10. CENTRIFUGAL SEPARATION OPERATIONS

10.1 Equilibrium in a centrifugal field

When solid particles are extremely fine then the time required to eliminate such particles from a fluid stream may prove to be very long. In that case, centrifugal separation is needed as it provides accelerations largely exceeding gravitational action. Such accelerations can reach or even exceed $10^3.g$. The same methods can be used to separate oil from water when the difference in their densities is low.

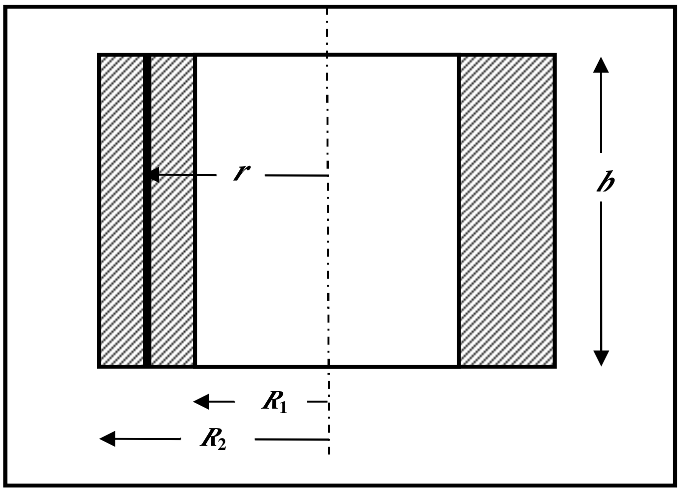


Fig (10.1) Liquid in a centrifugal bowl

Consider a liquid rotating in a bowl at an angular velocity ω. Let the inner and outer diameters of liquid annular zone be R_1 and R_2 respectively. Consider an annular ring at radius r, of thickness dr. The centrifugal force acting on that ring is:

$$dF = \omega^2.r.dm = \omega^2.r.\rho.2\pi r.b.dr$$

The corresponding pressure drop is:

$$dP = \frac{\omega^2.r.\rho.2\pi r.b.dr}{dA} = \frac{\omega^2.r.\rho.2\pi r.b.dr}{2\pi rb} = \omega^2.\rho.r.dr$$

Integrating we get:

$$\Delta P = \omega^2.\rho.\int_{R_1}^{R_2} r.dr$$

$$\Delta P = \frac{1}{2}.\rho.\omega^2.(R_2^2 - R_1^2) \qquad (10.1)$$

Example 10.1

25 liters of oil (Sp. Gr. = 0.876) is being centrifuged in a vertical bowl of 500 mm diameter×400 mm height to remove solid impurities. The bowl is rotated at 2400 rpm. Find the pressure exerted on the walls.

Estimate the required thickness of bowl walls (to the nearest mm) using the following equation:

$$t = \frac{P.D}{2.\sigma} + 3$$

Where,

t is the thickness (mm)

P is the applied pressure (Pa)

D is the bowl diameter (mm)

σ is the allowable stress, taken as 120 MPa

Solution:

Density of oil = 876 $kg.m^{-3}$

The radius of inner surface of liquid is calculated by equating the volume of oil to the volume of liquid annular space:

$$0.025 = \pi \times 0.4 \times (0.25^2 - R_1^2)$$

From which r_1 = 0.206 m

The pressure inside the bowl in the liquid free zone being atmospheric then the pressure drop = the gauge pressure on the walls.

Angular velocity = $\omega = \frac{2\pi \times 2400}{60}$ = 254.6 $rad.s^{-1}$

$$\Delta P = \frac{1}{2}.\rho.\omega^2.(R_2^2 - R_1^2) = \frac{1}{2} \times 876 \times 254.6^2.(0.25^2 - 0.206^2) = \mathbf{569650\ Pa}$$

The wall thickness is then obtained:

$$t = \frac{569650 \times 500}{2 \times 120 \times 10^6} + 3 \approx \mathbf{4.2\ mm}$$

10.2 Centrifugal decanters

As previously stated, when the difference between the densities of two liquids is small, it is customary to use centrifugal separation to achieve their separation in a reasonable time. Separation may be conducted in a liquid – liquid centrifuge, shown schematically in Figure (10.2). It usually consists of a vertical cylindrical bowl rotating at high speed about its central axis. As the bowl rotates, the heavy liquid forms a layer (A) close to the walls while the lighter liquid forms a layer (B) inside the layer of heavy liquid. A cylindrical vertical interface of radius r_i separates the two layers.

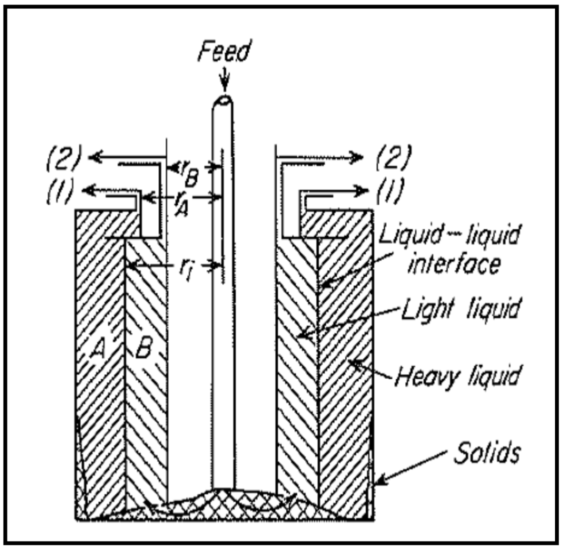


Fig (10.2) Centrifugal liquid – liquid separator

In operation, the feed is admitted continuously near the bottom of the bowl and the light liquid is discharged through ports (2) at a distance r_B from the axis of the bowl. Heavy liquid passes under a ring, inwards toward the axis of rotation and discharges over a weir at (1) at a distance r_A from axis. A hydrostatic balance is established between the two liquid layers so that:

$P_i - P_B = P_i - P_A$

From equation (1):

$$\frac{1}{2}.\rho_B.\omega^2.(r_i^2 - r_B^2) = \frac{1}{2}.\rho_A.\omega^2.(r_i^2 - r_A^2)$$

Hence:

$$\rho_B.(r_i^2 - r_B^2) = \rho_A.(r_i^2 - r_A^2) \quad (10.2)$$

Or

$$r_i = \sqrt{\frac{r_A^2 - (\rho_B / \rho_A).r_B^2}{1 - (\rho_B / \rho_A)}} \quad (10.3)$$

In practice, separators are constructed so as to be able to vary at will the positions of outlet ports so as to have clear cut separation.

The following example shows how this equation is of use.

Example 10.2

A centrifugal separator of internal diameter 250 mm rotates at 8000 rpm. It is used to separate water from chlorobenzene (Specific gravity = 1.1). The heavy liquid port is at 50 mm from the axis of rotation and that of the light liquid is at 40 mm from that axis. Find the position of the interface.

Solution:

r_A= 50 mm and r_B = 40 mm

ρ_B = 1000 kg.m^{-3}

ρ_A = 1100 kg.m^{-3}

Substituting in equation (3):

$$r_i = \sqrt{\frac{r_A^2 - (\rho_B / \rho_A).r_B^2}{1-(\rho_B / \rho_A)}} = \sqrt{\frac{50^2 - (1000/1100).40^2}{1-(1000/1100)}} = =108 \text{ mm}$$

We notice that the position of interface is close to the wall (bowl radius = 125 mm). This means that the amount of chlorobenzene has to be much lower than that of water. In the event of chlorobenzene being the predominant phase, the position of the heavy liquid port has to be modified. If it were decreased to 45 mm, then the interface radius = 80 mm allowing for a mixture containing much more chlorobenzene to be processed.

Figure (10.3) shows a tubular continuous centrifuge.

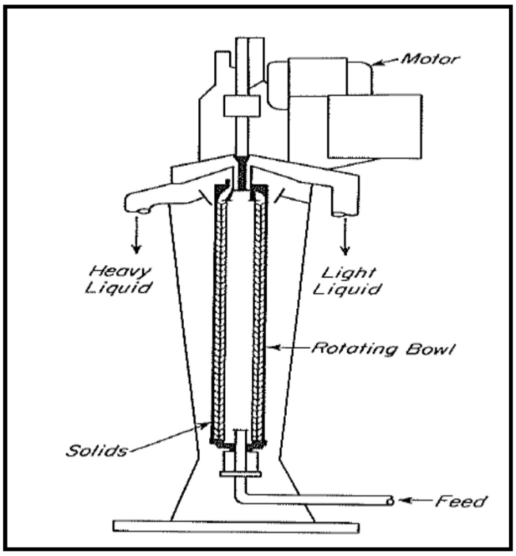


Fig (10.3) Tubular continuous centrifuge

10.3 Centrifugal sedimentation

10.3.1 Theory

In sedimentation centrifugation a liquid containing very fine solids is admitted into a cylindrical bowl that rotates at very high speed forcing the particles to impinge against the walls and drop to the bottom of the centrifuge.

In case of batch centrifugation, the static bowl contains a suspension of solids that holds a certain volume V_0 of the bowl. As the bowl rotates, the inner radius of liquid is r_1 while that of the bowl is R (Figure 10.4)

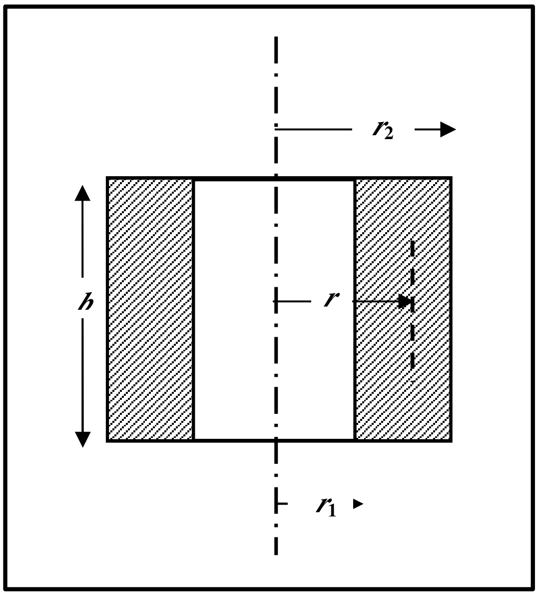


Fig (10.4) Motion of a particle in sedimentation centrifuge

A solid particle moves radially a maximum distance from r_1 to r_2. Particles reach very rapidly their terminal velocity so that their velocity can be predicted from a modified form of Stokes law substituting gravitational acceleration (g) by centrifugal acceleration ($\omega^2.r$):

$$v_T = \frac{\omega^2.r(\rho_p - \rho_f).D_p^2}{18.\mu} = \frac{dr}{dt} \qquad (10.4)$$

Integrating, we get:

$$t = \frac{18.\mu}{\omega^2.(\rho_p - \rho_f).D_p^2}.\int_{r_1}^{r_2}\frac{dr}{r}$$

$$t = \frac{18.\mu}{\omega^2.(\rho_p - \rho_f).D_p^2}.ln\frac{r_2}{r_1} \qquad (10.5)$$

In case of continuous centrifugation, the retention time of solids is equal to the ratio of volume of liquid to the flow rate.

$$t = \frac{\pi.(r_2^2 - r_1^2).b}{Q} \qquad (10.6)$$

Substituting in equation (5):

$$Q = \frac{\pi.(r_2^2 - r_1^2).b}{18\mu.\ln\frac{r_2}{r_1}} \times \omega^2(\rho_{p-}\rho_f)D_p^2 \qquad (10.7)$$

Example 10.3

A batch sedimentation centrifuge has a diameter of 600 mm and a height of 300 mm. It is used to separate 25 liters of an aqueous slurry containing suspended solids of average particle size 2 μm and density = 2600 kg.m^{-3}. The bowl is rotated at 1000 rpm. Find the time required to free the liquid from solid impurities. Compare with the time it would take for these particles to settle by gravity in that same bowl.

Solution:

The volume of suspension = 0.025 m^3 = $\pi \times 0.3 \times (0.32 - r_1^2)$

Hence, r_1 = 0.25 m (250 mm)

Angular speed $\omega = \frac{2\pi \times 1000}{60}$ = 104.7 $rad.s^{-1}$

Substituting in equation (10.5):

$$t = \frac{18 \times 10^{-3}}{104.7^2 \times (2600 - 1000) \times (2 \times 10^{-6})^2} . ln \frac{300}{250} = \mathbf{46.7\ s}$$

If the particles settle by gravity, then the height of liquid (h) has to be first calculated:

0.025 = $\frac{\pi}{4} \times (0.6)^2 \times h$, from which h = 0.088 m

In gravity settling, stokes law will be used so that:

$\frac{0.088}{t} = \frac{9.81 \times (2 \times 10^{-6})^2 \times (2600 - 1000)}{18 \times 10^{-3}}$, from which: t = 25230 s. ≡ **7 h**

Example 10.4

Fine silt is to be continuously separated from crude in a centrifugal sedimentation tank. The flow rate of crude is 5 $m^3.h^{-1}$, its specific gravity = 0.9 and its viscosity = 12 cP. The particle size analysis of silt shows that the particle size ranges from 8 to 100 μm. The density of silt is 2400 $kg.m^{-3}$. A centrifugal bowl of 600 mm diameter and 500 mm is available to this aim. The oil – silt suspension is continuously delivered through a slotted vertical shaft of 150 mm diameter. Specify the required speed of revolution to perform full separation of silt.

Solution:

Equation (10.7) will be applied using the given data as follows:

Q = 5 $m^3 \cdot h^{-1}$ ≡ 0.0014 $m^3.s^{-1}$

b = 0.5 m, r_2 = 0.3 m, r_1 = 0.075 m (slotted shaft radius)

ρ_p = 2400 $kg.m^{-3}$ ρ_f = 900 $kg.m^{-3}$ $\mu = 12 \times 10^{-3}$ Pa.s

$D_p = 8 \times 10^{-6}$ m

Substituting in equation (10.7):

$$Q = \frac{\pi.(r_2^2 - r_1^2).b}{18\mu.\ln\frac{r_2}{r_1}} \times \omega^2 (\rho_p - \rho_f) D_p^2$$

$$0.0014 = \frac{\pi.(0.3^2 - 0.075^2)\times 0.5}{18\times 12\times 10^{-3}\ln\frac{0.3}{0.075}}\times\omega^2\times(8\times 10^{-6})^2.(2400-900)$$

This gives: ω = 181 s^{-1}

Since $\omega = \frac{2\pi.N}{60}$, we therefore get: ***N* = 1725 rpm**

10.3.2 Equipment

Equipment used for continuous separation of solids from a slurry involves the rapid rotation of the suspension so as to have solid particles collide against the walls while the liquid is directed to an outlet connection.

Two main types are used:

The nozzle discharge centrifuge: This device shown in Figure (10.4) consists of a vertical slotted shaft through which the suspension (oil + water + solids) is admitted to the rotating bowl. Owing to the difference in liquid densities the two liquids are separated and flow out from two different outlet connections. On the other hand, the solids after collision with the walls drop to discharge nozzles where they leave the bowl with some liquid.

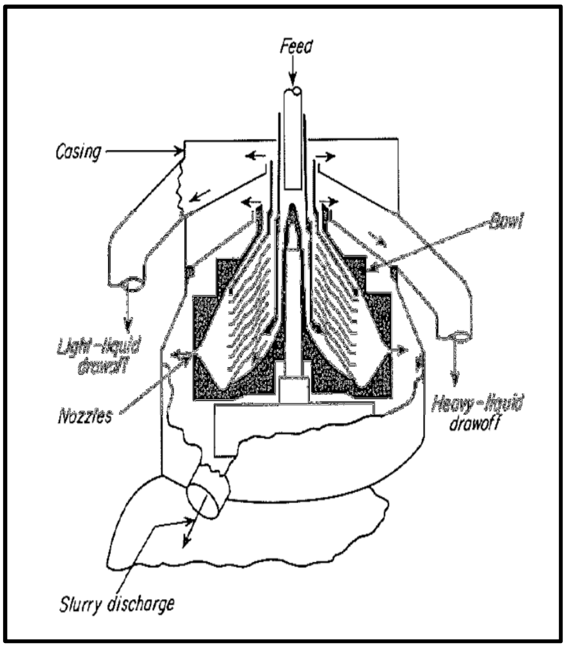


Fig (10.4) Nozzle discharge centrifuge

The helical conveying centrifuge: This is shown in Figure (10.5) and consists of a horizontal bowl rotating at a high speed, with a helical extraction screw placed coaxially. The screw perfectly fits the internal contour of the bowl, only allowing clearance between the bowl and the scroll. The differential speed between screw and scroll provides the conveying motion to collect and remove the solids, which accumulate at the bowl wall.

The product to be treated is introduced axially into the unit by appropriate distributor. It is propelled into the ring space formed by the internal surface of the bowl and the body of the scroll. The separation process basically takes place inside the cylindrical section of the bowl. The relative velocity of the scroll pushes the settled product along into the bowl. The conveyance of the solids into the length of the cone enables the sediment to pass out of the clarified liquid phase. As the feed is continuous a liquid level is established in the unit following a cylindrical surface that constitutes the internal surface of the liquid ring. Once the solids have passed out of the liquid ring the remaining section of the cone all the way up to the ejector provide the final draining: this section is known as the drying zone. The clarified liquid is collected at the other end of the bowl by flowing through the adjustable threshold, which restricts the liquid ring of the unit. A cover that enables the clarified liquid as well as the sediments to be collected protects the rotor.

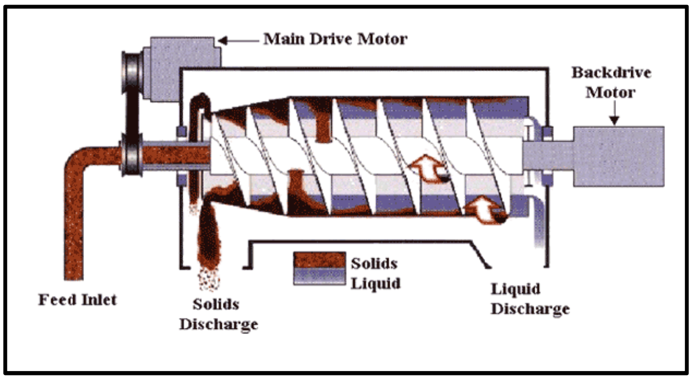


Fig (10.5) Helical conveyor centrifuge

10.4 Centrifugal filtration

10.4.1 Operating principle

In chemical industries it is sometimes necessary to filter off a very fine solid from a slurry. If the particle size of solids is in the micron range then conventional filtration will be a tedious time consuming and uneconomical operation. In that case centrifugal filters can be used. They are also used, though rarely, in ridding crude from suspended solids whenever their level exceeds few percent. In such equipment, centrifugal force substitutes the conventional forces of gravity in forcing the clarified liquid to pass through a layer of deposited cake.

10.4.2 Theory of centrifugal filtration

The principles of constant pressure filtration can be accommodated to apply to centrifugal filtration. Figure (10.6) shows a cylindrical bowl of inner radius r_2 and height b, on the periphery of which a filter medium has been fixed.

Let r_1 be the radius of inner liquid surface and r_i the radius of inner surface of cake.

The following assumptions are made:

- The effect of gravitational forces on liquid is neglected
- The flow of liquid across the cake is laminar
- The cake is always saturated with liquid.

- The cake is nearly incompressible owing to the high pressure exerted by the rotating liquid.

The following analysis, strictly speaking, applies for cake washing. It is therefore assumed that the cake thickness remains constant and that the slurry composition is that of pure wash liquid (water).

Despite of that this analysis can be taken as first approximation for calculating operating parameters in centrifugal filtration.

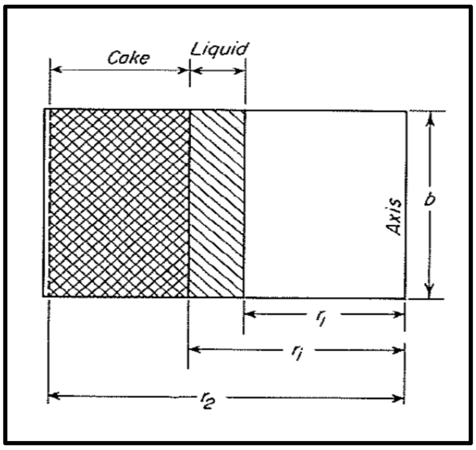


Fig (10.6) centrifugal filter

The basic equation for constant pressure filtration is:

$$\frac{dt}{dV}=\frac{\alpha.\mu.c}{A^2.\Delta P}.V+\frac{R_m.\mu}{A.\Delta P}$$

Where:

V is the volume of filtrate m^3

α is the specific cake resistance (assumed constant) kg/m

c is the solid concentration = m_c/V kg.m^{-3}

ΔP is the pressure drop across cake and filter medium Pa

A is the area of filtration m^2

Denoting by Q the flow rate and substituting $c.V$ by m_c, we get:

$$\frac{1}{Q}=\frac{\alpha.\mu.m_c}{A^2.\Delta P}+\frac{R_m.\mu}{A.\Delta P}$$

This can be re-arranged to the following form:

$$\Delta P=Q.\mu.\left(\frac{\alpha.m_c}{A^2}+\frac{R_m}{A}\right) \qquad (10.8)$$

It has been shown that the pressure drop due to a centrifugal field is:

$$\Delta P=\frac{1}{2}.\rho.\omega^2.(r_1^2-r_2^2) \qquad (10.1)$$

Equating (10.1) and (10.8), we get:

$$Q = \frac{\rho.\omega^2.(r_2^2 - r_1^2)}{2.\mu.(\frac{\alpha.m_c}{A^2} + \frac{R_m}{A})} \qquad (10.9)$$

Where $A = 2\pi r_2.b$

Equation (10.9) assumes a thin layer of cake that does not significantly alter the value of r_2. Sometimes this is not the case, so that the previous equation has to be modified to include the radius of inner cake (r_i) as follows:

$$Q = \frac{\rho.\omega^2.(r_2^2 - r_1^2)}{2.\mu.(\frac{\alpha.m_c}{\overline{A}_L.\overline{A}_m} + \frac{R_m}{A_2})} \qquad (10.10)$$

Where $\overline{A}_L$ is the mean logarithmic area between the inner bowl area ($A_2 = 2\pi r_2.b$) and the inner cake area ($A_i = 2\pi r_i.b$) and $\overline{A}_m$ their arithmetic mean

$$\overline{A}_L = \frac{A_2 - A_i}{ln\frac{A_2}{A_i}} \quad \text{and} \quad \overline{A}_m = \frac{A_2 + A_i}{2}$$

10.4.3 Equipment

The most common continuous filter centrifuge is the suspended basket centrifuge shown in Figure (10.7). It consists of a vertical basket rotating by means of a vertical shaft powered by a motor. The slurry is continuously introduced through a feed pipe and the clarified liquid flows through a side duct. After performing filtration and washing, the basket is made to rotate slowly allowing for a knife to remove the solid cake which is then discharged through a special opening.

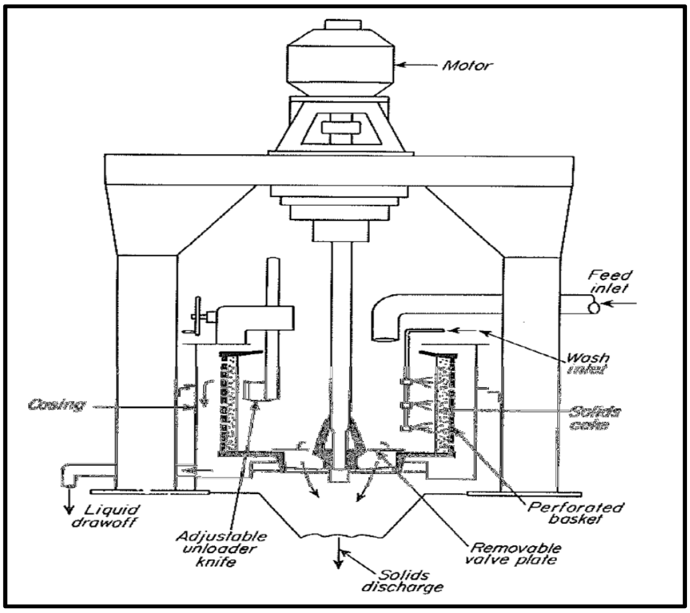


Fig (10.7) Suspended basket centrifuge

This type of centrifugal filters has bowl diameter in the range 750 – 1200 mm, a height of 500 – 750 mm and revolves at a speed ranging from 600 to 1800 rpm. The cake thickness usually lies in the range 1 – 6".

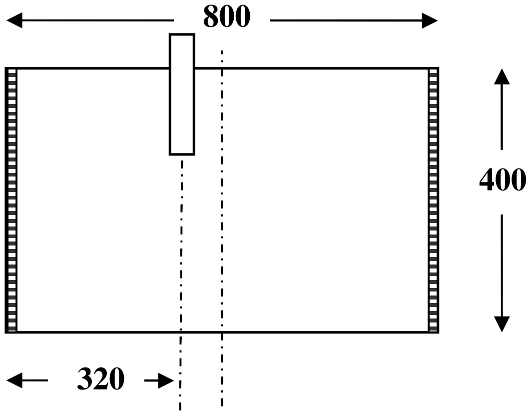
Example 10.5

A suspended basket centrifuge is used to filter fine solid particles from aqueous slurry. The percent solids by weight = 20%. The basket dimensions are shown in the accompanying figure (mm). The density of solids = 2000 kg.m^{-3}. If it is estimated that the cake porosity = 0.3, find:

1- The cake thickness and mass

2- The required speed of revolution for a flow rate of 60 m^3.h^{-1}.

(Compressibility coefficient of cake assumed constant = 1.2×10^9 m.kg^{-1}, filter medium resistance = 10^8 m^{-1})



Solution

For a 100 kg basis of slurry: Since the mass fraction of solids = 0.2, then its volume = $\dfrac{20}{2000}$ = 0.01 m^3

The volume of water = $\dfrac{80}{1000}$ = 0.08 m^3.

Hence the volume fraction of solids = $\dfrac{0.01}{0.09}$ = 0.11 and that of water = 0.810.

The volume fraction of water in cake = 0.3. Hence:

$$0.3 = \frac{V_{water}}{V_{cake}} = \frac{V_{water}}{V_{water} + V_{solids}} \qquad \text{(i)}$$

From which: $\dfrac{V_{water}}{V_{solids}} = \dfrac{0.3}{0.7} = 0.428$

Hence, 100 kg slurry will produce 0.01 m^3 solids capturing 0.00428 m^3 water. So, the volume of cake = 0.01428 m^3.

On the other hand, the volume of water = 0.08 – 0.00428 = 0.0757 m^3

Cake density = $0.3\times1000 + 0.7\times2000$ = 1700 kg/m^3

The slurry enters from the feed pipe at a radius of 0.4 – 0.32 = 0.08 m. Hence if r_i is the inner cake radius, then:

$V_{\text{cake}} = \pi.b.(r_2^2 - r_i^2) = \pi \times 0.4 \times (0.4^2 - r_i^2)$ (ii)

$V_{\text{water}} = \pi.b.(r_i^2 - r_1^2) = \pi \times 0.4 \times (r_i^2 - 0.08^2)$ (iii)

Since it is assumed that the rotating liquid has the composition of the filtrate (water), then we can divide (iii) by (ii) and equate the result to 0.0757/0.01428 we get: r_{i} =0.368 m

Cake thickness = 0.4 – 0.368 = 0.032 m ≡ **32 mm (1.26")**

The volume of cake from equation (ii) is therefore: $\pi \times 0.4 \times (0.4^2 - 0.368^2)$ = 0.0308 m^3.

Since the density of cake = 1700 kg/m^3, hence its mass = **52.36 kg**

The inner area of bowl surface = $2.\pi.b.r_2 = 2.\pi \times 0.4 \times 0.4 \approx 1$ m^2

The speed of revolution is obtained by substituting all given or calculated data in equation (10.9). This equation has been chosen rather than (10.10) owing to the low value of cake thickness.

$$\frac{60}{3600} = \frac{1000 \times \omega^2.(0.4^2 - 0.08^2)}{2 \times 10^{-3}.(\frac{1.2 \times 10^9 \times 52.36}{1^2} + \frac{10^8}{1})}$$

This gives ω = 117 s^{-1}

This corresponds to an rpm = $\frac{60 \times 117}{2\pi}$ = **1117 rpm**

10.4.4. Calculation of power required to rotate centrifugal devices

When a particle moves in rotational motion about a fixed axis, its velocity is tangential to the circular path. Its value is obtained by:

$v = \omega r$

Where: ω is the angular speed (s^{-1}) and r the radius of circular path

Its acceleration consists of two components:

A radial component: $a_r = \omega^2 r$ and a tangential component $\alpha = \frac{dv}{dt}$

If the angular speed ω is kept constant then $\alpha = 0$

Consider a cylinder rotating about its central axis on two bearings. If the bearings are **frictionless** then the cylinder only needs an initial power to initiate rotation from rest up to a specified speed ω. Once that speed is reached, the cylinder needs no more power to sustain its rotation.

The value of this power is calculated from the relation:

$P = T.\omega$ (10.11)

Where T is the torque obtained from:

$T = I.\alpha$ (10.12)

Where: I is the moment of inertia of rotating body about the axis of rotation.

In practice, however, owing to the friction developed at bearings and other factors, a certain power is required to sustain the speed reached. **This power amounts to 50 to 70% of the calculated power.**

Consider the simplest case of a bowl having a thin wall (thickness = x) and internal radius = R_2 containing a liquid with an interface radius = R_1 (Figure 10.8). It is required to rotate the bowl at an angular speed = ω. The bowl has to be accelerated from rest to that speed in a time = t. The constant angular acceleration $\alpha = \omega/t$.

In equation (3.12), I is the moment of inertia of rotating body about the axis of rotation. It is the sum of two terms: the moment of inertia of the rotating bowl I_1 and that of the rotating liquid I_2: $I = I_1 + I_2$

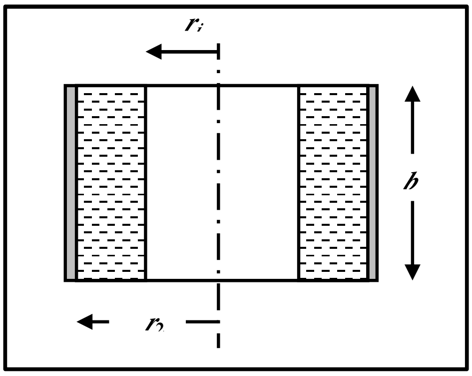


Fig (10.8) Rotating bowl

For a thin walled cylinder, the value of I is simply calculated from:

$$I_1 = MR_2^2 = 2\pi\rho x r_2^3 b \qquad (10.13)$$

Here, ρ is the density of wall material.

The moment of inertia of the rotating liquid is calculated from:

$$I_2 = \frac{M'}{2}(r_i^2 + r_2^2) \qquad (10.14)$$

Where M' is the mass of liquid calculated from:

$$M' = \pi\rho_f .b.(r_2^2 - r_i^2) \qquad (10.15)$$

The power is calculated by combining equations (10.11) through (10.15). The power transmission efficiency is usually elevated since the motor is directly connected to the basket. ($\eta = 0.85 - 0.9$)

Example 10.6

Calculate the required motor power to operate the bowl in Example (10.1) so as to reach the required speed in 4 seconds. ($\rho_{steel} = 7800$ kg.m^{-3})

Solution

Referring to example (10. 1): $R_1 = 0.206$ m $R_2 = 0.25$ m $b = 0.4$ m

$\omega = 188.4$ s^{-1} $\rho_f = 876$ kg/m^3 $x = 0.007$ m

From equation (10.13): $I_1 = 2\pi \times 7800 \times 0.007 \times 0.25^3 \times 0.4 = 2.14$ kg.m^2

From equation (10.15): $M' = \pi\times876\times0.4\times(0.25^2 - 0.206^2) = 22.086$ kg
From equation (10.14): $I_2 = 0.5\times22.086\times(0.25^2 + 0.206^2) = 1.16$ kg.m^2
Hence $I = 2.14 + 1.16 = 3.3$ kg.m^2
The angular acceleration = $\alpha = 188.4 / 4 = 47.1$ s^{-2}.
From equation (10.12) the torque = 3.3×47.1 = 155.43 N.m
Finally, from equation (10.11):
$P = 155.43\times188.4 = 29283$ W
Considering an efficiency of 0.85, we get: $P = 48 \rightarrow$ **50 hp**

The power required to sustain centrifugation at the specified speed will range from 25 to 35 hp.

10.5 Gas – solid separation: cyclones

10.5.1 Cyclone operation

In many industries it is required to separate fine solid particles from a gas stream. If the particle size of solids is relatively high (above 100 μm), settling chambers can be used. A settling chamber consists of a large rectangular duct where unclean gas is introduced. The dimensions of this chamber are such that all solid particles will have settled by gravity by the time the gas outlets the chamber.

If, however, the particle size is smaller (10 – 100 μm), centrifugal separation is more efficient. This is usually performed using a simple design, known as **cyclone collector**. For smaller sizes other devices such as bag filters or electrostatic precipitators have to be used.

Figure (10.9) shows the operation of a cyclone. Unclean gas is led (usually under negative pressure) to a rectangular inlet connection at the top cyclone walls. The gas is admitted tangentially so that a vortex is formed that moves downward. During its motion, solid particles are projected by centrifugal action toward the walls where after impacting upon them, they drop to ac conical bottom. The gas reaching the bottom of the cyclone then forms an upward moving vortex that ends as the cleaned gas leaves the cyclone from the top opening. The downward moving vortex is the outer vortex while the upward one is the inner vortex.

The pressure drop across cyclones involves five distinct components and is usually difficult to predict theoretically. However, common practice has shown that its value ranges from 4" water in low efficiency cyclones to 10" in cyclones of high efficiency.

The power of the circulating fan can be calculated once the pressure drop is known from the equation:

$$P = \frac{Q.\Delta p}{735.\eta} \text{ hp} \qquad (10.16)$$

Where, Q is the gas flow rate, m^3.s^{-1}
Δp is the pressure drop, Pa
η is the fan efficiency, (0.75 to 0.85)

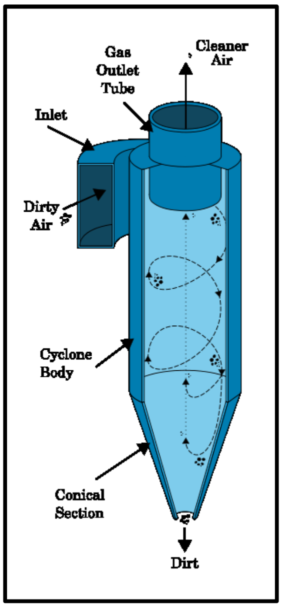


Fig (10.9) A cyclone

10.5.2 Cyclone calculations

The design of cyclones usually relies on preserving certain dimensional ratios known to maximize its efficiency. Such ratios are shown in Figure (10.10).

The basis of cyclones design was set by Lapple. He was able to obtain the following approximate relation for the number of spirals in the outer vortex:

$$N = \frac{1}{H_i} \cdot \left[H_1 + \frac{H_2}{2} \right] \tag{10.17}$$

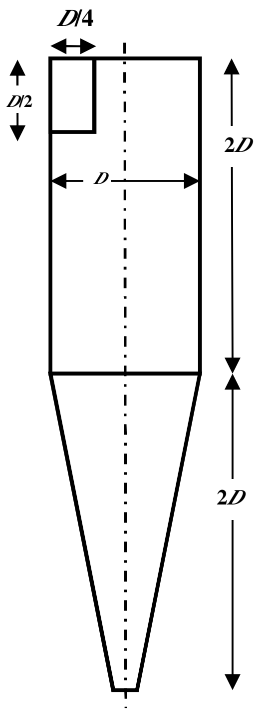


Fig (10.10) Relative dimensions of high efficiency cyclone

Where, H_i is the height of entrance duct, H_1 the height of cylindrical section and H_2 the height of conical section.

For a high efficiency cyclone having the dimensions shown in Figure (10.10):
$H_i = 0.5D$, $H_1 = 2D$ and $H_2 = 2D$, so that $\boldsymbol{N = 6}$.

The gas residence time in outer vortex t_R = the length of path divided by the gas velocity. Hence:

$$t_R = \frac{\pi.D.N}{v_i} \qquad (10.18)$$

Where v_i is the entrance velocity of gas (10 – 30 m.s^{-1})

We note that the maximum distance that solid particles entrained with the gas can cover will be the width of the inlet duct (W_i), so that the terminal velocity v_t of solids will be:

$$v_t = \frac{W_i}{t_R} = \frac{W_i}{\pi.D.N} \times v_i \qquad (10.19)$$

Now, Stokes law can be applied to predict the terminal velocity of solids using the centrifugal acceleration (v_i^2 / r_i) instead of (g). Here (r_i) is taken as $D/2$. This way, Stokes law reads:

$$v_t = \frac{v_i^2.D_p^2.(\rho_p - \rho_f)}{9.\mu.D} \qquad (10.20)$$

Eliminating v_t from equations (10.19) and (10.20), we get:

$$D_p^2 = \frac{9.\mu.W_i}{\pi.N.(\rho_p - \rho_f).v_i} \qquad (10.21)$$

Since:

$$v_i = \frac{Q}{W_i.H_i} \qquad (10.22)$$

Where, H_i is the breadth of cyclone inlet duct.

Then from equations (10.15) and (10.16), we get:

$$D_{p\,min}^2 = \frac{9.\mu.W_i^2.H_i}{\pi.N.(\rho_p - \rho_f).Q} \qquad (10.23)$$

This equation allows calculating the minimum particle size to be eliminated in a cyclone of known dimensions. This size is however only theoretically correct since it has been assumed in deriving the equation that particles travel at most a distance from the inside edge of the inlet duct to the cyclone walls. Some particles with size > D_{pmin} will travel greater distances owing to flow enlargement at the entrance duct; so even though their size exceeds the minimum they will not be totally eliminated. Also, as can be seen in Figure

(10.8), the outer vortex may extend below the cylindrical region allowing for particles with $D_p < D_{pmin}$ to be eliminated.
That is why; equation (10.17) is primarily used as a first step in designing a cyclone whenever it is required to eliminate a specific minimum size.

10.5.3 Separation efficiency of a cyclone of known dimensions

The separation efficiency of a cyclone mainly depends on the particle size to be separated and to lesser extent on the rate of gas flow. Figure (10.11) shows this situation.
This figure shows that for a flow rate of 5000 $ft^3.min^{-1}$, all particles of particle size greater than 15 μm are eliminated. This figure increases to about 40 μm for a flow rate of 25000 $ft^3.min^{-1}$, while the maximum size eliminated at a flow rate of 1000 $ft^3.min^{-1}$ decreases to about 10 μm.

Actually, the relations illustrated in that figure can be generalized for any cyclone dimensions and air flow rate. In this respect, a particle diameter, known as the **cut diameter D_{pc}**, is defined as that particle size corresponding to 50% separation efficiency.
This diameter is related to the flow rate and cyclone dimensions by the following formula elaborated by Lapple:

$$D_{pc}^2 = \frac{9.\mu.W_i^2.H_i}{2\pi.N.(\rho_p - \rho_f).Q} \qquad (10.24)$$

Note that this equation is very similar to (10.23) except for the (2) present in denominator. Thus:

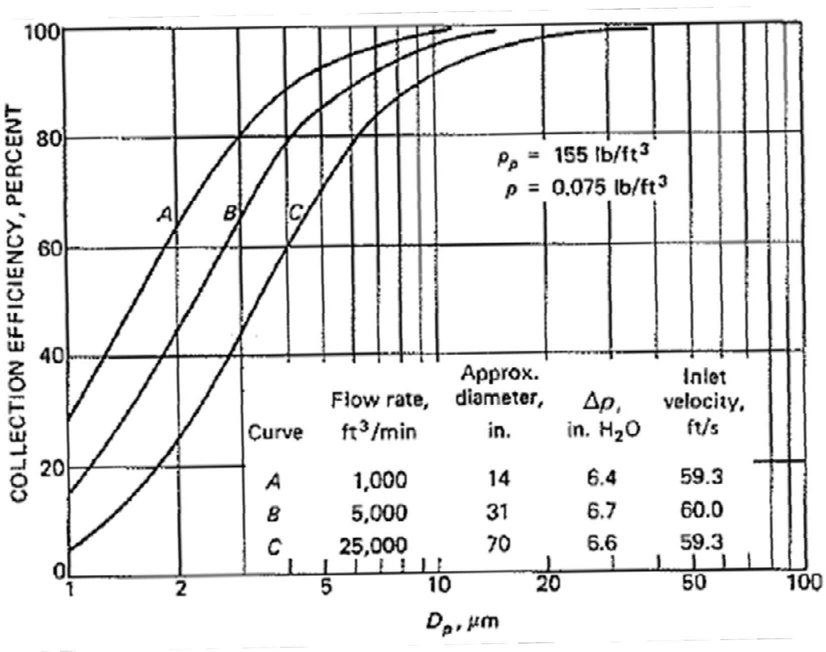


Fig (10.11) Cyclone efficiency

$$D_{pc} = \frac{D_{p\,min}}{\sqrt{2}} \quad (10.25)$$

If the cyclone dimensions and gas flow rate are known, then it is possible to calculate the value of cut diameter. The efficiency of collection for any size D_{pi} is then obtained by the following formula worked out by Theodore and DePaola:

$$\eta_i = \frac{1}{1+\left(\frac{D_{pc}}{D_{pi}}\right)^2} \quad (10.26)$$

The overall cyclone efficiency can then be obtained as weighted average between the different sizes as follows:

$$\eta_{ov} = \sum_{i=1}^{n} \eta_i . x_i \quad (10.27)$$

Where: x_i is the mass fraction of particles of diameter D_{pi}.

Example 10.7

Air laden dust enters the cyclone of shown dimensions (in mm) at the rate of 1800 $m^3.h^{-1}$ at 30°C and 1.1 atm.

(1) Assuming a pressure drop of 10" water, calculate the power of the suction fan required to move the gas across the cyclone.

(2) Calculate the retention time of air in the downward vortex.

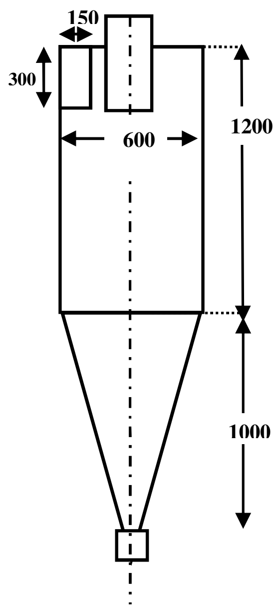
(3) If the particle density is 800 $kg.m^{-3}$, calculate the minimum particle size to be separated.

(4) If the screen analysis of inlet dust is as shown in the following table, calculate the separation efficiency of this cyclone.

(Air viscosity = 2×10^{-5} Pa.s, density = 1.1 $kg.m^{-3}$)

D_p **µm**	50	40	20	10	5	3
x_i	0.00	0.15	0.45	0.20	0.15	0.05

Solution:



(1) Calculation of fan power:

$$P = \frac{Q.\Delta p}{735.\eta} \text{ hp}$$

Δp = $10\times0.0254\times9.81\times1000$= 2492 Pa

Assuming fan efficiency = 0.8:

$$P = \frac{1800\times2492}{3600\times735\times0.8} = 2.11$$

Taken as **2.5 hp**

(2) Calculation of retention time:

Substituting with the dimensions of the figure in equation (10.17), we get:

$N = 5.7$

Area of inlet connection = 0.15×0.3 m^2

Inlet velocity = $\dfrac{1800}{3600\times 0.15\times 0.3}$ = 11.1 m.s^{-1}

From equation (10.18), we get: $\boldsymbol{t_R}$ **= 0.97 s**

(3) Minimum size to be separated

The air density is given as 1.1 kg.m^{-3} and its viscosity as 2×10^{-5} Pa.s

$W_i = 0.250.6 = 0.15$ m and $H_i = 0.5\times 0.6 = 0.3$ m

From equation (10.23):

$$D_p^2 = \frac{9\times 2\times 10^{-5}\times 0.15^2\times 0.3}{\pi\times 5.7\times (800-1.1)\times 0.5} = 1.7 \qquad \times 10^{-10}$$

Hence $D_{pmin} = 1.3\times 10^{-5}$ m ≡ **13 µm**

(4) Calculation of overall separation efficiency:

To get the separation efficiency of this cyclone, we first obtain the cut diameter from equation (10.25). We get: $\mathbf{D_{pc}}$ **= 10.2 µm**

The efficiency of separation for each size fraction is then calculated from equation (10.26) and the overall efficiency obtained as shown in the following table by applying equation (10.27).

D_p **µm**	50	40	20	10	5	3	
x_i	0	0.15	0.45	0.2	0.15	0.05	
η_i	0.967	0.9497	0.825	0.542	0.228	0.096	**total**
$\eta_i.x_i$	0	0.142	0.371	0.108	0.0342	0.0048	**0.66**

The overall separation efficiency is therefore **66%**

10.5.4 Design of a cyclone

The design of an efficient cyclone needs specifying the following information: Gas flow rate – Particle size analysis – Overall separation efficiency to be achieved.

To this aim, one starts by assuming a value for the minimum size to be eliminated and get the value of D_{pc} from equation (10.25).The value of the cut diameter is then used to calculate the collection efficiency out of the particle size distribution of the solids. If the value is smaller than the target value then a smaller minimum size is chosen and the procedure repeated. When the targeted

collection efficiency is reached for a certain minimum size, then $W^2.H_i$ can be calculated. Once this value is calculated, one can evaluate the cyclone diameter from Figure (10.10); the number spirals can be taken as 6 for high efficiency cyclones. Finally, the entrance velocity of gas should be calculated to fall between the limits 10 – 30 m/s. Too low a velocity will not impart enough centrifugal acceleration for particles while too high a velocity means higher pressure drop and hence motor power.
Such calculations are displayed in the following example.

Example 10.8
It is required to design a cyclone of the highly efficient type to rid dusty air at 20°C and 1.06 atm from dust having the particle size distribution shown in table (specific gravity = 2.3). The expected air flow rate is 8000 $m^3.h^{-1}$.
The targeted separation efficiency = 90%

D_p µm	40	30	20	10	5	2
Fraction	0.15	0.2	0.25	0.2	0.15	0.05

Solution:
From the given table, a first guess about the minimum size to be eliminated can be made by obtaining the particle size corresponding to a cumulative fraction = 0.10. This is about 6 µm.

The cut diameter is then calculated from equation (10.26) as D_{pc} = 4.25 µm

The following table is then established to calculate the collection efficiency

D_p µm	40	30	20	10	5	2	
x_i	0.15	0.2	0.25	0.2	0.15	0.05	
η_i	0.9888	0.9803	0.9568	0.847	0.5806	0.1813	**Total**
$\eta_i.x_i$	0.1483	0.1961	0.2392	0.1694	0.0871	0.0091	**0.8491**

The calculated overall efficiency is slightly lower than the targeted value of 0.10. This necessitates assuming a new minimum diameter, say 5 µm. The corresponding cut diameter is then calculated as 3.54 µm. The previous calculations are then repeated to obtain the following table:

D_p µm	40	30	20	10	5	2	
x_i	0.15	0.2	0.25	0.2	0.15	0.05	
η_i	0.9922	0.9863	0.9696	0.8886	0.6661	0.242	**Total**
$\eta_i.x_i$	0.1488	0.1973	0.2424	0.1777	0.0999	0.0121	**0.8782**

The obtained efficiency ≈ 0.88 is acceptable.

Now, $Q = \frac{8000}{3600} = 2.22\ m^3.s^{-1}$

$$\rho_f = \frac{1.06 \times 29}{0.082 \times 293} = 1.28 \text{ kg.m}^{-3}$$ and $\mu = 1.8 \times 10^{-5}$ Pa.s

Substituting in equation (10.23), we get:

$$(5 \times 10^{-6})^2 = \frac{9 \times 1.8 \times 10^{-5} \times W^2.H_i}{\pi \times 6 \times (2300 - 1.28) \times 2.22}.$$

From which we get: $W^2.H_i = 0.015$ m^3

From the relative dimensions of high efficiency cyclone, we have: $W_i = 0.25D$ and $H_i = 0.5\ D$, so that: $0.03125\ D^3 = 0.015$

The design cyclone diameter is then: $\boldsymbol{D}$ **= 0.78 m**. $\Rightarrow$ **0.8 m (800 mm)**

We then check on the inlet air velocity: $v_i = \frac{Q}{W.H_i} = \frac{2.22}{(0.25 \times 0.8).(0.5 \times 0.8)}$

Hence $\boldsymbol{v_i}$ **= 27.75 m.s^{-1} (checks)**