8. FILTRATION

8.1 Introduction

Filtration is a unit operation in which a mixture of solids and liquid called the feed or the suspension or the slurry or the influent or the dispersion is forced through a porous medium in which the solids are deposited or entrapped. The porous filter medium is the permeable material that separates particles from liquids and is known as the filter. The solids retained on the filter are called the residue. The solids form a cake on the surface of the medium and the clarified liquid known as effluent or filtrate is discharged from the filter.

Filtration processes can be broadly classified into three categories. If recovery of solids is desired, the process is called cake filtration. The term clarification is applied when the solids do not exceed 1 % w/w and the filtrate is the primary product. The third type of filtration is called cross-flow filtration in which the liquid flows in a tangential direction with respect to the filtration medium. Cross-flow filtration is mainly used for membrane filtration. In this section only cake filtration will be discussed.

8.2 Theory

As filtration of a slurry proceeds, a porous wet cake is deposited on the filter medium and the total pressure drop is therefore the sum of the pressure drop across the cake Δp_c and the pressure drop across the filter medium Δp_m:

$$\Delta p = \Delta p_c + \Delta p_m \qquad (8.1)$$

Referring to Fig.(8.1), the pressure drop across an element of cake thickness dl can be predicted from the laminar term of the laminar term of Ergun equation as follows:

$$\Delta p_c = \frac{150\mu L v(1-\varepsilon)^2}{\Phi^2 \varepsilon^3 D_p^2} \qquad (8.2)$$

Hence:

$$\frac{dp_c}{dL} = \frac{150\mu v(1-\varepsilon)^2}{\Phi^2 \varepsilon^3 D_p^2} \qquad (8.3)$$

The mass of the element of cake considered is $dm_c = \rho_p .A.(1-\varepsilon).dL$

Hence substituting for dL in Eq. (8.3), we get:

$$\frac{dp_c}{dm_c} = \frac{150\mu v(1-\varepsilon)}{A\Phi^2 \rho_p \varepsilon^3 D_p^2} \qquad (8.4)$$

We have now to differentiate between two types of cakes:

Incompressible cakes: Where the values of s and ε remain constant throughout the filtration process. In this case, the term $\frac{150(1-\varepsilon)}{\Phi^2 \rho_p \varepsilon^3 D_p^2}$ remains constant and is termed the **local specific cake resistance,** α_L. Equation (8.4) then takes the form:

$$dp_c = \frac{\mu.v.\alpha_L}{A} dm_c \quad (8.5)$$

The coefficient of dm_c in the above equation being constant, this equation can be readily integrated to give:

$$\Delta p_c = \frac{\mu v \alpha_L}{A} m_c \quad (8.6)$$

Where the constant value of α_L is given by:

$$\alpha_L = \frac{150(1-\varepsilon)}{\Phi^2 \rho_p \varepsilon^3 D_p^2} \quad (8.7)$$

The above equations may be applied in the filtration of incompressible cakes such as crushed limestone or dolomite, ground quartz or feldspar and any rigid cake that will not yield to the external pressure.

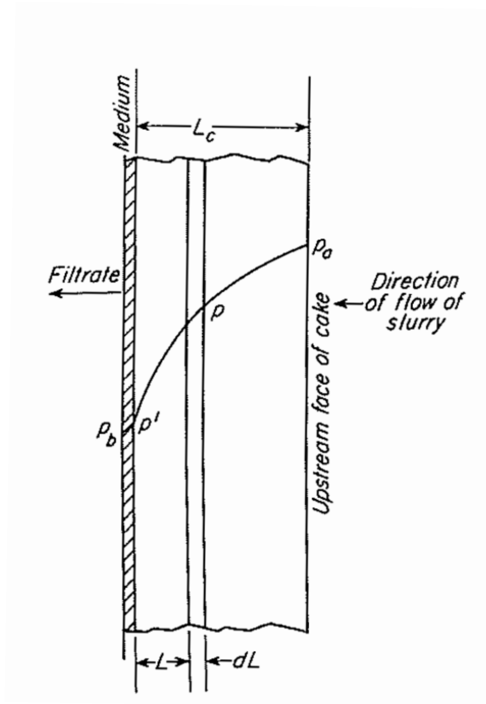

Fig (8.1) Mechanism of filtration

Compressible cakes: This is the case of most industrial cakes such as phosphate cakes, hydrated alumina etc… In that case the value of α_L is not constant as the porosity decreases during filtration. Integration of Eq. (8.5) gives:

$$\int dp_c = \frac{\mu . v}{A} \int \alpha_L . dm_c \qquad (8.8)$$

It is not easy to evaluate the RHS of the above equation by direct integration and usually a mean value of α_L is introduced as being:

$$\alpha = \frac{\int \alpha_L . dm}{\int dm} \qquad (8.9)$$

Since $m_c = \int dm$, hence combining (8.8) and (8.9), we get:

$$\Delta p_c = \frac{\mu v \alpha}{A} m_c \qquad (8.10)$$

The value of α is termed the **mean specific cake resistance**.

Besides, the filter medium offers a resistance to the flow of filtrate that appears in the form of a pressure drop which can be written in a way similar to equation (8.10):

$$\Delta p_m = \mu . v . R_m \qquad (8.11)$$

Where R_m is termed the **filter medium resistance.**

In the above equations, in the SI units, α has for unit m.kg^{-1} while the unit of R_m is m$^{-2.}$

Recalling that the superficial velocity, $v = \frac{1}{A}\frac{dV}{dt}$ and the total pressure drop can be obtained by summing up the two above equations. We get:

$$\Delta p = \Delta p_c + \Delta p_m = \frac{\mu}{A}\frac{dV}{dt}\left[\frac{m_c \alpha}{A} + R_m\right]$$

The value of m_c can be obtained by knowing the volume of filtrate V and the solid concentration in the slurry $c = \frac{m_c}{V}$. The above equation becomes:

$$\Delta p = \frac{\mu}{A}\frac{dV}{dt}\left[\frac{cV\alpha}{A} + R_m\right]$$

Rearranging, we get:

$$\frac{dt}{dV} = \frac{\mu}{A \Delta p}\left[\frac{cV\alpha}{A} + R_m\right] \qquad (8.12)$$

There are **two ways of performing filtration:**

8.3 Constant pressure filtration

If the pressure is held constant throughout the whole filtration period, then due to the continuous deposition of the cake, the rate of filtration will eventually decrease. In that case Eq. (8.12) is usually written in the following form:

$$\frac{dt}{dV} = K_p.V + B \qquad (8.13)$$

Where:

$$K_p = \frac{\mu.c.\alpha}{A^2.\Delta p} \qquad (8.14)$$

$$B = \frac{\mu.R_m}{A.\Delta p} \qquad (8.15)$$

Eq. (8.13) can be integrated to give:

$$t = K_p.\frac{V^2}{2} + B.V \qquad (8.16)$$

It is usually needed for design purpose to determine α and R_m from equations (8.14) and (8.15). This is done by collecting the filtrate at fixed time intervals so as to have values of *V*, the cumulative volume of filtrate against time *t*.

Dividing the two sides of Eq. (8.16) by *V*, we get:

$$\frac{t}{V} = K_p.\frac{V}{2} + B \qquad (8.17)$$

Hence, a plot of t/V against *V* should give a straight line of slope $K_p/2$ and intercept *B*. After obtaining those two constants, equations (8.14) and (8.15) can be applied to get the mean specific cake resistance and the filter medium resistance.

The Compressibility Coefficient

For a compressible cake, the value of α is variable and increases with the increase in the pressure drop across the cake. An empirical equation that is usually applied for the variation of the mean specific cake resistance is:

$$\alpha = \alpha_0\, (\Delta p_c)^s \qquad (8.18)$$

Where: *s* is a constant called the *compressibility coefficient*. For incompressible cakes $s = 0$, while for compressible cakes $0.2 < s < 0.8$.

In most cases, the pressure drop across the filter medium can be neglected compared to the pressure drop across the cake so that we may write:

$$\alpha = \alpha_0\, (\Delta p)^s \qquad (8.19)$$

8.4 Constant rate filtration

If the filtration rate is to be kept constant, then the pressure drop across the cake will increase throughout the filtration process. In this case:

Recalling that the filtration velocity is $v = \frac{V}{A.t}$

Eq. (8.10) can be rewritten as:

$$\Delta p_c = \frac{\alpha \mu c}{A} \frac{V^2}{At} \tag{8.20}$$

Combining the above equation with Eq. (8.19), we get:

$$\Delta p_c = \frac{\alpha_0 \mu c}{t} \frac{V^2}{A^2} (\Delta p)^s$$

Or:

$$(\Delta p_c)^{1-s} = \alpha_0 \mu c \left(\frac{V}{At}\right)^2 t \tag{8.21}$$

The value of $\frac{V}{At}$ is the constant rate of filtration per unit area (or the superficial velocity v).

Hence, we may write:

$$(\Delta p_c)^{1-s} = K_r.t \tag{8.22}$$

Where K_r is a constant defined by:

$$K_r = \mu.c.\alpha_o.v^2 \tag{8.23}$$

The values of s and K_r can be obtained by taking the logarithms of both sides of equation (8.22):

$$(1-s).\log \Delta p_c = \log K_r + \log t \tag{8.24}$$

A plot of log t against log $(\Delta p - \Delta p_m)$ gives a straight line of slope $(1 - s)$. The value of Δp_m can be obtained by first plotting Δp against time. At zero time the value of Δp equals Δp_m.

8.5 Filtration practice

8.5.1 Modes of filtration

In practical operation, it is customary to have neither constant pressure nor constant rate filtration. However, in case of filters run under positive pressure, a centrifugal pump is used to drive the slurry to the filter. In this case the mode of operation is that of constant rate during the early stages of filtration and of constant pressure in the later stages.

Also, it is common practice to wash the cake after filtration has been effected. This is simply done by having the wash liquid (very often water), flowing through the cake. The initial rate of washing is usually that of the final rate of filtration. However, **the overall rate of washing is usually about one fourth of the overall rate of filtration**. This latter is defined as being the ratio between the filtrate volume collected during the total filtration period t and the total time of the filtration cycle which consists of the sum $(t + t_T)$ where t_T is known as the **Tare**

time which is the time taken to dismantle the filter (in case of batch filters) , to remove the cake etc…On the other hand, a cycle including both filtration and washing will have an overall rate of filtration defined by:

$$R = \frac{V}{(t + t_T + t_w)} \tag{8.25}$$

Where t_w is the washing time.

If filtration is effected under constant pressure, then from equation (8.16), R is related to V by:

$$R = \frac{V}{(K_p.\frac{V^2}{2} + B.V + t_T + t_w)} \tag{8.26}$$

The maximum overall rate of filtration will be obtained by equating $\frac{dR}{dt}$ to zero. Once the value of V corresponding to that maximum rate is obtained, we can calculate the value of t. **In case the filter medium resistance is negligible**, then on writing $\frac{dR}{dt} = 0$ we finally get $t = t_T + t_w$

8.5.2 Filter media

The choice of a proper filter medium can often be the most important consideration in assuring satisfactory operation of a filter. It should be capable of properly retaining the solids that are to be separated from the liquid, with suitable length of life. The following criteria are usually used in the selection of a filter medium:

a- Ability to bridge solids across its pores quickly after feed is started.
b- The solids should not blind the filter openings.
c- Minimum filter medium resistance.
d- Resistance to chemical attack.
e- Acceptable resistance to mechanical wear.
f- Sufficient strength to support the filtration pressure.
g- Ability to discharge the cake easily and cleanly.
h- Relatively low cost.

Filter media are manufactured from textiles woven of cotton, synthetic fibers and sometimes, in case fine crystals are filtered, from metal fabric of about 400 mesh opening.

8.5.3 Filter aids

Filtration of some solids can be accompanied with problems of slow filtration rate or unsatisfactory filtrate clarity. This may be improved by using a filter aid. This consists of a granular or fibrous material capable of forming a highly permeable filter cake within which the troublesome solids will be incorporated. They are usually used when the solids consist of very fine particles. The particles of a good filter aid should be light and porous and chemically inert to the filtrate. The most used types are: **Diatomite** which is a variety of extremely porous silica and **Expanded Perlite** which is a highly porous alkali aluminosilicate. Their bulk density is in the range of 200 - 400 kg.m^{-3}.

Filter aids are used in two ways: (1) Either as a pre-coat to protect the filter medium and prevent the escape of occasional fine particles in the filtrate (the coat is applied at about 0.5 kg.m^{-2} of filter area), or (2) they can be mixed with the slurry to trap the difficult filterable particles in a permeable cake.

Example 8.1

The following data belong to lab tests undertaken on a calcium carbonate slurry. The filter area equals 0.045 m² and the solid concentration in the slurry is 24 kgm^{-3}. Evaluate the mean specific cake resistance and the filter medium resistance at a fixed pressure of 50 kPa.

***V* L**	0.5	1.0	1.5	2.0	2.5	3
***t* s**	17.3	42.3	72.0	108.3	152	202.7

Solution:

Fig. (8.2) shows a plot of *t/V* against *V* (equation (8.17))

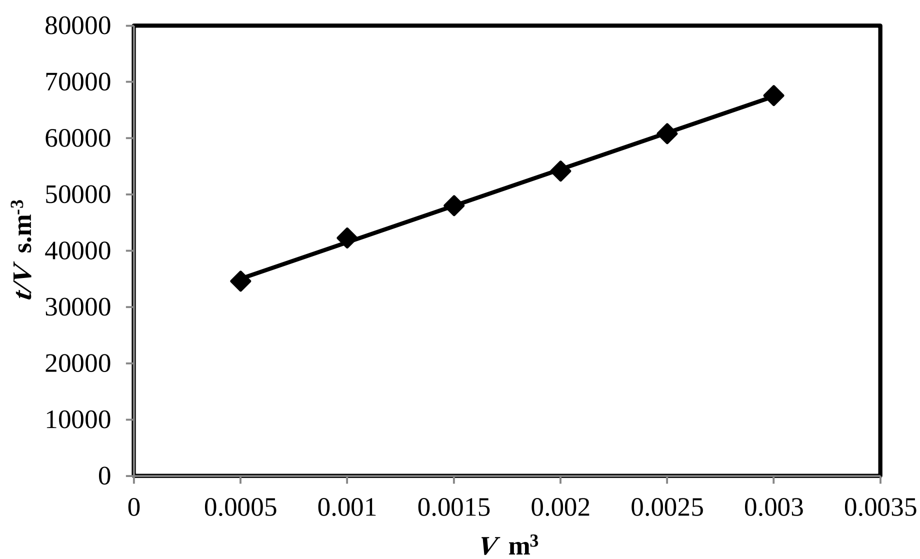

Fig (8.2) Plot between *t*/*V* and *V*

The slope of this plot is 12.9×10^6 s.m^{-4}

Hence $K_p/2 = 12.9\times10^6$ and $K_p = 2.58\times10^7$.

From Eq. (8.14)

$$\frac{\mu.c.\alpha}{A^2.\Delta p} = 2.58\times10^7 \text{ and } \alpha = \frac{(0.045)^2.50000}{0.001\times24} \times 2.58\times10^7 \text{ m.kg}^{-1}$$

Hence, $\boldsymbol{\alpha = 1.09\times10^{11}}$ **mkg^{-1}**

On the other hand, the intercept is 28600 s.m^{-3}

From equation (8.15):

$\frac{\mu R_m}{A\Delta p}$ = 28600 Hence, $\boldsymbol{R_m = 6.435\times10^{10}\ \mathrm{m}^{-1}}$

Example 8.2

The following data belong to a filtration test conducted under a constant rate of 0.05 m^{3}h^{-1}. The filtration area is 0.05 m^2 and the solid concentration in the slurry is 25 kg.m^3. From these data, deduce the values of R_m, α_o , K_r and s.

Δp **kPa**	30	34.1	43.6	52.1	59.3	69.5	80.5	92	103.6
***t* s**	10	20	30	40	50	60	70	80	90

Solution:

From a plot of Δp against time, extrapolating to time = 0, we get:

$\Delta p_m \approx 24000$ Pa Fig.(8.3)

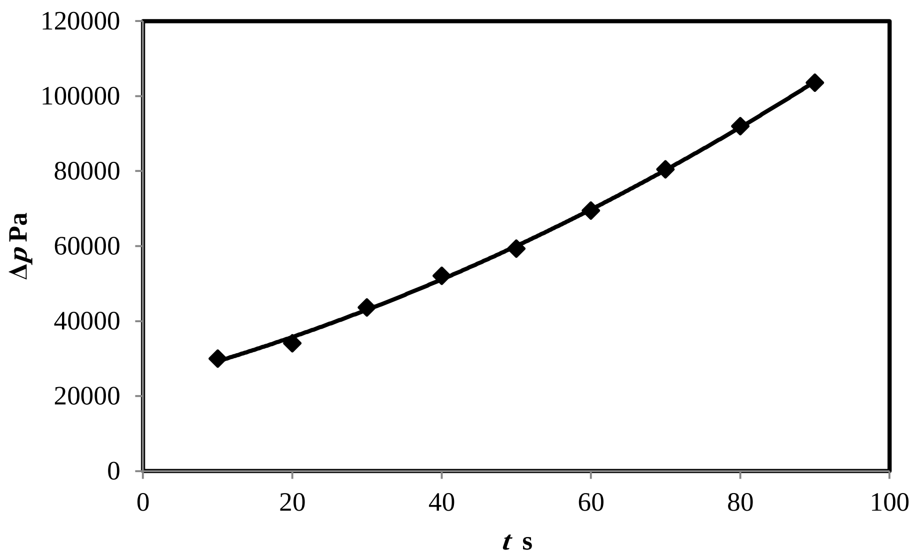

Fig (8.3) Plot of pressure drop against time

Since $\Delta p_m = R_m.\mu.v$, and v = 0.05/.05 m.h^{-1} = 1/3600 = 0.000278 m.s^{-1}

We get: $R_m = \frac{24000}{0.001\times0.000278}$ = $\boldsymbol{8.63\times10^{10}\ \mathrm{m}^{-1}}$

To get the other parameters, we plot log t against log (Δp - 24000)

log *t*	1.00	1.30	1.48	1.60	1.70	1.78	1.85	1.90	1.95
log (Δp - 24000)	3.78	4.00	4.29	4.45	4.55	4.66	4.75	4.83	4.90

The slope is (1 – s) = 0.81 (Figure2.3) hence $\boldsymbol{s = 0.19}$

Also, $K_r = \mu.c.\alpha_o.v^2$ Equation (8.23)

To get the value of K_r, we substitute a value of t against a value of $(\Delta p - \Delta p_m)$ in Eq. (8.24).

From the table in the heading of the problem: t = 60 s corresponds to $(\Delta p - \Delta p_m)$ = 69500 – 24000 = 45500 Pa , hence in Eq. (8.24):

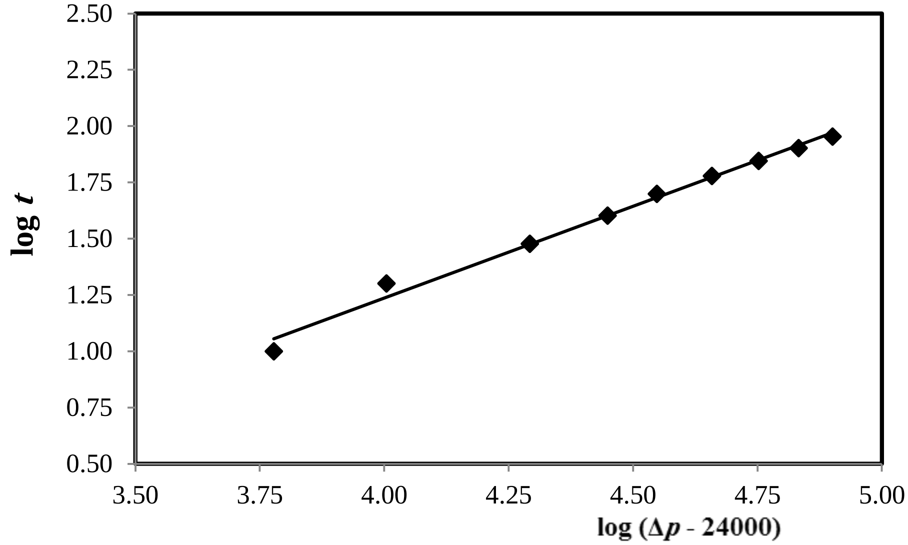

Fig (8.4) Plot of log *t* against log (Δ*p* – 24000)

$45500^{0.81} = K_r \times 60$, from which $K_r = 98.8$

Substituting in Eq. (8.25):

$98.8 = 0.001 \times 25 \times (0.000278)^2 . \alpha_0$

From which $\boldsymbol{\alpha_0 = 5.11 \times 10^{10}}$ **m.kg^{-1}**

8.6 Types of industrial filters

8.6.1 Pressure filters versus vacuum filters:

Pressure filters are operated under super atmospheric pressures. The feed slurry is introduced to the filter at pressures ranging from 2 to 5 atm gauge reaching in some cases figures as high as 35 atm. The most commonly used pressure filters are of the batch type. They have the following advantages:

(a) They provide high filtration rates and proper separation of fine solids.
(b) They usually occupy a small floor space per unit area of filtration.
(c) Their initial cost is low.
(d) They are flexible in their operation, allowing to filter a wide variety of solids.

On the other hand, such filters have also some disadvantages:

(a) They are difficult to operate.
(b) Their operating cost is high since they require a lot of labor.

Vacuum filters are operated using a vacuum pump or alternatively a jet ejector. They are usually more adapted for continuous operation. They offer the following advantages:

(a) They use low labor compared to batch filters

(b) Their surface is opened to the atmosphere allowing easy inspection and repair.

(c) The maintenance cost is relatively low.

Drawbacks of using such filters are:

(a) A vacuum system has to be maintained throughout the filtration process

(b) Due to the low pressure used , this is unsuitable for use in the case of volatile filtrates

(c) Such systems cannot handle difficult solids

(d) They lack flexibility as to the type of solid and the rate of filtration

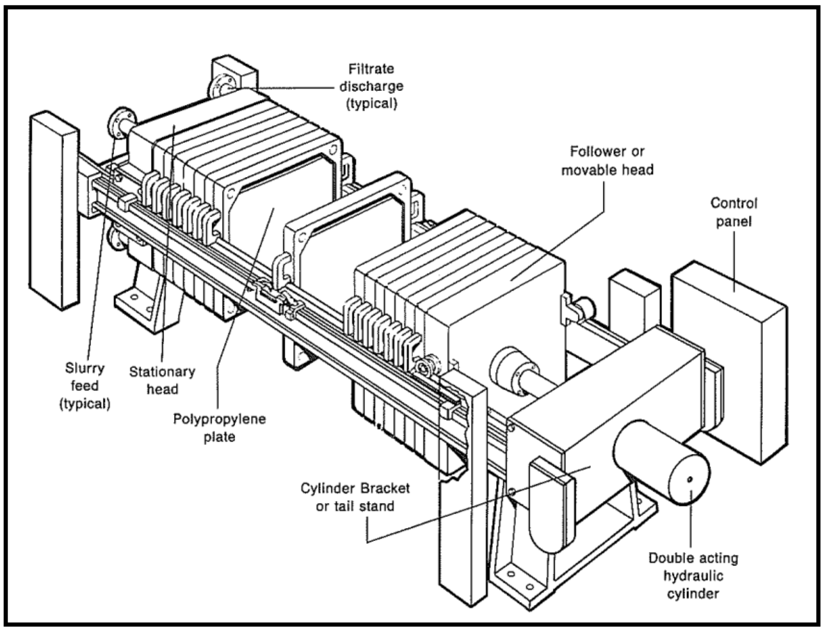

Fig (8.5) Filter press equipped for automatic operation

8.6.2 Pressure filters: The plate and frame filter press

A plate and frame filter press is an assembly of alternate *solid plates* the faces of which are studded, grooved or perforated to permit drainage and *hollow frames* in which the cake collects during filtration. A filter cloth covers both faces of each plate. Their shape is usually rectangular although circular plates and frames are sometime used. They are hung in a vertical position on a pair of parallel supporting bars. During filtration they are compressed between two end half-plates (one of which is fixed) using a screw or by a hydraulic ram.

The slurry is fed to the press under a pressure ranging from 3 to 10 atm. and moves along a duct formed by the alignment of slots present in the top corner of the plates and the frames. These slots are connected to grooves into the frames so as to direct the slurry into the frames towards the two plates sandwiching the frame. The cake is thus allowed to deposit on the filter clothes and the discharged filtrate flows through cock valves situated at the bottom of each plate. During filtration, these valves are open.As the cake builds up to cover from 70 to 90% of the frame thickness filtration is stopped and washing begun. The wash liquid is admitted through a duct formed by slots contained in the top corner of the plates and frames other than those used for the flow of slurry during the filtration period. These slots are connected with each other plate (known as wash plates), the valves of which are closed. The liquid therefore has to pass through the entire cake thickness until it reaches an open valve down a non-wash plate. Fig. (8.5(b))

In designing a filter press, we first determine the required area of filtration. Then, the number of plates is determined depending on the chosen dimensions of plates. These are usually of square shape. Their dimensions range from 150 mm to more than one meter. The thickness of plates ranges from 6 mm to 50 mm while that of frames ranges from 6 to 200 mm.

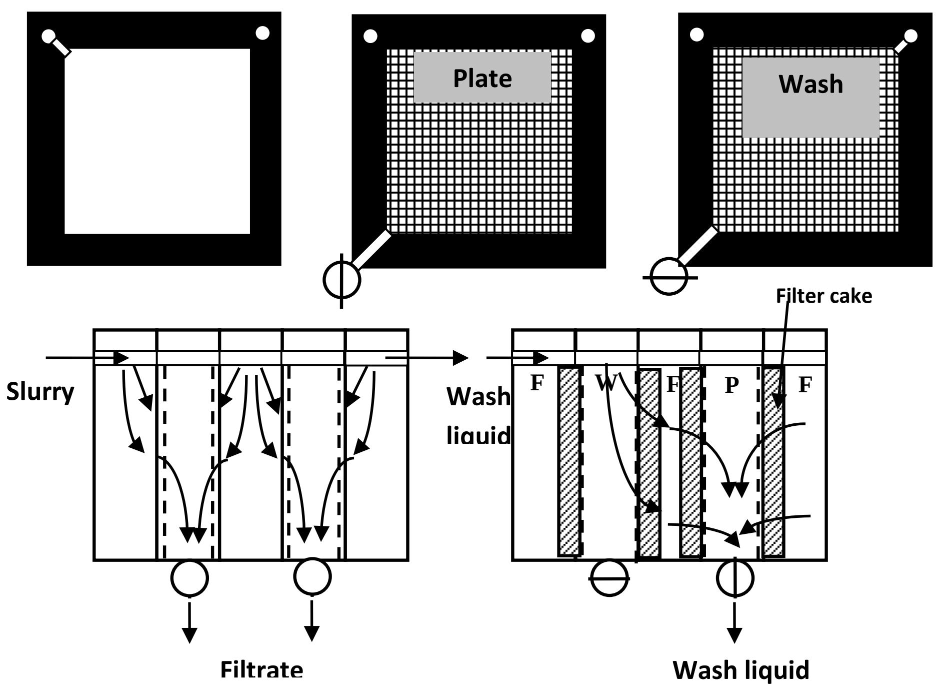

Fig (8.6) Filling and washing patterns in a filter press

Example 8.3:
A plate and frame filter press is to be designed for the filtration of a slurry consisting of 250 kg magnesite in 10 m³ water. Filtration is conducted under a constant pressure of 200 kPa. The specific cake resistance is 3×10^{10} m.kg^{-1} and the filter medium resistance is 10^6 m$^{-1.}$ The time of filtration is taken as 2 h and the cake density is 1400 kg solid/m^3 of wet cake. Find the number of plates if their filter dimensions measure 12”×12”. Also estimate the thickness of frames.

Solution:

We first calculate the area of filtration from Eqs. (8.14) to (8.16)

$$7200 = \frac{0.001\times25\times3\times10^{10}\times100}{2\times A^2\times200000} + \frac{0.001\times10^{6}\times10}{A\times200000}$$

$187500\times A^{-2} + 5\times10^{-2}\times A^{-1} = 7200.$

Solving for A, we get: $A = 5.1\ m^2$

The area of filtration per plate = $2\times144\times0.0254^2 = 0.185\ m^2$

The number of plates = 5.1 / 0.185 = **27 plates**

The area per frame is $(12\times0.0254)^2 = 0.094\ m^2$

The total mass of solids being 250 kg, then the mass of solids per frame is **250/27 = 9.26 kg.**

Since the cake density is 1400 kg solids per cubic meter wet cake then the volume of wet cake deposited per frame is 9.26 / 1400 = 0.00661 m^3.

Therefore the thickness of cake is 0.00661/0.094 = 0.07 m or 70 mm.

Assuming the cake to fill 80% of the frame then, the thickness of the frame is 70/0.8 ≈ **88 mm.**

8.6.3 Vacuum filters: The rotary filter

The most common type of continuous vacuum filters is the *Rotary drum filter*. This is shown in Fig. (8.6) while the continuous filtration setup is shown in Fig. (8.7).The filter consists of a horizontal drum with a slotted face turning at 0.2 - 2 RPM in an agitated slurry trough. The face of the drum is covered by a filter medium (usually a cloth known as canvas) and the drum is partly submerged in the slurry. Under the slotted cylindrical face of the main drum is a second smaller drum with a solid surface. Between the two drums are radial partitions dividing the annular space into separate compartments, each connected by an internal pipe to one hole in the rotating plate of a rotary valve .A strip of filter cloth covers the exposed face of each compartment to form a succession of panels.

Consider the panel A in Fig.(8.7). It is about to enter the slurry in the trough. As it dips under the surface of the liquid, vacuum is applied through the rotary valve. A layer of cake builds up on the face of the panel as liquid is sucked through the cloth into the compartment, through the internal pipe, through the valve into a collecting tank. As this panel leaves the drum it enters into the washing zone. Vacuum is applied to the panel from a separate system sucking wash liquid and air through the cake of solids. After the cake has dried vacuum is ceased and the cake removed by a horizontal knife blade. A little air is blown in under the cake to ease its removal from the filter cloth. Once the cake is taken away, the panel reenters the trough and the cycle is repeated (Figure 2.8).

Often a precoat is continuously supplied on the filter surface.

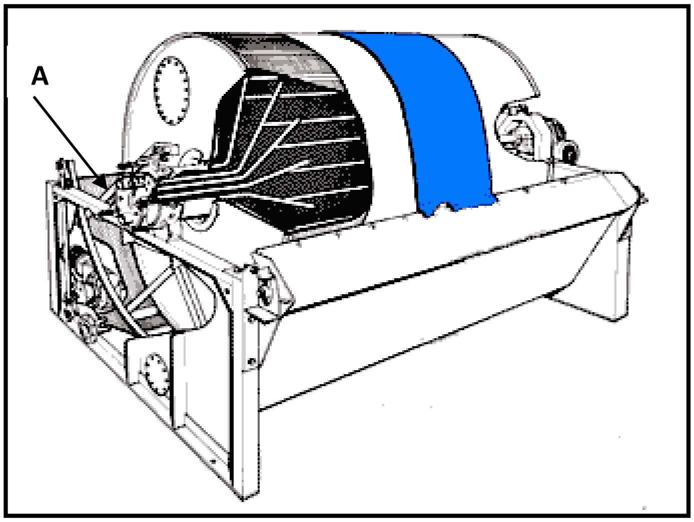

Fig (8.7) Continuous rotary vacuum filter

Consider the panel A in Fig.(8.7). It is about to enter the slurry in the trough. As it dips under the surface of the liquid, vacuum is applied through the rotary valve. A layer of cake builds up on the face of the panel as liquid is sucked through the cloth into the compartment, through the internal pipe, through the valve into a collecting tank. As this panel leaves the drum it enters into the washing zone. Vacuum is applied to the panel from a separate system sucking wash liquid and air through the cake of solids. After the cake has dried vacuum is ceased and the cake removed by a horizontal knife blade. A little air is blown in under the cake to ease its removal from the filter cloth. Once the cake is taken away, the panel reenters the trough and the cycle is repeated (Figure 2.8).

Calculations of continuous filters

The calculation of the filtration area in case of a drum filter is very similar to that of a batch filter. The time required for filtration is therefore obtained by Eq. (8.16). Usually, the filter medium resistance can be neglected with respect to that of cake, so that:

$$t = \frac{\alpha\mu c}{2A^2\Delta p} V^2$$

Noting that the flow rate of filtrate is $Q = V/t$ the previous equation can be written in the form:

$$Q^2 = \frac{2A^2\Delta p}{\alpha\mu c t}$$

The time of filtration is lower than the actual time of a cycle t' by a factor ranging from 2.5 to 3.5, hence $t = f \times t'$, where $f = 0.3 - 0.4$. Therefore we may write the above equation in the modified form:

$$Q^2 = \frac{2A^2\Delta p}{\alpha\mu c f t'} \qquad (8.27)$$

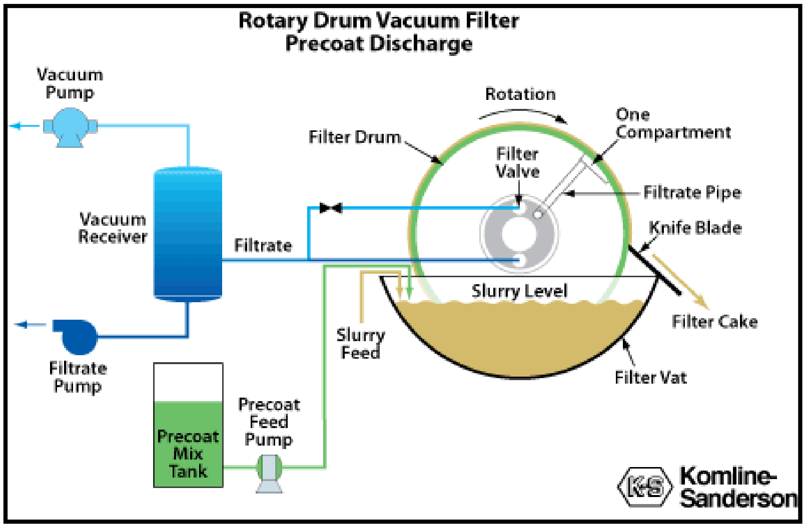

Fig (8.8) Flow Sheet for Continuous Vacuum Filtration

Note that the time of a cycle is the reciprocal of the speed of rotation of the drum (n rps). Also, the effective area of filtration A is $A_T.f$, where A_T is the total area of the filter medium. Equation (8.27) becomes:

$$Q^2 = \frac{2A_T^2 \Delta p f}{\alpha \mu c t'} \qquad (8.28)$$

Once the area A_T is known we may choose the drum dimensions from the standard data of Table (8.1) shown below:

Table (8.1) Dimensions of drum filters of known area

	Length, ft										
***D*, ft**	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**
6	76	113	151	189	226						
8			200	250	300	350	400				
10				310	372	434	496	558	620		
12					456	532	608	684	760	836	912

Example 8.4

It is required to design a rotary filter with 30% submergence fed with 3.3 $m^3.h^{-1}$ of a slurry containing 236 kg solids per m^3 of filtrate. The absolute pressure maintained during filtration is 68 kPa and the mean specific cake resistance is 5×10^{10} mkg^{-1}. The speed of rotation of drum = 0.2 rpm.

Calculate the rate of dry cake production.

Solution:

This is a direct application of Eq. (8.28) with $t' = \frac{60}{0.2}$ = 300 s and f = 0.3

$$\left(\frac{6.6}{3600}\right)^2 = \frac{2A^2 \times 68000 \times 0.3}{5\times10^{10}\times0.001\times236\times300}$$

From which: A = 8.55 m² which is equivalent to about 92 ft²

The nearest dimensions from Table (8.1) are: ***D* = 6 ft and *L* = 6 ft**

The rate of cake production is $Q.c$ = 3.3×236 = **779 kg.h^{-1}**

8.7 Filtration of solids from gases: Bag filters

8.7.1 Operating principle

Conventional bag filters consist of an arrangement of vertical bags where dusty air (or gases) is allowed to pass through the woven fabrics which filter the air. Actually, dust removal is rather performed by the coat of dust deposited on the fabric rather than by the fabric itself. Figures (8.9) and (8.10) show a typical arrangement of vertical bags in a rectangular housing.

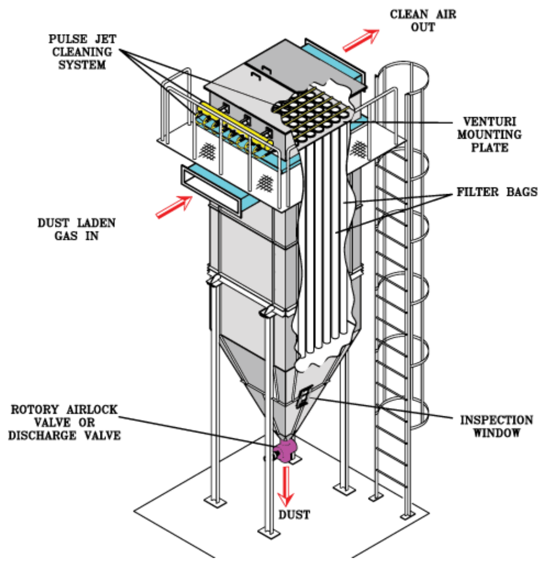

Fig (8.9) Bag house arrangement

Commonly, dusty air is circulated on the outer periphery of the bags, thereby depositing the dust on the outer surface of the bags.

Cleaning of the bags is done periodically by closing the incoming air valve to a set of bags and either mechanically shaking these bags or introducing compressed clean air in a reverse direction to make the dust particles drop in the collecting funnel. In that case, cleaning is done by passing clean air inside the bags thus expelling the dust to the bottom funnel.

The bags are usually made from cotton, Nylon, Dacron, Polyethylene for normal operation, and from fiberglass for high temperature uses.

8.7.2 Design principles

The pressure drop across a bag is the sum of two terms: The pressure drop across the fabric Δp_f and the pressure drop across the dust Δp_d.

Since the motion of air (or gases) across the fabric usually falls in the laminar flow regime, it is customary to keep the first term in the Ergun equation governing fluid flow through porous media:

$$\frac{\Delta P}{L} = \frac{150.\bar{v}.\mu.(1-\varepsilon)^2}{\Phi^2.D_p^2.\varepsilon^3} + \frac{1.75\rho_f.\bar{v}^2.(1-\varepsilon)}{\Phi.D_p.\varepsilon^3}$$

So that:

$$\frac{\Delta P}{L} \approx \frac{150.\bar{v}.\mu.(1-\varepsilon)^2}{\Phi^2.D_p^2.\varepsilon^3}$$

Written in permeability form, this equation reads:

$$\Delta P_f \approx \frac{\bar{v}.\mu.L}{K} \qquad (8.29)$$

Where: K is the Darcy permeability of the fabric and L its thickness.

Practically, the fabrics used industrially have fixed values of L/K so that the above equation simplifies to:

$$\Delta P_f = K_f.\mu.v \qquad (8.30)$$

Where,

ΔP_f: is in inch water

μ: is in cP

v: is the peripheral velocity of air, $ft.min^{-1}$.

The values of the constant K_f are listed for some woven fabrics in Table (8.2).

Table (8.2) Values of the constant K_f

Fabric	Cotton	Wool	Nylon	Asbestos	Fiberglass	Dacron	Teflon
K_f	2.4-2.5	0.3-0.5	2.7-3.7	0.56	2.6	0.3-0.8	2.4

The value of the pressure drop across the accumulated dust increases with time. It has been established that its value is proportional to the square of gas velocity as shown by the following equation:

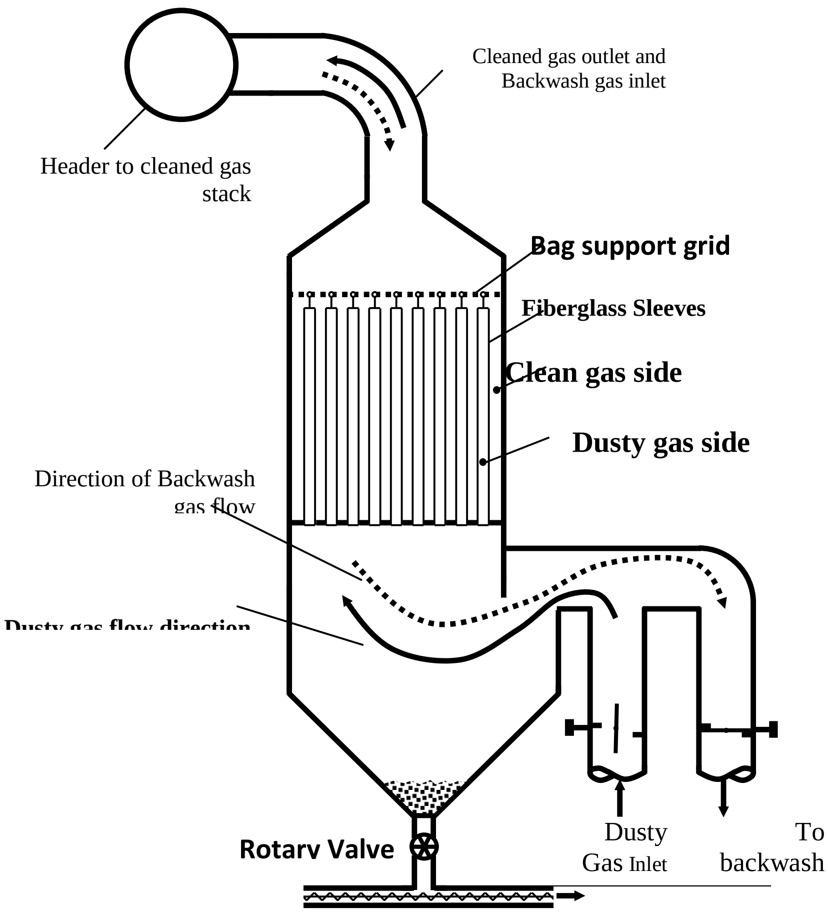

Fig (8.10) A typical bag house compartment

$$\Delta P_d = 0.5\, K_d.\mu.c_d.t.v^2 \qquad (8.31)$$

Where,

ΔP_d: is in inch water

μ: is in cP

c_d: is the concentration of dust, g.m^{-3}

v: is the peripheral velocity of air, ft.min^{-1}.

t: is the time of filtration, min.

The value of K_d has to be determined experimentally. For particle sizes less than 10 microns (which is a common case where bag filters are used) it ranges from 0.2 to 0.4.

Bag filters operate on pressure drops ranging from 2 to 6 inch water. The value of the filtration time t is determined from practice as the filter has to be cleaned when the pressure drop exceeds its maximum value. In practice the values of t range from 2 to 15 min.

By adding up the two above equations and equating the total pressure drop to a maximum value of 6 in. water, the value of the superficial velocity v can be deduced. From knowledge of the dusty air flow rate, one can calculate the total filtration area. The value of v should range from 1 to 8 $ft.min^{-1}$.

Example 8.5

Dusty air containing 20 $g.m^{-3}$ solids of average particle size = 5 μm is to be filtered from dust in a bag filters. The available bags are made of cotton fabric. They consist of cylinders of diameter = 0.25 m and length = 3 m. Air is at a temperature of 70°C, at which temperature its viscosity = 0.02 cP. The flow rate of air is 47000 $Nm^3.h^{-1}$. Estimate the necessary number of bags.

Solution

From equation (8.30):

$\Delta P_f = K_f.\mu.v = 2.4\times0.02\, v = 0.048\, v$

The pressure drop across the dust cake will be calculated from Eq. (8.31) by taking t = 10 min., $K_d = 0.3$

$\Delta p_d = 0.5\, K_d.\mu.c_d.t.v^2 = 0.5\times0.3\times0.02\times20\times10\, v^2$, hence

$\Delta p_d = 0.6\, v^2$

The total pressure drop is taken as 5 " water, hence:

$0.048\, v + 0.6\, v^2 = 5$

Solving for v we get: v = 2.85 $ft.min^{-1}$. (note that $1 < v < 8$)

This is equivalent to: v = 0.0145 $m.s^{-1}$

The actual flow rate of air at 70°C has to be obtained from ideal gas rules:

$\frac{Q}{343} = \frac{47000}{273+25}$, from which: Q = 55097 $m^3.h^{-1}$= 15 $m^3.s^{-1}$

The total area of bags is therefore A = 15/0.0145 = 1035 m^2.

The area of one bag is $\pi.D.L = \pi\times0.25\times3 = 2.36\ m^2$.

Hence the number of bags required is: 1035/2.36 = **439 bags**