymbol{E} = \mathbf{54245}$ **J.mol^{-1}**

6. $A = 0.2 \times 0.6 = 0.12$ m^2

$\frac{\Delta v}{\Delta y} = \frac{0.3-0}{0.003} = 100$ s^{-1}

$F = 0.12 \times 0.120 \times 100 = \mathbf{1.44}$ **N**

SHEET 2

1. $A = \frac{\pi}{4} \times 0.045^2 = 0.00159$ m²

   $V = \frac{0.75}{780} = 0.000961$ m³

   $h = \frac{0.000961}{0.00159} = 0.6045$ m

   $p = 780 \times 9.81 \times 0.6045 = 5930.9$ Pa $\equiv$ **59.31 millibar**

2. $V_{water} = \frac{1060}{1000} = 1.06$ m³ $\quad\quad h_{water} = \frac{1.06}{0.64} = 1.656$ m

   $V_{oil} = \frac{275}{830} = 0.3313$ m³ $\quad\quad h_{oil} = \frac{0.3313}{0.64} = 0.518$ m

   $p = 1000 \times 9.81 \times 1.656 + 830 \times 9.81 \times 0.518 = 20463$ Pa $\equiv 0.202$ atm

   $p = 0.202 \times 14.7 =$ **2.97 psi**

3. $\Delta p = 250$ Pa $= 1000 \times 9.81h \rightarrow \boldsymbol{h = 0.0254}$ **m**

4. $\Delta p = 0.1$ atm $= 10130$ Pa $= 1000 \times 9.81h \rightarrow \boldsymbol{h = 1.015}$ **m**

5. $p_{water} = 1000 \times 9.81 \times 0.28 + p_{applied} = 2746.8 + p_{applied} = 93000$

   $p_{applied} = 90253$ Pa $\equiv$ **90.25 kPa**

   $p_{bottom} = 93000 + 13600 \times 9.81 \times 0.08 = 103673$ Pa $\equiv$ **103.67 kPa**

6. When the pressure is 50000 Pa, it differs from the pressure on the free liquid surface by 100000 – 50000 = 50000 Pa

   $50000 = 1000 \times 9.81h \rightarrow h = 5.097$ m

   When the pressure is 75000 Pa, the difference with the level at free surface = 100000 – 75000 = 25000 Pa

   $25000 = 1000 \times 9.81h' \rightarrow h' = 2.548$ m

   The level **decreases** by $5.097 - 2.548 =$ **2.548 m**

7. $r = 1.75$ m $\rightarrow V = \frac{4\pi}{3} \times 1.75^3 = 22.45$ m³

   $\rho = \frac{26}{22.45} = 1.158$ kg.m^{-3}.

   $1.158 = \frac{p \times 17}{10^3 \times 8.314 \times 298} \rightarrow p = 2869439\ Pa \equiv$ **28.7 MPa**

8. The pressure at the 1" opening = the pressure exerted by the load:

   $F = 2000 \times 0.454 = 908$ Kg$_f$ $\equiv 908 \times 9.81 = 8907.5$ N

Pressure exerted by the load = $\frac{8907.5}{\frac{\pi}{4}\times(3\times 0.0254)^2}$ = 1953233 Pa

Force at the 1" opening = $1953233 \times \frac{\pi}{4} \times (1 \times 0.0254)^2 = 989.7$ N

Applying the lever rule: $989.7 \times 1 = F \times 15 \rightarrow F = 66$ N ≡ 6.72 Kg_f ≡ $\mathbf{14.81}\ \boldsymbol{lb_f}$

9. When the gauge reads full, this means that the tank is full of fuel. This means that the pressure exerted on the bottom of the tank = $680 \times 9.81 \times 0.3 = 2001.24$ Pa

   The actual height of fuel = $0.3 - 0.02 - h = 0.28 - h$ m.

   The actual pressure = $680 \times 9.81 \times (0.28 - h) + 0.02 \times 9.81 \times 1000 = 2064.024 - 6670.8h$

Hence: $2064.024 - 6670.8h = 2001.24 \rightarrow \boldsymbol{h} = \mathbf{0.0094}\ \boldsymbol{m} \equiv \mathbf{9.4\ mm}$

10. Total pressure = $1000 \times 9.81 \times 0.5 + 800 \times 9.81 \times 0.5 = 8829$ Pa

    $8829 = 1000 \times 9.81 \times 1.1 \sin\theta \rightarrow \boldsymbol{\theta} \approx \mathbf{55^o}$

SHEET 3

1. $A = 0.25\pi \times (12 \times 0.0254)^2 = 0.07296$ m^2, $v_1 = \frac{62000}{3600 \times 0.07296 \times 920}$ = **0.256 m.s^{-1}.**

2. $A_1 = 0.25\pi \times (1.5 \times 0.0254)^2 = 0.00114$ m^2.
   $v_1 = 1.6 \rightarrow Q = A.v = 1.6 \times 0.00114 = \mathbf{0.001824\ m^3.s^{-1}}$
   $D_1^2.v_1 = D_2^2.v_2 \rightarrow 1.5^2 \times 1.6 = 1^2 v_2 \rightarrow \boldsymbol{v_2} = \mathbf{3.6\ m.s^{-1}}$

3. $A.v = A_1.v_1 + A_2.v_2$ $\qquad \frac{1}{4}\pi D^2 v = \frac{1}{4}\pi D_1^2 v_1 + \frac{1}{4}\pi D_2^2 v_2$
   Hence: $(0.0254 \times 8)^2 \times 0.8 = (0.0254 \times 6)^2 \times v_1 + (0.0254 \times 4)^2 \times 1.2 \rightarrow$
   $\boldsymbol{v_1} = \mathbf{0.89}$ **m.s^{-1}**

4. $A = 0.25\pi \times (2 \times 0.0254)^2 = 0.00203$ m^2, $v_1 = \frac{6}{3600 \times 0.00203}$ = **0.822 m.s^{-1}**
   $\frac{1}{2}\rho v_1^2 + \rho g h_1 + p_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + p_2$
   Since the velocity is the same and $h_1 = 0, p_2 = 0$, therefore:
   $p_1 = \rho g h_2 = 1000 \times 9.81 \times 5 = 40950$ Pa $\equiv \mathbf{0.49\ bar}$

5. $p = 700 \times 9.81 \times 4 = 27468$ Pa
   Since the nozzle is on the same horizontal level of the oil – water interface, hence, applying Bernoulli equation between interface and the top of the jet:
   $27468 + 0 + 0 = 0 + 0 + 1000 \times 9.81h \rightarrow \mathbf{2.8\ m}$

6. Applying Bernoulli equation between interface and nozzle:
   $27468 + 0 + 0 = 0 + \frac{1}{2} \times 1000v^2 \rightarrow v = 7.41$ m.s^{-1} (At nozzle).
   If (1) denotes the horizontal pipe and (2) the nozzle, then:
   $D_1^2.v_1 = D_2^2.v_2 \rightarrow 0.2^2 \times v_1 = 0.1^2 \times 7.41 \rightarrow v_1 = 1.853\ \mathrm{m.s^{-1}}$
   Applying Bernoulli equation between the level of the horizontal pipe in the tank and inside this pipe:
   $27468 + 0 + 1000 \times 9.81 \times 1 = p + \frac{1}{2} \times 1000 \times 1.853^2 + 0 \rightarrow \boldsymbol{p} = \mathbf{35561\ Pa}$

7. $\frac{d}{D} = \frac{1}{2} \rightarrow \left(\frac{d}{D}\right)^4 = \frac{1}{16}$
   $$\frac{2(\rho_l - \rho)}{\rho} gh = \frac{2 \times (13600 - 1000)}{1000} \times 9.81 \times 0.12 = 29.67$$
   $$Q = 0.98 \times \frac{\pi}{4}(1 \times 0.0254)^2 \sqrt{\frac{29.67}{\left(1 - \frac{1}{16}\right)}} = 0.00279\ \mathrm{m^3.s^{-1}} \equiv \mathbf{10.06\ m^3h^{-1}}$$

8. $\sqrt{H} - \sqrt{h} = \frac{C_d}{2}\left(\frac{d}{D}\right)^2 \sqrt{2g}.t$ $\quad$ $\sqrt{2.3} - 0 = \frac{0.65}{2}\left(\frac{0.06}{1.2}\right)^2 \sqrt{2 \times 9.81}.t$

$\boldsymbol{t = 421.4\ \mathbf{s}}$

9. $A_1 = 0.25\pi \times (2.05 \times 0.0254)^2 = 0.00213\ \text{m}^2$

$A_2 = 0.25\pi \times (3.71 \times 0.0254)^2 = 0.00697\ \text{m}^2$

$v_1 = \frac{Q}{0.00213} = 469.5Q$

$v_1 = \frac{Q}{0.00697} = 143.47Q$

$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + h_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + h_2$

$\frac{(469.5)^2 Q^2}{2\times 9.81} + \frac{56.3\times 1.0132\times 10^5}{14.7\times 1000\times 9.81} = \frac{(143.47)^2 Q^2}{2\times 9.81} + \frac{58.2\times 1.0132\times 10^5}{14.7\times 1000\times 9.81}$

$10197Q^2 = 40.89 - 39.56 = 0.0113\ \text{m}^3.\text{s}^{-1} \equiv \mathbf{41\ m^3.h^{-1}}$

10. Let h be the final equilibrium height in both tanks. The total volume of water in the two tanks is constant:

$\frac{1}{4}\pi \times 1.2^2 h_1 + \frac{1}{4}\pi 1^2 h_2 = \frac{1}{4}\pi \times 1.2^2 \times 1.5 + \frac{1}{4}\pi 1^2 \times 2.5$

$\mathbf{1.44}\boldsymbol{h_1} + \boldsymbol{h_2} = \mathbf{4.66}$

When $h_1 = h_2 = h$: $2.44h = 4.66 \rightarrow \boldsymbol{h} = \mathbf{1.91\ m}$

11. $\frac{120}{3600} = 0.033 = \frac{1}{4}\pi D_1^2 v_1 + \frac{1}{4}\pi D_2^2 v_2 + \frac{1}{4}\pi D_3^2 v_3$ $\quad$ Multiply both sides by $\frac{4}{\pi}$:

$0.042 = 0.04^2 \times 5 + 0.05^2 \times 4 + 0.06^2 v_3 \rightarrow \boldsymbol{v_3} = \mathbf{6.67\ m.s^{-1}}$

$0.033 = 0.25\pi \times 0.09^2 v_4 \rightarrow \boldsymbol{v_4} = \mathbf{5.19\ m.s^{-1}}$

12. $Q_{in} = 0.25\pi \times 0.12^2 \times 2.5 = 0.0283$ $\quad$ $Q_{out} = 0.25\pi \times 0.12^2 \times 1.9 = 0.0215$

Net flow to the tank = $0.0283 - 0.0215 = 0.0068\ \ \text{m}^3.\text{s}^{-1}$

Volume to be filled in the tank = $0.25\pi \times 0.75^2 \times (1 - 0.3) = 0.3092\ \text{m}^3$

Time required to fill the tank = $\frac{0.3092}{0.0068} = \mathbf{45.5\ s}$

SHEET 4

1. $A = 0.25\pi \times (12 \times 0.0254)^2 = 0.07296 \text{ m}^2$

$v = \frac{860}{3600 \times 0.07296}$ = 3.274 m.s^{-1}.

$\mu = 42 \times 10^{-6} \times 900 = 0.0378$ Pa.s

$Re = \frac{900 \times 3.274 \times 12 \times 0.0254}{0.0378}$ = $\mathbf{23760}$

2. $\Delta p = \frac{1}{2}\rho_2 v_2^2 + \rho_2 g h_2 + p_2 - \left(\frac{1}{2}\rho_1 v_1^2 + \rho_1 g h_1 + p_1\right) + \Delta p_{losses}$

$= \frac{1}{2} \times 1000 \times 3.5^2 + 1000 \times 9.81 \times 30 - \left(\frac{1}{2} \times 1000 \times 1^2\right) + 8 \times 1000 \times 9.81$

= 378405 Pa

$Q = 0.25\pi \times (1.5 \times 0.0254)^2 \times 3.5 = 0.004 \text{ m}^3.\text{s}^{-1}$

Minimum power requirement = $\frac{0.004 \times 378405}{0.65 \times 735}$ = $3.16 > 3$ **The pump is not adequate**

3. $A = \frac{\pi}{4} \times (18 \times 0.0254)^2 = 0.1641 \text{m}^2$

$v = \frac{\frac{250}{3600}}{0.1641}$ = 0.423 m.s^{-1}

$\frac{\Delta p}{L} = \frac{2 f \rho v^2}{D}$ $\quad \frac{\Delta p}{5000} = \frac{2 \times 0.003 \times 880 \times 0.423^2}{18 \times 0.0254} \rightarrow \Delta p = 10331.5$ Pa

Other head losses result in a pressure drop $= 35 \times 880 \times 9.81 = 302148$ Pa

Total pressure loss = $10331.5 + 302148 = 312479.5$ Pa

$\mathscr{P} = \frac{Q.\Delta P}{\boldsymbol{\eta}} = \frac{\frac{250}{3600} \times 312479.5}{\mathbf{0.75}}$ = 28933.3 W $\equiv$ **29 kW**

4. Total vertical head = $4 + 9 + 12\sin 25 = 18.07$ m

$A = \frac{\pi}{4} \times (2 \times 0.0254)^2 = 2.026 \times 10^{-3} \text{m}^2$

$v = \frac{\frac{24}{3600}}{0.002026}$ = 3.29 m.s^{-1}

$\Delta p = \frac{1}{2}\rho_2 v_2^2 + \rho_2 g h_2 + p_2 - \left(\frac{1}{2}\rho_1 v_1^2 + \rho_1 g h_1 + p_1\right) + \Delta p_{losses}$

$= \frac{1}{2} 1000 \times 3.29^2 + 1000 \times 9.81 \times 18.07 + 0 - (0 + 0 + 0) + 1000 \times 9.81 \times 1.2$

$= 194459$ Pa

$\mathscr{P} = \frac{Q.\Delta P}{\eta} = \frac{\frac{24}{3600}\times 194459}{0.65}$ = **1994 W ≡ about 2 kW**

5. $A = \frac{\pi}{4} \times (8 \times 0.0254)^2 = 0.0324\text{m}^2$

$v = \frac{\frac{108}{3600}}{0.0324} = 0.926$ m.s^{-1}

$\frac{\Delta p}{L} = \frac{2f\rho v^2}{D}$ $\quad \frac{\Delta p}{1000} = \frac{2\times 0.02\times 1080\times 0.926^2}{8\times 0.0254} \rightarrow \Delta p = 182298$ Pa

Head loss due to valves = $2\times\, 10 \times \frac{0.926^2}{2\times 9.81}$ = 0.874 m

Head loss due to elbows = $4\times\, 0.3 \times \frac{0.926^2}{2\times 9.81}$ = 0.0524 m

Δp due to valves + elbows = $(0.874 + 0.0524) \times 1000 \times 9.81 = 9088$ Pa

Δp_{total} = $182298 + 9088 = 191386$ Pa ≡ $\mathbf{191.4}$ **kPa**

6. The pressure loss due to contraction is calculated based on the highest fluid velocity, that is at the smaller of the two diameters.

$A = \frac{\pi}{4} \times (4 \times 0.0254)^2 = 0.0081\text{m}^2$

$v = \frac{\frac{98}{3600}}{0.0081} = 3.36$ m.s^{-1}

Head loss due to gradual contraction = $0.6\times \frac{3.36^2}{2\times 9.81}$ = 0.345 m

(At maximum value of K=0.6)

$\Delta p = 0.345 \times 1000 \times 9.81 = \mathbf{3384.5}$ **Pa**

7. $A = \frac{\pi}{4} \times (4 \times 0.0254)^2 = 0.0081\text{m}^2$

$v = \frac{\frac{14.2}{3600}}{0.0081} = 0.487$ m.s^{-1}

Since the friction coefficient is not given, the Reynolds number must be determined.

$Re = \frac{1200\times 0.487\times 4\times 0.0254}{0.23} = 258 < 2000 \rightarrow$ Laminar flow

$\frac{\Delta p}{L} = \frac{32\mu v}{D^2}$ $\quad \frac{\Delta p}{200} = \frac{32\times 0.23\times 0.487}{(4\times 0.0254)^2} \rightarrow \Delta p = 69446$ Pa

$\mathscr{P} = Q.\Delta p = A.v.\Delta p = 0.0081 \times 0.487 \times 69446 = \mathbf{274\ W}$

8. $\mu = 850 \times 6 \times 10^{-6} = 0.0051 \text{ Pa.s}$

$A = \frac{\pi}{4} \times (12 \times 0.0254)^2 = 0.07296 \text{ m}^2$ $\quad v = \frac{265}{3600 \times 0.07296} = 1 \text{ m.s}^{-1}$

$\text{Re} = \frac{850 \times 1 \times (12 \times 0.0254)}{0.0051} = 50800$

At $Re \approx 50000$, and roughness = 0.01, $f = 0.039$

$\frac{\Delta p}{L} = \frac{2f\rho v^2}{D}$ $\quad \frac{\Delta p}{2000} = \frac{2 \times 0.039 \times 850 \times 1^2}{12 \times 0.0254} \rightarrow \Delta p = 435040 \text{ Pa}$

Head loss due to valves = $3 \times 10 \times \frac{1^2}{2 \times 9.81} = 1.529$ m

Head loss due to elbows = $2 \times 0.3 \times \frac{1^2}{2 \times 9.81} = 0.0306$ m

Δp due to valves + elbows = $(1.529 + 0.0306) \times 850 \times 9.81 = 13005$ Pa

$\Delta p_{total} = 435040 + 13005 = 448045$ Pa

$\mathscr{P} = \frac{265 \times 448045}{3600 \times 0.75} = \mathbf{43974\ W \equiv 43.974\ kW}$

9. $A = \frac{\pi}{4} \times (0.005)^2 = 1.9635 \times 10^{-5} \text{ m}^2$ $\quad v = \frac{0.071}{3600 \times 1.9635 \times 10^{-5}} = 1 \text{ m.s}^{-1}$

$\frac{\Delta p}{L} = 375000 \text{ Pa.m}^{-1}$. Assume laminar flow:

$\frac{\Delta p}{L} = \frac{32\mu v}{D^2}$ $\quad 375000 = \frac{32\mu \times 1}{0.005^2} \rightarrow \boldsymbol{\mu = 0.29\ \text{Pa.s}}$

Check on Re: $\quad Re = \frac{13600 \times 1 \times 0.005}{0.29} = 234 < 2000 \rightarrow$ Laminar flow

10. $A = \frac{\pi}{4} \times (0.007)^2 = 3.8485 \times 10^{-5} \text{ m}^2$

$v = \frac{0.692}{3600 \times 3.8485 \times 10^{-5}} = 5 \text{ m.s}^{-1}$

For mercury, $\rho = 13600 \text{ kg.m}^{-3}$, $\mu = 1.55 \times 10^{-3} \text{ Pa.s}$

$Re = \frac{13600 \times 5 \times 0.007}{1.55 \times 10^{-3}} = 307096 \gg 4000$ Highly turbulent

From Moody charts, at $Re \approx 300000$, for smooth pipes, $f = 0.014$

$\frac{\Delta p}{L} = \frac{2f\rho v^2}{D}$ $\quad \frac{\Delta p}{4} = \frac{2 \times 0.014 \times 13600 \times 5^2}{0.007} \rightarrow \Delta p = 5.44 \times 10^6 \text{ Pa} \equiv \mathbf{5.44\ MPa}$

SHEET 5

1. $A_p = \frac{1}{4}\pi D^2 = 0.25 \times \pi \times 0.05^2 = 0.0019635\ \text{m}^2$

   $Re = \frac{1000\times3.6\times0.05}{0.001} = 180000 > 10^5 \rightarrow C_d = 0.44$

   $F_d = \frac{1}{2} C_d \rho_f v^2 A_p = \frac{1}{2} \times 0.44 \times 1000 \times 3.6^2 \times 0.0019635 = \mathbf{5.6\ N}$

2. $A = 1.2 \times 0.02 = 0.024\ \text{m}^2$

   $v = \frac{72}{3600\times0.024} = 0.833\ \mathbf{m.s^{-1}}$

   Projected area of cylinder = $0.05 \times 0.12 = 0.006\ \text{m}^2$

   $Re = \frac{1000\times0.833\times0.05}{0.001} = 41650 \rightarrow C_d \approx 1$

   $F_d = \frac{1}{2} C_d \rho_f v^2 A_p = \frac{1}{2} \times 1 \times 1000 \times 0.833^2 \times 0.006 = \mathbf{2.08\ N}$

3. $A_p = 0.01 \times 0.015 = 1.5 \times 10^{-4}\ \text{m}^2$

   $Re_L = \frac{1000\times5\times0.25}{0.001} = 1.25 \times 10^6 \rightarrow C_d = 1$

   $F_d = \frac{1}{2} C_d \rho_f v^2 A_p = \frac{1}{2} \times 1 \times 1000 \times 5^2 \times 1.5 \times 10^{-4} = \mathbf{1.875\ N}$

4. Since the front face is rectangular, then:

   $A_p = 2.1 \times 1.75 = 3.675\ \text{m}^2$

   The actual velocity of air is the relative velocity between the wind and the car = $120 + 30 = 150\ \text{km.h}^{-1} = \frac{150000}{3600} = 41.67\ \text{m.s}^{-1}$

   $\rho = \frac{pM}{10^3 RT} = \frac{1.1\times10^5\times29}{1000\times8.314\times(273+15)} = 1.332$ kg.m^{-3}

   $F_d = \frac{1}{2} \times 0.7 \times 1.332 \times 41.67^2 \times 3.675 = \mathbf{2975\ N}$

5. Maximum height to be covered by oil droplets = height of emulsion = $\frac{V}{A} = \frac{0.68}{0.25\pi\times1^2}$

   $h = 0.866\ \text{m}$

   Assume Stokes law to apply:

   $v = \frac{9.81\times(10^{-4})^2\times(1000-790))}{18\times0.001} = 0.001145$ m.s^{-1}

   Time = $\frac{0.866}{0.001145} = \mathbf{756\ s}$ (Check: Re = $\frac{1000\times0.001145\times10^{-4}}{0.001} = 0.114 < 1$)

SHEET 6

1. Bingham fluid: $\tau = \tau_0 + k.\dot{\gamma}$

   $560 = \tau_0 + 40k \qquad (i)$

   $980 = \tau_0 + 80k \qquad (ii)$

   Subtracting: $40k = 420 \rightarrow \boldsymbol{k = 10.5}$ **Pa. s**

   Substituting in (i): $560 = \tau_0 + 40 \times 10.5 \rightarrow \boldsymbol{\tau_0 =}$ **140 Pa**

2. Shear thinning: $\tau = k.\dot{\gamma}^n$

   $130 = k.25^n \qquad (i)$

   $320 = k.150^n \qquad (ii)$

   Dividing: $\frac{320}{130} = \left(\frac{150}{25}\right)^n \rightarrow 2.4615 = 6^n \rightarrow n \ln 6 = \ln 2.4615 \rightarrow \boldsymbol{n = 0.503}$

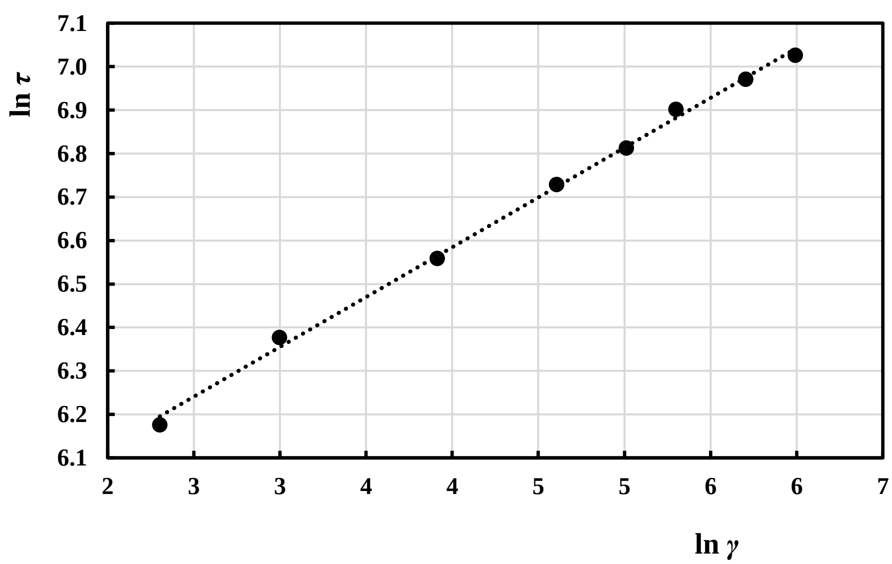
3. The given table is completed

Shear rate s^{-1}	10	20	50	100	150	200	300	400
Shear stress Pa	481	588	705	836	909	994	1065	1125
ln shear rate	2.303	2.996	3.912	4.605	5.011	5.298	5.704	5.991
ln shear stress	6.176	6.377	6.558	6.729	6.812	6.902	6.971	7.026

The plot between $\ln \tau$ and $\ln \dot{\gamma}$ is shown. We get a straight line of slope $\boldsymbol{n = 0.23}$

Substituting in a point from the table, like $\dot{\gamma} = 50, \tau = 705$:

$705 = k.50^{0.23} \rightarrow \boldsymbol{k = 287}$



4. $A = 0.25\pi \times (3 \times 0.0254)^2 = 0.00456 \text{ m}^2$

$v = \frac{Q}{A} = \frac{1.3}{3600 \times 0.00456} = 0.079\ \text{m. s}^{-1}$

$r = 0.5 \times 3 \times 0.0254 = 0.0381\ \text{m}$

$$\frac{\Delta p}{L} = \frac{2k}{r}\left[\frac{v.(3n+1)}{n.r}\right]^n$$

$$\frac{\Delta p}{120} = \frac{2 \times 287}{0.0381}\left[\frac{0.079 \times (3 \times 0.23 + 1)}{0.23 \times 0.0381}\right]^{0.23} \rightarrow \Delta p = 3382410\ \text{Pa}$$

Pumping power = $Q \times \Delta p = 0.0003611 \times 3382410 = \mathbf{1222\ W}$

5. Shear thickening: $\tau = k.\dot{\gamma}^n$

$195 = k.100^n \qquad (i)$

$505 = k.200^n \qquad (ii)$

Dividing: $\frac{505}{195} = \left(\frac{200}{100}\right)^n \rightarrow 2.59 = 2^n \rightarrow n \ln 2 = \ln 2.59 \rightarrow \boldsymbol{n = 1.373}$

Substituting in (i): $195 = k.100^{1.373} \rightarrow \boldsymbol{k = 0.35}$

$\dot{\gamma}_w = \frac{8v}{D} = \frac{8 \times 0.15}{0.01} = 120\ \text{s}^{-1}$

$\tau = k.\dot{\gamma}^n \rightarrow \tau_w = 0.35 \times 120^{1.373} = \mathbf{250.5\ Pa}$

6. $\tau_0 = 0.228\ \text{Pa}$ and $k = 0.12\ \text{Pa. s}$

Density of suspension = $0.15 \times 2600 + 0.85 \times 1000 = 1240\ \text{kg. m}^{-3}$

$A = 0.25\pi \times (4 \times 0.0254)^2 = 0.008107\ \text{m}^2$

$v = \frac{Q}{A} = \frac{16}{3600 \times 0.008107} = 0.5482\ \text{m. s}^{-1}$

$Re = \frac{1240 \times 0.5482 \times 4 \times 0.0254}{0.12} = 575.537 \qquad He = \frac{0.228 \times 1240 \times 4 \times 0.0254}{0.12^2} = 1994.75$

To prove that $f \approx 0.02325$, we replace f by this value in the RHS of the following equation and get the LHS.

$$f = \frac{46}{Re}\left[1 + \frac{He}{6Re} - \frac{64He^4}{3f^3Re^7}\right]$$

$$f = \frac{46}{575.537}\left[1 + \frac{1994.75}{6 \times 575.537} - \frac{64 \times 1994.75^4}{3 \times 0.02325^3 \times 575.537^7}\right] = 0.0234 \approx 0.02325.$$

$$\frac{\Delta p}{120} = \frac{2 \times 0.02325 \times 1240 \times 0.5482^2}{4 \times 0.0254}$$

Hence $\Delta p = 20466\ \text{Pa} \equiv \mathbf{20.466\ kPa}$

7. The shear stress is calculated for each value of shear strain from: $\tau = \mu.\dot{\gamma}$

Shear rate s^{-1}	20	40	60	100	200	300	500
Viscosity cP	180	115	94	81	66	62	55
Shear stress Pa	3.6	4.6	5.64	8.1	13.2	18.6	27.5

A plot of τ against $\dot{\gamma}$ is then carried out of slope = k, the plastic viscosity and intercept = τ_0, the yield stress.

First, we get the slope by choosing two points almost on the line: $(200{,}13.2)$ and $(40{,}4.6)$

Slope = k = $\frac{13.2-4.6}{200-40}$ = $\mathbf{0.0504}$ **Pa.s**

Then replace in the Bingham fluids equation: $\tau = \tau_0 + k.\dot{\gamma}$ with any of the two chosen points: $13.2 = \tau_0 + 0.0504 \times 200 \rightarrow \tau_0 = \mathbf{3.12}$ **Pa**