Partial Redundancy Elimination for Multi-threaded Programs

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Summary
Multi-threaded programs have many applications which are widely used such as operating systems. Analyzing multi-threaded programs differs from sequential ones; the main feature is that many threads execute at the same time. The effect of all other running threads must be taken in account. Partial redundancy elimination is among the most powerful compiler optimizations: it performs loop-invariant code motion and common subexpression elimination. We present a type system with optimization component which performs partial redundancy elimination for multi-threaded programs.

Key words:
Partial redundancy elimination, Type systems, Multi-threaded programs, Operational semantics, Compiler optimization.

1. Introduction

There are many methods for compiler optimizations; a powerful one of them is partial redundancy elimination (PRE). PRE eliminates redundant computations on some but not necessarily all paths of programs. PRE is a complex optimization as it consists of loop invariant code motion and common subexpression elimination. PRE was established by Morel and Renvoise [17] where they introduce a more general problem (as a system of Boolean equations). Xue and Cai formulated a speculative PRE as a maximum flow problem [27]. Xue and Knoop proved that the classic PRE is a maximum flow problem [28]. Saabas and Ustaluu use the type-systems framework to approach this problem [22]. Some optimizations have been added to PRE such as strength reduction [11] and global value numbering [3]. All methods mentioned above are established to operate on sequential programs.

In the present paper, we achieve partial redundancy elimination for multi-threaded programs which are widely used. Operating system is an example of a system software that depends on multi-threading. You can write your document in a word processor while running an audio file, downloading a file from the internet, and/or scanning for viruses (each of these tasks is considered a thread of computations). Web browser as an example can explore your e-mail, while downloading a file in the background.

The key feature of multi-threaded programs is that many threads can be executed at the same time. Consequently, when executing a thread there is an effect that comes from executing other threads. In general, when analyzing multi-threaded programs, the effect of all threads at the same time must be taken in account. Hence, analyzing multi-threaded programs completely differs from sequential ones. Deducing and stating properties of programs can be done using type systems as well as program analysis. Program analysis has algorithmic manner while type systems are more declarative and easy to understand with type derivations that provide human-friendly format of justifications. We present a type system for optimizing multi-threaded programs. Our type system depends on a new analysis, namely *modified analysis,* and a function called *concurrent modified*, rather than on *anticipability* analysis and *conditional partial availability* analysis used for the while language.

Organization of this paper is as follow. In section 2 we introduce an operational semantics for the language we study. Section 3 presents the concepts of modified analysis and concurrent modified function. Also the soundness of modified analysis, the anticipability analysis, and conditional partial availability analysis for multi-threaded language are discussed in this section. In section 4, we present the type system including the optimization component and prove its soundness. Section 5 and 6 outline related and future work, respectively.

2. Motivation

In this section we introduce the language we study (*FWHILE*), a motivating example, and a natural semantics of *FWHILE*.

2.1 *FWHILE* Language

We assume that our reader is familiar with data flow analysis. We introduce a motivating example to show the importance and obstacles of applying PRE on multi-threaded programs. We use a simple language which we call *FWHILE*. The basic building blocks of *FWHILE* are literals $l \in \text{Lit}$, statements $s \in \text{Stmt}$, arithmetic expressions $a \in \text{AExp}$, and Boolean expressions $b \in \text{BExp}$. These blocks are defined over a set of

program variables $x \in \mathbf{Var}$ and numerals $n \in \mathbf{Z}$ in the following way:

$$l ::= x \mid n$$

$$a ::= l \mid l_0 + l_1 \mid l_0 * l_1 \mid \ldots$$

$$b ::= l_0 = l_1 \mid l_0 \leq l_1 \mid \ldots$$

$$s ::= x := a \mid \mathbf{skip} \mid s_0 ; s_1 \mid \mathbf{if}\ b\ \mathbf{then}\ s_t\ \mathbf{else}\ s_f$$

$$\mid \mathbf{while}\ b\ \mathbf{do}\ s_t \mid \mathbf{fork}\{s_1, s_2, \ldots, s_n\}.$$

We use the following notation $\mathbf{AExp}^+$ for the non-trivial arithmetic expressions (i.e $\mathbf{AExp}^+ = \mathrm{AExp} \setminus \mathrm{Lit}$ **).**

2.2 Motivating Example

The following is an example that motivates our research**.**

$$v := a - c; u := a + b;$$

$$\mathbf{fork}\{\{y := a+b; c = 2; z := a-c;\};$$

$$\{x := a+b; z := a-c;\};\}$$

In this example, expressions $a + b$ and $a - c$ are evaluated before reaching the fork statement, hence we can use their values in the fork statement. But, one of the threads modifies the value of c hence we cannot use expressions containing c; because we do not know when the thread contains this modification will be executed. After applying our analysis the optimized version of the program will be:

$$v := a - c;$$

$$t_1 := a + b;$$

$$u := t_1;$$

$$\mathbf{fork}\ \{\{y := t_1; c = 2; z := a - c;\};$$

$$\{x := t_1; z := a - c;\};\}$$

2.3 Natural Semantics

We use the semantics introuced by Mohamed El-Zawawy in [9]. We review this semantics in this section.

a *state* as a function from a set of variables to integers: $\sigma \in \mathrm{State}$, $\sigma : \mathrm{Var} \to \mathrm{Z}$. The state assigns a value for each variable. Expressions (arithmetic and boolean) are defined by semantic functions $[\![-]\!] \in \mathbf{AExp} \cup \mathbf{BExp} \to Z \cup \{tt, ff\}$ in denotational style. For $a \in \mathbf{AExp}$ and $b \in \mathbf{BExp}$ we write $[\![a]\!]\sigma$ *and* $[\![b]\!]\sigma$ to denote the evaluations of

expressions a and b in a state σ , respectively. We write $\sigma \models b$ to denote that $[\![b]\!] = tt$ (i.e evaluation of b in σ is true). Statements are written in the form of evaluation relation $\succ - - \to \subseteq \mathbf{State} \times \mathbf{Stm} \times \mathbf{State}$. The notation $\sigma[x \mapsto [\![b]\!]\sigma]$ denotes that the state is σ rather than $\sigma(x) = [\![b]\!]\sigma$. Inference rules of the semantics are:

$$\frac{}{\sigma \succ x := a \to \sigma[x \mapsto [\![a]\!]\sigma]}\ {}^{:=}ns \qquad \frac{}{\sigma \succ skip \to \sigma}\ skip_{ns}$$

$$\frac{\sigma \succ s_0 \to \sigma'',\ \sigma'' \succ s_1 \to \sigma'}{\sigma \succ s_0 ; s_1 \to \sigma'}\ comp_{ns} \qquad \frac{\sigma \models b\ ,\ \sigma \succ s_t \to \sigma'}{\sigma \succ \mathbf{if}\ b\ \mathbf{then}\ s_t\ \mathbf{else}\ s_f \to \sigma'}\ if_{ns}^{tt}$$

$$\frac{\sigma \not\models b\ ,\ \sigma > s_f \to \sigma'}{\sigma \succ \mathbf{if}\ b\ \mathbf{then}\ s_t\ \mathbf{else}\ s_f \to \sigma'}\ if_{ns}^{ff} \qquad \frac{\sigma \not\models b}{\sigma \succ \mathbf{while}\ b\ \mathbf{do}\ s_t \to \sigma}\ while_{ns}^{ff}$$

$$\frac{\sigma \models b\ ,\ \sigma \succ s_t \to \sigma'',\ \sigma'' \succ \mathbf{while}\ b\ \mathbf{do}\ s_t \to \sigma'}{\sigma \succ \mathbf{while}\ b\ \mathbf{do}\ s_t \to \sigma'}\ while_{ns}^{tt}$$

$$\frac{\sigma_i \succ s_{\theta(i)} \to \sigma_{i+1}\ ,\qquad \forall i \in \{1,2,\ldots\}}{\sigma_1 \succ \mathbf{fork}\{s_1, s_2, \ldots s_n\} \to \sigma_{n+1}}\ fork_{ns}$$

where θ is a permutation on $\{1, 2, .., n\}$

The rule $fork_{ns}$ depends on the the order of executing the threads. We assume that order named θ which is a permutation on n (number of threads). In this rule we assume that the threads will execute one by one.
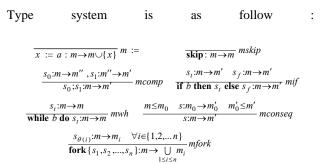
3. Program Analysis

In this section we will introduce the analysis of the multithreaded programs. We introduce type systems to help optimizing programs. Firstly, we introduce the modified analysis which tells which variables are modified. Secondly, we introduce a concurrent modified function. Also, we introduce traditional anticipability analysis and conditional partial availability analysis which are generalizations of the work of [22] (with additional rules for multi-threaded statements).

3.1 Modified Analysis

Modified analysis computes for each program point which variables have been modified. The type system is simple. It gathers the modified variables along the path to the point. Type $m \subseteq \mathbf{Var}$ is a set of variables. Modified analysis is a must forward analysis. The subtyping is the revered set inclusion (i.e $\leq\ =\ \supseteq$).

Definition 1 *For any program point, any state σ and a type $m \subseteq \mathbf{Var}$, e $\sigma \models m$ (σ entails m) iff $m \subseteq dom(\sigma)$.*

Type   system   is   as   follow   :

$$\frac{}{x := a : m \to m \cup \{x\}} \; m :=$$
$$\frac{}{\textbf{skip} : m \to m} \; mskip$$

$$\frac{s_0 : m \to m'' \;,\; s_1 : m'' \to m'}{s_0 ; s_1 : m \to m'} \; mcomp$$
$$\frac{s_t : m \to m' \quad s_f : m \to m'}{\textbf{if } b \textbf{ then } s_t \textbf{ else } s_f : m \to m'} \; mif$$

$$\frac{s_t : m \to m}{\textbf{while } b \textbf{ do } s_t : m \to m} \; mwh$$
$$\frac{m \leq m_0 \quad s : m_0 \to m_0' \quad m_0' \leq m'}{s : m \to m'} \; mconseq$$

$$\frac{s_{\theta(i)} : m \to m_i \quad \forall i \in \{1,2,\dots n\}}{\textbf{fork}\{s_1, s_2, \dots, s_n\} : m \to \bigcup_{1 \leq i \leq n} m_i} \; mfork$$

The type system is clear and simple. The rule $m :=$ adds the assigned variable to pre type. Rules $mskip, mconseq, mif$ and mwh are direct and similar to operational semantics. Rule $mconseq$ for strengthen the pre-type. Rule $mfork$ is for threading. This rule computes the modified variables along **fork** statement by collecting all modified variables over all threads.

The following two lemmas state properties about modified analysis:-

Lemma 1   *Suppose* $s : m \to m'$,   *where* $m, m' \subseteq \textbf{Var}$. *Then* $m \subseteq m'$.

Proof :   It is clear that the statement which actually changes the set m is the assignment statement where $m' = m \cup \{x\}$ *i.e* $m \subseteq m'$.

Lemma 2   *Suppose* $s : m' \to m''$ *and* $\sigma \succ s \to \sigma'$, *where* $m, m', m'' \subseteq \textbf{Var}$. *Then* $s : m' \cup m \to m'' \cup m$.

Proof :   We have $m \subseteq dom(\sigma) \Rightarrow m' \subseteq dom(\sigma')$.
Then it is clear that :-
$$m \cup m'' \subseteq dom(\sigma) \Rightarrow m' \cup m'' \subseteq dom(\sigma')$$

3.2 Soundness of modified analysis

The following theorem proves the soundness of modified analysis.

Theorem 1

Suppose $s : m \to m'$ *and* $\sigma \succ s \to \sigma'$. *Then*

if $\sigma \models m$ *then* $\sigma' \models m'$.

Proof : The proof is by structure induction of type derivation. We will prove only main rules.

• Type derivation is $m :=$ and corresponding
   operational semantic is $:=_{ns}$:

We have $\sigma \models m$ and $\sigma' = \sigma[x \mapsto [\![a]\!]\sigma]$, which implies
$$dom(\sigma') = dom(\sigma) \cup \{x\}.$$
$$\because m \subseteq dom(\sigma) \Rightarrow m \cup \{x\} \subseteq dom(\sigma) \cup \{x\}$$
$$\Rightarrow m' \subseteq dom(\sigma') \Rightarrow \sigma' \; m'$$

• Type derivation is $mconseq$ and $\sigma \succ s \to \sigma'$:

$$\sigma \models m \Rightarrow \sigma \models m_0 \quad (m \leq m_0)$$
$$\Rightarrow \sigma' \models m_0' \quad (s : m_0 \to m_0' \;\; \sigma \succ s \to \sigma')$$
$$\Rightarrow \sigma' \models m' \quad (m_0' \leq m')$$
$$i.e \quad\quad \sigma \models m \Rightarrow \sigma' \models m'$$

• Type derivation is $mfork$ and the corresponding operational semantic is $fork_{ns}$. We prove that :
$$\sigma_1 \models m_1 \Rightarrow \sigma_{n+1} \models \bigcup_{1 \leq i \leq n} m_i$$

From premises we have $s_{\theta(i)} : m \to m_i$, which by lemma 1 implies $m \subseteq m_i, \forall i \in \{1, 2 \dots, n\}$.

From lemma 2 and lemma 1 we can get:

$s_{\theta(1)} : m \to m_1$

$s_{\theta(2)} : m_1 \to m_1 \cup m_2$

$s_{\theta(3)} : m_1 \cup m_2 \to m_1 \cup m_2 \cup m_3$

$$\vdots$$

$s_{\theta(n)} : m_1 \cup m_2 \dots \cup m_{n-1} \to m_1 \cup m_2 \dots \cup m_n$

To simplify the notations we let

$M_0 = m,$

$M_1 = m_1,$

$M_2 = m_1 \cup m_2$

$$\vdots$$

$M_n = m_1 \cup m_2 \dots \cup m_n = \bigcup_{1 \leq i \leq n} m_i.$

The previous sequence can be written as:-

$s_{\theta(i)} : M_{i-1} \to M_i$, also we have $\sigma_i \succ s_{\theta(i)} \to \sigma_{i+1}$.

Then we get:

$$\sigma_1 \models M_0 \Rightarrow \sigma_2 \models M_1$$
$$\sigma_2 \models M_1 \Rightarrow \sigma_3 \models M_2$$
$$\vdots$$
$$\sigma_n \models M_{n-1} \Rightarrow \sigma_{n+1} \models M_n$$

From last sequence, we conclude $\sigma_1 \models m \Rightarrow \sigma_{n+1} \models M_n$.

3.3 Concurrent modified function C

In this section we will present concurrent modified function C. We start by defining *sub-statement* relation between statements.

Definition 2 *For any two statements s and t, we say that t is a sub-statement of s written as $(t \prec s)$ iff :-*

1. $t = s$ or

2. $s = s_1 ; s_2$ and $(t \prec s_1)$ or $(t \prec s_2)$

We mean by $t = s$, that t is identically (syntactically) equivalent to s.

Definition 3 *The modified concurrent function assigns a set of variables for each program point,* i.e. $C : \mathbf{Stm} \rightarrow \mathbf{Var}$:

1. For $\mathbf{fork}\{s_1, \ldots, s_n\}$ statement:

$$C(s_i) = \bigcup_{i \neq j} m_j, \quad where \quad s_i : m \rightarrow m_i$$

$$C(t) = C(s_i) \qquad for \qquad t \prec s_i$$

2. *otherwise*

$$C(s) = \varphi$$

3.4 Anticipability analysis

We now present anticipability analysis. For each program point it computes which non-trivial arithmetic expressions will be evaluated on all paths before any of their operands are modified. In the typing rule we use $eval(a)$ to denote $\{a\}$ if a is non-trivial expression and φ otherwise.

For $a \in \mathbf{AExp}^+$ we define :-

$$mod(x) =_{\mathbf{df}} \{a \mid x \in FV(a)\} \quad and$$

$$mce(s) =_{\mathbf{df}} \{a \mid a \in mod(y). \forall y \in C(s)\}$$

We use $\underline{s}$ to denote the full type derivation of $s : m \rightarrow m'$. Inference rules of the type system include:

$$\overline{x := a : (ant' \setminus mod(x) \cup eval(a)) \setminus mce(x := a) \rightarrow ant'}$$

$$\frac{s_i : ant_i \rightarrow Ant' \setminus mce(s_i) \qquad \forall i \in \{1,2,\ldots n\}}{\mathbf{fork}\{s_1, s_2, \ldots s_n\} : \bigcup_{1 \leq i \leq n} ant_i \rightarrow Ant'.}$$

The rules for other program statements (if statement, while statement,..) follow the same line of corresponding rules

introduced in [22] for the while langauge. The novelty of our work comes from fitting the fork statement into the type system of [22] and making necessary changes. We note that if no thread exists then for each statement s in the program $mce(s) = \varphi$. Besides the rule of **fork** statement which characterizes the multi-threaded concept. These rules prevent any modified expression *(i.e modified in concurrent threads)* from being used from the start of **fork** statement. As anticipability analysis is backward analysis, we remove modified expressions from Ant' of each statement. Also, in each assignment statement we remove modified expressions. All of these removals are guided by the set $mce(s)$.

3.5 Partial availability analysis

It computes for each program point which non-trivial arithmetic expressions has already been evaluated and later not modified on some path through this program point and also anticipable.
Inference rules of the type system include:

$$\frac{s_i : ant_i, CPAV \setminus mce(s_i) \rightarrow Ant' \setminus mce(s_i), cpav_i' \quad \forall i \in \{1,2,\ldots,n\}}{\mathbf{fork}\{\underline{s_1}, \underline{s_2}, \ldots, \underline{s_n}\} : \bigcup_{1 \leq i \leq n} ant_i, CPAV \rightarrow Ant', \bigcup_{1 \leq i \leq n} cpav_i'}$$

The rules for other program statements (if statement, while statement,..) follow the same line of corresponding rules introduced in [22] for the while langauge. The novelty of our work comes from fitting the fork statement into the type system of [22]. Here we excluded the expressions in concurrent modified of all threads of **fork** statement to avoid using these expressions after exiting **fork** statement.

4. Optimization Component

In this section we introduce a type system with optimization components for multithreaded programs. We mean by the notation $\underline{s} : ant, cpav \rightarrow ant', cpav' \rhd s_*$ that, the statement s with complete type system m, ant and $cpav$ is optimized to s_*. Inference rules of the type system include:

$$\frac{s_i : ant_i, CPAV \setminus mce(s_i) \rightarrow Ant' \setminus mce(s_i), cpav_i' \rhd s* \quad \forall i \in \{1,2,\ldots,n\}}{\mathbf{fork}\{\underline{s_1}, \underline{s_2}, \ldots, \underline{s_n}\} \bigcup_{1 \leq i \leq n} ant_i, CPAV \rightarrow Ant', \bigcup_{1 \leq i \leq n} cpav_i' \rhd \mathbf{fork}\{s_1^*, s_2^*, \ldots s_n^*\}} fork_{pre}$$

The rules for other program statements (if statement, while statement,..) follow the same line of corresponding rules introduced in [22] for the while langauge. The novelty of our work comes from fitting the fork statement into the type system of [22]. Our new analysis also affects the

component of optimizations as in the rule $\mathbf{fork_{pre}}$, which achieves optimizion via optimizing each thread of its components.

The soundness of the above system is proved in [22] except of course for the fork rule which is at the heart of our contribution. We discuss this case here. The definition of the similarity relation $\sigma \sim_{cpav} \sigma_*$ is in [22]. Soundness of the fork statement is as follows:

Suppose $s:ant,cpav \rightarrow ant',cpav',\sigma \sim_{cpav} \sigma_*$ and $\sigma \succ\!\!- s \rightarrow \sigma'$. We have to find σ_*' such that $\sigma' \sim_{cpav'} \sigma_*'$ and $\sigma_* \succ\!\!- s_* \rightarrow \sigma_*'$. The proof is by induction on the typing derivation. We have $s_i:ant_i,cpav \setminus mce(s_i) \rightarrow ant'\setminus mce(s_i),cpav_i' \triangleright s_i^*$.

If $\mathbf{fork}\,\{\underline{s}_1,\underline{s}_2,...,\underline{s}_n\}:a,c \rightarrow a',c' \triangleright \mathbf{fork}\,\{s_1^*,s_2^*,...,s_n^*\}$ and $\sigma_1 \sim_c \sigma_1^*$, where $a=\bigcup\limits_{1\leq i\leq n} ant_i, c=cpav, a'=Ant', c'=\bigcup\limits_{1\leq i\leq n} cpav_i$ then we have to find β such that :

if $\sigma_1 \succ\!\!- \mathbf{fork}\{s_1,s_2,...,s_n\} \rightarrow \sigma_{n+1}$ then $\sigma_{n+1} \sim_{c'} \beta$ and $\sigma_i^* \succ\!\!- \mathbf{fork}\{s_1^*,s_2^*,...,s_n^*\} \rightarrow \beta$. The following is given: if $s_i:ant_i,cpav \setminus mce(s_i) \rightarrow ant'\setminus mce(s_i),cpav_i' \triangleright s_i^*$ and $\sigma_j \sim_{c_i} \sigma_j^*$ (where $c_i = cpav \setminus mce(s_i)$ and $j = \theta(i)$) then $\sigma_i \succ\!\!- s_j \rightarrow \sigma_{i+1}$ implies the existence of γ_{i+1}^* such that $\gamma_{i+1}^* \sim_{cpav_j} \sigma_{i+1}$ and $\sigma_i^* \succ\!\!- s_j \rightarrow \gamma_{i+1}^*$.

Choosing $\gamma_{i+1}^* = \sigma_{i+1}$, our choice is amenable where, it leads to the following:

$$\sigma_i^* \succ\!\!- s_j^* \rightarrow \sigma_{i+1}^* \Rightarrow \sigma_1^* \succ\!\!- \mathbf{fork}\{s_1^*,s_2^*,...,s_n^*\} \rightarrow \sigma_{n+1}^* \cdots i$$

$$\sigma_{i+1} \sim_{cpav_j} \sigma_{i+1}^* \Rightarrow \sigma_{n+1}^* \sim_{c'} \sigma_{n+1} \qquad \cdots ii$$

from i and ii and choosing $\beta = \sigma_{n+1}^*$ the proof is done.

5. Related Work

There are three aspects of work that are related to our work:-

Partial Redundanc elimination (PRE)
PRE [8,18,25] was originated by Morel and Renvoice[17]. They applied PRE using static analysis and presented PRE as a general problem of global optimization using boolean system of equations. [17] also presents an algorithm for global optimization, which does not need *a control flow graph* (CFG). Efforts have been done to improve the formulation of PRE [7]. The work in [28,7] formulates classical and commutative PRE as a maximum flow problem. PRE is used as a framework and is extended to do more optimizations as strength reduction [11]

Type systems
Analyzing a program using type systems rather than *control flow graph* was the idea of work in [14]. More accurately, it proves that types can describe results of an analysis of a program if and only if this type is a supertype of a result of applying the analysis. The work in [1,9,10,14,18,19,21,22] uses type systems to accomplish the program analysis in similar way of ours. The work in [21] introduces type systems for PRE and proves the correctness of optimizations. Dead code elimination was treated using type systems. [1] presents type systems for dead code elimination and constant folding. Also [22] introduces type systems for dead code elimination and common subexpression elimination and proves a relational soundness.

Analysis of Multi-threaded Programs
The field of program analysis has been extended to treat multi-threaded programs besides sequential ones. We can use these analyses in compiler optimizations. The main obstacle to apply traditional compiler optimizations is that, we do not know which order of threads will be executed [15]. Generalizing standard program presentations, analysis, and transformations are used to optimize multi-threaded programs in the presence of access to shared data [12,13,24,26]. Another way to use analysis occurs when two threads access the same data without synchronization (one of them is write): data races[5,6,16]. Yet another aspect to use analysis is called dead lock detection (which occurs when threads are permanently blocked waiting for resources)[2,4]. Pointer analysis [9,20,23] of multi-threaded programs attracts attention of researchers. None of works above deal with PRE. The present paper is the first work that uses type systems as a framework to implement PRE to multi-threaded programs.

6 Conclusions and Future Work

In this paper the main contribution is the application of PRE to a multi-threaded programming language. Up to our knowledge, this paper is the first to deal with this problem. We use type systems as a tool to solve the problem. We designed a simple type system for optimizing multi-threaded programs. We approach the problem in a simple way; we use usual PRE with simple modifications. We look for variables that have been modified in other threads and exclude the expressions that contain any of the modified variables. For future work, we study more complicated optimization and consider using other tools. Many modifications can be applied.

References

[1] Nick Benton, "Simple relational correctness proofs for static analyses and program transformations", In *Proceedings of 31st ACM SIGPLAN-SIGAT symposium on Principles of programming languages,* POPL'04, pages 14-25, New York, USA 2004 ACM.

[2] Johann Blieberger, Bernd Burgstaller, and Bernhard Scholz. "Symbolic data flow analysis for detecting deadlocks ada tasking programs", In *Proceedings of the 5th Ada-Eurpe International Conference on Reliable Software Technologies*, Ada-Europe '00, pages 225-237, London, UK, 2000, Springer-Verlag.

[3] Perston Briggs and Keith D. Cooper, "Effective partial redundancy elimination", In *Proceedings of the ACM SIGPLAN 1994 conference on Programming language design and implementation*, PLDI '94, pages 159-170, New York, USA, 1994, ACM.

[4] Eric Bruneton and Jean-Francois Pradat-Peyre, "Automatic verification of concurrent ada programs", In *Proceedings of the 1999 Ada-Europe International Conference on Reliable Software Technologies,* Ada-Europe '99, pages 146-157, London, UK, 1999, Springer-Verlag.

[5] D. Callahan, K. Kennedy, and J. Subhlok, " Analysis of event synchronization in a parallel programming tool", In *Proceedings of the second ACM SIGPLAN symposium on Principles & practice of parallel programming,* PPOPP '90, pages 21-30, New York, NY, USA, 1990, ACM.

[6] David Callahan and Jaspal Sublok, "Static analysis of low-level synchronization ", In *Proceedings of the 1988 ACM SIGPLAN and SIGOPS workshop on Parallel and distributed debugging,* PADD '88, pages 100-111, New York, NY, USA, 1988, ACM.

[7] Dhananjay M. Dhamdhere, " E-path-pre: Partial redundancy elimination made easy", *SIGPLAN Not.,*37:53-65, August 2002.

[8] Karl-Heinz Drechsler and Manfred P. Stadel, " A solution to a problem with morel and renvoise's global optimization by suppression of partial redundancies", *ACM Trans. Program. Lang. Syst.,* 10:635-640, October 1988.

[9] Mohamed A. El-Zawawy, " Flow sensitive-insensitive pointer analysis based memory safety for multithreaded programs", In Beniamino Murgante, Osvaldo Gervasi, Andrs Iglesias, David Taniar, and Bernady Apduhan, editors, *Computational Science and Its Applications - ICCSA 2011*, volume 6786 of *Lecture Notes in Computer Science,* pages 355-369. Springer Berlin/ Heidelberg, 2011.

[10] Mohamed A. El-Zawawy, "Program optimization based pointer analysis and live-stack heap analysis", *International Journal of Computer Science Issues*,8, March 2011.

[11] Robert Kennedy, Fred C. Chow, Peter Dahl, Shin-Ming Liu, Raymond Lo, and Mark Streich, " Strength reduction via ssapre", In Kai Koskimies, editor, *CC*, volume 1383 of *Lecture Notes in Computer Science*, pages 144-158, Springer, 1998.

[12] Jens Knoop and Bernhard Steffen, "Code motion for explicitly parallel programs", In *Proceedings of the seventh ACM SIGPLAN symposium on Principles and practice of parallel programming,* PPoPP '99, pages 13-24, New York, NY, USA, 1999. ACM.

[13] Jens Knoop, Bernhard Steffen, and Jurgen Vollmer, "Parallelism for free: efficient and optimal bitvector analyses for parallel programs", ACM *Trans. Program. Lang. Syst.,* 18:268-299, May 1996.

[14] Peeter Laud, Tarmo Uustalu, and Varmo Vene, "Type systems equivalent to data-flow analyses for imperative languages", *Theor. Comput. Sci.*, 364:292-310, November 2006.

[15] S. Midkiff and D. Padua, "Issues in the optimization of parallel programs", In *Proceedings of 1990 International Conference on Parallel Processing*, pages 105-113, 1990.

[16] S. P. Midkiff and D. A. Padua, "Compiler algorithms for synchronization", *IEEE Trans. Comput.,* 36:1485-1495, December 1987.

[17] E. Morel and C. Renvoise, "Global optimization by suppression of partial redundancies", *Commun. ACM,* 22:96-103, February 1979.

[18] Rocco De Nicola, Daniele Gorla, Ren Rydhof Hansen, Flemming Nielson, Hanne Riis Nielson, Christian W. Probst, and Rosario Pugliese, "From flow logic to static type systems for coordination languages", *Science of Computer Programming,* 75(6):376-397, 2010. 10th International Conference on Coordination Models and Languages COORD'08.

[19] Hanne Riis Nielson and Flemming Nielson, "Flow logic: a multi-paradigmatic approach to static analysis", pages 223-244. Springer-Verlag New York, Inc., New York, NY, USA, 2002.

[20] Radu Rugina and Martin C. Rinard, "Pointer analysis for structured parallel programs", *ACM Trans. Program. Lang. Syst.*, 25:70-116, January 2003.

[21] Ando Saabas and Tarmo Uustalu, "Program and proof optimizations with type systems", Journal *of Logic andAlgebraic Programming*, 77(1-2):131-154, 2008. The 16th Nordic Workshop on the Programming Theory (NWPT 2006).

[22] Ando Saabas and Tarmo Uustalu, " Proof optimization for partial redundancy elimination", *Journal of Logic and Algebraic Programming,* 78(7):619-642, 2009. The 19th Nordic Workshop on Programming Theory (NWPT 2007).

[23] Alexandru Salcianu and Martin Rinard, "Pointer and escape analysis for multithreaded programs", *In Proceedings of the eighth ACM SIGPLAN symposium on Principles and practices of parallel programming*, PPoPP '01, pages 12-23, New York, NY, USA, 2001. ACM.

[24] Vivek Sarkar and Barbara Simons, "Parallel program graphs and their classification", In *Proceedings of the 6th International Workshop on Languages and Compilers for Parallel Computing,* pages 633-655, London, UK, 1994. Springer-Verlag.

[25] Arthur Sorkin, "Some comments on morel and renvoise's global optimization by suppression of partial redundancies. *SIGPLAN Not.*, 31:69-72,December 1996.

[26] Harini Srinivasan, James Hook, and Michael Wolfe, "Static single assignment for explicitly parallel programs", In *Proceedings of the 20th ACM SIGPLAN-SIGACT symposium on Principles of programming languages*, POPL '93, pages 260-272, New York, NY, USA, 1993. ACM.

[27] Jingling Xue and Qiong Cai, "A life time optimal algorithm for speculative pre", *ACM Trans. Archit. Code Optim.*, 3:115-155, June 2006.

[28] Jingling Xue and Jens Knoop, "A fresh look at pre as a maximum flow problem", In Alan Mycroft and Andreas

Zeller, editors, *Compiler Construction*, volume 3923 of *Lecture Notes in Computer Science*, pages 139-154.Springer Berlin / Heidelberg, 2006.

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