-3-

Reliability Analysis

3.1 Introduction

In industry, the problem of assuring and maintaining reliability depends on a set of factors among which one may list:

- Original equipment design
- Quality control during production
- Acceptance inspection
- Field trials
- Design modifications
- Cost, size and weight of equipment, etc…

Despite this complicated nature, one can simply define reliability as follows:
Reliability is the probability that an equipment (or a product) will function within specified limits for at least a specified period of time (under specified environmental conditions).

Being a probability, reliability can thus be calculated in the same way used in Chapter 3. In this connection, one has to differentiate between two types of systems:

- **Series systems**, where all components are in series so that the failure of any of them will cause failure of the entire system.
- **Parallel systems,** where the failure of any component will only affect partly the performance of the system. For such systems to fail it takes all its components to fail.

3.2 Reliability of different systems

3.3.1 Series systems

If all components of a system in series are independent, then the reliability as defined above will follow equation (3.13) for independent events, that is, the reliability of the system will equal the product of reliability of its components.

$$R = R_1 \times R_2 \times R_3 \times \ldots \times R_n \qquad (3.1)$$

This equation shows that the reliability rapidly drops as the number of components increases. For example, if the individual reliability of all components are assumed to be constant = 0.98, then the reliability of a system consisting of 10 components will be $(0.98)^{10} = 0.817$, while in case of 20 components it becomes: $(0.98)^{20} = 0.667$.

3.3.2 Parallel systems

In parallel systems, the system will fail if all its components fail so that we rather deal with the **failure probability** defined by:

$$F_i = 1 - R_i \qquad (3.2)$$

The law of independent events can now be applied to failure probabilities so that the failure probability of the system will be:

$$F = F_1 \times F_2 \times F_3 \times \ldots \times F_n \qquad (3.3)$$

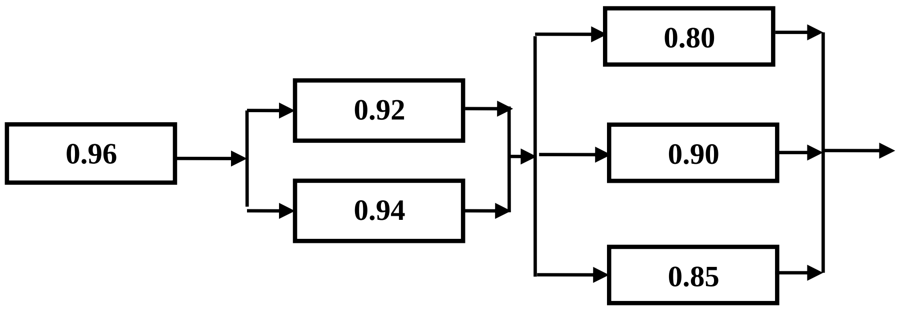
So that the reliability of a parallel system will be given by:

$R = 1 - F_1 \times F_2 \times F_3 \times ... \times F_n = 1 - (1 - R_1) \times (1 - R_2) \times \dots (1 - R_n)$ (3.4)

For example, in the example dealt with in section (3.3.1), the failure probability of any component will be 0.03. Hence in a 5 components system F = $(0.02)^5 = 3.2\times10^{-9}$. This extremely low probability corresponds to an extremely high reliability. This is expected in view of the fact that for such a system to fail it takes all its 5 components to fail.

Example 3.1

The following diagram shows a mixed system (Series + parallel) with the reliability of each component shown. Calculate the reliability of the system.



Solution:

There are two parallel systems, the first consists of 2 components. According to equation (3.4), the reliability of this system is:

$1 - (1 - 0.92)\times(1 - 0.94) = 0.9952$

The second parallel system consists of three components. Its reliability is:

$1 - (1 - 0.8)\times(1 - 0.9)\times(1 - 0.85) = 0.997$

Therefore the reliability of the three component series system will be:

R = 0.96×0.9952×0.997 = **0.9525**

3.3 Failure – time distribution

3.3.1 The exponential model

By failure – time distribution is meant the distribution of the time to failure of a component under working conditions. If we assume that this distribution is continuous with a density function $f(t)$, then the **probability that failure will occur in the interval of time (0, *t*)** will be given by an equation in the form:

$$F(t) = \int_0^t f(t).dt \qquad (3.5)$$

The reliability function is therefore $R(t) = 1 - F(t)$

The probability that failure will occur in the interval $(t, t + \Delta t)$ is:

$P(A) = F(t + \Delta t) - F(t)$

The probability that the equipment has properly functioned up to time t is its reliability = $R(t)$.

Therefore the probability that failure occurs in the interval ($t,\ t + \Delta t$) **provided it has not occurred before that** is the conditional probability:

$$\frac{P(A)}{R(t)} = \frac{F(t+\Delta t) - F(t)}{R(t)}$$

The average rate of failure is then defined as the ratio:

$$\frac{P(A)}{R(t).\Delta T} = \frac{F(t+\Delta t) - F(t)}{R(t).\Delta t}$$

As $t \rightarrow 0$, the instantaneous rate of failure at time t (for the first time) is therefore:

$$Z(t) = \frac{F'(t)}{R(t)} = \frac{f(t)}{R(t)} \qquad (3.6)$$

Or, $$Z(t) = \frac{f(t)}{1 - F(t)} \qquad (3.7)$$

The failure rate usually varies with time as shown in Figure (3.1), which is shown to consist of three distinct stages:
The first stage is a decreasing rate curve which covers the initial period of time through which poorly manufactured items are eliminated.
Throughout the second stage, the rate remains constant. This represents the constant failure rate period where failure would only occur on a random basis, being exclusively due to chance.

Finally, the third increasing rate period is typical of items beginning to wear out.

The main period of interest in this curve is the constant failure rate period (Z = constant) since it represents the useful lifetime of the equipment.
Since $F(t) = 1 - R(t)$, then $f(t) = F'(t) = -R'(t)$

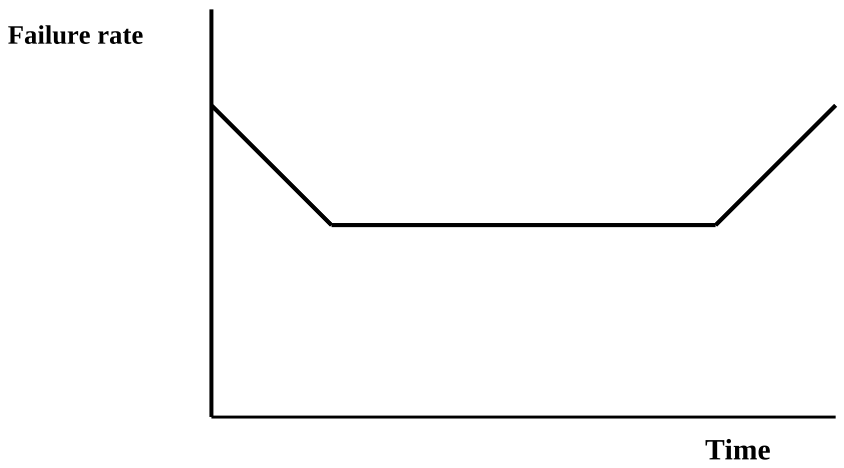


Fig. 3.1 Failure rate curve

This way, equation (3.7) reads:

$Z = \frac{R'(t)}{R(t)}$ Integrating, one gets:

$$R(t) = e^{-\int_0^t Z.dt}$$

Denoting the constant failure rate by α, we get:

$R(t) = e^{-\alpha.t}$ (3.8)

And since $f(t) = -R'(t)$, one gets:

$f(t) = \alpha.e^{-\alpha.t}$ (3.9)

This special continuous distribution is known as **the exponential distribution.** It can be deduced that the mean time of failure is $1/\alpha$.

This time is known as the mean time between failures **MTBF**.
So that if this parameter is denoted by τ

τ = MTBF = $1/\alpha$ (3.10)

3.3.2 Application to reliability analysis

For systems **in series**, the combination of equations (3.1) and (3.8) gives the following result

$$R(t) = e^{-(\sum_{i=1}^{n} \alpha_i).t} \quad (3.11)$$

The failure rate of the system is obviously the sum of failure rates of all its components. For constant failure rates of α_1, α_2, α_3, …, from equation (3.10) the failure rate of the system is:

$$\alpha = \sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \frac{1}{\tau_i} \quad (3.12)$$

Therefore, from equation (3.10), the mean time of failure of the system is:

$$\tau_m = \frac{1}{\sum_{i=1}^{n} \frac{1}{\tau_i}} \quad (3.13)$$

For systems **in parallel**, the problem is more complicated. From equation (3.4), the rate of failure of a system assuming exponential distribution will be:

$F(t) = (1 - e^{-\alpha_1.t}).(1 - e^{-\alpha_{3.}t}).(1 - e^{-\alpha_{3.}t}).... (1 - e^{-\alpha_n.t})$ (3.14)

If all components have equal failure rate = α, then the above equation simplifies to:

$F(t) = (1 - e^{-\alpha.t})^n$ (3.15)

In this case it can be proved that the mean time between failures for the system is:

$$\tau_m = \frac{1}{\alpha} \times \left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}\right) \quad (3.16)$$

Example 3.2

In example (3.1), find the constant rate of failure of the 6 components constituting the system assuming the given reliabilities to be for 10 months operation. Then, estimate the mean time of failure of the entire system.

Solution:

Assuming exponential distribution of failure rate, we use equation (3.8) to calculate the different values of α.
For example, the first item has a reliability of 0.96. Hence:
$0.96 = e^{-\alpha_1 \cdot 10}$ from which $\alpha_1 = 4.08\times10^{-3}$ failures per hour.
The following table summarizes the obtained values of α for the 6 components.

Component	1	2	3	4	5	6
$\boldsymbol{\alpha}$	4.08×10^{-3}	8.34×10^{-3}	6.19×10^{-3}	3.23×10^{-2}	1.02×10^{-2}	1.6×10^{-2}

The first item has a mean failure time = $1 / 4.08\times10^{-3}$ = 245.1 month
The first two items in parallel (2 and 3) don't have equal values of α. So we use a mean value = $(8.34\times10^{-3} + 6.19\times10^{-3}) / 2 = 7.26\times10^{-3.}$
From equation (3.16): $\tau = (1 + 1/2) / 7.26\times10^{-3} \approx 206.6$ month.
Similarly, for the last three items we get a mean failure rate value of about 1.62×10^{-2}, corresponding to a mean failure time of:

$(1 + 1/2 + 1/3) / 1.62\times10^{-2} \approx 113.1$ h

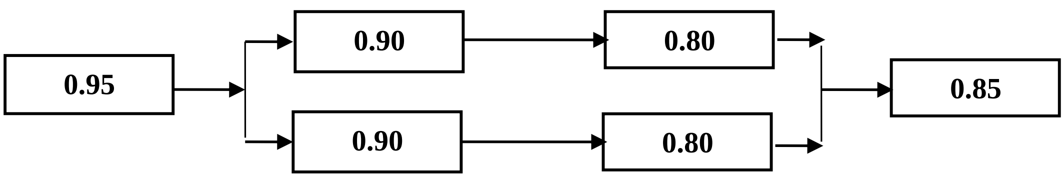
From equation (3.13), the mean time of failure for the entire system is:

$\tau_m = 1 / (1 / 245.1 + 1 / 206.6 + 1 / 113.1)$, from which: $\boldsymbol{\tau_m}$ **= 56.3 month**

3.4 Exercise problems

(1) If 10 items placed in parallel have an overall reliability of 0.95, what is the individual reliability of each item provided all 10 of them have equal failure probabilities.

(2) The following figure shows an arrangement of exchangers to each of which some reliability is assigned. Find the overall reliability of the system.



(3) If in the previous problem, reliability data were collected over a 300 days period, find the mean time between failures of the system assuming the rate of failure is exponentially distributed.