-2-
Probabilities

2.1 The statistical definition of probability

Consider 10 samples taken at random from a production line of insulating panels. Following standards, the density of the product should not exceed 1000 kg/m^3. Let the measured densities be represented by the set: *S* = {995, 985, 995, 1025, 995, 985, 1010, 975, 990, 1030}.
The probability of getting a defective sample will simply be 3 out of 10, that is 0.3.

In general, the set of all samples under consideration is called the **sample space *S***. Any number of samples belonging to *S* is called a sub-set of *S*. This sub-set is called an **event**. For example, in the above case, the event: "The chosen item is defective" corresponds to a sub-set:

A = {1025, 1010, 1030}

Let the number of elements of the sample space = *N*(*S*), and that of the event *A* = *N*(*A*). Then the probability of occurrence of *A* is:

$$P(A) = \frac{N(A)}{N(S)} \qquad (2.1)$$

2.2 Calculation of *N*(*A*)

2.2.1 Elements of combinatory algebra

To calculate *N*(*S*) or *N*(*A*), we recall some basic definitions:

(a) The **factorial *n*!** of a positive integer *n*

$$n! = 1 \times 2 \times 3 \times 4 \ldots n \qquad (2.2)$$

This represents the number of ways one can choose *n* items out of *n*, the choice being done **one by one without replacement.**

(b) A **combination C_r^n** represents the number of ways one can choose r items out of n, the choice being done **once at a time.**

The value of C_r^n can be calculated from the following formula:

$$C_r^n = \frac{n!}{r!\, n-r!} \qquad (2.3)$$

For example, out of 8 different brands of paint, by how many methods one can choose 3 brands?

$$C_3^8 = \frac{8!}{3!\, 8-3!} = \frac{40320}{6\times 720} = 56$$

2.2.2 Types of choice

Consider an event *A* consisting of choosing *r* items out of *n*. there are several methods by which this choice can be accomplished:

(a) Items are chosen **one by one with replacement**:

$$N(A) = n^r \qquad (2.4)$$

(b) Items are chosen **one by one without replacement**:

$$N(A) = \frac{n!}{n-r!} \tag{2.5}$$

(c) Items are chosen **once, at a time**

$$N(A) = C_r^n \tag{2.6}$$

For example, we need to choose 3 samples out of 10.

If drawing of samples was done one by one with replacement, then:

$N(A) = 10^3 = 1000$

If drawing of samples was done one by one without replacement, then:

$N(A) = \frac{10!}{10-3!} = 720$

If drawing of samples was done once, at a time, then

;

$N(A) = \frac{10!}{3!10-3!} = 120$

2.3 Elements of event algebra

Consider the sample space $S = \{a_1, a_2, a_3, \ldots, a_n\}$. This consists of n single events. Any sub-set of S, like A usually consists of either one **elementary event** such as $\{a_3\}$ or a **compound event** such as $\{a_2, a_3\}$.

For example, let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and let a random number be chosen from that set.

Then the event A = "An odd number is obtained", $A = \{1, 3, 5, 7, 9\}$

The event B: "the number obtained is ≥ 6", $B = \{6, 7, 8, 9, 10\}$

The event: "A multiple of 6 is obtained" is $C = \{6\}$

These events are represented in Figure (2.1) by the **Venn diagram**.

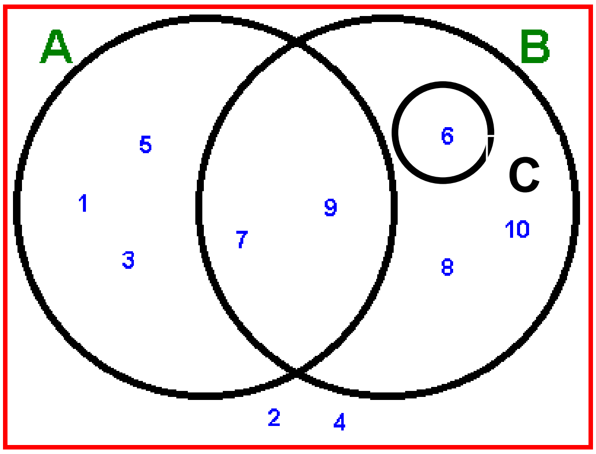


Fig (2.1) Venn diagram

- A **complementary event** A' is the set of single events (elements) that do not belong to A. In the present case: $A' = \{2, 4, 6, 8, 10\}$
- An **impossible event** is an event containing zero elements, like throwing a die and obtaining the number 7. It is written Ø.

- The **intersection** of two events is the set of elements belonging to both A and B. It is written $A \cap B$. In the present case: $A \cap B = \{7, 9\}$, and $B \cap C = \{6\}$
- The **union** of two events is the set of elements belonging to either A or B or both. It is written $A \cup B$. In the present case: $A \cup B = \{1, 3, 5, 6, 7, 8, 9, 10\}$ and $A \cup C = \{1, 3, 5, 6, 7, 9\}$
- The **difference** between two events A and B is the set of elements present in$A$ but not in B: $A - B = A \cap \bar{B} = \{1 , 5 , 3\}$
- Two events A and C are said to be **mutually exclusive** if $A \cap C = \emptyset$
- Finally if C is **sub-set** of B, $C \subseteq B$, then $B \cap C = C$ and $B \cup C = B$

Example 2.1

A pump house contains 4 rotary and 6 centrifugal pumps. If 2 pumps are chosen at random, what is the probability that both will be of the latter type?

Solution

$N(S) = C_2^{10} = 45$

Let A be the event: The two chosen pumps are of the centrifugal type.

Hence, $N(A) = C_2^{6} = 15$

From equation (2.1): $P(A) = \frac{15}{45} = \frac{1}{3}$

2.4 The axiomatic definition of probability

Consider the sample space S consisting of a finite number of elementary events $\{A_1, A_2, A_3, \ldots, A_i, \ldots, A_N\}$.

If A is a sub-set of S, then the probability function $P(A)$ can be defined according to the following axioms:

(1) $P(A) > 0 \quad (A \neq \emptyset)$

(2) $P(S) = 1$

(3) If A and B are two mutually exclusive events, then:

$$P(A \cup B) = P(A) + P(B)$$

Applying these axioms on the elementary mutually exclusive events of S, the following two conditions are obtained:

(1) $P(A_i) > 0$

(2) $\sum P(A_i) = 1$

For example, the following table shows the probability that out of 100items chosen from a production line, there will be (n) defective items.

n	0	1	2	3	4	5	6
Probability	0.2	0.25	0.3	0.15	0.06	0.03	0.01

It is easy to notice that the two previous conditions are fulfilled.

2.5 Basic laws of probability

The following laws represent the basic laws of probability. They are given here without proof.

(1) $P(\emptyset) = 0$ (2.7)

(2) $P(A') = 1 - P(A)$ (2.8)

(3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (2.9)

Note that if A and B are mutually exclusive, then $A \cap B = \emptyset$ and this law reduces to the third axiom.

Example 2.2

In a statistic involving the origin of the main raw material used by 30 factories, it was found that 18 of them draw their raw materials from source A, 16 from source B, and 7 from both sources. If a factory is chosen at random, what is the probability that:

(1) The factory gets its raw materials from either A or B

(2) The factory gets its raw materials from a totally different source.

Solution:

$P(A) = \frac{18}{30}$, $P(B) = \frac{16}{30}$ and $P(A \cap B) = \frac{7}{30}$

(1) This is the event $A \cup B$ the probability of which can be obtained from equation (2.9):

$$P(A \cup B) = \frac{18}{30} + \frac{16}{30} - \frac{7}{30} = \mathbf{\frac{9}{10}}$$

(2) This is the complementary event $(A \cup B)'$

Its probability is calculated from equation (2.8):

$$P(A \cup B)' = 1 - P(A \cup B) = \mathbf{\frac{1}{10}}$$

Example 2.3

In a research facility there are 8 chemists and 4 engineers. Two people were chosen at random. What is the probability that both are chemists? That both are engineers? One is a chemist and the other an engineer.

Solution:

$N(S) = C_2^{12} = 66$

Let A be the event: Both persons are chemists, then $N(A) = C_2^{8} = 28$

Hence $P(A) = \mathbf{\frac{28}{66}}$ = **0.424**

Let B be the event: Both persons are engineers, then $N(B) = C_2^{4} = 6$.

Hence $P(B) = \mathbf{\frac{6}{66}}$ = **0.091**

The remaining situation (One chemist and one engineer) is the complementary event of both A and B:

Its probability = 1 – (0.424 + 0.091) = **0.485**

2.6 Conditional probability

2.6.1 Basic concept

Consider the following experiment: Out of a bag containing 7 white balls and 3 black balls are drawn 2 balls, one after the other, without replacement. $N(S) = 10 \times 9 = 90$.

Let event A be: the two balls are black. There will be 3 chances in the first draw and 2 in the second **if the first was black**. The probability of getting two successive black draws will be: $\frac{\mathbf{3}}{\mathbf{10}} \times \frac{\mathbf{2}}{\mathbf{9}} = \frac{\mathbf{1}}{\mathbf{15}}$

The first term in the above product is the probability of getting a black ball in the first draw $P(A_1)$ while the second term represents the probability of drawing a second black ball if the first one was black. This is written: $P(A_2/A_1)$ and is termed **conditional probability**. The general rule governing such situation is:

$$P(A_1 \cap A_2) = P(A_1).P(A_2/A_1) \qquad (2.10)$$

2.6.2 The total probability – Bayes theorem

Consider the following situation: In a factory there are three production lines. The first, A, produces 30% of the total output stock, B produces 20% and C produces 50%. It is known from past practice that 5% of products from line A are defective, while the percentage is 4% from B and 6% from C. We wish to calculate the probability that a sample selected at random will be defective $P(D)$. This probability will be:

$$P[(A \cap D)U(B \cap D)U(C \cap D)] = P(A \cap D) + P(B \cap D) + P(C \cap D)$$

Following equation (2.10), this can be written as:

$$P(D) = P(A).P(D/A) + P(B).P(D/B) + P(C).P(D/C)$$

$$P(D) = 0.3 \times 0.05 + 0.2 \times 0.04 + 0.5 \times 0.06 = 0.053$$

Now, one may ask: If an item was found to be defective, what is the probability that it would have come from line C? It will be required to calculate $P(C/D)$.

From equation (2.10), we may write: $P(C \cap D) = P(D).P(C/D)$

Hence, $0.5 \times 0.06 = 0.053 \times P(C/D)$, from which $P(C/D) = 0.566$

The steps undertaken to solve the latter problem can be written in a more general form as follows:

$$P(B) = \sum_{i=1}^{n} P(A_i).P(B/A_i) \qquad (2.11)$$

This is **the law of total probability.**

And,

$$P(A_i/B) = \frac{P(A_i).P(B/A_i)}{\sum_{i=1}^{n} P(A_i).P(B/A_i)} \qquad (2.12)$$

This formula is known as **Bayes theorem**.

Example (2.4)

In a plant, there are four major departments: Technical (A), sales (B), financial (C) and R&D (D). The percentage personnel in these departments represent 40%, 15%, 30% and 15% of the plant working power respectively. The percent of women in these departments is: 25%, 20%, 45% and 30% respectively.

An employee was chosen at random. What is the probability that it was a woman? And if it turned out to be a woman, what is the probability that it would have come from department C?

Solution:

The following probabilities are readily calculated:

$P(A) = 0.4, P(B) = 0.15, P(C) = 0.3$ and $P(D) = 0.15$.

$P(W/A) = 0.25, P(W/B) = 0.2, P(W/C) = 0.45, P(W/D) = 0.3$

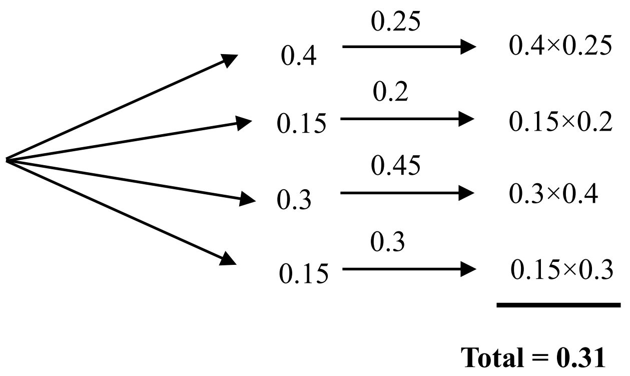
The law of total probability is applied:

$P(W) = \sum P(A_i).P(W/A_i)$

$= 0.4 \times 0.25 + 0.15 \times 0.2 + 0.3 \times 0.45 + 0.15 \times 0.3 = \mathbf{0.31}$

Bayes theorem can then be written as:

$P(C/W) = P(C \cap W)/P(W) = \frac{0.3\times 0.45}{0.31} =$ **0.435**



2.7 Independent events

Consider once more the following experiment: Out of a bag containing 7 white balls and 3 black balls are drawn 2 balls, one after the other, but this time with replacement. $N(S)$ = 10×10 = 100

Let event A be: the two balls are black. There will be 3 chances in the first draw and 3 also in the second. The probability of getting two successive black draws will be: $\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$.

In this case, the probability of the second draw does not depend on the outcome of the first. Such events are said to be independent.

In this case, $P(A_2/A_1)$=$P(A_2)$, and equation (2.10) can be written as:

$$P(A_1 \cap A_2) = P(A_1).P(A_2) \qquad (2.13)$$

This is the law of **independent events.**

Example 2.5
The probability that an experiment will produce a positive result is 0.8. How many times do we have to repeat that experiment so that the probability of obtaining at least one positive result exceeds 0.99?

Solution:
The sequence of experiments is assumed to represent independent events. Let the probability of obtaining a positive result = $P(A) = 0.8$. Hence the probability of failure = $P(A') = 0.2$.

If the experiment is repeated n times, then the probability of all n experiments to fail = 0.2^n.

Therefore the probability of having at least one positive result will be $1 - 0.2^n$.

This reduces to solving the inequality $1 - 0.2^n > 0.99 \rightarrow 0.2^n < 0.01$.

Applying Goal – Seek technique, we get $n > 2.85 \rightarrow \boldsymbol{n = 3}$

Example 2.6
In a lot containing 100 samples of a certain commercial product, 5 are known to be defective. If 3 samples are drawn, one after one with replacement, calculate the following probabilities:
(1) The three samples are defective
(2) At least one sample is not defective
(3) The three samples are not defective

Solution:
Since drawing has been done one after the other with replacement, then we are in presence of independent events.

(1) $P(D \cap D \cap D)$ = $(0.05)^3$ = $\mathbf{0.000125}$

(2) This is the complementary event of $D \cap D \cap D$: Its probability is therefore:

$$1 - P(D \cap D \cap D) = 1 - 0.000125 = \mathbf{0.999875}$$

(3) $P(D' \cap D' \cap D')$ = $(1 – 0.05)^3$ = $(0.95)^3$ = $\mathbf{0.875}$

2.8 Markov Chains

2.8.1 Regular Markov chains

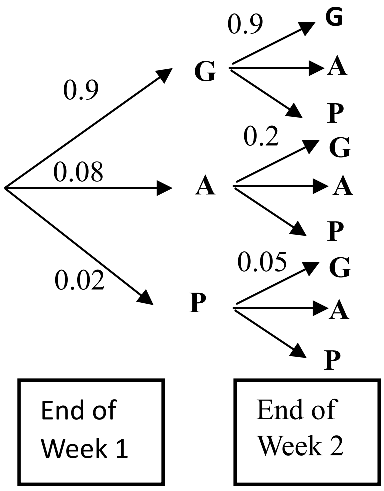
Consider the following situation:
A production line includes a set of control valves regulating its operation. Their performance is rated as Good (G), average (A) or poor (P). The probability that any of such performances as established during the first week of operation will affect the second week's performance is given in the following table by following up quality control charts for the first year of operation.

	G	**A**	**P**
G	0.9	0.08	0.02
A	0.2	0.7	0.1
P	0.05	0.25	0.7

The matrix M so defined is called the **transition matrix**:

$$\begin{pmatrix} 0.9 & 0.08 & 0.02 \\ 0.2 & 0.7 & 0.1 \\ 0.05 & 0.25 & 0.7 \end{pmatrix}$$

Assume that we began in the first week with a control system rated G then, the first column in the following tree diagram shows the probabilities of the corresponding performance at the end of this first week.



The probability of a good performance at the end of the second week is the sum of the following products:

0.9×0.9 + 0.08×0.2 + 0.02×0.05 = 0.827

Had we ended with an average performance, this product would have been:

0.9×0.08 + 0.08×0.7 + 0.02×0.25 = 0.133

And, finally, in case of ending with poor performance:

0.9×0.02 + 0.08×0.1 + 0.02×0.7 = 0.04

Note that the sum of these probabilities is 1.

These represent the first row of the matrix $M^2 = M*M$
Upon matrix multiplication we get the following expression for M^2:

$$\begin{pmatrix} 0.827 & 0.133 & 0.04 \\ 0.325 & 0.531 & 0.144 \\ 0.13 & 0.354 & 0.516 \end{pmatrix}$$

To follow up the performance after any number of weeks we keep on finding the expressions of the matrices M^n for increasing values of n:

For example, M^4 and M^8 will respectively show as:

$$\begin{pmatrix} 0.7324 & 0.1948 & 0.4601 \\ 0.4601 & 0.3762 & 0.1638 \\ 0.2896 & 0.3879 & 0.3224 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0.6471 & 0.2442 & 0.1088 \\ 0.5574 & 0.2946 & 0.1479 \\ 0.484 & 0.3274 & 0.1886 \end{pmatrix}$$

The following matrices are M^{16}, M^{32} and M^{64} respectively:

$$\begin{pmatrix} 0.6074 & 0.2656 & 0.127 \\ 0.5965 & 0.2714 & 0.1321 \\ 0.587 & 0.2764 & 0.1366 \end{pmatrix}$$

$$\begin{pmatrix} 0.6025 & 0.2682 & 0.1293 \\ 0.6012 & 0.2689 & 0.1299 \\ 0.6 & 0.2695 & 0.1305 \end{pmatrix}$$

$$\begin{pmatrix} 0.6019 & 0.2685 & 0.1296 \\ 0.6018 & 0.2685 & 0.1296 \\ 0.6018 & 0.2685 & 0.1296 \end{pmatrix}$$

We note that as $n \rightarrow \infty$, the entries of the columns stabilize to constant values, meaning that the performance after enough time does not depend on the initial state.

In the present example, let the initial performance of purchased valves be as follows: 95% *G*, 4% *A*, 1% *P*. These data are represented by the row matrix $A_0 = (0.95, 0.04, 0.01)$.
The performance after n weeks is calculated from the row matrix:

$$A_n = A_0.M^n \qquad (2.14)$$

Then:

$$A_n = (0.95, 0.04, 0.01) \times \begin{pmatrix} 0.6019 & 0.2685 & 0.1296 \\ 0.6018 & 0.2685 & 0.1296 \\ 0.6018 & 0.2685 & 0.1296 \end{pmatrix}$$

Hence, $A_n = (0.6019, 0.2685, 0.1296)$

This means that whether the initial performance was good or average or poor, then after about one year, the probability of having a good performance will equal 0.6019, whereas an average performance will have a probability of 0.2685 and there will be 0.1296 probability of having a poor performance.

This chain of matrices is called a **regular Markov chain** and establishes an extremely important result regarding the performance of different equipment in a production line. That is: **The performance after enough time does not depend on the initial state.**

Generally, let the steady state Markov matrix = $M^n \times \begin{pmatrix} a & b & c \\ a & b & c \\ a & b & c \end{pmatrix}$

And the initial status be $A_0 = (x_0, y_0, z_0)$

Then, from equation (2.14):

$A_n = (a.(x_0 + y_0 + z_0) + b.(x_0 + y_0 + z_0) + c.(x_0 + y_0 + z_0))$

Since $x_0 + y_0 + z_0 = 1$, then $A_n = (a, b, c)$, regardless of the values of x_0, y_0, z_0.

This result is made use of in predicting the long term performance based on statistical field data gathered during the initial period of operation or from similar production lines that have been operating for some time.

Here is another interesting example:

Let us now assume that there was no initial probability that poor performance on a certain day would give way to good performance the next day. However, there is a very small probability that good performance might result in a subsequent poor performance.

The transition matrix reads:

M =

	G	**P**
G	0.999	0.001
P	0	1

The values of matrix entries will then stabilize very slowly owing to the high probability of the a_{11} cell. For example:

M^{8192} =

	G	***P***
G	0.0003	0.9997
P	0	1

This result is very interesting: It means that as long as there is even an infinitesimal probability that a good performance might lead to a poor one, the long-term probability of having a good performance will approach zero!!

This result is used to classify the performance of some critical pieces of equipment. The value of n in M^n must exceed the lifetime of the piece for a predetermined probability of accepted performance.

For example, if the accepted probability of good performance in this example is 0.99, then we calculate M^n until the a_{11} term decreases below 0.99.
This will take place for M^{11} as M^{10} shows as:

$$\begin{pmatrix} 0.99005 & 0.00995 \\ 0 & 1 \end{pmatrix}$$

This shows that the performance will get below the acceptable level after only 10 days, which is obviously unacceptable.

To decide about the initial level of performance we set the general form of the transition matrix as:

$$M = \begin{pmatrix} a & 1-a \\ 0 & 1 \end{pmatrix}$$

It can easily be shown that:

$$M^n = \begin{pmatrix} a^n & 1-a^n \\ 0 & 1 \end{pmatrix}$$

If the expected lifetime of the equipment is, say, 5 years (1500 working days), then the condition for good performance is:

$a^{1500} > 0.99$

Yielding: $a = 0.999993$
This represents the safe limit for the initial equipment performance that would guarantee an acceptable performance for five years. In practice, this corresponds to an initial failure probability of $1 – a = 0.0000067$.

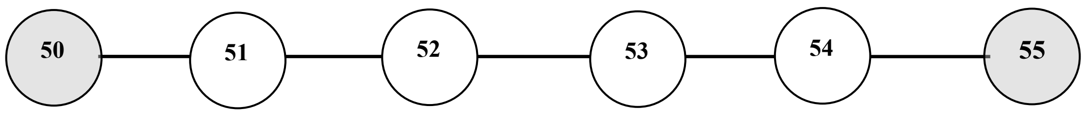
The **logarithm of the reciprocal** of this figure is known as the **SIL value** (Safety Integrity Level). In this example it is in the range of 5 although in practice it rarely exceed 4.

Using EXCEL:
To perform matrix multiplication using excel:

- First: determine the order of the product matrix
- Second: Choose a region of cells having the above order
- Third: in this region, write: = MMULT (Range 1st matrix, range 2nd matrix), then CTRL SHIFT =
- The result will be displayed within the chosen region

2.8.2 Absorbing Markov chains

Consider the following process:
A two – level on – off controller is used to maintain the temperature inside a reactor between 50 and 55°C. This means that the controller is off if $50 < T < 55$. Let us also assume that temperature variations are erratic so that if the temperature is, say, 53°C it may in the next instant either decrease to 52 or increase to 54°C with equal probabilities of 0.5 each.
time. Once it either reaches 50 or 55°C, the control action sets in. If we assume that as one degree temperature transition takes for example 5 seconds, then the question is: how long would it take, on average, for the controller to be in an idle state?
The problem can be put in the following form. In the accompanying figure, if temperature is at any state from 51 to 54, then it has an equal probability to shift either to a higher or to a lower value. Whenever it reaches the end values of 50 or 55, it is "absorbed", meaning that the controller will now operate to maintain temperature between these two levels. In this state, no more erratic behavior is expected.



Let the four positions of temperature be labeled: P_1, P_2, P_3 and P_4 and the two absorbing positions A_1 and A_2.

Since there is an equal probability of 0.5 for temperature at any position to shift to a neighboring position, we can form the following transition matrix M:

`	P_1	P_2	P_3	P_4	A_1	A_2
P_1	0	0.5	0	0	0.5	0
P_2	0.5	0	0.5	0	0	0
P_3	0	0.5	0	0.5	0	0
P_4	0	0	0.5	0	0	0.5
A_1	0	0	0	0	1	0
A_2	0	0	0	0	0	1

This is usually written in the following canonical form:

M =

Q	*R*
0	*I*

Where, Q is a 4×4 matrix, R is a 4×2 matrix, 0 the zero matrix and I the unit matrix.

It can be proved that the most probable number of temperature transitions that can take place for any initial position is calculated from the sum of the corresponding rows of the matrix $(I - Q)^{-1}$.
The following show the matrix $I - Q$ and $(I - Q)^{-1}$

$$\begin{pmatrix} 1 & -0.5 & 0 & 0 \\ -0.5 & 1 & -0.5 & 0 \\ 0 & -0.5 & 1 & -0.5 \\ 0 & 0 & -0.5 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1.6 & 1.2 & 0.8 & 0.4 \\ 1.2 & 2.4 & 1.6 & 0.8 \\ 0.8 & 1.6 & 2.4 & 1.2 \\ 0.4 & 0.8 & 1.2 & 1.6 \end{pmatrix}$$

Summing up the rows of the last matrix, we get the following column vector V:

$$V = \begin{bmatrix} 4 \\ 6 \\ 6 \\ 4 \end{bmatrix}$$

This means that if the temperature was initially 51ºC, it will be absorbed on the average in 4 steps, while if it was initially 52ºC, it would take it on the average 6 steps to be absorbed.
If now the matrix $(I - Q)^{-1}$ is multiplied by the matrix R, we get the probability of absorption at any initial position of temperature:

$$\begin{pmatrix} 1.6 & 1.2 & 0.8 & 0.4 \\ 1.2 & 2.4 & 1.6 & 0.8 \\ 0.8 & 1.6 & 2.4 & 1.2 \\ 0.4 & 0.8 & 1.2 & 1.6 \end{pmatrix} \times \begin{bmatrix} .5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & .5 \end{bmatrix}$$

This product yields:

$$\begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \\ 0.4 & 0.6 \\ 0.2 & 0.8 \end{bmatrix}$$

The interpretation of the obtained matrix is as follows:

If the temperature is at position P_1 (51ºC), then there will be a 80% probability that it will reach 50ºC and 20% that it will reach 55ºC and so on…

On the other hand, referring to the column vector V, the average number of steps to attain absorption, that is, for the controller to function, is the mean value of its elements, which consists of 5 steps. If each step takes about 5 second, then the controller will be off, on the average, for 25 seconds.

Using EXCEL:
To obtain an inverse matrix using EXCEL, highlight the region of M^{-1}, then write = MINVERSE (Range of M) and CTRL SHIFT =
The result is directly displayed.

2.9 Exercise problems

(1) Polypropylene is mainly produced by two processes: process (A) and process (B). Out of 40 factories it was found that 30 adopt the first process, 8 the second process and 5 factories contain production lines of both types. If a factory is randomly chosen, calculate the following probabilities:
   (a) The factory chosen adopts either process
   (b) All the production lines of the chosen factory are of type (A)
   (c) The factory uses a totally different process than either (A) or (B).

(2) A plant engineer looks for a certain type of fuse (A). He knows that 80 fuses have been purchased of which 30 are of the A type. All fuses were mixed up in a drawer. How many fuses should he choose at a time so as to ensure that he will get at least one proper fuse with at least 99.9% chance?

(3) An importing company purchases chemicals from 3 different sources: A, B and C. The annual purchased amounts from these three sources are 150 tons, 200 tons and 250 tons respectively. Of these, the amounts of grade "ANALAR" reagents are 5 tons, 8 tons and 6 tons respectively. A product is chosen at random. Calculate the probability that it is of the ANALAR type. And if turns out to be from that type, what is the probability that it originates from source B?

(4) A certain product was found to have two types of minor defects. The probability that an item of the product has only a type A defect is 0.2, and the probability that it has only a type B defect is 0.15. The probability that it has both defects is 0.1 Find the probabilities of the following events:
(a) An item has either a type A or type B defect.
(b) An item does not have any of these defects.
(c) An item has defect A, but not defect B.
(d) An item has exactly one of the two defects.

(5) Companies manufacturing LWPE get their raw materials from two suppliers only: X and Y; out of 100 factories, 55 get it from X and 65 from Y. What is the probability that a factory chosen at random would get its raw materials from both sources?

(6) A plant installs temperature controllers that were purchased with the following initial performance: (E = excellent, G = good, F = fair) corresponding to the row vector $\begin{pmatrix} 0.94 & 0.05 & 0.01 \end{pmatrix}$.
Long time experience has permitted to set the following Markov transition matrix based on weekly performance

$$\begin{pmatrix} 0.92 & 0.07 & 0.01 \\ 0.05 & 0.93 & 0.02 \\ 0.01 & 0.06 & 0.93 \end{pmatrix}$$

Estimate the performance:

(a) After 8 weeks
(b) After 10 weeks
(c) The ultimate performance

(7) The following matrix shows the probability that a student enrolled at the faculty of Engineering will access any higher level. For example, the first row means that a student in preparatory year has a 0.2 probability of remaining in the same preparatory level next year. He has a probability of 0.1 of being rejected out of the faculty next year etc. The two absorbing states are: rejection and graduation.
Find the probability of finally being rejected or graduating for a student at any academic level.

	Prep	1	2	3	4	Rejected	Graduated
Prep	0.2	0.7	0	0	0	0.1	0
1	0	0.2	0.75	0	0	0.05	0
2	0	0	0.18	0.8	0	0.02	0
3	0	0	0	0.15	0.85	0	0
4	0	0	0	0	0.05	0	0.95
Rejected	0	0	0	0	0	1	0
Graduated	0	0	0	0	0	0	1

(8) The grades transition matrix of students attending a certain program is assumed to remain constant as follows:

	A	B	C	D	F
A	0.7	0.2	0.08	0.02	0
B	0.2	0.65	0.05	0.08	0.02
C	0.02	0.18	0.67	0.1	0.03
D	0	0.02	0.22	0.66	0.1
F	0	0	0.05	0.8	0.15

Find their status after 6 semesters if their original status was as follows:

$$[0.15 \quad 0.25 \quad 0.35 \quad 0.25 \quad 0]$$

(9) A company performs a semiannual assessment of the performance of its employees. They are graded as: E (excellent), G (good), and U (unsatisfactory). The biannual transition matrix is assumed to be constant:

$$\begin{pmatrix} 0.7 & 0.25 & 0.05 \\ 0.3 & 0.6 & 0.1 \\ 0.1 & 0.6 & 0.3 \end{pmatrix}$$

Determine the ultimate performance status.

(10) In January 2020, a company purchased several electric bulbs of different brands. Experience has shown that brand A bulbs have a superior lifetime than those of brands B or C. As time elapses, the lifetime of each type generally decreases to obtain the following semi-annual transition matrix:

$$\begin{pmatrix} 0.3 & 0.65 & 0.05 \\ 0.1 & 0.55 & 0.35 \\ 0.0 & 0.1 & 0.9 \end{pmatrix}$$

In January 2023, the status of the bulbs was given by the row vector:

$$[0.044378 \quad 0.25732 \quad 0.6983].$$

Determine the original status of the bulbs.