9. FLOW OF FLUIDS PAST IMMERSED BODIES (Theory and Applications)

9.1 Free settling

By free settling is meant the vertical motion of a single solid particle in a fluid. In case of numerous particles, free settling can be assumed to occur if the motion of any particle will not influence that of a neighboring particle. This will normally happen when the volumetric fraction of solids is very low.

9.1.2 Equation of motion of a single particle

When a single solid particle moves in a fluid under the effect of gravity, then the forces acting will consist of:

- The weight of the body $\rho_p.V_p.g$
- The force of buoyancy $\rho_f.V_p.g$
- The drag force due to fluid colliding with the body F_d

A force balance shows that:

$$\rho_p.V_p.g - \rho_f.V_p.g - F_d = m.\frac{dv}{dt} \qquad (9.1)$$

Where,

ρ_p is the density of solid particle

ρ_f is the fluid density

V_p is the volume of particle

The drag force is defined as:

$$F_d = \frac{1}{2}.C_D.A_p.v^2 \qquad (9.2)$$

Where,

v is the settling velocity

A_p is the projected area perpendicular to the direction of motion

C_D is a dimensionless parameter known as the **drag coefficient.**

As the particle descends across the fluid, its velocity rapidly reaches a maximum constant value known as the **terminal velocity v_t**. Under such conditions, the RHS of equation (9.1) will vanish and the steady state equation will then read:

$$\rho_p.V_p.g - \rho_f.V_p.g - \frac{1}{2}.C_D.A_p.\rho_f.v_t^2 = 0 \qquad (9.3)$$

Finally, we get the following expression for terminal velocity:

$$v_t = \sqrt{\frac{2.(\rho_p - \rho_f).V_P.g}{\rho_f.C_D.A_p}} \qquad (9.4)$$

9.1.3 Motion of spherical particles

In case of a spherical particles, the above equation simplifies to the following form, taking into account that:

$$V_p = \frac{\pi . D_p^3}{6} \text{ and } A_p = \frac{\pi . D_p^2}{4}$$

$$v_t = \sqrt{\frac{4.(\rho_p - \rho_f).D_P.g}{3.\rho_f.C_D}} \tag{9.5}$$

a- The drag coefficient

The drag coefficient C_D is somewhat similar to a friction factor the value of which is related to the Reynolds number.

A modified Reynolds number is used defined as:

$$\text{Re} = \frac{\rho_f.v_t.D_p}{\mu_f} \tag{9.6}$$

Accordingly, it has been possible to describe the dependence of the drag coefficient on *Re* by the following chart.

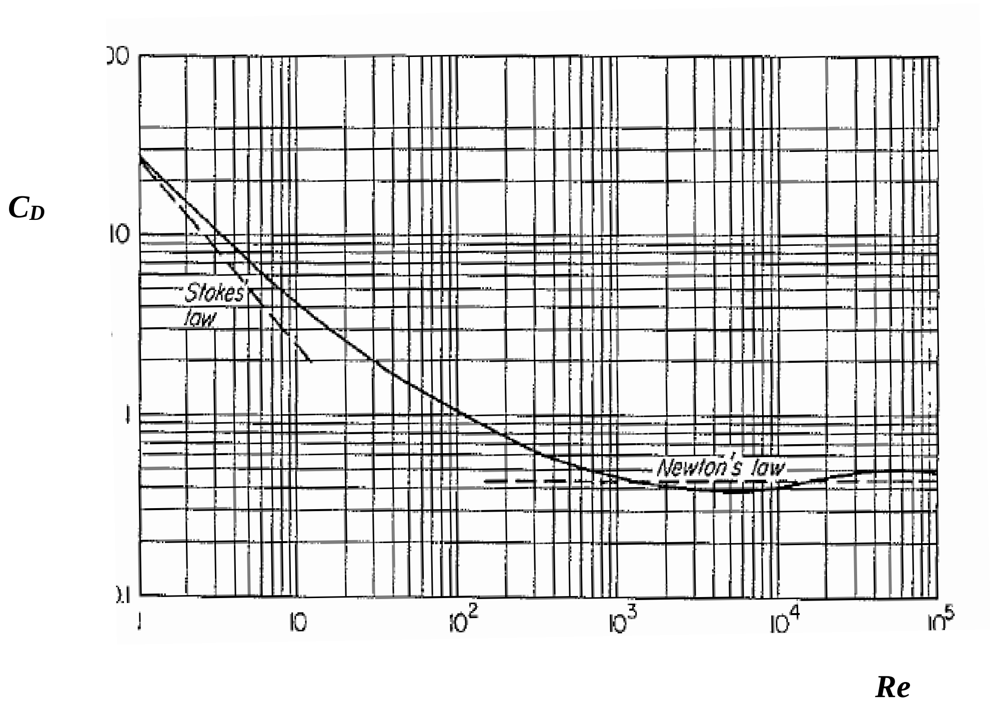


Fig (9.1) Dependence of Drag coefficient on Reynolds number

This curve shows that dependence of the drag coefficient on Reynolds number can be roughly divided into three stages:

a-1 The laminar regime (Stokes law)

This region is very limited and only extends to values of *Re* < 1. In that case, the dependence takes the form:

$$C_D = \frac{24}{\text{Re}} = \frac{24.\mu_f}{v_t.D_p.\rho_f} \tag{9.7}$$

Substituting in equation (9.5), we get the famous Stokes law:

$$v_t = \frac{g.D_p^2.(\rho_p - \rho_f)}{18.\mu_f} \tag{9.8}$$

a-2 The intermediate regime

This region extends from *Re* = 1 to *Re* = 10^3. The relation between C_D and *Re* reads as follows:

$$C_D = \frac{18.7}{\mathrm{Re}^{0.61}} = \frac{18.7 \times \mu_f^{0.61}}{v_t^{0.61}.D_p^{0.61}.\rho_f^{0.61}} \tag{9.9}$$

Substituting in equation (9.5), we get:

$$v_t^{1.39} = \frac{g.D_p^{1.61}.(\rho_p - \rho_f)}{14.\mu_f^{0.61}.\rho_f^{0.39}} \tag{9.10}$$

a-3 The turbulent regime (Newton's law)

For values of *Re* > 10^3, the drag coefficient is practically constant and equals about 0.44. Substituting in equation (9.5), we get:

$$v_t = \sqrt{\frac{3.(\rho_p - \rho_f).D_P.g}{\rho_f}} \tag{9.11}$$

b- The *K* criterion for determination of settling regime

The determination of the settling regime requires the knowledge of *Re*, which is unknown since the terminal velocity is unknown as well.

A suitable combination of the limiting values of *Re* together with the equations of terminal velocity is usually used to assess the regime of settling.

b-1 Stokes law region:

From the condition $\mathrm{Re} = \frac{\rho_f.v_t.D_p}{\mu_f} < 1$, we get:

$$v_t < \frac{\mu_f}{\rho_f.D_p}$$

Substituting in equation (9.8), we get the following condition:

$$K = D_p.[\frac{g.\rho_f.(\rho_p - \rho_f)}{\mu_f^2}]^{1/3} < 2.62 \tag{9.12}$$

b-2 Intermediate region:

The condition related to that region is difficult to derive by eliminating the terminal velocity term from equation (9.10) and the condition 1 < *Re* < 10^3; so, this will be deduced from the third region of settling.

b-3 Newton's law region

From the condition $\mathrm{Re} = \frac{\rho_f.v_t.D_p}{\mu_f} > 10^3$, we get:

$$v_t > \frac{10^3.\mu_f}{\rho_f.D_p}$$

Substituting in equation (9.11), we get the following condition:

$$K = D_p.[\frac{g.\rho_f.(\rho_p - \rho_f)}{\mu_f^2}]^{1/3} > 69.3 \qquad (9.13)$$

To conclude, the following table can be used to decide about the settling regime using the calculated values of *K*.

$K < 2.62$	Stokes law range
$2.62 < K < 69.3$	Intermediate law range
$K > 69.3$	Newton's law range

9.1.4 Settling of non – spherical particles

In case of non – spherical particles, the prediction of settling velocity has to rely on experimental data since the shape of particles will have a direct effect on its settling behavior.

Several methods were suggested to account for the non – sphericity of particles, by defining different shape factors. However, the easiest method among these, although not the most accurate, defines an equivalent diameter as the diameter of a sphere that would have the same volume as the particle:

$$D_e = \sqrt[3]{\frac{6.V_p}{\pi}} \qquad (9.14)$$

Once this equivalent diameter has been calculated it can be substituted for D_p in any of the equations (9.8) to (9.13).

9.2 Application to continuous fluidization

Continuous fluidization of a solid particle will take place if the velocity of the fluidizing fluid exceeds the terminal velocity of the solid as it settles through this fluid. This velocity is related to the minimum fluidization superficial velocity $\overline{v_M}$ in the following way:

9.2.1 Laminar flow (Stokes law range)

The general equation used to predict the velocity at onset of fluidization is obtained by equating the pressure drop obtained from force balance to that predicted by the Ergun equation:

$$\frac{\Delta P}{L} = \frac{150.\mu.\overline{v_M}.(1-\varepsilon_M)^2}{D_p^2.\varepsilon_M^3} + \frac{1.75\rho_f.\overline{v_M}^2.(1-\varepsilon_M)}{D_p.\varepsilon_M^3} = g.(\rho_p - \rho_f).(1-\varepsilon_M)$$

In case of laminar flow, only the first term in the Ergun equation is kept:

$$\bar{v}_M = \frac{g.D_p^2.(\rho_p - \rho_f)}{18.\mu_f}.\frac{3.\varepsilon_M^3}{25.(1-\varepsilon_M)} = v_t.\frac{3.\varepsilon_M^3}{25.(1-\varepsilon_M)}$$

The value of minimum porosity of fluidization usually ranges from 0.4 to 0.45 which gives the following result:

$v_t = (50 – 78).\ \overline{v_M}$ (9.15)

9.2.2 Turbulent flow (Newton's law range)

In that case, only the second term in the Ergun equation is kept. We obtain:

$$\frac{1.75\rho_f.\overline{v_M}^2.(1-\varepsilon_M)}{D_p.\varepsilon_M^3} = g.(\rho_p - \rho_f).(1-\varepsilon_M)$$

Which reduces to:

$$\overline{v_M} = \sqrt{\frac{3.(\rho_p - \rho_f).D_P.g}{\rho_f}} \times 0.43 \times \varepsilon^{3/2} \;=\; v_t \times 0.43.\varepsilon_M{}^{3/2}$$

For values of minimum fluidization porosity ranging from 0.4 to 0.45, we get:

$v_t = (7.6 – 9).\ \overline{v_M}$ (9.16)

Comparing equations (9.15) with (9.16) we reach the following important conclusion:

Whenever fluidization is performed in laminar regime, usually in case of fine particles, the sensitivity of the operation is low, so that it takes a very large increase in velocity to reach continuous fluidization. On the other hand, if fluidization is performed with large particles, the flow will be turbulent. This way, the sensitivity is high, so that a slight increase in velocity can move the operation from onset to continuous fluidization.

9.3 Hindered settling

When the concentration of solids in suspension exceeds 10% by volume, it is no more possible to assume that the settling behavior of any particle will not be affected by that of its neighboring particle. This situation is known as **hindered settling.** Usually, the settling regime is determined from the *K* criterion assuming free settling conditions.

In case of Stokes law applying, the terminal settling velocity will be calculated from a modified stokes law equation in the following form:

$$U = \frac{g.D_p^2.(\rho_p - \rho_m)}{18.\mu_m}.(1-f_v)^n \qquad (9.17)$$

Where *U* is the actual terminal settling velocity, and ρ_m and μ_m are the density and viscosity of the settling medium (suspension); f_v is the volumetric fraction related to porosity by: $f_v = 1 - \varepsilon$

The parameter (*n*) appearing in the above equation is related to the modified Reynolds number by the following empirical formula valid for $f_v > 0.1$

$n = 4.Re^{-0.07}$ (9.18)

The calculation of the value of n needs knowledge of v_t which is unknown. That is why; it is common in case of settling in the Stokes law region, where $Re < 1$, to use a value of n ranging from 4.3 to 4.6

The medium density is calculated from the equation:

$$\rho_m = f_v.\rho_p + (1 - f_v).\rho_f \qquad (9.19)$$

Finally, there are several empirical formulas that can be used to calculate the medium viscosity; one of them is the following:

$$\mu_m = \frac{1 + 0.5 f_v}{(1 - f_v)^4}.\mu_f \qquad (9.20)$$

Example 9.1

Dust particles settle in air at 25ºC in a chamber 2 m. high. Their shape is assumed to be spherical and their size about 10 µm. The density of fibers = 700 kg/m^3. How long it would take for these particles to settle? (Viscosity of air = 0.018 cP)

Solution:

$$\rho_f = \frac{1 \times 29}{0.082 \times 298} = 1.186 \text{ kg/m}^3$$

$$K = 10^{-5}.[\frac{9.81 \times 1.186 \times (700 - 1.186)}{(1.8 \times 10^{-5})^2}]^{1/3} = 0.292 < 2.62 \text{ (Stokes law range)}$$

$$v_t = \frac{g.D_p^2.(\rho_p - \rho_f)}{18.\mu_f} = \frac{9.81 \times (10^{-5})^2.(700 - 1.186)}{18 \times 1.8 \times 10^{-5}} = 0.00211 \text{ m/s}$$

Hence the time required for settling = $\frac{2}{0.00211}$ = **948 s**.

Example 9.2

Quartz grains settle in water in free settling regime. Their shape approximates cuboids of dimensions $20 \times 30 \times 35$ mm. The particle density = 2.65 g/cm^3. Estimate their final settling velocity.

Solution:

The equivalent diameter is calculated from equation (9.14):

$$D_e = \sqrt[3]{\frac{6.V_p}{\pi}} = \sqrt[3]{\frac{6 \times 20 \times 30 \times 35}{\pi}} = 34.23 \text{ mm}$$

Then, we calculate the value of K

$$K = 34.23 \times 10^{-3} \times [\frac{9.81 \times (2650 - 1000) \times 1000}{(10^{-3})^2}]^{1/3} = 84.8 > 69.3 \text{ (Newton's law)}$$

Substituting in equation (11):

$$v_t = \sqrt{\frac{3.(\rho_p - \rho_f).D_P.g}{\rho_f}} = \sqrt{\frac{3.(2650 - 1000) \times 34.23 \times 10^{-3} \times 9.81}{1000}} = \mathbf{1.29 \text{ m/s}}$$

Example 9.3

Oil drops of density 600 kg/m^3 and viscosity 15 cP rise in a water tank 3 m. high. The average drop size is about 50 µm and the volume concentration of oil is 15%. Calculate the time required to free water from oil.

Solution:
First, we calculate the value of K assuming free settling.

$$K = 50\times10^{-6}\times[\frac{9.81\times(1000-600)\times1000}{(10^{-3})^2}]^{1/3} = 0.79 < 2.62 \text{ (Stokes law range)}$$

Hindered settling: $f_v = 0.15$

$$\rho_m = 0.15\times600+(1-0.15)\times1000 = 940 \text{ kg/m}^3$$

$$\mu_m = \frac{1+0.5\times0.15}{(1-0.15)^4}.1 = 2.06 \text{ cP} \equiv 2.06\times10^{-3} \text{ Pa.s}$$

Substituting in equation (9.17) with n = 4.5, we get:

$$U = \frac{9.81\times(50\times10^{-6})^2.(940-600)}{18\times2.06\times10^{-3}}.(1-0.15)^{4.5} = 0.000108 \text{ m/s}$$

Hence the time required for oil drops to rise = $\frac{3}{0.000108}$ = 27777 s**. ≡ 7.71 h**

9.4 Application: Gas - liquid continuous separators

9.4.1 Types of separators

In upstream operations, crude oil is associated with high pressure gas stream. Separation of these two phases is accomplished in either horizontal or vertical separators. Figure (9.2) shows a horizontal separator while Figure (9.3) shows a vertical type.

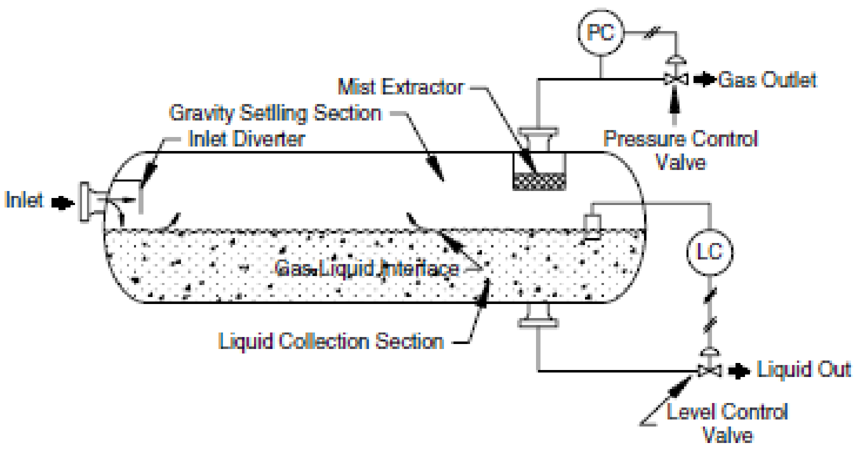


Fig (9.2) Horizontal gas – oil separator

A horizontal separator consists of an elongated pipe where the two phase mixture flows from one end to impinge against a diverter that forces oil to move towards the bottom so while gas is removed at the upper end of the separator. During the motion of gas towards the separator outlet, oil drops are eliminated by settling vertically through the gas phase. A mist extractor is placed ahead of the gas exit duct to filter off remaining oil drops.

A vertical separator, on the other hand, admits the two phase mixture at an opening at its upper third and the gas leaves the separator at its top.
In the following are summarized the steps undertaken to obtain the dimensions of a horizontal separator.

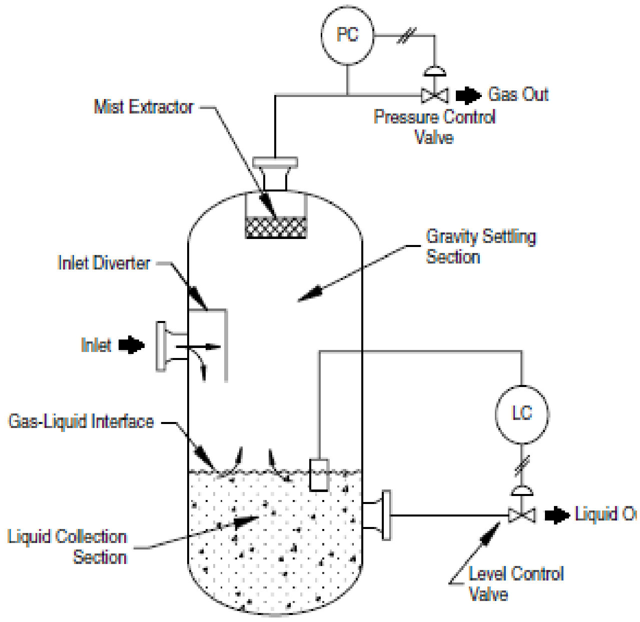
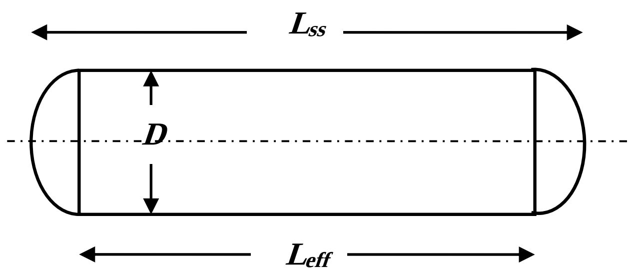


Fig (9.3) Vertical gas – oil separator

9.4.2 Design of horizontal gas – liquid separator

In the design of such separators the following assumptions are usually made:

1. Liquid holdup equals half the cylinder volume
2. Effective length L_{eff} = (3 – 4). D
3. Size of eliminated oil droplets ranges from 100 – 140 μm.
4. The seam to seam length = $L_{eff} + D$
5. The residence time in drum ranges from 3 – 4 minutes.



A main design criterion is that oil droplets should get eliminated from the gaseous phase by settling during the time it takes for the gas to cover horizontally the effective length. This is the **gas phase criterion**.
If the gas velocity is v, then its flow rate $Q_g = A.v$, where A is half the cross sectional area of the drum (Since liquid fills half of it). This way, we get:

$$v = \frac{8.Q_g}{\pi.D^2} \qquad (9.21)$$

If the residence time of gas in drum = t_r, then:

$$L_{eff} = v.t_r \qquad (9.22)$$

On the other hand, if the terminal settling velocity of a gas droplet is v_t, then it will settle as it covers one half the drum diameter. The residence time being t_r, we get:

$$\frac{D}{2} = v_t.t_r \qquad (9.23)$$

Equating t_r from equations (9.22) and (9.23), we get:

$$D.L_{eff} = \frac{4.Q_g}{\pi.v_t} \qquad (9.24)$$

On the other hand, enough time has to be given for the liquid to fill half the drum so that if the flow rate of oil is Q_l, then:

$$Q_l = \frac{V}{t_r} = \frac{\pi.D^2.L_{eff}}{8.t_r} \qquad (9.25)$$

Hence,

$$D^2.L_{eff} = \frac{8.Q_l.t_r}{\pi} \qquad (9.26)$$

This is the liquid phase criterion.

The **design steps** are as follows:

1. Specify the conditions of gas and liquid and their flow rates
2. Evaluate the terminal settling velocity of droplets using equation (9.8) to (9.11) according to the value of *K*.
3. Calculate the value of $D.L_{eff}$ from equation (9.24)
4. Calculate the product from $D^2.L_{eff}$ from equation (9.26)
5. Start substituting in each of these two equations with values of *D* corresponding to **standard steel vessels**. For each value of *D* take the **largest value obtained for L_{eff}.**
6. Calculate the seam – to – seam length from $L_{ss} = L_{eff} + D$
7. Calculate each time the **slenderness ratio** SR = L_{ss}/D. This should lie in the range **3 to 5**.

Example 9.4

Design a horizontal gas – liquid separator from the following data:

- Gas flow rate = 13500 sm^3/h at 22^{o}C and 45 atm.
- Viscosity of gas = 0.023 cP
- Oil flow rate = 9.6 m^3/h – Density = 850 kg/m^3
- Consider the gas to consist of: 85% methane and 15% ethane (By volume)
- Compressibility coefficient of gas = 0.9
- Oil droplet diameter to be removed = 100 μm

Solution

Gas phase criterion:

We first get the properties of the gas:

Molecular weight = (0.85).(16) + (0.15).(30) = 18.1

Density of gas: $\rho_g = \frac{M.P}{Z.RT} = \frac{18.1 \times 45}{0.9 \times 0.082 \times 295}$ = 37.4 kg/m^3

The actual flow rate of gas is obtained by converting from normal to actual conditions:

$\frac{Q_g \times 45}{0.9 \times (22 + 273)} = \frac{13500 \times 1}{15.6 + 273}$ From which: $\mathbf{Q_g = 276\ m^3/h}$

We now proceed to calculate the terminal velocity of oil drops assuming D_p = 100 µm. we first determine the settling regime of oil droplets:

$10^{-4}.[\frac{9.81 \times 37.4 \times (825 - 37.4)}{(2.3 \times 10^{-5})^2}]^{1/3}$ = 8.17 > 2.65 (Intermediate region)

The settling velocity is hence calculated from equation (9.10):

$$v_t^{1.39} = \frac{g.D_p^{1.61}.(\rho_p - \rho_f)}{14.\mu_f^{0.61}.\rho_f^{0.39}} = \frac{9.81 \times (10^{-4})^{1.61}.(825 - 37.4)}{14 \times (2.3 \times 10^{-5})^{0.61} \times 37.4^{0.39}} = 0.032$$

$\mathbf{v_t}$ **= 0.084 m/s = 302 m/h**

From equation (9.24): $D.L_{eff} = \frac{4 \times 276}{\pi \times 302}$ = 1.16 m^2 (i)

Liquid holdup criterion:

For a liquid holding time of 3 min., from equation (9.26):

$D^2.L_{eff} = \frac{8.Q_l.t_r}{\pi} = \frac{8 \times 9.6 \times 3}{60 \times \pi}$ = 1.22 m^3 (ii)

We then substitute in equations (i) and (ii) with standard pipe diameters starting with D = 12", obtain L_{eff} each time and choose the **largest value**. We calculate then L_{ss} and obtain the slenderness ratio. The following table summarizes calculations.

A reasonable slenderness ratio has been obtained on using a **30" diameter** vessel of **seam – to – seam length = 2.86 m**

D"	***D* m**	***L***			
12	0.3048	3.8058	13.13	13.44	44.08
14	0.3556	3.2621	9.65	10.00	28.13
16	0.4064	2.8543	7.39	7.79	19.18
18	0.4572	2.5372	5.84	6.29	13.77
20	0.508	2.2835	4.73	5.24	10.31
24	0.6096	1.9029	3.28	3.89	6.39
30	**0.762**	**1.5223**	**2.10**	**2.86**	**3.76**

9.5 Sedimentation

9.5.1 Batch sedimentation curves

When a relatively concentrated suspension of solid particles is allowed to settle in a batch container, different zones are formed as sedimentation proceeds. Usually the mode of settling is of the hindered type and all particles present at a certain level settle at equal velocities. In Figure (9.4) is shown the pattern appearing as sedimentation proceeds.

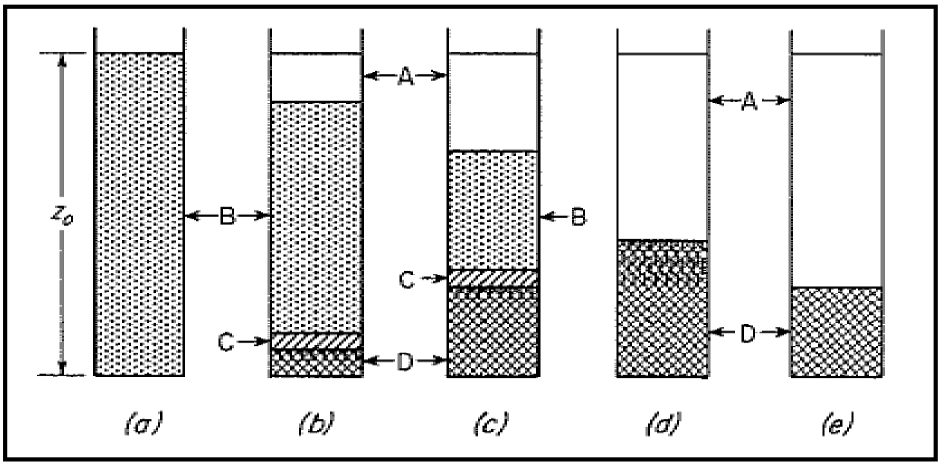


Fig (9.4) Batch sedimentation

At first, the solids are uniformly distributed in the liquid (a). After some time elapses, four layers appear: A clear upper layer (A), a second layer (B) where the concentration of solids is equal to its initial concentration, a lower layer (D) where solid particles have settled and an intermediate layer (C) where there is a concentration gradient from that of (B) to that of (D).

As settling proceeds the depth of zones (A) and (D) increases. Eventually zone (B) disappears and all the solids are in zones (C) and (D). Also, the gradual accumulation of solids in zone (D) puts pressure on the material at the bottom, thus compressing this zone forcing liquid to flow from this layer upwards. Finally, settling stops as the height of the compression zone stabilizes. The relation between the height of the clear liquid interface and time is shown in Figure (9.5).

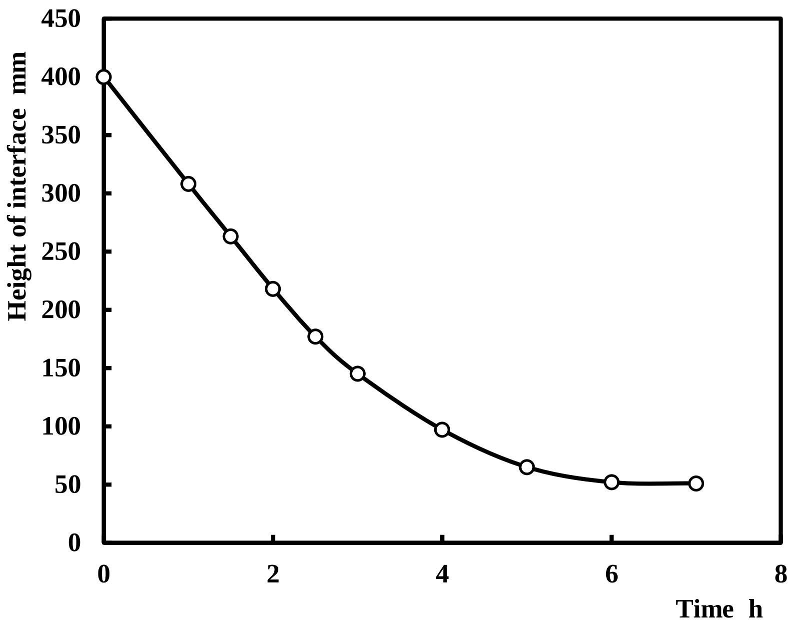


Fig (9.5) Batch sedimentation curve

9.4.2 Continuous sedimentation equipment: Thickeners

a- Features of the equipment

Industrially the above operation is conducted on large scale equipment known as thickener. Sometimes, a batch thickening tank can be used to separate solids of high settling rate. However, continuous operation is more common. A continuous thickener consists of a large shallow tank with slow moving radial rakes driven by a central shaft. Its bottom is usually in the form of a shallow cone. Dilute slurry flows from an inclined trough to a central feed well where it is distributed into the center of the thickener (Figure 9.6).

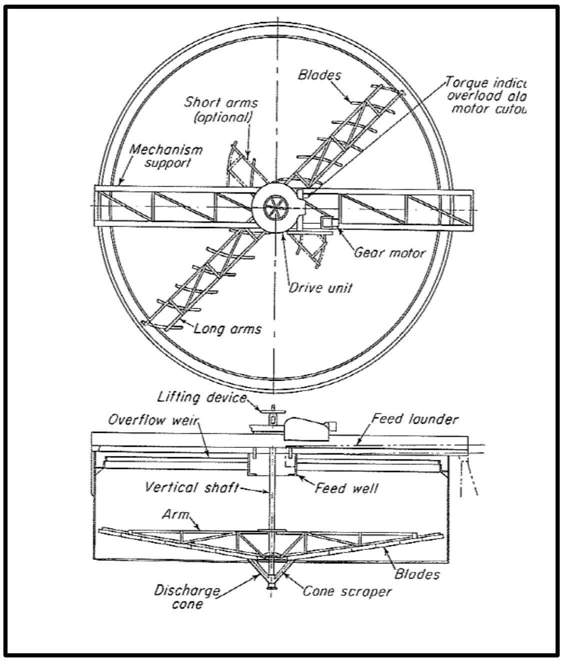


Fig (9.6) Continuous thickener

The feed slurry being denser than water tends to flow downwards until its density stabilizes. It then moves radially outwards at decreasing speed and the flow gradually divides into a downward moving suspension and an upward moving clear liquid. Bottom rakes slowly agitate the solid sludge moving it towards a central opening at the conical bottom while the clear liquid flows over a weir situated in the upper part of the thickener.
Such thickeners can be very large of diameters reaching 100 m. and depths up to 4 m and the rakes move at about 2 rpm.

b- Kynch method: Settling rate plot
The design of such thickeners is primarily concerned with finding the area required to perform a certain duty. This in turns requires the elaboration of a settling rate curve. To this aim, the Kynch method is a simple method that relies on one single batch sedimentation experiment. The settling rate curve is a curve relating the settling rate dz/dt against solid concentration at the solids at the top of the settling zone inside the thickener. Kynch has elaborated a simple technique to obtain such concentration a function of height:

Consider any point on the on the settling curve in Figure (3.4) like (t, z). A tangent is drawn at that point and extended to intersect the height axis at z_i. If the initial height of suspension is z_0 and its initial concentration is c_0, then the expected concentration at height z can be calculated from:

$$c_i = \frac{c_0 . z_0}{z_i} \qquad (9.27)$$

The procedure is repeated for several values of z and a plot made between the slope of tangent $\left(\frac{dz}{dt}\right)$ and c_i. (Rate of settling curve, Figure (9.8)

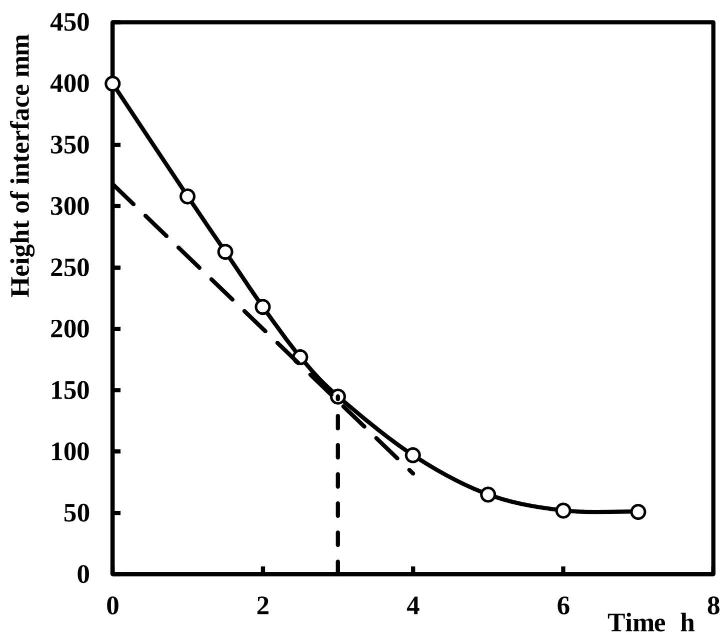


Fig (9.7) Kynch construction

Practically, it is more convenient to determine the equation of the decreasing rate portion of the curve and perform analytical differentiation of the equation obtained.

c- Application to the design of continuous thickeners

As opposed to a batch sedimentation tank, in a continuous thickener, there is a downward flow of liquid with solids at an almost constant velocity *v*. The mass rate of solids transported with the liquid per unit area is called the **transport flux** G_t ($kg.m^{-2}.h^{-1}$). On the other hand, a second solid flux is associated with settling solids and is called **settling flux** G_s.

The transport flux $G_t = c.v$ whereas the settling flux = $c.\frac{dz}{dt}$, so that the total flux is:

$$G = G_t + G_s = c.v + \frac{dz}{dt}.c \qquad (9.28)$$

Using the construction shown in Figure (9.7) and knowing the value of the transport velocity *v*, we can plot the total flux *G* against c_i. The total flux passes through a minimum value G_{min} as shown in Figure (9.9).

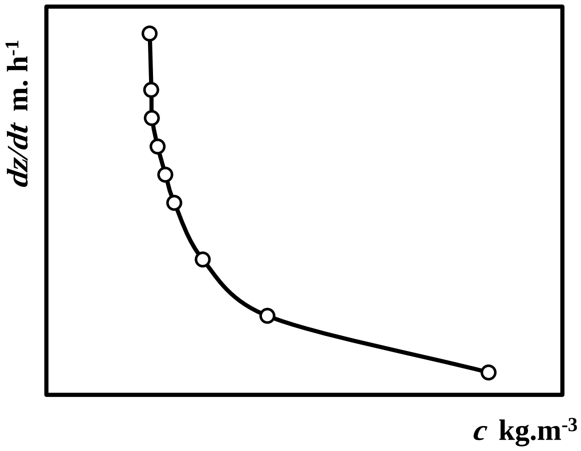


Fig (9.8) Plot of rate of settling against concentration

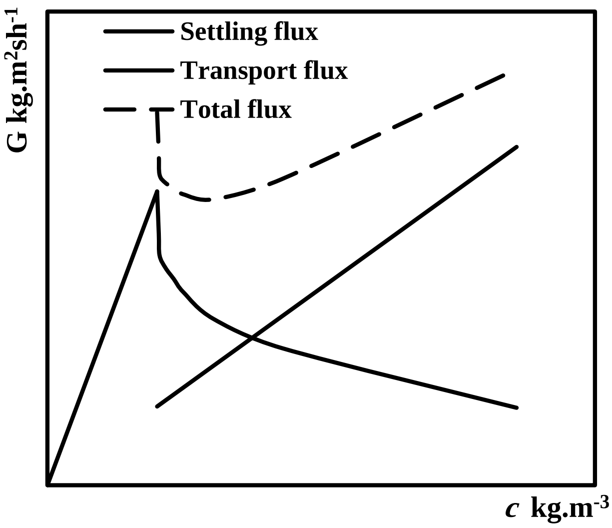


Fig (9.9) Total flux plot

If the inlet rate of solids is $F.c_o$ kg/h (where F is the feed rate in m^3/h), then the area of the thickener can be calculated from the following relation:

$$F.c_o = A.G_{min} \qquad (9.29)$$

Example 9.8

In a water treatment unit, 500 m^3.h^{-1} containing 250 kg.m^{-3} silt is to be clarified in a continuous thickening tank. Data for batch sedimentation are shown in the table below. Assuming the transport flux to equal 1.8 m.h^{-1}, calculate the diameter of the required thickener.

Time min.	0	20	40	60	80	100	120	140	∞
Height mm	475	350	260	200	160	135	120	110	85

Solution:

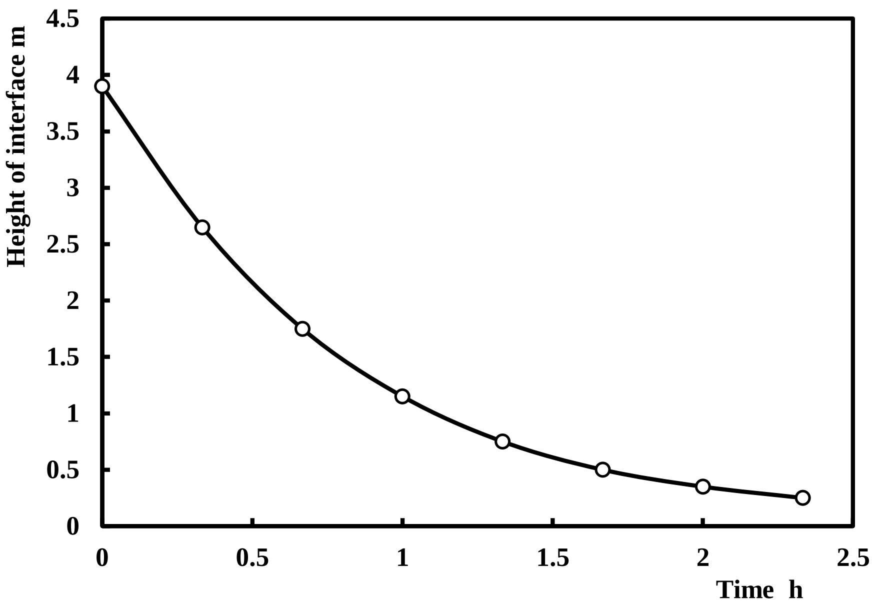


Fig (9.10) Sedimentation curve

An equation has been obtained for the settling curve after transforming time units to hours and height units to meters, taking into account that the asymptotic value of height = 85 mm:

$z = 0.085 + 3.856e^{-1.2t}$

Differentiating, we obtain: $\left|\frac{dz}{dt}\right| = 4.627e^{-1.2t} = 1.2(z - 0.085)$

The intercepts of tangents drawn at that curve at different values of c_i are obtained by letting $t = 0$ in the equation of tangents.

$z_i = z - |\text{slope}|.t$

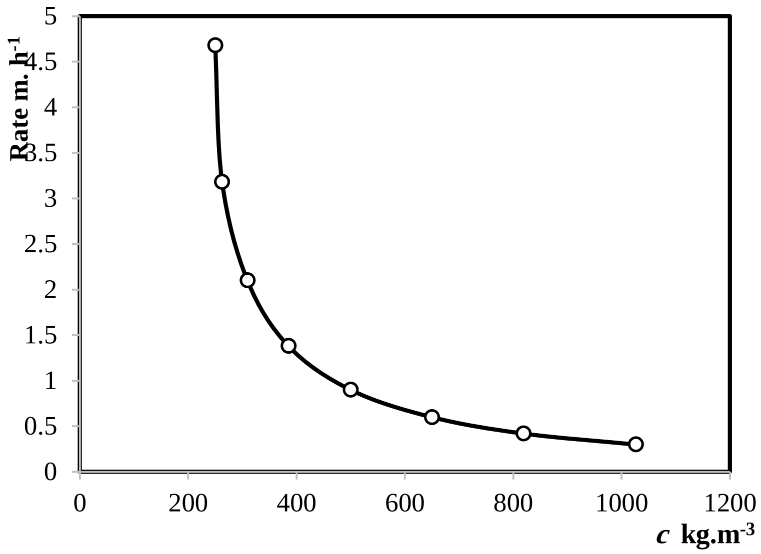


Fig (9.11) Settling rate – concentration curve

The values of c_i are then obtained from equation (9.27).

The values of $\frac{dz}{dt}$ are plotted against c_i to obtain the **settling rate – concentration plot** (Figure 9.11)

The settling flux is calculated at each value of c_i as $c_i \times \frac{dz}{dt}$

The transport flux is calculated from 1.8*c*

The following table shows all calculations:

t h	*z* m	*dz/dt*	*z**	*c*	*c×dz/dt*	*c×v*	Total G
0	9.9	4.578	9.9	250	1144.5	450	1594.5
0.33333	2.65	9.078	9.676	265.234	816.39	477.421	1299.81
0.66667	1.75	1.998	9.082	316.353	632.073	569.435	1201.51
1	1.15	1.278	2.428	401.565	519.2	722.817	1236.02
1.33333	0.75	0.798	1.814	537.486	428.914	967.475	1396.39
1.66667	0.5	0.498	1.33	739.083	365.075	1319.55	1684.62
2	0.35	0.318	0.986	988.844	314.452	1779.92	2094.37
2.33333	0.25	0.198	0.712	1369.38	271.138	2464.89	2736.03

The total minimum flux = 1201.51 $kg.m^{-2}.h^{-1}$ is obtained at a concentration of about 316.53 $kg.m^{-3}$.

Hence from equation (9.29), F = 500 $m^3.h^{-1}$, c_0 = 250 $kg.m^{-3}$, we get:

Area of thickener = $F_0.c_0/G_{\min} = \frac{500 \times 250}{1201.5}$ = 109.94 m^2

Thickener diameter ≈ 11.5 m

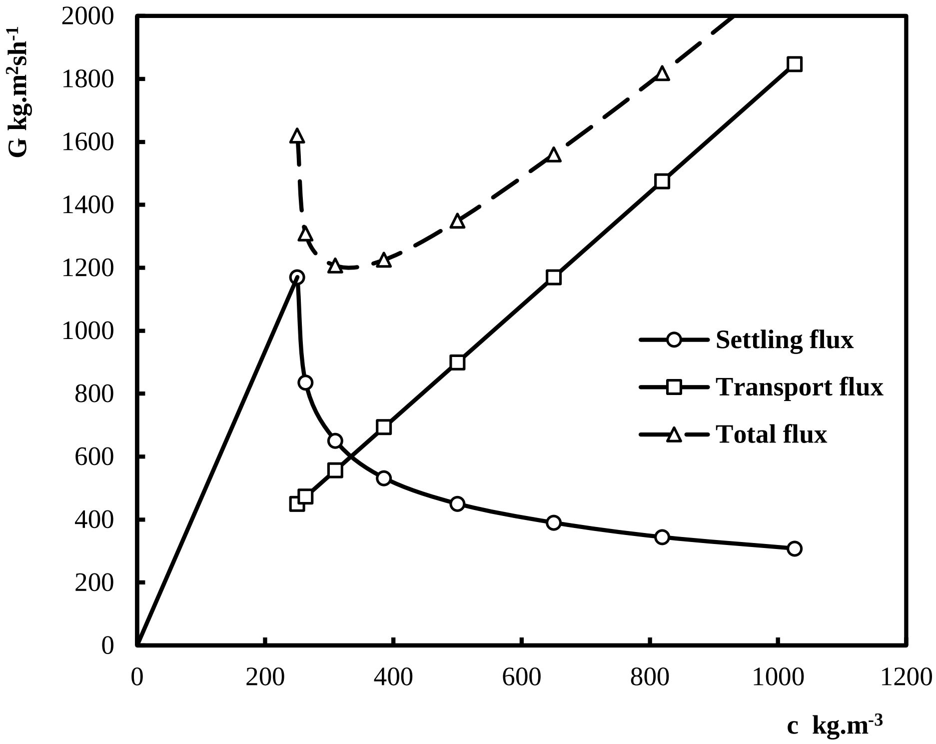


Fig (9.12) Total flux / concentration curve