1. REVISION (UNIT OPERATIONS)

1.1 Particle size analysis

1.1.1 Particle size and shape

For spherical particles, the diameter is considered to be the defining dimension. For irregular particles, it is the second largest dimension, provided the greatest dimension does not exceed twice the smallest. Otherwise, the greatest dimension is taken as the defining dimension D_p.

The ratio of total surface area to volume = $s = \frac{A_p}{V_p} = \frac{6}{\Phi.D_p}$

Where Φ is the sphericity (< 1 for irregular particles and = 1 for spheres).

1.1.2 Specific surface area

For particles having the same diameter D_p:

The specific surface area = $A_w = \frac{6}{\Phi.\rho_p.D_p}$ $m^2.kg^{-1}$ (1.1)

For a particle size distribution where the mass fraction retained between two standard sieves of diameters = D_i and D_{i+1} is x_i, the specific surface area is:

$$A_w = \frac{6}{\Phi.\rho_p}.\sum_{i=1}^{n}\frac{x_i}{D_{av\,i}} \; m^2.kg^{-1} \tag{1.2}$$

The cumulative fraction φ_i is defined as: $\varphi_i = \sum_{k=1}^{k=i} x_i$

If an analytical relation of the type $\varphi_i = f(D_p)$ is available, then the specific surface area can be obtained from:

$$A_w = \frac{6}{\Phi.\rho_p}\int_0^1 \frac{d\varphi}{D_p} \tag{1.3}$$

1.1.3 Mean particle size

The volume – surface mean diameter is defined by:

$$\frac{1}{D_{vs}} = .\sum_{i=1}^{n}\frac{x_i}{D_{av\,i}} = \int_0^1 \frac{d\varphi}{D_p} \tag{1.4}$$

1.2 Flow through porous media

The pressure drop per unit length is obtained from the Ergun equation:

$$\frac{\Delta P}{L} = \frac{150.\bar{v}.\mu.(1-\varepsilon)^2}{\Phi^2.D_{vs}^2.\varepsilon^3} + \frac{1.75\rho_f.\bar{v}^2.(1-\varepsilon)}{\Phi.D_{vs}.\varepsilon^3} \tag{1.5}$$

In case of laminar flow ($Re < 1$), the second term can be omitted and the first term can be written as:

$$\frac{\Delta P}{L} = \frac{\bar{v}.\mu}{K} \tag{1.6}$$

Where, K is the Darcy permeability (m^2).

2. FILTRATION

2.1 Introduction

Filtration is a unit operation in which a mixture of solids and liquid called the feed or the suspension or the slurry or the influent or the dispersion is forced through a porous medium in which the solids are deposited or entrapped. The porous filter medium is the permeable material that separates particles from liquids and is known as the filter. The solids retained on the filter are called the residue. The solids form a cake on the surface of the medium and the clarified liquid known as effluent or filtrate is discharged from the filter.
Filtration processes can be broadly classified into three categories. If recovery of solids is desired, the process is called cake filtration. The term clarification is applied when the solids do not exceed 1 % w/w and the filtrate is the primary product. The third type of filtration is called cross-flow filtration in which the liquid flows in a tangential direction with respect to the filtration medium. Cross-flow filtration is mainly used for membrane filtration. In this section only cake filtration will be discussed.

2.2 Theory

As filtration of a slurry proceeds, a porous wet cake is deposited on the filter medium and the total pressure drop is therefore the sum of the pressure drop across the cake Δp_c and the pressure drop across the filter medium Δp_m:

$$\Delta p = \Delta p_c + \Delta p_m \qquad (2.1)$$

Referring to Fig. (2.1), the pressure drop across an element of cake thickness dl can be predicted from the laminar term of the laminar term of Ergun equation as follows:

$$\Delta p_c = \frac{150\mu L v(1-\varepsilon)^2}{\Phi^2 \varepsilon^3 D_p^2} \qquad (2.2)$$

Hence:

$$\frac{dp_c}{dL} = \frac{150\mu v(1-\varepsilon)^2}{\Phi^2 \varepsilon^3 D_p^2} \qquad (2.3)$$

The mass of the element of cake considered is $dm_c = \rho_p .A.(1-\varepsilon).dL$

Hence substituting for dL in Eq. (2.3), we get:

$$\frac{dp_c}{dm_c} = \frac{150\mu v(1-\varepsilon)}{A\Phi^2 \rho_p \varepsilon^3 D_p^2} \qquad (2.4)$$

We have now to differentiate between two types of cakes:
Incompressible cakes: Where the values of s and ε remain constant throughout the filtration process. In this case, the term $\frac{150(1-\varepsilon)}{\Phi^2 \rho_p \varepsilon^3 D_p^2}$ remains constant and is termed the **local specific cake resistance,** α_L. Equation (2.4) then takes the form:

$$dp_c = \frac{\mu . v . \alpha_L}{A} dm_c \tag{2.5}$$

The coefficient of dm_c in the above equation being constant, this equation can be readily integrated to give:

$$\Delta p_c = \frac{\mu v \alpha_L}{A} m_c \tag{2.6}$$

Where the constant value of α_L is given by:

$$\alpha_L = \frac{150(1-\varepsilon)}{\Phi^2 \rho_p \varepsilon^3 D_p^2} \tag{2.7}$$

The above equations may be applied in the filtration of incompressible cakes such as crushed limestone or dolomite, ground quartz or feldspar and any rigid cake that will not yield to the external pressure.

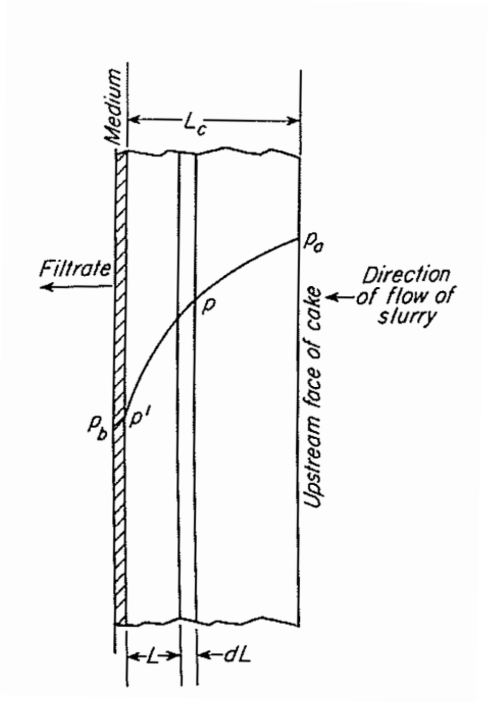


Fig (2.1) Mechanism of filtration

Compressible cakes: This is the case of most industrial cakes such as phosphate cakes, hydrated alumina etc… In that case the value of α_L is not constant as the porosity decreases during filtration. Integration of Eq. (2.5) gives:

$$\int dp_c = \frac{\mu . v}{A} \int \alpha_L . dm_c \tag{2.8}$$

It is not easy to evaluate the RHS of the above equation by direct integration and usually a mean value of α_L is introduced as being:

$$\alpha = \frac{\int \alpha_L . dm}{\int dm} \tag{2.9}$$

Since $m_c = \int dm$, hence combining (2.8) and (2.9), we get:

$$\Delta p_c = \frac{\mu v \alpha}{A} m_c \tag{2.10}$$

The value of α is termed the **mean specific cake resistance**.

Besides, the filter medium offers a resistance to the flow of filtrate that appears in the form of a pressure drop which can be written in a way similar to equation (2.10):

$$\Delta p_m = \mu . v . R_m \tag{2.11}$$

Where R_m is termed the **filter medium resistance.**

In the above equations, in the SI units, α has for unit m.kg^{-1} while the unit of R_m is m$^{-2.}$

Recalling that the superficial velocity, $v = \frac{1}{A}\frac{dV}{dt}$ and the total pressure drop can be obtained by summing up the two above equations. We get:

$$\Delta p = \Delta p_c + \Delta p_m = \frac{\mu}{A}\frac{dV}{dt}\left(\frac{m_c \alpha}{A} + R_m\right)$$

The value of m_c can be obtained by knowing the volume of filtrate V and the solid concentration in the slurry $c = \frac{m_c}{V}$. The above equation becomes:

$$\Delta p = \frac{\mu}{A}\frac{dV}{dt}\left(\frac{cV\alpha}{A} + R_m\right)$$

Rearranging, we get:

$$\frac{dt}{dV} = \frac{\mu}{A\Delta p}\left(\frac{cV\alpha}{A} + R_m\right) \tag{2.12}$$

There are **two ways of performing filtration:**

2.3 Constant pressure filtration

If the pressure is held constant throughout the whole filtration period, then due to the continuous deposition of the cake, the rate of filtration will eventually decrease. In that case Eq. (2.12) is usually written in the following form:

$$\frac{dt}{dV} = K_p . V + B \tag{2.13}$$

Where:

$$K_p = \frac{\mu . c . \alpha}{A^2 . \Delta p} \tag{2.14}$$

$$B = \frac{\mu . R_m}{A . \Delta p} \tag{2.15}$$

Eq. (2.13) can be integrated to give:

$$t = K_p . \frac{V^2}{2} + B . V \tag{2.16}$$

It is usually needed for design purpose to determine α and R_m from equations (2.14) and (2.15). This is done by collecting the filtrate at fixed time intervals so as to have values of V, the cumulative volume of filtrate against time t.

Dividing the two sides of Eq. (2.16) by V, we get:

$$\frac{t}{V} = K_p.\frac{V}{2} + B \qquad (2.17)$$

Hence, a plot of t/V against V should give a straight line of slope $K_p/2$ and intercept B. After obtaining those two constants, equations (2.14) and (2.15) can be applied to get the mean specific cake resistance and the filter medium resistance.

The Compressibility Coefficient

For a compressible cake, the value of α is variable and increases with the increase in the pressure drop across the cake. An empirical equation that is usually applied for the variation of the mean specific cake resistance is:

$$\alpha = \alpha_0 \, (\Delta p_c)^s \qquad (2.18)$$

Where: s is a constant called the *compressibility coefficient*. For incompressible cakes $s = 0$, while for compressible cakes $0.2 < s < 0.8$.

In most cases, the pressure drop across the filter medium can be neglected compared to the pressure drop across the cake so that we may write:

$$\alpha = \alpha_0 \, (\Delta p)^s \qquad (2.19)$$

2.4 Constant rate filtration

If the filtration rate is to be kept constant, then the pressure drop across the cake will increase throughout the filtration process. In this case:

Recalling that the filtration velocity is $v = \frac{V}{A.t}$

Eq. (2.10) can be rewritten as:

$$\Delta p_c = \frac{\alpha \mu c}{A} \; \frac{V^2}{At} \qquad (2.20)$$

Combining the above equation with Eq. (2.19), we get:

$$\Delta p_c = \frac{\alpha_0 \mu c}{t} \; \frac{V^2}{A^2} \; (\Delta p)^s$$

Or:

$$(\Delta p_c)^{1-s} = \alpha_0 \mu c \left(\frac{V}{At}\right)^2 t \qquad (2.21)$$

The value of $\frac{V}{At}$ is the constant rate of filtration per unit area (or the superficial velocity v).

Hence, we may write:

$$(\Delta p_c)^{1-s} = K_r.t \qquad (2.22)$$

Where K_r is a constant defined by:

$$K_r = \mu.c.\alpha_o.v^2 \qquad (2.23)$$

The values of s and K_r can be obtained by taking the logarithms of both sides of equation (2.22):

$$(1\text{-}s).\log \Delta p_c = \log K_r + \log t \qquad (2.24)$$

A plot of log t against log $(\Delta p - \Delta p_m)$ gives a straight line of slope (1 - s). The value of Δp_m can be obtained by first plotting Δp against time. At zero time the value of Δp equals Δp_m.

2.5 Filtration practice

2.5.1 Modes of filtration

In practical operation, it is customary to have neither constant pressure nor constant rate filtration. However, in case of filters run under positive pressure, a centrifugal pump is used to drive the slurry to the filter. In this case the mode of operation is that of constant rate during the early stages of filtration and of constant pressure in the later stages.

Also, it is common practice to wash the cake after filtration has been effected. This is simply done by having the wash liquid (very often water), flowing through the cake. The initial rate of washing is usually that of the final rate of filtration. However, **the overall rate of washing is usually about one fourth of the overall rate of filtration**. This latter is defined as being the ratio between the filtrate volume collected during the total filtration period t and the total time of the filtration cycle which consists of the sum ($t + t_T$) where t_T is known as the **Tare time** which is the time taken to dismantle the filter (in case of batch filters) , to remove the cake etc…On the other hand, a cycle including both filtration and washing will have an overall rate of filtration defined by:

$$R = \frac{V}{(t + t_T + t_w)} \qquad (2.25)$$

Where t_w is the washing time.

If filtration is effected under constant pressure, then from equation (2.16), R is related to V by:

$$R = \frac{V}{(K_p.\frac{V^2}{2} + B.V + t_T + t_w)} \qquad (2.26)$$

The maximum overall rate of filtration will be obtained by equating $\frac{dR}{dt}$ to zero. Once the value of V corresponding to that maximum rate is obtained, we can calculate the value of t.

In case the filter medium resistance is negligible, then on writing $\frac{dR}{dt} = 0$ we finally get $t = t_T + t_w$

2.5.2 Filter media

The choice of a proper filter medium can often be the most important consideration in assuring satisfactory operation of a filter. It should be capable of properly retaining the solids that are to be separated from the liquid, with suitable length of life. The following criteria are usually used in the selection of a filter medium:

a- Ability to bridge solids across its pores quickly after feed is started.

b- The solids should not blind the filter openings.
c- Minimum filter medium resistance.
d- Resistance to chemical attack.
e- Acceptable resistance to mechanical wear.
f- Sufficient strength to support the filtration pressure.
g- Ability to discharge the cake easily and cleanly.
h- Relatively low cost.

Filter media are manufactured from textiles woven of cotton, synthetic fibers and sometimes, in case fine crystals are filtered, from metal fabric of about 400 mesh opening.

2.5.3 Filter aids

Filtration of some solids can be accompanied with problems of slow filtration rate or unsatisfactory filtrate clarity. This may be improved by using a filter aid. This consists of a granular or fibrous material capable of forming a highly permeable filter cake within which the troublesome solids will be incorporated. They are usually used when the solids consist of very fine particles. The particles of a good filter aid should be light and porous and chemically inert to the filtrate. The most used types are: **Diatomite** which is a variety of extremely porous silica and **Expanded Perlite** which is a highly porous alkali aluminosilicate. Their bulk density is in the range of 200 - 400 kg.m^{-3}.

Filter aids are used in two ways: (1) Either as a pre-coat to protect the filter medium and prevent the escape of occasional fine particles in the filtrate (the coat is applied at about 0.5 kg.m^{-2} of filter area), or (2) they can be mixed with the slurry to trap the difficult filterable particles in a permeable cake.

Example 2.1

The following data belong to lab tests undertaken on a calcium carbonate slurry. The filter area equals 0.045 m² and the solid concentration in the slurry is 24 kgm^{-3}. Evaluate the mean specific cake resistance and the filter medium resistance at a fixed pressure of 50 kPa.

***V* L**	0.5	1.0	1.5	2.0	2.5	3
***t* s**	17.3	42.3	72.0	108.3	152	202.7

Solution:

Fig. (2.2) shows a plot of *t/V* against *V* (Equation (2.17))

The slope of this plot is 12.9×10^6 s.m^{-4}

Hence $K_p/2 = 12.9\times10^6$ and $K_p = 2.58\times10^7$.

From Eq. (2.14)

$$\frac{\mu . c . \alpha}{A^2 . \Delta p} = 2.58\times10^7 \quad \text{and } \alpha = \frac{(0.045)^2 .50000}{0.001\times24} \times 2.58\times10^7 \text{ m.kg}^{-1}$$

Hence, $\boldsymbol{\alpha}$ **= 1.09×10^{11} mkg^{-1}**

On the other hand, the intercept is 28600 s.m^{-3}

From equation (2.15):

$$\frac{\mu R_m}{A\Delta p} = 28600$$

Hence, $\boldsymbol{R_m}$ **= 6.435×10^{10} m^{-1}**

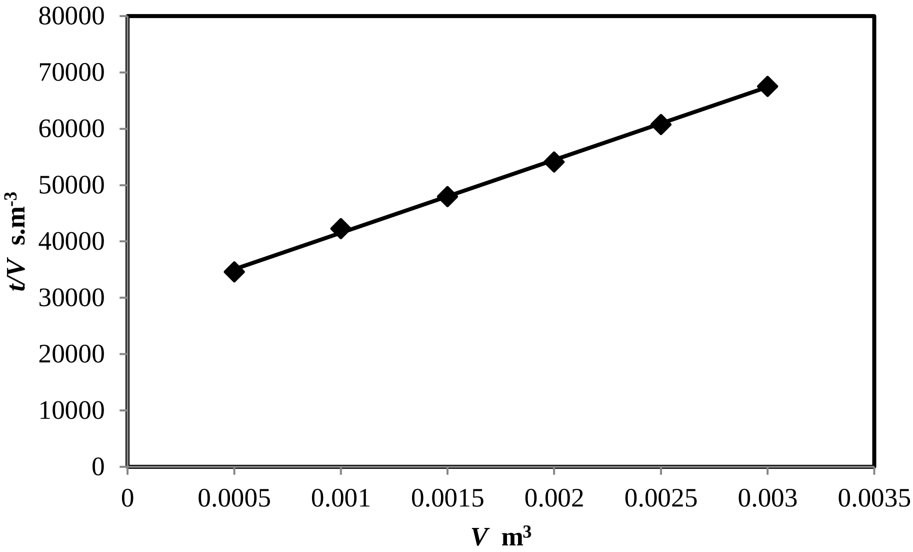


Fig (2.2) Plot between *t*/*V* and *V*

Example 2.2

The following data belong to a filtration test conducted under a constant rate of 0.05 m^3h^{-1}. The filtration area is 0.05 m^2 and the solid concentration in the slurry is 25 $kg.m^{-3}$. From these data, deduce the values of R_m, α_o, K_r and s.

Δp kPa	30	34.1	43.6	52.1	59.3	69.5	80.5	92	103.6
***t* s**	10	20	30	40	50	60	70	80	90

Solution:

From a plot of Δp against time, extrapolating to time = 0, we get:
$\Delta p_m \approx 24000$ Pa from Figure (2.3)

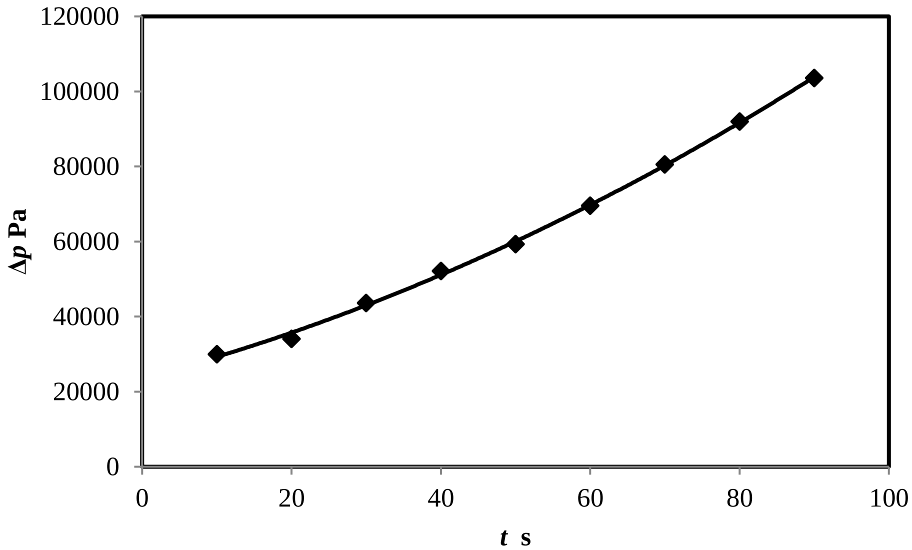


Fig (2.3) Plot of pressure drop against time

Since $\Delta p_m = R_m.\mu.v$, and $v = 0.05/.05$ $m.h^{-1}$ = 1/3600 = 0.000278 $m.s^{-1}$

We get: $R_m = \frac{24000}{0.001\times0.000278}$ = $\mathbf{8.63\times10^9\ m^{-1}}$

To get the other parameters, we plot log t against log (Δp - 24000)

log *t*	1.00	1.30	1.48	1.60	1.70	1.78	1.85	1.90	1.95
log (Δ*p* - 24000)	3.78	4.00	4.29	4.45	4.55	4.66	4.75	4.83	4.90

The slope is $(1 - s) = 0.81$ (Figure 2.3) hence $\boldsymbol{s = 0.19}$

Also, $K_r = \mu.c.\alpha_o.v^2$ Equation (2.23)

To get the value of K_r, we substitute a value of t against a value of $(\Delta p - \Delta p_m)$ in Eq. (2.24). From the table in the heading of the problem: t = 60 s corresponds to $(\Delta p - \Delta p_m)$ = 69500 – 24000 = 45500 Pa, hence in Eq. (2.24):

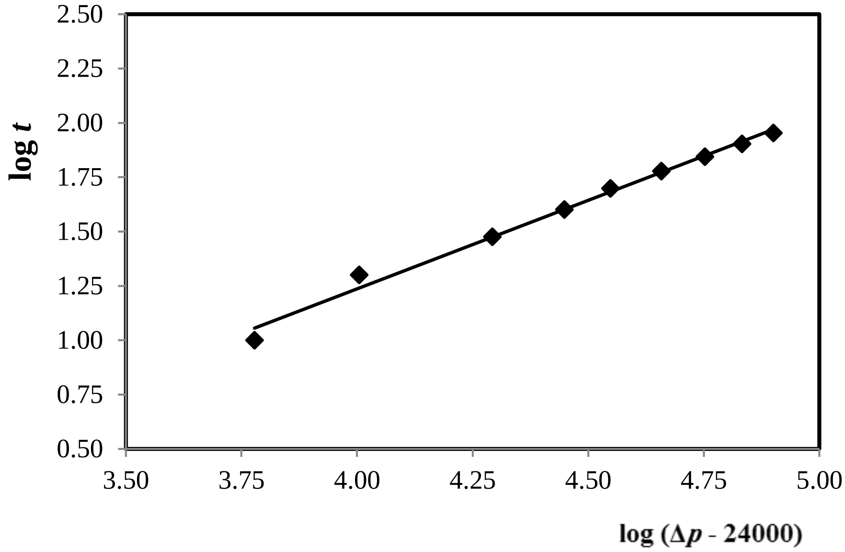


Fig (2.4) Plot of log *t* against log (Δ*p* – 24000)

$45500^{0.81} = K_r\times60$, from which $K_r = 98.8$

Substituting in Eq. (2.25):

$98.8 = 0.001\times25\times(0.000278)^2.\alpha_0$

From which $\boldsymbol{\alpha_0 = 5.11\times10^{10}\ m.kg^{-1}}$

2.6 Types of industrial filters

2.6.1 Pressure filters versus vacuum filters:

Pressure filters are operated under super atmospheric pressures. The feed slurry is introduced to the filter at pressures ranging from 2 to 5 atm gauge reaching in some cases figures as high as 35 atm. The most commonly used pressure filters are of the batch type. They have the following advantages:

(a) They provide high filtration rates and proper separation of fine solids.
(b) They usually occupy a small floor space per unit area of filtration.
(c) Their initial cost is low.
(d) They are flexible in their operation, allowing to filter a wide variety of solids.

On the other hand, such filters have also some disadvantages:

(a) They are difficult to operate.
(b) Their operating cost is high since they require a lot of labor.

Vacuum filters are operated using a vacuum pump or alternatively a jet ejector. They are usually more adapted for continuous operation. They offer the following advantages:

(a) They use low labor compared to batch filters.
(b) Their surface is open to the atmosphere allowing easy inspection and repair.
(c) The maintenance cost is relatively low.

Drawbacks of using such filters are:

(a) A vacuum system must be maintained throughout the filtration process.
(b) Due to the low pressure used, this is unsuitable for use in the case of volatile filtrates.
(c) Such systems cannot handle difficult solids.
(d) They lack flexibility as to the type of solid and the rate of filtration.

2.6.2 Pressure filters: The plate and frame filter press

A plate and frame filter press is an assembly of alternate *solid plates* the faces of which are studded, grooved or perforated to permit drainage and *hollow frames* in which the cake collects during filtration. A filter cloth covers both faces of each plate. Their shape is usually rectangular although circular plates and frames are sometime used. They are hung in a vertical position on a pair of parallel supporting bars. During filtration they are compressed between two end half-plates (one of which is fixed) using a screw or by a hydraulic ram.

The slurry is fed to the press under a pressure ranging from 3 to 10 atm. and moves along a duct formed by the alignment of slots present in the top corner of the plates and the frames. These slots are connected to grooves into the frames to direct the slurry into the frames towards the two plates sandwiching the frame. The cake is thus allowed to deposit on the filter clothes and the discharged filtrate flows through cock valves situated at the bottom of each plate. During filtration, these valves are open. Fig. (2.5) and (2.6)

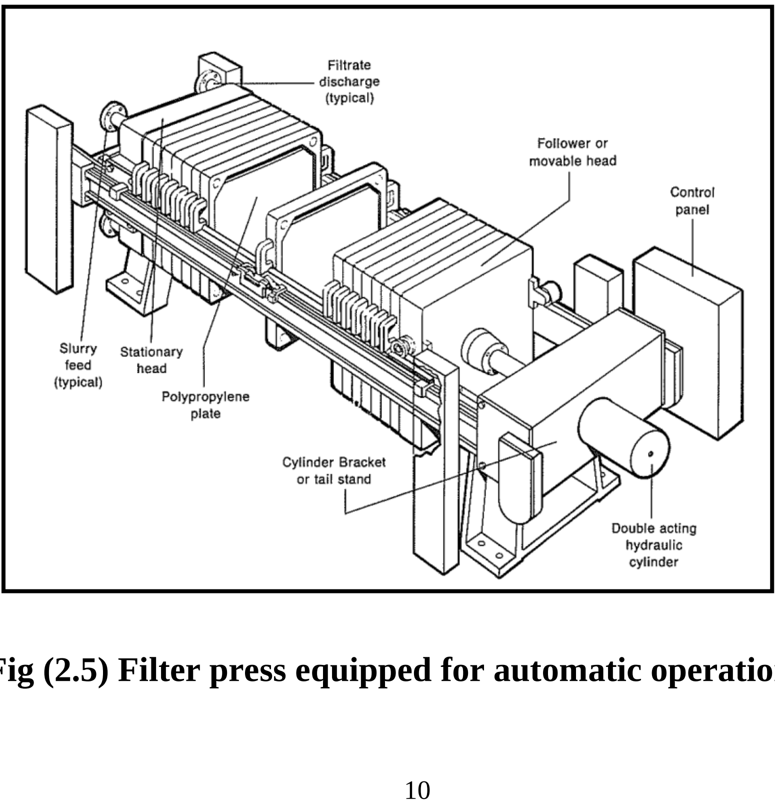


Fig (2.5) Filter press equipped for automatic operation

As the cake builds up to cover from 70 to 90% of the frame thickness filtration is stopped and washing begun. The wash liquid is admitted through a duct formed by slots contained in the top corner of the plates and frames other than those used for the flow of slurry during the filtration period. These slots are connected with each other plate (known as wash plates), the valves of which are closed. The liquid therefore must pass through the entire cake thickness until it reaches an open valve down a non-wash plate. Fig. (2.5(b))

In designing a filter press, we first determine the required area of filtration. Then, the number of plates is determined depending on the chosen dimensions of plates. These are usually square shaped. Their dimensions range from 150 mm to more than one meter. The thickness of plates ranges from 6 mm to 50 mm while that of frames ranges from 6 to 200 mm.

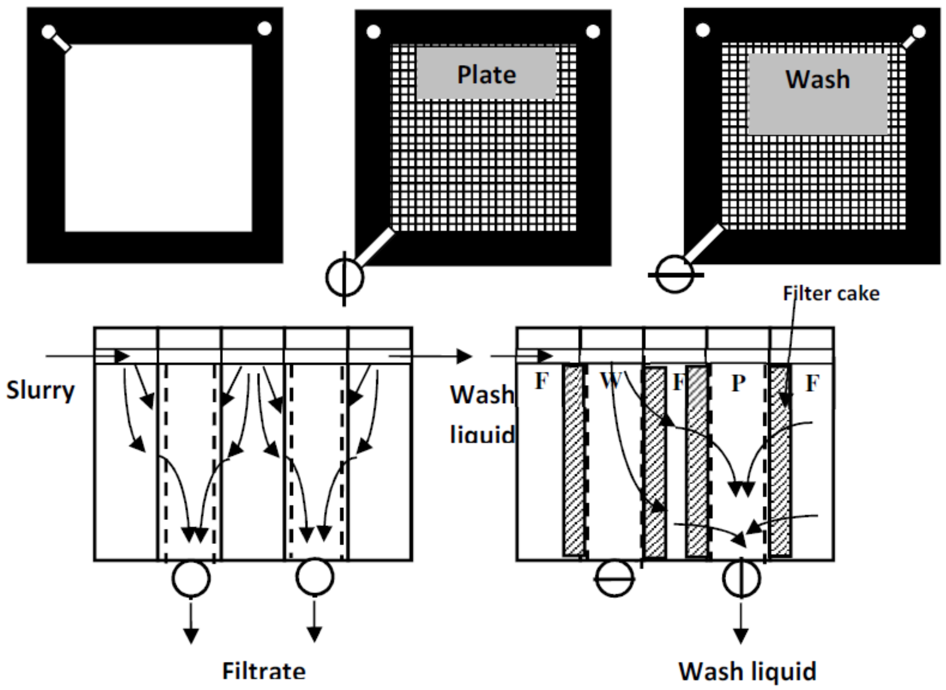


Fig (2.6) Filtering and washing patterns in a filter press

Example 2.3
A plate and frame filter press is to be designed for the filtration of a slurry consisting of 250 kg magnesite in 10 m³ water in 2 hours. Filtration is conducted under a constant pressure of 200 kPa. The specific cake resistance is 3×10^{10} m.kg^{-1} and the filter medium resistance is 10^6 m$^{-1.}$ Find the number of plates if their filter dimensions measure 12”×12”.

Solution:

We first calculate the area of filtration from Equations. (2.14) to (2.16)

$$7200 = \frac{0.001\times25\times3\times10^{10}\times10^{2}}{2\times A^{2}\times20000} + \frac{0.001\times10^{6}\times10}{A\times20000}$$

$187500\times A^{-2} + 5\times10^{-2}\times A^{-1} = 7200.$

Solving for A, we get: $A = 5.1$ m²

Area of filtration per plate = $2\times144\times0.0254^2 = 0.185$ m^2

Number of plates = 5.1 / 0.185 = **27 plates**

2.6.3 Vacuum filters: The rotary filter

The most common type of continuous vacuum filter is the *Rotary drum filter*. This is shown in Fig. (2.6) while the continuous filtration setup is shown in Fig. (2.7). The filter consists of a horizontal drum with a slotted face turning at 0.2 - 2 RPM in an agitated slurry trough. The face of the drum is covered by a filter medium (usually a cloth known as canvas) and the drum is partly submerged in the slurry. Under the slotted cylindrical face of the main drum is a second smaller drum with a solid surface. Between the two drums are radial partitions dividing the annular space into separate compartments, each connected by an internal pipe to one hole in the rotating plate of a rotary valve. A strip of filter cloth covers the exposed face of each compartment to form a succession of panels.

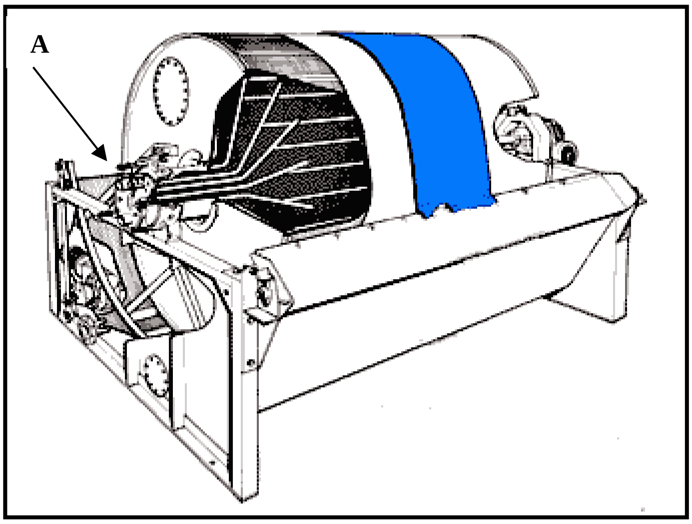


Fig (2.7) Continuous rotary vacuum filter

Consider panel A in Fig.(2.7). It is about to enter the slurry in the trough. As it dips under the surface of the liquid, vacuum is applied through the rotary valve. A layer of cake builds up on the face of the panel as liquid is sucked through the cloth into the compartment, through the internal pipe, through the valve into a collecting tank. As this panel leaves the drum it enters the washing zone. Vacuum is applied to the panel from a separate system sucking wash liquid and air through the cake of solids. After the cake has dried, vacuum is ceased and the cake removed by a horizontal knife blade. A little air is blown in under the cake to ease its removal from the filter cloth. Once the cake is taken away, the panel reenters the trough, and the cycle is repeated (Figure 2.8).

Often a precoat is continuously supplied on the filter surface.

Calculations of continuous filters

The calculation of the filtration area in the case of a drum filter is very similar to that of a batch filter. The time required for filtration is therefore obtained by Eq. (2.16). Usually, the filter medium resistance can be neglected with respect to that of cake, so that:

$$t = \frac{\alpha\mu c}{2A^2\Delta p} V^2$$

Noting that the flow rate of filtrate is $Q = V/t$ the previous equation can be written in the form: $Q^2 = \dfrac{2A^2\Delta p}{\alpha\mu c t}$

The time of filtration is lower than the actual time of a cycle t' by a factor ranging from 2.5 to 3.5, hence $t = f \times t'$, where $f = 0.3 - 0.4$. Therefore, we may write the above equation in the modified form:

$$Q^2 = \frac{2A^2 \Delta p}{\alpha \mu c f t'} \quad (2.27)$$

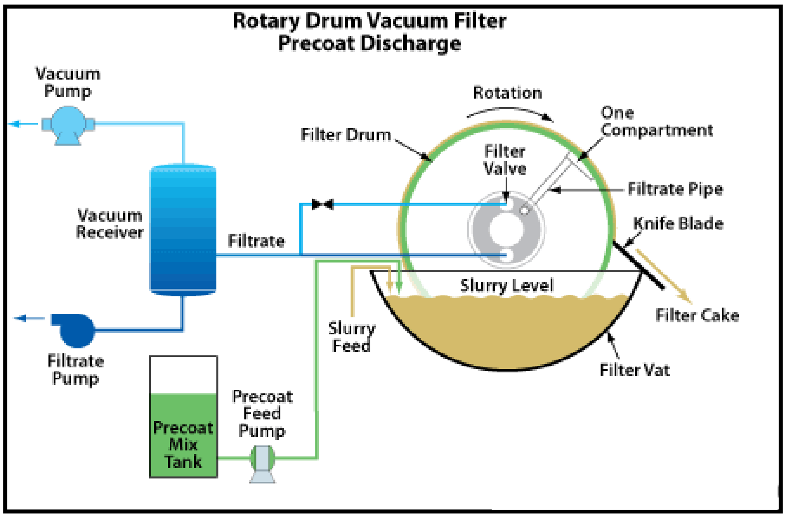


Fig (2.8) Flow Sheet for Continuous Vacuum Filtration

Note that the time of a cycle is the reciprocal of the speed of rotation of the drum (n rps). Also, the effective area of filtration A is $A_T.f$, where A_T is the total area of the filter medium. Equation (8.27) becomes:

$$Q^2 = \frac{2A_T^2 \Delta p f}{\alpha \mu c t'} \quad (2.28)$$

Once the area A_T is known we may choose the drum dimensions from the standard data of Table (8.1) shown below:

Table (2.1) Dimensions of drum filters of known area

	Length, ft										
***D*, ft**	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**
6	76	113	151	189	226						
8			200	250	300	350	400				
10				310	372	434	496	558	620		
12					456	532	608	684	760	836	912

Example 2.4

It is required to design a rotary filter with 30% submergence fed with 3.3 $m^3.h^{-1}$ of a slurry containing 236 kg solids per m^3 of filtrate. The absolute pressure maintained during filtration is 68 kPa and the mean specific cake resistance is 5×10^{10} mkg^{-1}. The speed of rotation of drum = 0.2 rpm.

Also, calculate the rate of dry cake production.

Solution:

This is a direct application of Eq. (2.27) with $t' = \frac{60}{0.2}$ = 300 s and f = 0.3

$$\left(\frac{3.3}{3600}\right)^2 = \frac{2A_T^2\times 68000\times 0.3}{5\times 10^{10}\times 0.001\times 236\times 300}$$

From which: A = 8.54 m² which is equivalent to about 92 ft²

The nearest dimensions from Table (2.1) are: ***D* = 6 ft and *L* = 6 ft**

The rate of cake production is $Q.c$ = 3.3×236 = **779 kg.h^{-1}**

2.7 Filtration of solids from gases: Bag filters

2.7.1 Operating principle

Conventional bag filters consist of an arrangement of vertical bags where dusty air (or gases) is allowed to pass through the woven fabrics which filter the air. Actually, dust removal is performed by the coat of dust deposited on the fabric rather than by the fabric itself. Figures (2.9) and (2.10) show a typical arrangement of vertical bags in a rectangular housing.

Commonly, dusty air is circulated on the outer periphery of the bags, thereby depositing the dust on the outer surface of the bags.

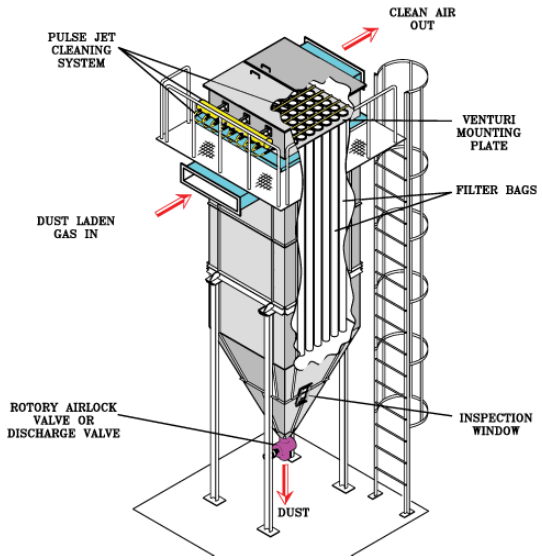


Fig (2.9) Bag house arrangement

Cleaning of the bags is done periodically by closing the incoming air valve to a set of bags and either mechanically shaking these bags or introducing compressed clean air in a reverse direction to make the dust particles drop in the collecting funnel. In that case, cleaning is done by passing clean air inside the bags thus expelling the dust to the bottom funnel. The bags are usually made from cotton, Nylon, Dacron, Polyethylene for normal operation, and from fiberglass for high temperature uses.

2.7.2 Design principles

The pressure drop across a bag is the sum of two terms: The pressure drop across the fabric Δp_f and the pressure drop across the dust Δp_d.

Since the motion of air (or gases) across the fabric usually falls in the laminar flow regime, it is customary to keep the first term in the Ergun equation governing fluid flow through porous media:

$$\frac{\Delta P}{L} = \frac{150.\bar{v}.\mu.(1-\varepsilon)^2}{\Phi^2.D_p^2.\varepsilon^3} + \frac{1.75\rho_f.\bar{v}^2.(1-\varepsilon)}{\Phi.D_p.\varepsilon^3}$$

So that:

$$\frac{\Delta P}{L} \approx \frac{150.\bar{v}.\mu.(1-\varepsilon)^2}{\Phi^2.D_p^2.\varepsilon^3}$$

Written in permeability form, this equation reads:

$$\Delta P_f \approx \frac{\bar{v}.\mu.L}{K} \qquad (2.29)$$

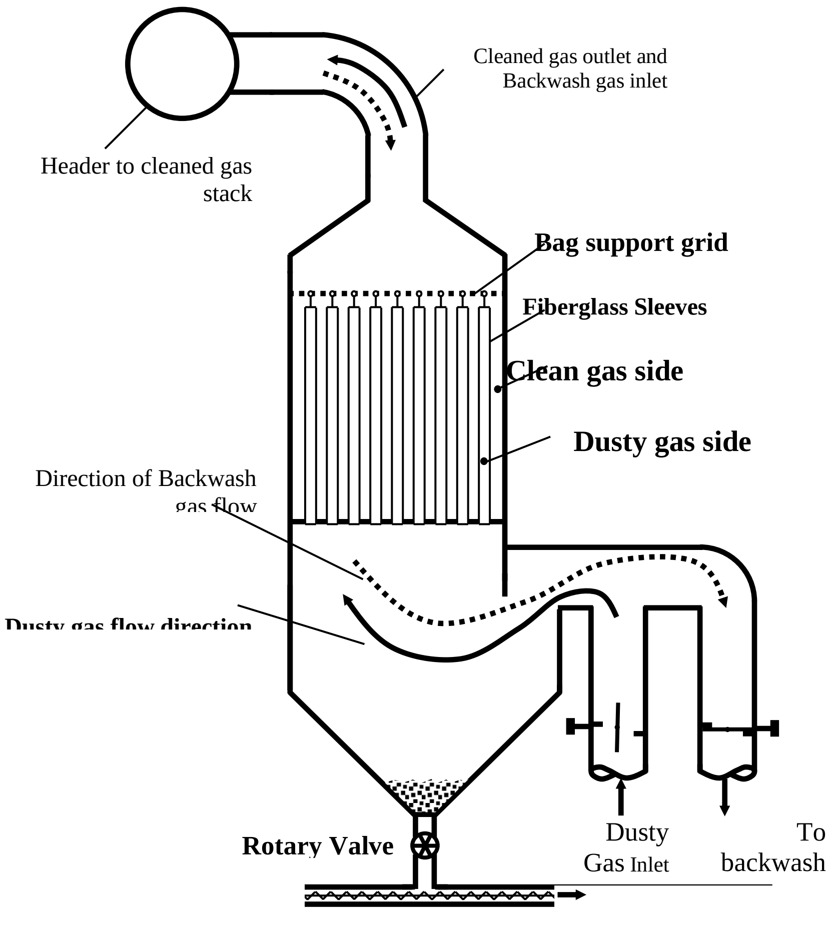


Fig (2.10) A typical bag house compartment

Where: *K* is the Darcy permeability of the fabric and *L* its thickness

Practically, the fabrics used industrially have fixed values of *L/K* so that the above equation simplifies to:

$$\Delta P_f = K_f.\mu.v \qquad (2.30)$$

Where,

ΔP_f: is in inch water

μ: is in cP

v: is the peripheral velocity of air, $ft.min^{-1}$.

The values of the constant K_f are listed for some woven fabrics in Table (2.2)

Table (2.2) Values of the constant K_f

Fabric	Cotton	Wool	Nylon	Asbestos	Fiberglass	Dacron	Teflon
K_f	2.4-2.5	0.3-0.5	2.7-3.7	0.56	2.6	0.3-0.8	2.4

The value of the pressure drop across the accumulated dust increases with time. It has been established that its value is proportional to the square of gas velocity as shown by the following equation:

$$\Delta P_d = 0.5\, K_d.\mu.c_d.t.v^2 \qquad (2.31)$$

Where,

ΔP_d: is in inch water

μ: is in cP

c_d: is the concentration of dust, $g.m^{-3}$

v: is the peripheral velocity of air, $ft.min^{-1}$.

t: is the time of filtration, min.

The value of K_d has to be determined experimentally. For particle sizes less than 10 microns (which is a common case where bag filters are used) it ranges from 0.2 to 0.4.

Bag filters operate on pressure drops ranging from 2 to 6 inch water. The value of the filtration time t is determined from practice as the filter has to be cleaned when the pressure drop exceeds its maximum value. In practice the values of t range from 2 to 15 min.

By adding up the two above equations and equating the total pressure drop to a maximum value of 6 in. water, the value of the superficial velocity v can be deduced. From knowledge of the dusty air flow rate, one can calculate the total filtration area. The value of v should range from 1 to 8 $ft.min^{-1}$.

Example 2.5

Dusty air containing 20 $g.m^{-3}$ solids of average particle size = 5 μm is to be filtered from dust in a bag filters. The available bags are made of cotton fabric. They consist of cylinders of diameter = 0.25 m and length = 3 m. Air is at a temperature of 70°C, at which temperature its viscosity = 0.02 cP. The flow rate of air is 47000 $sm^3.h^{-1}$. Estimate the necessary number of bags.

Solution

From equation (2.30):

$\Delta P_f = K_f.\mu.v = 2.4 \times 0.02v = 0.048v$

The pressure drop across the dust cake will be calculated from Eq. (2.31) by taking t = 10 min., K_d = 0.3

$\Delta p_d = 0.5\, K_d.\mu.c_d.t.v^2 = 0.5 \times 0.3 \times 0.02 \times 20 \times 10.\, v^2$, hence $\Delta p_d = 0.6\, v^2$

The total pressure drop is taken as 5'' water, hence:

$0.048\, v + 0.6\, v^2 = 5$

Solving for v we get: $v = 2.85$ ft.min^{-1}. (note that $1 < v < 8$)
This is equivalent to: $v = 0.0145$ m.s^{-1}

The actual flow rate of air at 70°C is obtained from ideal gas rules:

$\frac{Q}{343} = \frac{47000}{273+15.6}$, from which: $Q = 55859$ m^3.h^{-1}= 15.5 m^3.s^{-1}

The total area of bags is therefore A = 15.5/0.0145 = 1069 m^2.

The area of one bag is $\pi.D.L = \pi \times 0.25 \times 3 = 2.36$ m^2.

Hence the number of bags required is: 1069/2.36 = **435 bags**

2.8 Problems

(1) It is required to evaluate the characteristics of the cake obtained on filtering a slurry containing 120 kg.m^{-3} water using canvas filter medium. To this effect, a one plate filter of area 0.09 m^2 is used. The following data were obtained for the variation of volume of filtrate with time under a pressure of 6 bar.

***V* liter**	0.5	1	1.5	2	2.5	3	3.5	4
Time min	7	19	35	53	76	102	131	163

Estimate the cake mean specific resistance and the filter medium resistance.

(2) It is required to design a plate and frame filter press for the separation of water from mud. To this aim, a laboratory experiment was performed under two constant pressures of 6.7 and 16.2 psig using a similar filter cloth to that intended to use in the filter. The filtration area = 0.05 m^2.
The data are shown in the table.
It is intended to filter 6 m^3 in three hours of the slurry that contains 35 kg solids per m^3 water under a pressure of 12 psig.
The time of filtration is taken as two hours. Find the number of plates if their filter dimensions measure 15”x15”.

***V* liter**	0.5	1	1.5	2	2.5	3	3.5	4
Time s (6.7 psig)	17.3	41.3	72	108	152	202	----	----
Time s (16 psig)	6.8	19	34.6	53.4	76	102	131	163

(3) A rotary drum filter with 30% submergence is used to filter concentrated slurry containing 236 kg.m^{-3} water under a pressure drop of 50 mm Hg. Calculate the filter area required to filter 50 lit.min^{-1} of slurry when the filter cycle is 5 minutes. The filter cake contains 50% water (on wet basis). Assume the specific filter cake resistance to be the same that would be obtained from the previous problem under the specified pressure drop. (Neglect the filter medium resistance).

(4) The filter in problem (2) is operated at constant rate = 15 lit.m^{-2}.min^{-1} from the start of the run until the pressure reaches 70 psig. Then a constant pressure is applied until 5000 L of filtrate are collected. Calculate the total time required for filtration.

(5) An industrial filter press consists of 35 plates (and an equal number of frames) of dimensions 700×700 mm^2. It is used to filter off 5 m^3 of aluminous mud containing 280 kg solid per m^3 filtrate. The specific cake resistance is related to the applied pressure by the relation:

$\alpha = 2.35 \times 10^9 (\Delta p)^{0.48}$ $m.kg^{-1}$ (Pressure in Pa)

Filtration is conducted at constant rate for 6 hours until the pressure reaches 6 bar, after which filtration proceeds at that same pressure. Evaluate the total time required to carry out the filtration process.

(6) Air laden dust at 25 $g.m^{-3}$ and with an average particle size of 5 µm is filtered in a filter bag house. The bags consist of cotton cylinders of 2.5 m length and 300 mm diameter. Air is admitted at 60°C and its viscosity is 0.018 cP. The air flow rate is 45000 Nm^3/h. Estimate the number of bags in the house. (Take K_f = 1.5 and K_d = 0.3 and consider the filtration time to be 5 min.).

(7) A bag house contains 400 filters of 250 mm diameter and 3 m length. Flue gases are admitted to that filter at the rate of 35000 $Nm^3.h^{-1}$ at 115°C and 1.5 bar. They contain 15 $g.m^{-3}$ dust. Calculate the power required to drive the gases through the bag house. (Viscosity under prevailing conditions = 2.5×10^{-2} cP. Take K_f = 1.25 and K_d = 0.35, time of filtration = 8 min).

3. SEDIMENTATION

3.1 Batch sedimentation curves

When a relatively concentrated suspension of solid particles is allowed to settle in a batch container, different zones are formed as sedimentation proceeds. Usually, the mode of settling is of the hindered type and all particles present at a certain level settle at equal velocities. In Figure (3.1) is shown the pattern appearing as sedimentation proceeds.

First, the solids are uniformly distributed in the liquid (a). After some time elapses, four layers appear: A clear upper layer (A), a second layer (B) where the concentration of solids is equal to its initial concentration, a lower layer (D) where solid particles have settled and an intermediate layer (C) where there is a concentration gradient from that of (B) to that of (D).

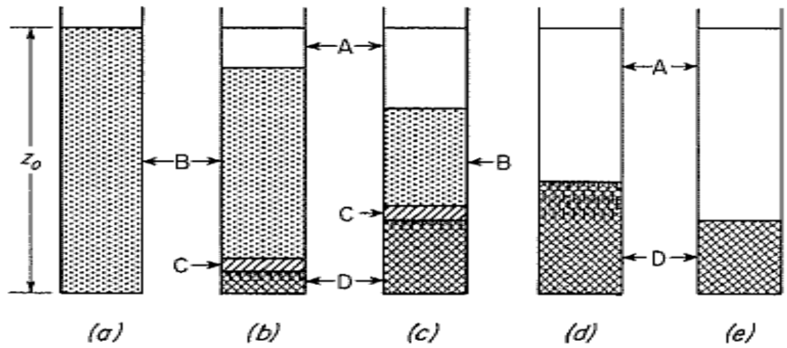


Fig (3.1) Batch sedimentation

As settling takes place the depth of zones (A) and (D) increases. Eventually zone (B) disappears, and all the solids are in zones (C) and (D). Also, the gradual accumulation of solids in zone (D) puts pressure on the material at the bottom, thus compressing this zone forcing liquid to flow from this layer upwards. Finally, settling stops as the height of the compression zone stabilizes. Figure (3.2) shows the relation between the height of the clear liquid interface and time.

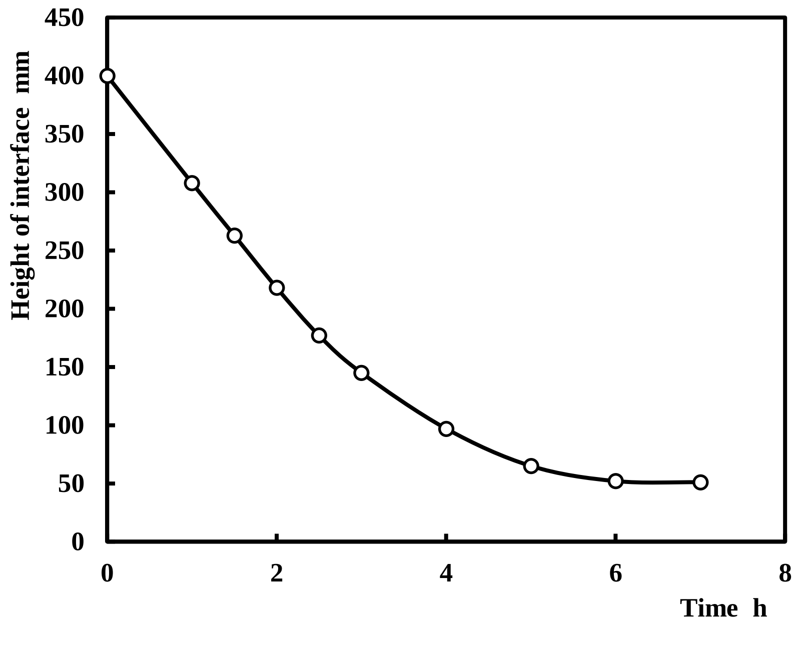


Fig (3.2) Batch sedimentation curve

3.2 Continuous sedimentation equipment: Thickeners

3.2.1 Features of the equipment

Industrially the above operation is conducted on large scale equipment known as thickener. Sometimes, a batch thickening tank can be used to separate solids of high settling rate. However, continuous operation is more common. A continuous thickener consists of a large shallow tank with slow moving radial rakes driven by a central shaft. Its bottom is usually in the form of a shallow cone. Dilute slurry flows from an inclined trough to a central feed well where it is distributed into the center of the thickener (Figure 3.3).
The feed slurry being denser than water tends to flow downwards until its density stabilizes. It then moves radially outwards at decreasing speed and the flow gradually divides into a downward moving suspension and an upward moving clear liquid. Bottom rakes slowly agitate the solid sludge moving it towards a central opening at the conical bottom while the clear liquid flows over a weir situated in the upper part of the thickener.
Such thickeners can be very large of diameters reaching 100 m. and depths up to 4 m and the rakes move at about 2 rpm.

3.2.2 Kynch method: Settling rate plot

The design of such thickeners is primarily concerned with finding the area required to perform a certain duty. This in turn requires the elaboration of a settling rate curve. To this aim, the Fynch method is a simple method that relies on one single batch sedimentation experiment. The settling rate curve is a curve relating the settling rate $\frac{dz}{dt}$ against solid concentration at the solids at the top of the settling zone inside the thickener. Fynch has elaborated a simple technique to obtain such concentration a function of height:

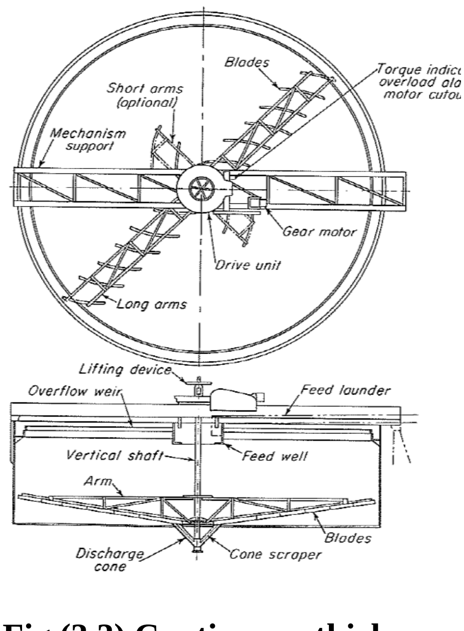


Fig (3.3) Continuous thickener

Consider any point on the on the settling curve in Figure (3.4) like (t, z). A tangent is drawn at that point and extended to intersect the height axis at z_i. If the initial height of suspension is z_o and its initial concentration is c_o, then the expected concentration at height z can be calculated from:

$$c_i = \frac{c_0 . z_0}{z_i} \qquad (3.1)$$

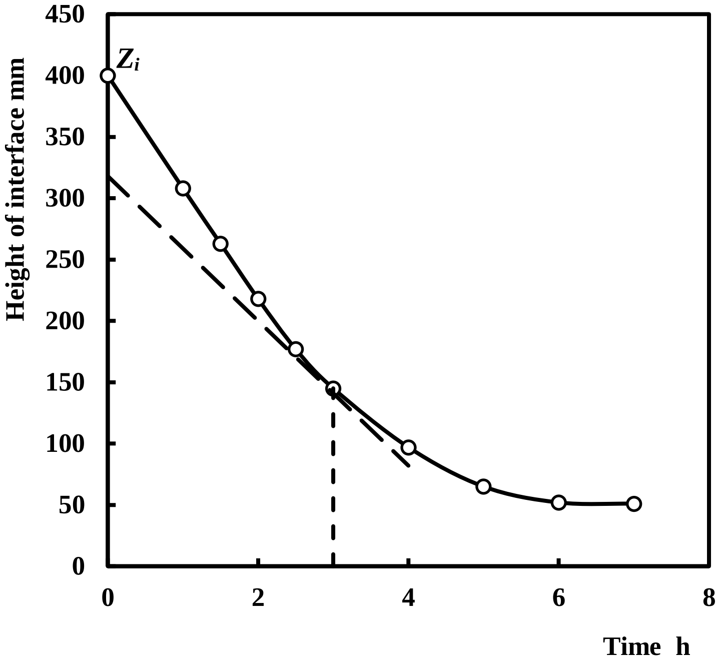


Fig (3.4) Kynch construction

The procedure is repeated for several values of z and a plot made between the slope of tangent ($\frac{dz}{dt}$) and c_i. (Figure 3.5)

Practically, it is more convenient to determine the equation of curve and perform analytical differentiation of the equation obtained. This is best understood by the following example.

3.2.3 Application to the design of continuous thickeners

As opposed to a batch sedimentation tank, in a continuous thickener, there is a downward flow of liquid with solids at an almost constant velocity v. The mass rate of solids transported with the liquid per unit area is called the **transport flux** G_t (kg/m^2.h). Also, a second solid flux is associated with settling solids and is called **settling flux** G_s.

The transport flux $G_t = c.v$ whereas the settling flux = $c.\frac{dz}{dt}$, so that the total flux is:

$$G = G_t + G_s = c.v + \frac{dz}{dt}.c \qquad (3.2)$$

Using the construction shown in Figure (3.5) and knowing the value of the transport velocity v, we can plot the total flux G against c_i. The total flux passes through a minimum value G_{min} as shown in Figure (3.6).

If the inlet rate of solids is $F.c_o$ kg/h (where F is the feed rate in m^3/h), then the area of the thickener can be calculated from the following relation:

$$F.c_o = A.G_{min} \qquad (3.3)$$

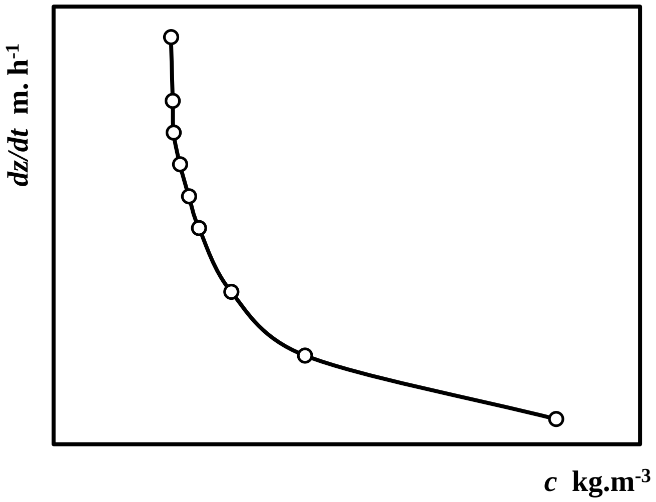


Fig (3.5) Plot of rate of settling against concentration

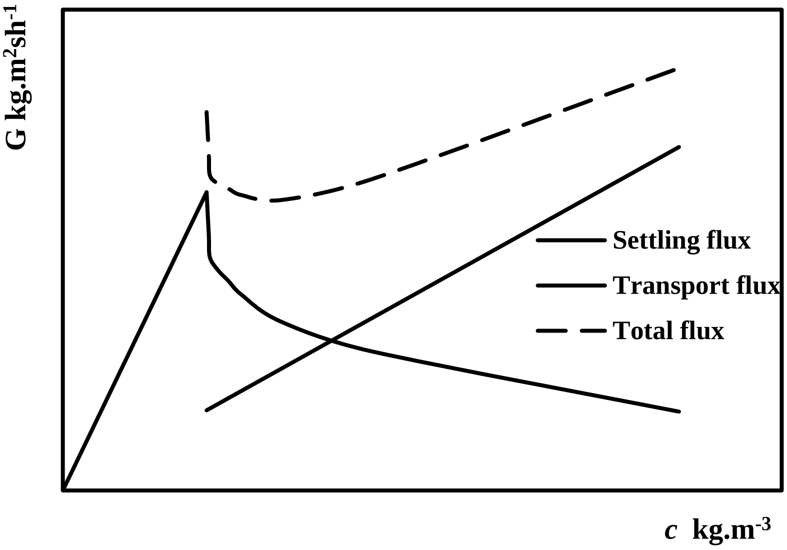


Fig (3.6) Flux – concentration curves

Example 3.1

In a water treatment unit, 200 $m^3.h^{-1}$ containing 250 $kg.m^{-3}$ silt is to be clarified in a continuous thickening tank. Data for batch sedimentation are shown in the table below. Assuming the transport flux to equal 0.5 $m.h^{-1}$, calculate the diameter of the required thickener.

Time min.	0	20	40	60	80	100	120	140	∞
Height mm	475	350	260	200	160	135	120	110	85

Solution:

An equation has been obtained for the settling curve after transforming time units to hours and height units to meters, taking into account that the asymptotic value of height = 85 mm:

$z = 0.085 + 3.856e^{-1.2t}$ (Figure 3.7)

Differentiating, we obtain: $\left|\frac{dz}{dt}\right| = 4.627e^{-1.2t} = 1.2\,(z - 0.085)$

The intercepts of tangents drawn at that curve at different values of c_i are obtained from the equation of tangents.

$z_i = z - \text{slope}.t$

The values of c_i are then obtained from equation (3.1)

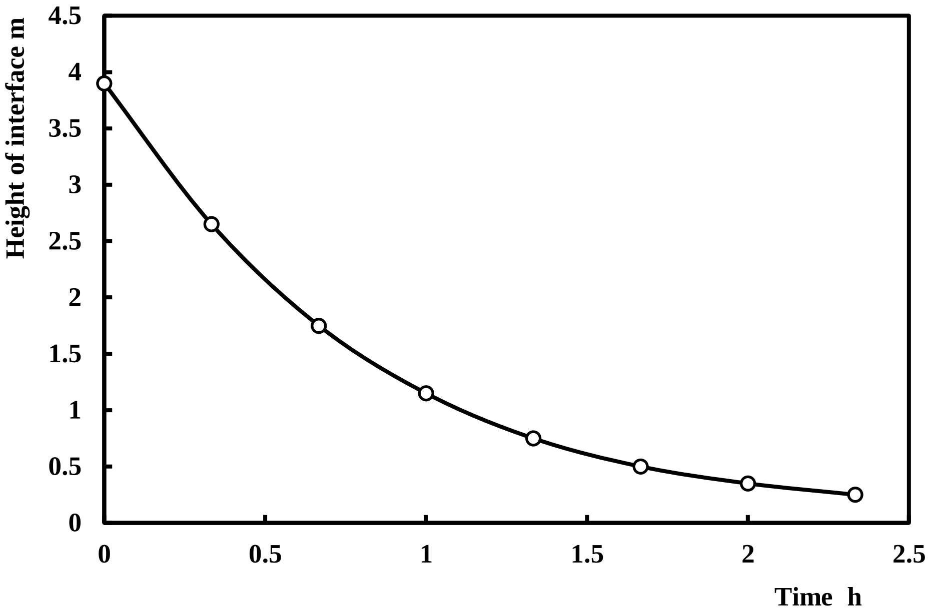


Fig (3.7) Sedimentation curve

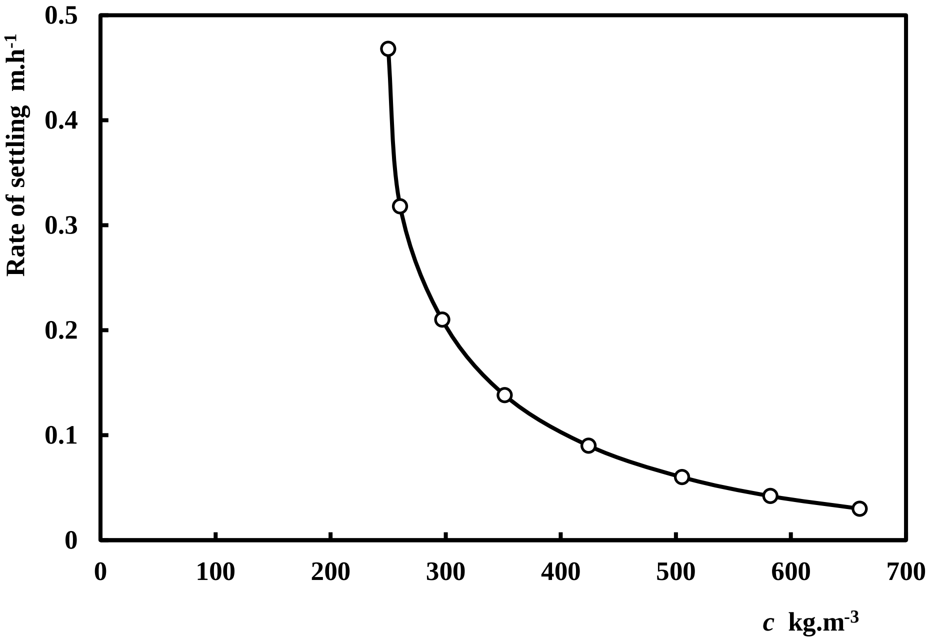


Fig (3.8) Settling rate – concentration curve

The values of $\frac{dz}{dt}$ are plotted against c_i to obtain the **settling rate – concentration plot** (Figure 3.8)

The settling flux is calculated at each value of c_i as $c_i \times \frac{dz}{dt}$

The transport flux is calculated from 0.5*c*

The following table shows all calculations:

t min	0	20	40	60	80	100	120	140	∞
z mm	475	350	260	200	160	135	120	110	85
z - 0.085 m	0.39	0.265	0.175	0.115	0.075	0.05	0.035	0.025	0
abs rate m/h	0.468	0.318	0.21	0.138	0.09	0.06	0.042	0.03	
z^* m	0.475	0.456	0.4	0.338	0.28	0.235	0.204	0.18	
c kg/m^3	250	260.417	296.875	351.331	424.107	505.319	582.108	659.722	
G_s kg/m^2.h	117	82.8125	62.3438	48.4837	38.1696	30.3191	24.4485	19.7917	
G_t kg/m^2.h	125	130.208	148.438	175.666	212.054	252.66	291.054	329.861	
Total flux	242	213.021	210.781	224.149	250.223	282.979	315.502	349.653	

The total minimum flux = 210 kg.m^{-2}.h^{-1} is obtained at concentration about 297 kg.m^{-3}. Hence from equation (3.3), F = 200 m^3.h^{-1}, c_0 = 250 kg.m^{-3}, we get:

Area of thickener = $F_0.c_0/G_{\min} = \frac{200\times250}{210.787}$ = 237.2 m^2

Thickener diameter ≈ 17.4 m

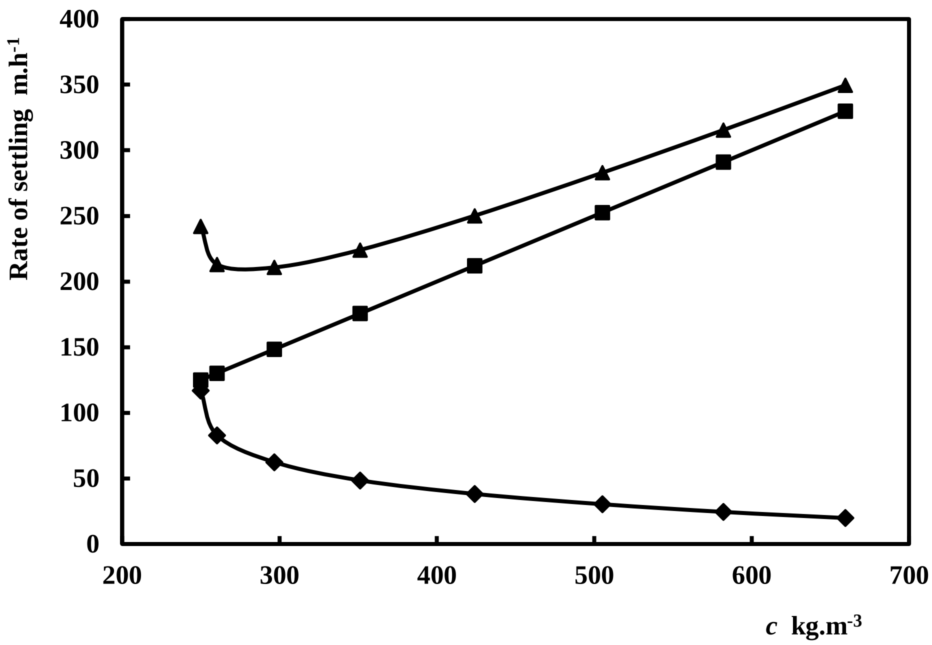


Fig (3.9) Flux curves

3.3 Problems

(1) Data were obtained in a single sedimentation test on a suspension of concentration 250 kg.m^{-3} placed in a graduated cylinder. The height of interface (z mm) was correlated to time (min) by the approximate expression:

$z = 100 + 300\, e^{-0.3t} \quad (0 \le t \le 7)$

(a) What is the final height of the compression zone?

(b) What is the initial height of suspension?

(c) Prove that the numerical value of the rate of settling ($mm.min^{-1}$) is related to the height of interface by: $\left|\frac{dz}{dt}\right| = 0.3z - 30$

(d) Plot the effective height of interface against concentration.

(e) Plot the settling flux ($kg.m^{-2}.h^{-1}$) as function of concentration.

(2) An equation was derived to correlate the height of interface (mm) to time (min) in a single sedimentation test:

$z = 380 - 38.6\sqrt{t} \quad (t < 60 \text{ min})$

The original concentration of suspension = 600 $kg.m^{-3}$.

For a solid transport rate of 0.7 $m.h^{-1}$, estimate the diameter of a continuous thickener that treats 400 $m^3.h^{-1}$ slurry.

4. AGITATION AND MIXING

4.1 Introduction

Many processing operations depend for their success on the effective agitation and mixing of fluids. Agitation differs from mixing in that the former usually refers to the circulatory motion of a fluid inside a vessel, while the latter is associated with the random distribution one into another of at least two phases (one of which may be a solid). The homogeneity of the mix differs from one case to another. For example, mixing gases always produces a homogeneous phase where the composition is constant throughout the mixture. On the other hand, mixing of cement and sand with water will not produce such homogeneous mix.

4.2 Agitation of liquids

4.2.1 Purpose of agitation

Liquids are agitated for a number of reasons:

- Suspending solid particles.
- Blending miscible liquids (such as ethanol and water).
- Dispersing gas in a liquid in bubble form.
- Dispersing a liquid into an immiscible liquid to form an emulsion.
- Increasing the heat transfer coefficient between a liquid and a cooling coil or jacket.

Sometimes more than one function can be achieved: In hydrogenation of a liquid, agitation serves to disperse hydrogen gas into the liquid besides helping to remove the heat of reaction through the external cooling coil or jacket.

4.2.2 Agitation equipment

A typical equipment for agitation is shown in Figure (4.1). it consists of a cylindrical vessel fitted with a central shaft ending with an impeller. The motor is either directly connected to the shaft or through a belt drive. The edges of the tank bottom are usually rounded to eliminate regions where fluid currents would not penetrate.

The tank may be fitted with an external jacket for cooling or heating. Baffles are often fixed to the vessel walls to prevent the formation of vortices.

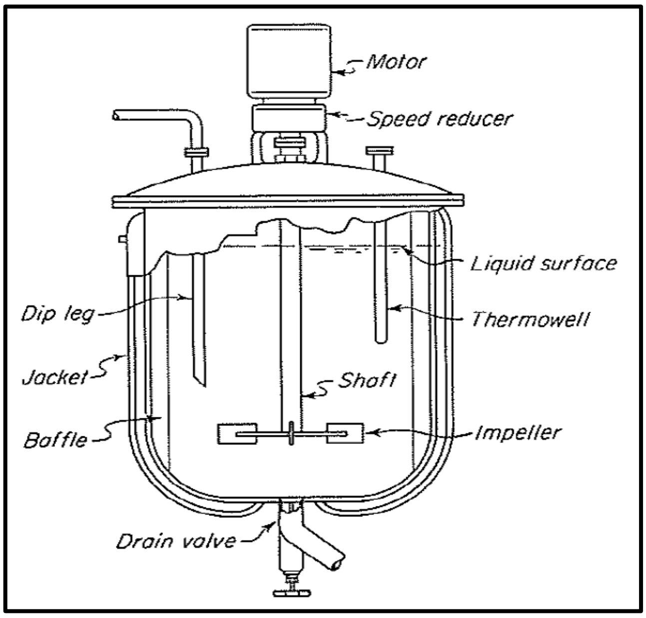


Fig (4.1) An agitated vessel

4.2.3 Impellers

In general, there are three main types of impellers for liquid agitation purposes.

Propellers:

A propeller is an axial flow, high speed impeller for liquids of low viscosity. Their speeds range from 400 rpm in case of large propellers to 1700 rpm for smaller sizes. The flow currents leaving the impeller continue through the liquid in a certain direction until deflected by the floor or walls. The propeller blades vigorously shear the liquid causing high turbulence to prevail. Such impellers are particularly effective in large vessels. Figure (4.2) shows a three-blade marine propeller. The size of such propellers does not usually exceed 18" regardless of the diameter of the vessel.

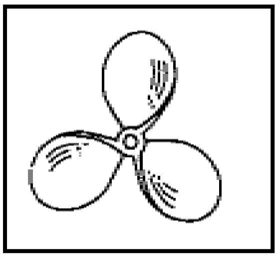


Fig (4.2)Three blade Propeller

Paddle agitators

These simply consist of a number of paddles (usually 2 or 4) mounted on a vertical shaft. They are used to slow agitate moderately viscous liquids. The pattern of currents is mainly radial with no vertical motion. Sometimes more than one set of paddles can be fitted to the central shaft. These impellers usually rotate at relatively low speeds averaging 80 rpm. Their size ranges from 60 to 80% of the vessel diameter while their width is about 10% of their length. It is usual to use unbaffled vessels for low speeds (up to 50 rpm), while for higher speeds vortices tend to form so that baffling is necessary. Sometimes the shape of the paddles may accommodate that of the bottom of the vessel to prevent scale deposition on heat transfer surfaces. One typical such type is the anchor agitator which is used in agitating viscous liquids while providing excellent heat transfer to an external cooling jacket. (Figure 4.3)

Fig (4.3) Anchor agitator

Turbine agitators

These resemble multibladed paddle agitators except that they can turn at higher speeds. They operate over a wide range of liquid viscosities. The blades may be straight, curved or pitched (Figure 4.4). The impeller diameter ranges from 30 to 50% of that of the vessel. In low viscosity liquids, the rapid currents formed possess both radial and tangential components. These latter components cause the formation of vortices which must be broken since they do not result in proper mixing. On the other hand, the zone near the impeller is a zone of high turbulence and intensive shear.

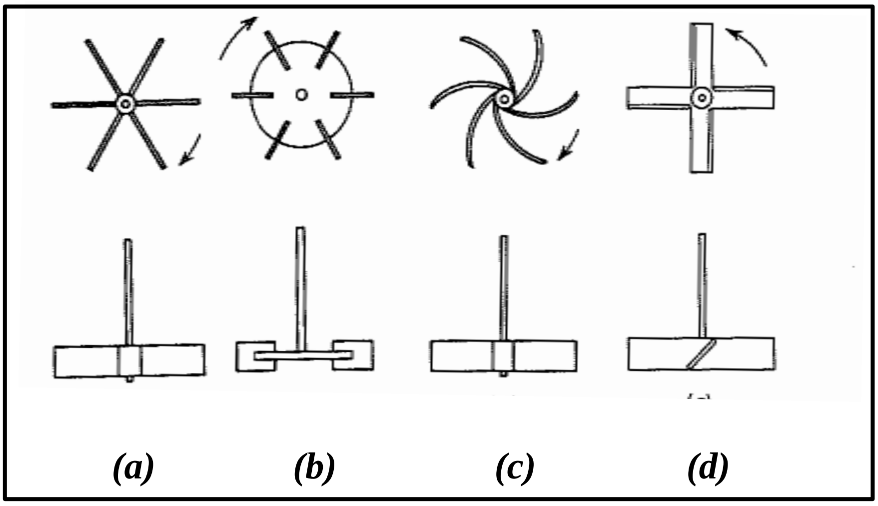


Fig (4.4) **(a) Open straight blade turbine impeller**
(b) Bladed disk turbine impeller
(c) Curved blade turbine impeller
(d) Pitched blade turbine impeller

Figure (4.5) shows details of curved and pitched blades turbine impellers.

Fig (4.5) (a) Curved blades and (b) Pitched blades turbines

4.2.4 Pattern of flow

The type of flow depends on the type of impeller, fluid characteristics, size and geometry of vessel and agitator.

The velocity of the fluid at any point has three components: A radial and a tangential component in a plane perpendicular on that of the shaft and a longitudinal component parallel to the direction of the shaft. The tangential component is responsible for the formation of a vortex with little or no mixing while the two other components are responsible for the mixing action.

In small vessels, this swirling action can be prevented by mounting the shaft off the center line of the vessel (Figure 4.6). In large vessels, on the other hand, it is customary to use up to four baffles welded to the vessel walls. The flow pattern in these cases changes radically since the swirling vortices are not formed and the efficiency of agitation is highly enhanced.

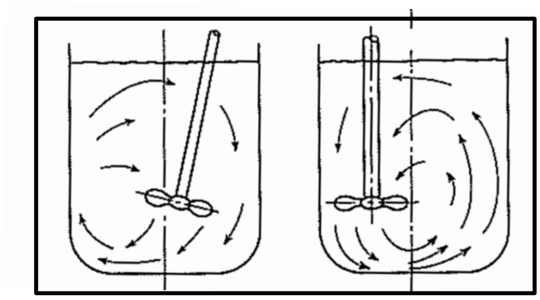
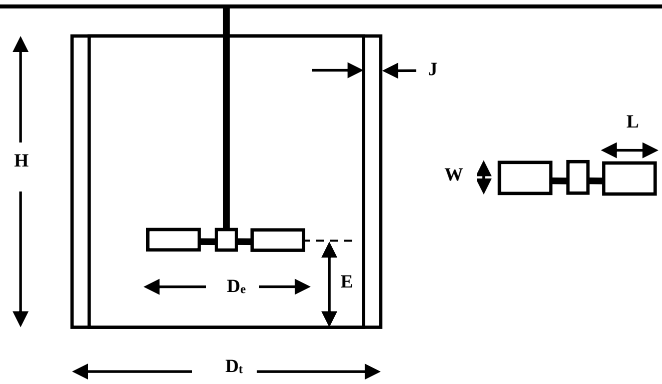


Fig (4.6) Eliminating swirling

4.2.5 Recommended dimensions for turbine mixers

Although the design of an agitated vessel varies considerably depending on the location of the impeller, the number of baffles (if any), the relative proportion of the vessel, etc., it is customary in case of turbine impellers to use some typical relative dimension. Figure (4.7) shows these ratios.



$$S_1 = \frac{D_e}{D_t} = \frac{1}{3} \qquad S_2 = \frac{E}{D_t} = \frac{1}{3} \qquad S_3 = \frac{L}{D_e} = \frac{1}{4}$$

$$S_4 = \frac{W}{D_e} = \frac{1}{5} \qquad S_5 = \frac{J}{D_t} = \frac{1}{12} \qquad S_6 = \frac{H}{D_t} = 1$$

Fig(4.7) Typical turbine proportions

4.3 Power consumption in agitation

4.4.1 Elaboration of an equation to predict power consumption

The power consumed in liquid agitation depends on several factors: speed of rotation (rps = n), impeller diameter (D_e), fluid properties: density (ρ), viscosity (μ) besides vessel geometry. This last factor is quantified by the relative ratios shown in Figure (4.7). A general relation can be written in the form:

$$P = f(n, D_e, \rho, \mu, g, S_1, S_2, S_3, ...) \qquad (4.1)$$

Where g = gravitational acceleration and S_1, S_2, ,… = dimensionless shape factors defined in Figure (4.7).

The elaboration of a suitable formula for the prediction of power consumption in a agitation operation, we use dimensional analysis as follows:

Let $P = k.\rho^a.\mu^b.n^c.g^d.D_e^h$ (4.2)

Now, denoting by Θ, L and M the basic dimensions of time, length and mass respectively, we get:

$$\frac{M.L^2}{\Theta^3} = \left(\frac{M}{L^3}\right)^a.\left(\frac{M}{L.\Theta}\right)^b.\left(\frac{1}{\Theta}\right)^c.\left(\frac{L}{\Theta^2}\right)^d.L^h$$

Hence:

$$\frac{M.L^2}{\Theta^3} = \Theta^{-b-c-2d}.L^{-3a-b+d+h}.M^{a+b}$$

Equating the powers of similar terms we get the following set of equations:

$b + c + 2d = 3$ (i)
$-3a - b + d + h = 2$ (ii)
$a + b = 1$ (iii)

Solving for a, c and h, considering b and d to be independent parameters, we get:

$a = 1 - b$ (iv)
$c = 3 - b - 2d$ (v)
$h = 5 - 2b - d$ (vi)

Replacing in equation (4.2):

$$P = k.\rho^{1-b}.\mu^b.n^{3-b-2d}.g^d.D_e^{5-2b-d}$$

Grouping alike powers, we get:

$$\frac{P}{\rho.n^3.D_e^5} = k.\left(\frac{\rho.n.D_e^2}{\mu}\right)^{-b}.\left(\frac{n^2.D_e}{g}\right)^{-d} \qquad (4.3)$$

The first dimensionless group $\left(\frac{\rho.n.D_e^2}{\mu}\right)$ in the RHS is a modified **Reynolds number Re** where the usual term of velocity (v) has been replaced by ($n.D_e$), which is a variation of the $\omega.R$ form.

The second term $\left(\frac{n^2.D_e}{g}\right)$ represents the ratio between the centrifugal acceleration $n^2.D_e$ (to replace $\omega^2.R$) and the gravitational acceleration (g). It is called the **Froude number *Fr***.

The LHS $\left(\frac{P}{\rho.n^3.D_e^5}\right)$ is a dimensionless group known as the **Power number *Po***

It is analogous to the friction factor in fluid flow.

Equation (4.1) can then be put in the form:

$Po = f\,(Re, Fr, S_1, S_2, S_3, \ldots)$ (4.4)

4.4.2 Power correlations for specific impellers

The various shape factors in equation (4.4) depend on the type and arrangement of the equipment. The shape factors S_1, S_2, S_3, S_4, S_5 and S_6 are defined as follows with reference to Figure (4.7):

$$S_1 = \frac{D_e}{D_t},\ S_2 = \frac{E}{D_t},\ S_3 = \frac{L}{D_e},\ S_4 = \frac{W}{D_e},\ S_5 = \frac{J}{D_t},\ S_6 = \frac{H}{D_t} \qquad (4.5)$$

In addition, the number of baffles and the number of impeller blades must be specified.

Figure (4.8) shows a typical plot of *Po* against *Re* for baffled tanks fitted with centrally located flat bladed turbines with 6 blades. Different values of shape factors are shown on the figure for different curves.

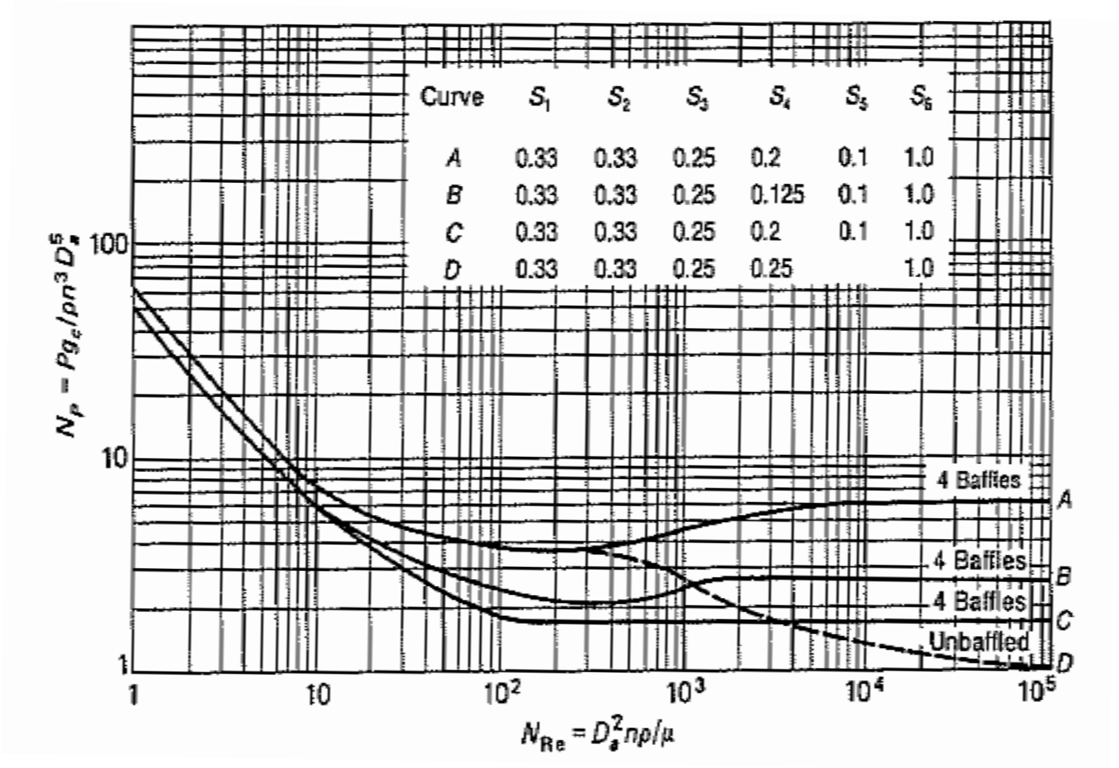


Fig (4.8) Power number - Reynolds number plots for 6 bladed turbines

We note that in all three baffled curves, the Froude number does not play any role in assessing the *Po – Re* relation. This is since the presence of baffles prevents the swirling motion which is associated with the appearance of Froude number. Curve D on the other hand, denoted the case of unbaffled tanks. The portion following $Re = 10^3$ is dotted. This is since the values of power number calculated thereof must be corrected to take into consideration vortex formation owing to high speed of revolution. However, the use of unbaffled tanks is very rare at high Reynolds numbers. That is why correlations involving the use of Froude number are not of interest.

It is to be noted that the difference between curves A and C is that C represents the case of pitched blade turbines while curve A corresponds to flat blades.

We also note that at low values of $Re < 10$, all curves have a slope = – 1 on the logarithmic scale corresponding to a relation in the form:

$$Po = \frac{K_L}{Re} \qquad (4.6)$$

Hence:

$$\frac{P}{\rho.n^3.D_e^5} = \frac{\mu.K_L}{\rho.n.D_e^2}$$

Leading to the following equation for power prediction:

$$P = K_L.n^2.D_e^3.\mu \qquad (4.7)$$

On the other hand, at high values of $Re > 10^4$, the values of Re tend to stabilize so that $Re = K_T$. Hence, the power can be calculated from the following equation:

$$P = K_T.n^3.D_e^5.\rho \qquad (4.8)$$

The values of the constants K_L and K_T are tabulated in Table (4.1) for different types of impellers for baffled tanks (4 baffles) with $S_4 \approx 0.2$.

Table 4.1: Values of K_L and K_T

Type of impeller	K_L	K_T
Propeller three blades	41	0.32
Turbine		
• Six blade disk	65	5.75
• Four pitched blade	44.5	2.27
Flat paddle (2 paddles)	34.5	2.7
Anchor	300	0.35

If the Reynolds number is in the range $10 - 10^4$, then the previous equations (4.7) and (4.8) can no more be of use and one has to predict the power consumption through charts such as that shown in Figure (4.8).

Example 4.1

A six blades disk turbine impeller is installed centrally in a vertical baffled tank 2 m in diameter. The turbine has a diameter = 670 mm and is positioned 670 mm above the bottom of the tank. The turbine blades are 134 mm wide. The tank is filled to a depth of 2 m with an aqueous solution of 50% NaOH at 65°C (Density = 1500 kg.m^{-3}, viscosity = 12 cP). Calculate the power required for a speed of revolution of impeller = 90 rpm. Then deduce the power of the motor assuming an efficiency = 0.85.

Solution:

We have: $n = 2.5$ rps, $D_e = 0.67$ m, $\rho = 1500$ kg.m^{-3}, $\mu = 12\times10^{-3}$ Pa.s

Hence: $Re = \dfrac{1500\times1.5\times0.67^2}{0.012} = 84169 > 10^4$

Hence from Table (4.1), we get: $K_T = 5.75$

From equation (4.8), we get:

$P = 5.75\times1.5^3\times0.67^5\times1500$ = **3930 W**.

Hence, motor power = $\dfrac{3930}{0.85\times735}$ = 6.29 → **7 hp**

Example 4.2
If the vessel of the previous example is used to mix a polymeric compound of viscosity 4.2×10^5 cP and density = 1200 kg.m^{-3}, what should be the power of the motor used in that case?

Solution:

$$Re = \frac{1200\times1.5\times0.67^2}{120} = 6.73 < 10$$

Hence equation (4.7) can be used with K_L = 65. We get:

$$P = 65\times1.5^2\times0.67^3\times120 = 5278 \text{ W}$$

Hence, motor power = $\dfrac{5278}{0.85\times735}$ = 8.44 $\rightarrow$ **9 hp**

Example 4.3
The same vessel is now used to mix a monomer of viscosity = 150 cP and density = 1200 kg.m$^{-3.}$ What should be the power of the motor used?

Solution:

$$Re = \frac{1200\times1.5\times0.67^2}{150\times10^{-3}} = 5387$$

This value of Re situate the flow pattern in neither laminar nor turbulent regime so that we cannot apply either corresponding equation.

Referring to equation (4.5), the given data show that:

$$S_1 = S_2 = \frac{0.67}{2} = 0.33,\ S_4 = \frac{134}{670} = 0.2,\ S_6 = \frac{2}{2} = 1$$

These values correspond to curve (A) in Figure (4.8). For a value of $Re \approx 5400$, we get: $Po \approx 6$. From the definition of Po, we get:

$$P = Po.\rho.n^3.D_e^5 = 6\times1200\times1.5^3.\times0.67^5 = 3280 \text{ W.}$$

The motor power will then be:

$\dfrac{3280}{0.85\times735}$ = 5.25 $\rightarrow$ **6 hp**

4.4 Mixing of liquids

4.4.1 Mixing of miscible liquids

The previous sections dealt with predicting the power necessary for agitation of liquids. In case of multiphase mixing, the operation is much more difficult to describe. The pattern of flow in single phase agitation is usually predictable given a certain mixer design, but in multiphase mixing the extent to which uniformity is reached often depends on such factors as color change or reaching uniform temperature. The main factor of interest in multiphase mixing is to predict the time required to get a uniform mix under the given operating conditions.

In case of miscible liquids blending is usually performed in relatively small process vessels by propellers or turbines that are usually centrally mounted. In case of large quantities to be mixed, the agitator may be idle most of the time and be turned on only to blend any formed stratified layers of liquid.

The prediction of the time required to get almost complete mixing is based on the following assumptions:

- Efficiency of mixing = 99%
- Number of circulation loops = 5
- ***Re* > 2000**

Given these assumptions it was possible to establish the following equation for predicting the required time of mixing for standard six blade turbine

$$t_M = 4.3 \times \frac{H.D_t}{n.D_e^2} \tag{4.9}$$

For lower values of *Re*, the mixing time is appreciably higher.

For ***Re* < 2000**, in case of standard six blade turbines, a friction factor has been defined by Norwood and Metzner that makes it possible to predict the mixing time over a large range of Reynolds numbers:

$$f = n.t_M.\left(\frac{D_e}{D_t}\right)^2.\left(\frac{D_t}{H}\right)^{\frac{1}{2}}.Fr^{-\frac{1}{6}} \tag{4.10}$$

The relation between this friction factor and Reynolds number for *Re* < 2000 takes the form shown in Figure (4.9)

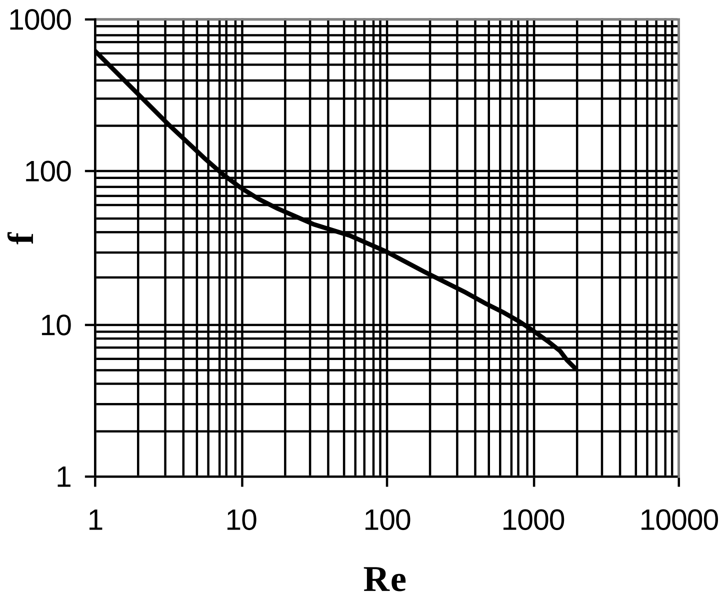


Fig.(4.9) Correlation of mixing time for miscible liquids in 6 blade turbine baffled vessels

Finally, it is worth mentioning that the power required for mixing can still be calculated from equations (4.7) for laminar regime (*Re* < 10) or (4.8) for turbulent regime (*Re* > 10^4) or from charts such as those shown in Figure (4.8).

Example 4.4

An agitated vessel 1.83 m in diameter contains a six blade turbine impeller 0.61 m in diameter rotating at 80 rpm. It is used to neutralize a solution of NaOH with concentrated nitric acid. The depth of solution = 1.83 m. How long would it take for neutralization to be complete? Estimate the required power. (Density of solution = 998 kg/m^3, viscosity = 0.98 cP)

(Density of solution = 998 kg.m^{-3}, viscosity = 0.98 cP)

Solution:

We have: n = 2.33 rps, D_e = 0.61 m, ρ = kg.m^{-3}, $\mu = 0.98\times10^{-3}$ Pa.s

$$Re = \frac{998\times1.33\times0.61^2}{0.00098} = 5\times10^5$$

Since Re > 2000, then equation (4.9) can be used. We get: $t_M = 4.3\times\frac{1.83\times1.83}{1.33\times0.61^2} \approx \mathbf{29\ s}$

Since $Re > 10^4$, then: $P = K_T.n^3.D_e^5.\rho = 5.75\times4.33^3\times0.61^5\times998$ = **1140 W**

Example 4.5

An agitated vessel 2 m in diameter contains a six blade turbine impeller 0.67 m in diameter rotating at 60 rpm. It is used to mix two miscible elastomers. The depth of solution = 2 m. How long would it take for blending to be complete? (Density of solution = 1100 kg.m^{-3}, viscosity = 500 cP)

Solution:

We have: n = 1 rps, D_e = 0.67 m, ρ = 1100 kg.m^{-3}, $\mu = 500\times10^{-3}$ Pa.s

$$Re = \frac{1100\times1\times0.67^2}{0.5} = 987 < 2000$$

From Figure (4.9), for $Re \approx 1000$, we get $f \approx 9$

And $Fr = \frac{1^2\times0.67}{9.81} = 0.068$

From equation (4.10)

$$f = n.t_M.\left(\frac{D_e}{D_t}\right)^2.\left(\frac{D_t}{H}\right)^{\frac{1}{2}}.Fr^{-\frac{1}{6}}\text{, hence:}$$

$$9 = 1\times t_M.\left(\frac{0.67}{2}\right)^2.\left(\frac{2}{2}\right)^{\frac{1}{2}}.\times0.068^{-\frac{1}{6}}$$

From which: **t ≈ 51 s**

4.4.2 Mixing of immiscible liquids

Various types of equipment are used to disperse one liquid in another immiscible liquid (e.g. benzene in water). A stirred tank can be commonly used to disperse a liquid in the form of droplets of size varying from 0.05 to 1 mm. if no agitation takes place these droplets will either rise or fall in the continuous liquid phase depending on the relative density of the two liquids.

In a stirred tank, the average drop size will depend on a balance between breakup of large drops in regions of high shear and coalescence of smaller drops in regions of lower shear. The applied stress tends to deform the drop while its surface tension will resist such deformation. To that aim, it is necessary to define an average drop size.

Let the volume of dispersed phase per unit volume of continuous phase = V_d. Hence for N spherical droplets of mean size D_s, we get:

$$V_d = N.\frac{\pi}{6}.D_s^3 \tag{4.11}$$

Denoting by A_d the total area of droplets, hence:

$$A_d = N.\pi.D_s^2 \tag{4.12}$$

In practice, A_d represents the interfacial area per unit volume of continuous phase.

Dividing equation (4.11) by (4.12), we get:

$$D_s = \frac{6.V_d}{A_d} \tag{4.13}$$

So, the mean droplet size can be obtained by determining the volume of the dispersed phase and the interfacial area.

Since the experimental determination of interfacial area is a difficult task, empirical correlations have been proposed that relate the mean droplet size D_s to the impeller diameter D_e and the volume of dispersed phase per unit volume of continuous phase V_d. These correlations include a dimensionless group known as the **Weber number *We*** which represents the ratio between the kinetic energy of liquid at impeller tip and its surface tension. It is defined as follows:

$$We = \frac{\rho_c.n^2.D_e^3}{\gamma_d} \tag{4.14}$$

Where, ρ_c is the density of continuous phase and γ_d the surface tension of dispersed phase ($N.m^{-1}$)

One such correlation applicable for the dispersion of low viscosity liquids in small tanks is:

$$\frac{D_s}{D_e} = 0.058\times(1+5.4V_d).We^{-0.6} \tag{4.15}$$

Example 4.6

8% cyclohexane (by volume) is dispersed in water at 25°C in a baffled vessel 300 mm in diameter with a depth of 350 mm. The agitator is a standard six blade turbine 100 mm in diameter. The stirrer is run at 360 rpm.
Estimate the power consumption and the mean droplet size.
(Density of cyclohexane = 760 $kg.m^{-3}$, viscosity ≈ 1 cP, surface tension = 0.046 $N.m^{-1}$)

Solution:

Mean density of mixture = $0.08\times760+0.92\times1000$ = 980.8 $kg.m^{-3}$
Speed of revolution = 360 rpm ≡ 6 rps
Hence:

$$Re = \frac{980.8\times6\times0.1^2}{10^{-3}} = 5.88\times10^5 > 10^4.$$

So, equation (4.8) can be used to predict the power with K_T = 5.75 (Table 4.1):

$$P = K_T.n^3.D_e^5.\rho = 5.75\times 6^3 \times 0.1^5 \times 980.8 = \mathbf{12\ W}$$

The mean droplet size is calculated from equation (4.15).

$$We = \frac{1000\times 6^2 \times 0.1^3}{0.046} = 783$$

$$\frac{D_s}{0.1} = 0.058\times(1+5.4\times 0.08)\times 783^{-0.6}$$, hence: $D_s = 4.52\times 10^{-4}$ m ≡ **0.152 mm**

4.5 Scale – up of mixers

One main problem associated with the industrial use of mixers and agitators is to scale – up laboratory results to industrial scale. The power required for a mixing operation that is calculated or determined experimentally is usually different from that that would be required for a larger agitated vessel.

In order to scale – up laboratory results, it has been suggested that some factors or combinations of factors have to be maintained constant throughout scaling up. This is known as similarity between small and large scale setups. There are several such similarities but the two major ones that are more frequently used are the equal tip velocities similarity and the equal power per unit volume similarity.

In the first case, we assume that the velocities at tip of impeller are similar. Denoting by (1) and (2) the values of the small scale and large scale setups respectively, we get the following condition:

$$n_1.D_{e1} = n_2.D_{e2} \qquad (4.16)$$

In the second case it is assumed that the power per unit volume in both cases is the same. Now the power can be obtained from the definition of power number where as the volume of liquid = $\frac{\pi}{4}.D_t^2.H$

The power per unit volume is therefore:

$$\frac{P}{V} = \frac{Po\times n^3.D_e^5.\rho}{\frac{\pi}{4}\times D_t^2.H} = \left[\frac{4.Po.\rho}{\pi}\times\left(\frac{D_e}{D_t}\right)^2\times\left(\frac{D_a}{H}\right)\right]\times(n^3.D_e^2)$$

The first bracket contains ratios that are usually kept constant whether in small or large tanks. Also the power number is sensibly the same so that equal power per unit volume similarity leads to the following equation:

$$n_1^3.D_{e1}^2 = n_2^3.D_{e2}^2 \qquad (4.17)$$

An important condition imposed in scaling – up is that the total number of revolutions to effect a certain blending should be kept constant. That is:

$$t_{M1}\times n_1 = t_{M2}\times n_2 \qquad (4.18)$$

Example 4.7

A pilot vessel 300 mm in diameter is agitated by a six blade turbine impeller of diameter = 100 mm. it is used to mix two miscible liquids of height 300 mm. When the speed of rotation is 320 rpm, the mixing time was found to be 15 s. Estimate the required mixing time. Use the two scaling – up similarities enunciated above.

Solution:

Since $n_1 = \dfrac{320}{60}$ = 5.33 rps and D_{e1} = 0.1 m, then assuming the ratio between vessel and impeller diameter to be the same, D_{e2} = 0.6 m.

(1) For equal tip speed similarity: From equation (4.16):

$5.33 \times 0.1 = n_2 \times 0.6$, so that n_2 = 0.89 rps

From equation (4.18):

$$15 \times 5.33 = t_{M2} \times 0.89$$

Hence: $\boldsymbol{t_{M2} \approx 90}$ **s**

(2) For equal power per unit volume: From equation (4.17):

$5.33^3 \times 0.1^2 = n_2^3 \times 0.6^2$ so that n_2 = 1.6 rps

From equation (4.18):

$$15 \times 5.33 = t_{M2} \times 1.6$$

Hence: $\boldsymbol{t_{M2} \approx 50}$ **s**

We note the discrepancy between the results obtained on using different similarity models. This is frequent in mixers scale – up. It is usually recommended to use the longest time obtained from either model or simply to rely on experimental data taken on the industrial setup.

4.6 Problems

(1) A disk turbine with 6 flat blades is installed centrally in a vertical baffled tank 1.5 m. in diameter. The turbine is 0.5 m in diameter and is positioned 0.5 m above tank bottom. The turbine blades are 110 mm wide. The mixed liquid has a density of 1150 kg.m^{-3} and a viscosity of 5 cP and fills the tank to a depth of 5 m. Calculate the theoretical power required for mixing at 120 rpm.

(2) An unbaffled tank 1.08 meters in diameter and 2 m high is filled with a viscous polymeric solution (specific gravity = 0.8 and viscosity = 1 Pa.s) to 1.2 m depth. A six blade 360 mm diameter turbine is installed in the tank at 360 mm from its bottom. The motor delivers a power of 3 kW. Is this power sufficient to drive the agitator at a speed of 360 rpm?

(3) In the previous problem, what would be the agitator speed if the viscosity of the polymer was 0.1 Pa.s?

(4) A six blades disk turbine impeller is installed centrally in a vertical baffled tank 2.4 m in diameter. The turbine has a diameter = 800 mm and is positioned 800 mm above the bottom of the tank. The turbine blades are 160 mm wide. The tank is filled to a depth of 2 m with an aqueous solution of density = 1200 kg.m^{-3}, viscosity = 6 cP. Calculate the power

required for a speed 120 rpm. Then deduce the power of the motor assuming an efficiency = 0.85.

(5) An anchor agitator is placed centrally in vertical baffled tank 2.1 m in diameter. The anchor diameter = 500 mm and is positioned 500 mm above the bottom of the tank. This is used to agitate a polymeric solution of density = 950 $kg.m^{-3}$ and viscosity = 41.5 Pa.s. Calculate the motor power required for a speed of revolution of impeller = 45 rpm assuming a motor efficiency = 0.7.

(6) An agitated vessel 1.2 m in diameter contains a six-blade turbine impeller 0.4 m in diameter rotating at 120 rpm. It is used to neutralize a solution of NaOH with dilute hydrochloric acid. The depth of solution = 1.2 m. How long would it take for neutralization to be complete? (Density of solution = 950 $kg.m^{-3}$, viscosity = 1.05 cP)

(7) An agitated vessel 1.5 m in diameter contains a six blade turbine impeller 0.5 m in diameter rotating at 90 rpm. It is used to neutralize a solution of sodium carbonate with dilute hydrochloric acid. The depth of solution = 1.5 m. How long would it take for neutralization to be complete? (Density of solution = 970 $kg.m^{-3}$, viscosity = 0.98 cP)

(8) An agitated vessel 1.5 m in diameter fitted with 4 baffles contains a six blade turbine impeller 0.5 m in diameter and 100 mm wide rotating at 45 rpm. It is used to mix two miscible viscous polymers. The depth of solution = 1.5 m. How long would it take for blending to be complete? (Density of solution = 1260 $kg.m^{-3}$, viscosity = 600 cP). Calculate the required power.

CHEN407

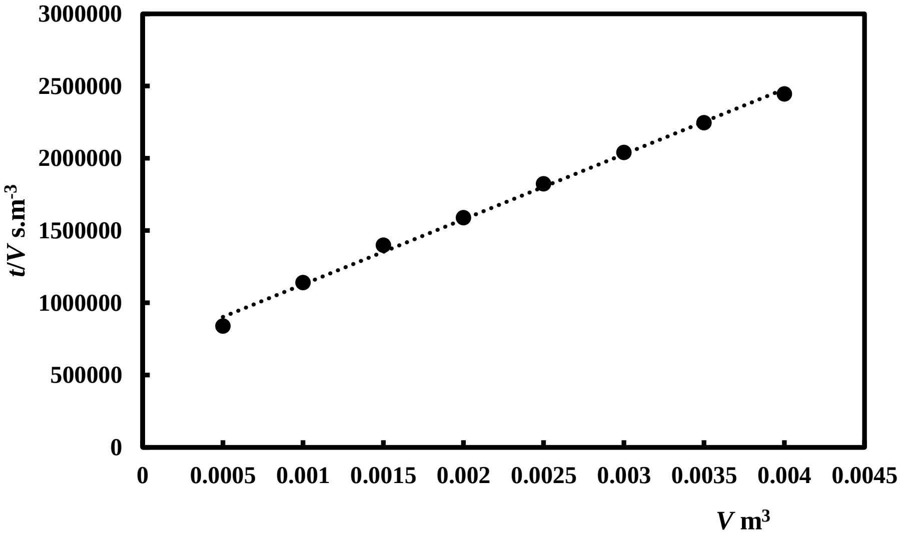
ADVANCED CHEM. ENG.DESIGN

MODEL ANSWERS

Sheet 1: Filtration

Question 1

***V* liter**	0.5	1	1.5	2	2.5	3	3.5	4
Time min	7	19	35	53	76	102	131	163
***V* m^3**	0.0005	0.001	0.0015	0.002	0.0025	0.003	0.0035	0.004
Time s	420	1140	2100	3180	4560	6120	7860	9780
***t*/V**	840000	1140000	1400000	1590000	1824000	2040000	2245714	2445000



Slope = $4.5 \times 10^8 \rightarrow \frac{\alpha\mu c}{2A^2\Delta p} = \frac{\alpha\times0.001\times120}{0.09^2\times6\times10^5} \rightarrow \alpha = \mathbf{3.645\times10^{13}\,m.kg^{-1}}$

Intercept = $6.8 \times 10^5 \rightarrow \frac{\mu R_m}{A\Delta p} = \frac{0.001R_m}{0.09\times6\times10^5} \rightarrow R_m = \mathbf{3.67\times10^{13}\,m^{-1}}$

Question 2

***V* liter**	0.5	1	1.5	2	2.5	3	3.5	4
***V*m3**	0.0005	0.001	0.0015	0.002	0.0025	0.003	0.0035	0.004
Time s (6.7 psig)	17.3	41.3	72	108	152	202	----	----
Time s (16 psig)	6.8	19	34.6	53.4	76	102	131	163
t/V (6.7 psig)	34600	41300	48000	54000	60800	67333	----	----
t/V (16 psig)	13600	19000	23066.6	26700	30400	34000	37428.5	40750

Draw t/V against V for the two sets of data and get each time the slope and intercept:

At $\Delta p_1 = 6.7$ psi (46170 Pa): $K_{p1} = 13038095$ $\quad B_1 = 28189$

At $\Delta p_2 = 16$ psi (110259 Pa): $K_{p2} = 7587925$ $\quad B_2 = 11045$

Hence $\alpha_1 = \frac{2\times(0.05)^2\times13038095\times46170}{0.001\times35} = 8.6 \times 10^{10}$ m.kg^{-1}

And $\alpha_2 = \frac{2\times(0.05)^2\times7587925\times110259}{0.001\times35} = 1.195 \times 10^{11}$ m.kg^{-1}

Hence $s = \frac{\ln\frac{1.195\times10^{11}}{8.6\times10^{10}}}{\ln\frac{110259}{46170}} = \mathbf{0.378}$

and $8.6 \times 10^{10} = \alpha_0(46170)^{0.378} \rightarrow \boldsymbol{\alpha_0 = 1.4838 \times 10^9}$

Also: $R_{m1} = \frac{28189\times0.05\times46170}{0.001} = 6.51 \times 10^{10}$ m^{-1}

And $R_{m2} = \frac{11045\times0.05\times110259}{0.001} = 6.09 \times 10^{10}$ m^{-1}

At pressure = 12 psig (82694 Pa):

$\alpha_3 = 1.4838 \times 10^9 \times 82694^{0.378} = 1.072 \times 10^{11}$ m.kg^{-1}

As for R_{m3} it is clear that the values of filter medium resistance are very close so that it would be reasonable, since $6.7 < 12 < 16$ psig, to take an average value of 6.3×10^{10} m^{-1}

$10800 = \frac{0.001\times35\times1.072\times10^{11}\times6^2}{2\times A^2\times82694} + \frac{0.001\times6.3\times10^{10}\times6}{A\times82694} \rightarrow \boldsymbol{A = 8.91}$ **m^2**

Area per filter plate = $2 \times (15 \times 0.0254)^2 = 0.29$ m^2 $\quad$ Hence, Number of plates = **31**

Question 3

$f = 0.3, c = 236, \Delta p = \frac{50}{760} \times 1.013 \times 10^5 = 6664$ Pa, $t = 300$s, $Q = \frac{1}{1200}$ m^3.s^{-1}

To calculate the value of Q. The amount of water retained in the cake must be considered:

Since $m_c = cV$. Therefore the mass of water in cake per second = $0.5m_c = 0.5cQ$ · The volume of water retained = $\frac{0.5cQ}{\rho_{water}} = 5 \times 10^{-4}cQ$

The actual flow rate of filtrate is therefore: $Q - 5 \times 10^{-4}cQ = Q(1 - 5 \times 10^{-4}c) = \frac{1}{1200}(1 - 5 \times 10^{-4} \times 236) = 0.000735$ m^3.s^{-1}

From Problem 2:

At $p = 82694$ Pa, $\alpha = 1.072 \times 10^{11}$ m.kg^{-1} and $s = 0.378, \alpha_0 = 1.4838 \times 10^9$

Hence, at $p = 6664$ Pa: $\alpha = 1.4838 \times 10^9 \times 6664^{0.378} = 4.137 \times 10^{10} \mathrm{m.kg^{-1}}$

$Q^2 = \frac{2A^2\Delta p f}{\alpha\mu c t'} \rightarrow \frac{2A^2\times6664\times0.3}{4.137\times10^{10}\times0.001\times236\times300} = (0.000735)^2 \rightarrow \boldsymbol{A \approx 19.9}$ **m^2**

Question 4

$\alpha_0 = 1.4838 \times 10^9$

$s = 0.378$

$c = 35$ kg.m^{-3}

$A = 8.91$ m^2

$v = \frac{0.0015}{60} = 0.00025$ m/s

$K_r = \mu c \alpha_0 v^2 = 3.24$

$\Delta p = 70$ psi (482380 Pa)

$\Delta p_m = R_m \, \mu v = 6.3 \times 10^{10} \times 0.001 \times 0.00025 = 15754.7$ Pa

$(\Delta p - \Delta p_m)^{1-s} = K_r.t \rightarrow (482380 - 15754.7)^{1-0.378} = 3.24\, t \rightarrow t = 1036.5$ s

Volume of collected filtrate = $v.A.t = 0.00025 \times 8.91 \times 1036.5 = 2.31$ m^3.

Remaining volume = $5 - 2.31 = 2.69$ m^3.

At a pressure = 70 psig $\equiv$ 482380 Pa, $\alpha = 1.4838 \times 10^9 \times 482380^{0.378} = 2.088 \times 10^{11}$

$t' = \frac{0.001 \times 35 \times 2.088 \times 10^{11} \times 2.69^2}{2 \times 8.91^2 \times 482380} + \frac{0.001 \times 6.3 \times 10^{10} \times 2.69}{8.91 \times 482380} = 1084.5$ s

Total filtration time = $1036.5 + 1084.5 = \mathbf{2121\ s}$

Question 5

$\alpha_0 = 2.35 \times 10^9$ $\qquad$ $s = 0.48$

$c = 280$ kg.m^{-3}

$A = 2 \times 0.7^2 \times 35 = 34.3$ m^2

$t_1 = 6 \times 3600 = 21600$ s

After 6 hours: $\Delta p = 6$ bar $\equiv 6 \times 10^5$Pa

$\Delta p_c \approx \Delta p = 6 \times 10^5 \rightarrow \Delta p^{1-s} = (6 \times 10^5)^{1-0.48} = K_r \times 21600 \rightarrow K_r = 0.04679$

$\alpha_0 c \mu v^2 = 0.04679 \rightarrow 2.35 \times 10^9 \times 280 \times 0.001 v^2 = 0.04679 \rightarrow$

$v = 2.67 \times 10^{-6}$ m.s^{-1}

$\frac{V}{At} = \frac{V}{34.3 \times 21600} = 2.67 \times 10^{-6} \rightarrow V = 1.98$ m^3

The remaining volume = $5 - 1.98 = 3.02$ m^3

Under a constant pressure of 6 bar, the mean specific cake resistance is:

$\alpha = 2.35 \times 10^9 \times (6 \times 10^5)^{0.48} = 1.395 \times 10^{12}$ m.kg^{-1}

$t = \frac{\alpha \mu c V^2}{2A^2 \Delta p} = \frac{1.395 \times 10^{12} \times 0.001 \times 280 \times 3.02^2}{2 \times 34.3^2 \times 6 \times 10^5} = 25233$ s $\equiv \mathbf{7\ h}$

Total filtration time = $6 + 7 = \mathbf{13\ h}$

Question 6

$c = 25, D = 0.3$ m, $L = 2.5$ m, $\mu = 0.018$ cP, t = 5 min, $Q_N = 45000$ m^3.h^{-1}

$Q = 45000 \times \frac{60+273}{25+273} = 50285$ m^3.h^{-1}

$\Delta p_f = K_f \mu v = 1.5 \times 0.018v = 0.027v$

$\Delta p_d = 0.5K_d \mu c_d t v^2 = 0.5 \times 0.3 \times 0.018 \times 25 \times 5v^2 = 0.3375v^2$

Taking Δp = 5" water, $0.3375v^2 + 0.027v = 5 \rightarrow v = 3.81$ ft.min^{-1} ≡ 69.7 m.h^{-1}

$Q = A.v \rightarrow 50285 = 69.7A \rightarrow A = 721.45$ m^2

Area per bag = $\pi DL = 3.1416 \times 0.3 \times 2.5 = 2.356$ m^2

Number of bags = $\frac{721.45}{2.356}$ = **307 bag**

Question 7

$c = 15, D = 0.25$ m, $L = 3$ m, $\mu = 0.025$ cP, t = 8 min, $Q_N = 35000$ m^3.h^{-1}

Area per bag = $\pi DL = 3.1416 \times 0.25 \times 3 = 2.356$ m^2 → $A = 400 \times 2.356 = 942.4$ m^2

$Q = 35000 \times \frac{115+273}{25+273} \times \frac{1.013}{1.5} = 30775$ m^3.h^{-1} → $v = \frac{30775}{942.4} = 32.66$ m.h^{-1} ≡ 1.78 ft.min^{-1}

$\Delta p_f = K_f \mu v = 1.25 \times 0.025v = 0.03125v = 0.03125 \times 1.78 = 0.0556"$

$\Delta p_d = 0.5K_d \mu c_d t v^2 = 0.5 \times 0.35 \times 0.025 \times 15 \times 8v^2 = 0.525v^2 = 1.663"$

Total pressure drop = $0.0556 + 1.663 = 1.72"$ ≡ $1.72 \times 248.84 = 428$ Pa

Power = $\frac{Q\Delta p}{735\eta} = \frac{30775 \times 428}{735 \times 3600 \times 0.75}$ = 6.63 hp → **7 hp**

Sheet 2:

Sedimentation

Question 1

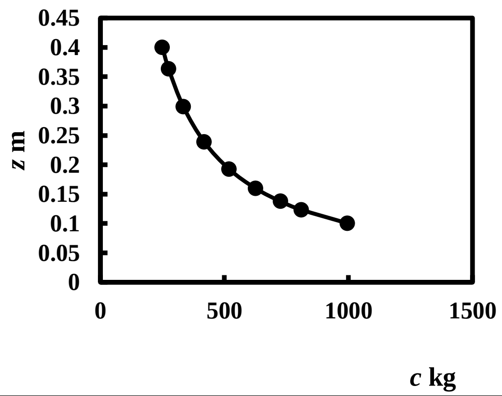
(a) Final height of compression zone $t \to \infty \to \boldsymbol{z}_\infty = \mathbf{100\ mm}$

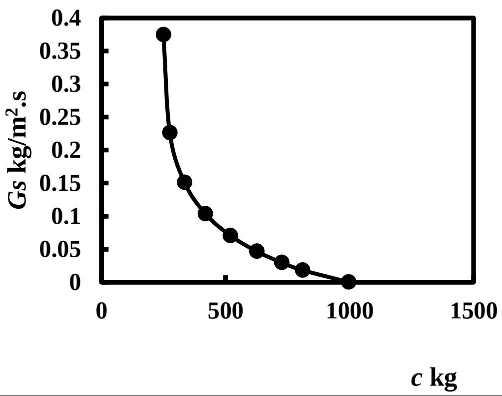
(b) Initial height of suspension $t = 0 \to \boldsymbol{z}_0 = \mathbf{400\ mm}$

(c) $\left|\frac{dz}{dt}\right| = 0.3 \times 300e^{-0.3t} = 0.3(z - 100) = 0.3z - 30$

(d) $z_i = z + \left|\frac{dz}{dt}\right| t = \mathrm{z} + (0.3z - 30)t$

***t* min**	0	5	10	15	20	25	30	35	40
***z* mm**	400	264.643	190.358	149.589	127.215	114.936	108.197	104.498	100.037
$\lvert dz/dt \rvert$ mm/min	90	49.3930	27.1075	14.8769	8.1646	4.4808	2.4591	1.3496	0.0111
$\lvert dz/dt \rvert$ m/s	0.0015	0.0008	0.0005	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
z_i mm	0.4	0.3634	0.2988	0.2389	0.1925	0.1597	0.1377	0.1234	0.1004
c kg/m^3	250	275.156	334.685	418.671	519.393	626	726.181	810.418	996.311
G_s kg/m^2.s	0.375	0.2265	0.1512	0.1038	0.0707	0.0468	0.298	0.0182	0.0002





Question 2

t	0	5	10	15	20	25	30	35	40
z	0.3800	0.2937	0.2579	0.2305	0.2074	0.1870	0.1686	0.1516	0.1359
t h	0.0000	0.0833	0.1667	0.2500	0.3333	0.4167	0.5000	0.5833	0.6667
$\lvert dz/dt \rvert$ mm/min		8.6312	6.1032	4.9832	4.3156	3.8600	3.5237	3.2623	3.0516
$\lvert dz/dt \rvert$ m/h		0.5179	0.3662	0.2990	0.2589	0.2316	0.2114	0.1957	0.1831
z*		0.337	0.319	0.305	0.294	0.284	0.274	0.266	0.258
c	kg/m^3	676.8714	714.8052	746.9253	776.3347	804.2328	831.2384	857.7244	883.9399
***Gs* kg/m^2.h**		350.534	261.756	223.326	201.022	186.260	175.741	167.889	161.846
***Gt* kg/m^2.h**		473.8100	500.3636	522.8477	543.4343	562.9630	581.8669	600.4071	618.7579
G_{total}		824.344	762.119	746.174	**744.456**	749.223	757.608	768.296	780.604

Area = $\frac{600 \times 400}{744.56} = 322.38$ m^2 Hence $\boldsymbol{D} = \mathbf{20.3\ m}$

Sheet 3: Mixing

Question 1

$n = 2, \rho = 1150 \text{ kg. m}^{-3}, \mu = 0.005 \text{ Pa. s}, D_e = 0.5 \text{ m}$

$$Re = \frac{1150 \times 2 \times 0.5^2}{0.005} = 115000 > 10^4$$

$$P = K_T n^3 D_e^5 \rho = 5.75 \times 2^3 \times 0.5^5 \times 1150 = \mathbf{1653\ W}$$

Question 2

$n = 6, \rho = 800 \text{ kg. m}^{-3}, \mu = 1 \text{ Pa. s}, D_e = 0.36 \text{ m}$

$$Re = \frac{800 \times 6 \times 0.36^2}{1} = 622$$

From Chart, $Po \approx 3$

$$P = Po.\rho.n^3.D_0^5 = 3 \times 800 \times 6^3 \times 0.36^5 = \mathbf{3134\ W > 3000\ W}$$

Hence, the provided motor power is insufficient to operate the mixer at that speed.

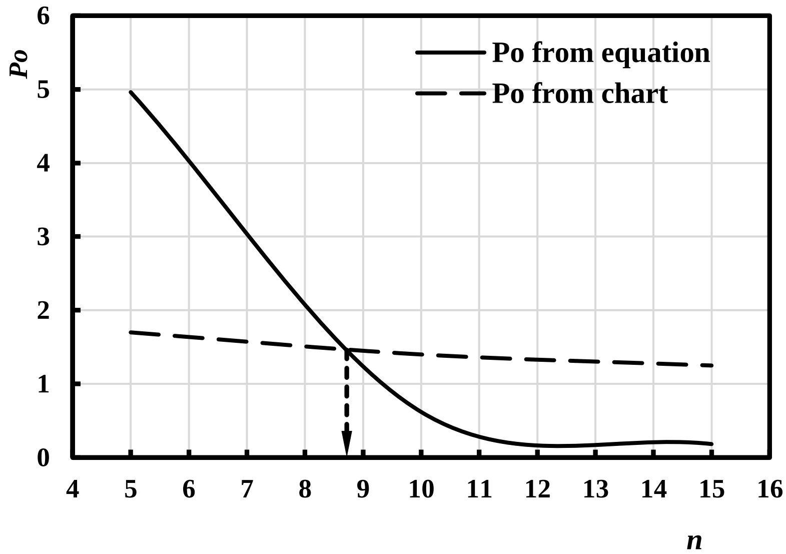
Question 3

In the previous problem, what would be the agitator speed if the viscosity of the polymer was 0.1 Pa.s?

$$Re = \frac{800 \times 0.36^2 n}{0.1} = 1036.8n$$

$$Po = \frac{3000}{800 \times 0.36^5 n^3} = \frac{620.18}{n^3}$$

For different values of n, obtain values of Re from the above equation and Po from the chart and from the above equations. Plot Po against n from both sources. At the intersection of the two curves, get the value of $\boldsymbol{n = 8.7}$



Question 4

$n = 2, \rho = 1200\ \text{kg.m}^{-3}, \mu = 0.006\ \text{Pa.s}, D_e = 0.8\ \text{m}$

$Re = \dfrac{1200 \times 2 \times 0.8^2}{0.006} = 256000 > 10^4$

$P = K_T n^3 D_e^5 \rho = 5.75 \times 2^3 \times 0.8^5 \times 1200 = \mathbf{18088\ W}$

$P_{actual} = \frac{18088}{735 \times 0.85} = 28.95 \rightarrow \mathbf{30\ hp}$

Question 5

$n = 0.75, \rho = 950\ \text{kg.m}^{-3}, \mu = 23.5\ \text{Pa.s}, D_e = 0.7\ \text{m}$

$Re = \dfrac{950 \times 0.75 \times 0.5^2}{41.5} = 4.29 < 10$

$P = K_L n^2 D_e^3 \mu = 65 \times 0.75^2 \times 0.5^3 \times 41.5 = \mathbf{189.7\ W}$

$P_{actual} = \frac{189.7}{735 \times 0.7} = 0.37 \rightarrow \mathbf{0.5\ hp}$

Question 6

$n = 2, \rho = 950\ \text{kg.m}^{-3}, \mu = 0.00105\ \text{Pa.s}, D_e = 0.4\ \text{m}$

$Re = \dfrac{950 \times 2 \times 0.4^2}{0.00105} = 289524 > 2000$

$t_M = \frac{4.3 \times 1.2}{2 \times 0.4^2} = \mathbf{16.13\ s}$

Question 7

$n = 1.5, \rho = 970\ \text{kg.m}^{-3}, \mu = 0.00098\ \text{Pa.s}, D_e = 0.5\ \text{m}$

$Re = \dfrac{970 \times 1.5 \times 0.5^2}{0.00098} = 371173 > 2000$

$t_M = \frac{4.3 \times 1.5}{1.5 \times 0.5^2} = \mathbf{17.2\ s}$

Question 8

$n = 0.75, \rho = 1200\ \text{kg.m}^{-3}, \mu = 0.6\ \text{Pa.s}, D_e = 0.5\ \text{m}$

$Re = \dfrac{1260 \times 0.75 \times 0.5^2}{0.6} = 1969 < 2000$ From chart: $f = 5$

$Fr = \frac{0.75^2 \times 0.5}{9.81} = 0.0287$

$$f = n.t_M . \left(\frac{D_e}{D_t}\right)^2 . \left(\frac{D_t}{H}\right)^{\frac{1}{2}} .Fr^{-\frac{1}{6}}$$

$5 = 0.75 t_M \times \left(\frac{0.5}{1.5}\right)^2 \times \left(\frac{1.5}{1.5}\right)^{1/2} \times 0.0287^{-\frac{1}{6}} \rightarrow \boldsymbol{t_M = 33.2\ s}$