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ORIGINAL PAPER

Effectiveness of porosity on transient generalized Couette flow with Hall effect and variable properties under exponential decaying pressure gradient

M A M Abdeen[1]*, H A Attia[2], W Abbas[3] and W Abd El-Meged[2]

[1]Department of Engineering Mathematics and Physics, Faculty of Engineering, Cairo University, Giza 12211, Egypt

[2]Department of Engineering Mathematics and Physics, Faculty of Engineering, El-Fayoum University, El-Fayoum 63514, Egypt

[3]Basic and Applied Science Department, College of Engineering and Technology, Arab Academy for Science, Technology and Maritime Transport (Cairo Branch), Cairo, Egypt

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Abstract: The transient generalized Couette flow with heat transfer through a porous medium between two infinite parallel porous plates is studied considering the Hall effect and temperature dependent physical properties. The upper plate is moving with a uniform velocity while the lower plate is kept stationary. An exponential decaying pressure gradient is imposed in the axial direction and an external uniform magnetic field as well as a uniform suction and injection are applied perpendicular to the horizontal plates. A numerical solution for the governing non-linear coupled set of equations of motion and the energy equation including the viscous and Joule dissipations is adopted. The effect of the porosity of the medium, the Hall current and the temperature dependent viscosity and thermal conductivity on both the velocity and temperature distributions is investigated.

Keywords: Couette flow; Variable properties; Hydromagnetics; Hall effect; Porous medium; Heat transfer; Numerical solution

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1. Introduction

The field of magnetohydrodynamic (MHD) flow and its applications have been widely studied [1–3]. The flow of an electrically conducting fluid between infinite horizontal parallel plates has important applications in MHD power generators and pumps, etc. Hartmann and Lazarus [4] have investigated the effect of a transverse uniform magnetic field on flow of a viscous incompressible electrically conducting fluid between two infinite parallel plates. Exact solutions for velocity fields have been developed [5–8] under different physical effects. Some exact and numerical solutions for the heat transfer problem have been derived [9]. Soundalgekar et al. [10], Soundalgekar and Uplekar [11] have examined the effect of Hall currents on the steady MHD Couette flow with heat transfer. The temperatures of the two plates are assumed constant [10] or varying along plates in the direction of the flow [11]. Attia [12] has studied the influence of the Hall current on velocity and temperature fields of an unsteady Hartmann flow with uniform suction and injection applied perpendicular to the plates.

In these studies the physical properties are assumed to be constant. However, it is known that some physical properties are function of temperature and assuming constant properties, in a good approximation as long as small differences in temperature are involved. More accurate prediction for flow and heat transfer have been achieved by considering the variation of the physical properties with temperature [13–16]. Klemp et al. [17] have studied the effect of temperature dependent viscosity on entrance flow in a channel in hydrodynamic case. Attia

*Corresponding author, E-mail: mostafa_a_m_abdeen@hotmail.com

and Kotb [18] have solved the steady MHD fully developed flow and heat transfer between two parallel plates with temperature dependent viscosity which has been extended to the transient state [19]. The influence of dependence of physical properties on temperature in the MHD Couette flow between parallel plates was studied [17, 18].

A modified Graetz method has been applied to investigate the thermal development of forced convection in a parallel plate channel filled by a saturated porous medium, with walls held either at a uniform temperature or at uniform heat flux and with the effects of axial conduction and viscous dissipation included [20, 21]. The problem is solved in steady state while the fluid is non-conducting and upper plate is kept stationary.

In this work, the transient generalized Couette flow through a porous medium of a viscous incompressible electrically conducting fluid is studied with heat transfer. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature while the Hall current is considered. The fluid is flowing between two electrically insulating porous plates and is acted upon by an exponential decaying pressure gradient. A uniform suction and injection and an external uniform magnetic field are applied normal to the surface of the plates. The two plates are kept at two constant but different temperatures and viscous and Joule dissipation terms are included in energy equation. The flow in the porous medium deals with analysis in which the differential equation governing the fluid motion is based on the Darcy's law which considers the drag exerted by the porous medium [22–24]. The coupled set of non-linear partial differential equations of motion and the energy equation are solved numerically using the method of finite difference to determine the velocity and temperature distributions for any instant of time. The effect of porosity of the medium, the Hall current and temperature dependent viscosity and thermal conductivity on both the velocity and temperature distributions is investigated.

2. Formulation of the problem

The fluid flows between two infinite horizontal parallel plates located at the $y = \pm h$ planes. The upper plate is moving with a uniform velocity U_o, while the lower plate is kept stationary. The two plates are porous and insulating and are kept at two constant temperatures: T_1 for the lower plate and T_2 for upper plate with $T_2 > T_1$. An exponential decaying pressure gradient is imposed in the axial x-direction and a uniform suction from above and injection from below, with velocity v_o, are applied impulsively at $t = 0$. A uniform magnetic field B_o, assumed unaltered, is applied perpendicular to the plates in the positive y-direction. The Hall effect is considered and accordingly, a z-component of the velocity is generated. The viscosity and thermal conductivity of the fluid depend on temperature exponentially and linearly respectively, while the viscous and Joule dissipations are not neglected in the energy equation. The flow is through a porous medium where the Darcy model is assumed [24]. The fluid motion starts from rest at $t = 0$, and the no-slip condition at the plates implies that fluid velocity has neither a z nor an x-component at $y = \pm h$. The initial temperature of the fluid is assumed to be equal to T_1 as the temperature of the lower plate. Since the plates are infinite in the x and z-directions, the physical quantities do not change in these directions which leads to one-dimensional problem. The flow of the fluid is governed by the Navier–Stokes equation:

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} p + \vec{\nabla}.(\mu \vec{\nabla} v) + \vec{J} \wedge \vec{B}_o \tag{1}$$

where, ρ is the density of the fluid, μ is the viscosity of the fluid, $\vec{J}$ is the current density, and $\vec{v}$ is the velocity vector of the fluid, which is given by

$$\vec{v} = u(y,t)\vec{i} + v_o\vec{j} + w(y,t)\vec{k}$$

If the Hall term is retained, the current density $\vec{J}$ is given by the generalized Ohm's law [7]

$$\vec{J} = \sigma(\vec{v} \wedge \vec{B}_o - \beta(\vec{J} \wedge \vec{B}_o)). \tag{2}$$

where, σ is the electric conductivity of the fluid and β is the Hall factor [7]. Equation (2) may be solved in $\vec{J}$ to yield:

$$\vec{J} \wedge \vec{B}_o = -\frac{\sigma B_o^2}{1+m^2}((u+mw)\vec{i} + (w-mu)\vec{k}) \tag{3}$$

where m is the Hall parameter and $m = \sigma \beta B_o$. Thus, the two components of the momentum Eq. (1) read:

$$\rho \frac{\partial u}{\partial t} + \rho v_o \frac{\partial u}{\partial y} = G e^{-\alpha t} + \mu \frac{\partial^2 u}{\partial y^2} + \frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y} - \frac{\sigma B_o^2}{1+m^2}(u+mw) - \frac{\mu}{\bar{K}} u, \tag{4}$$

$$\rho \frac{\partial w}{\partial t} + \rho v_o \frac{\partial w}{\partial y} = \mu \frac{\partial^2 w}{\partial y^2} + \frac{\partial \mu}{\partial y}\frac{\partial w}{\partial y} - \frac{\sigma B_o^2}{1+m^2}(w-mu) - \frac{\mu}{\bar{K}} w, \tag{5}$$

where, $\bar{K}$ is the Darcy permeability [24] and the last term in right sides of Eqs. (4) and (5) represent the porosity force in the x- and z-directions respectively. It is assumed that pressure gradient is applied at $t = 0$ and the fluid starts its motion from rest. Thus

$$t = 0 : u = w = 0. \tag{6a}$$

For $t > 0$, the no-slip condition at the plates implies that

$$y = -h : u = w = 0, \tag{6b}$$

$$y = h : u = U_o, \quad w = 0. \tag{6c}$$

The energy equation describing temperature distribution for the fluid is given by [23]

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p v_o \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \mu\left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right) + \frac{\sigma B_o^2}{1+m^2}(u^2 + w^2), \tag{7}$$

where, T is temperature of the fluid, c_p is the specific heat at constant pressure of fluid, and k is thermal conductivity of fluid. The last two terms in right side of Eq. (7) represent the viscous and Joule dissipations respectively.

Temperature of the fluid must satisfy the initial and boundary conditions,

$$t = 0 : T = T_1, \tag{8a}$$

$$t > 0 : T = T_1, \quad y = -h, \tag{8b}$$

$$t > 0 : T = T_2, \quad y = h. \tag{8c}$$

The viscosity of the fluid is assumed to vary with temperature and is defined as, $\mu = \mu_o f_1(T)$. By assuming the viscosity to vary exponentially with temperature, the function $f_1(T)$ takes the form, $f_1(T) = \exp(-a_1(T - T_1))$ [10]. In some cases a_1 may be negative, i.e. the coefficient of viscosity increases with temperature [10, 25]. Also thermal conductivity of the fluid is varying with temperature as $k = k_o f_2(T)$. We assume linear dependence for thermal conductivity upon the temperature in the form $k = k_o(1 + b_1(T - T_1))$ [26], where the parameter b_1 may be positive or negative [27].

Introducing the following non-dimensional quantities,

$$(\hat{x}, \hat{y}, \hat{z}) = \frac{(x, y, z)}{h}, \hat{t} = \frac{U_o t}{h}, \hat{G} = \frac{hG}{\rho U_o^2}, (\hat{u}, \hat{w}) = \frac{(u, w)}{U_o}, \theta = \frac{T - T_1}{T_2 - T_1},$$

$\hat{f}_1(\theta) = e^{-a_1(T_2 - T_1)\theta} = e^{-a\theta}$, a is the viscosity variation parameter, $\hat{f}_2(\theta) = 1 + b_1(T_2 - T_1)\theta = 1 + b\theta$, b is the thermal conductivity variation parameter, $S = v_o/U_o$ is the suction parameter. $Ha^2 = \sigma B_o^2 h^2/\mu_o$, Ha is the Hartmann number, $M = h\mu_o/(\rho U_o K)$, is the porosity parameter. $\Pr = \mu_o c_p/k_o$, is the Prandtl number, $Ec = \mu_o^2/h^2 c_p \rho^2 (T_2 - T_1)$, is the Eckert number. $Nu_L = (\partial T/\partial \hat{y})\hat{y} = -1$ is the Nusselt number at the lower plate, $Nu_U = (\partial T/\partial \hat{y})\hat{y} = 1$ is the Nusselt number at the upper plate.

Equations (4)–(8c) read (the hats are dropped for simplicity)

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = Ge^{-\alpha t} + f_1(\theta)\frac{\partial^2 u}{\partial y^2} + \frac{\partial f_1(\theta)}{\partial y}\frac{\partial u}{\partial y} - \frac{Ha^2}{1+m^2}(u + mw) - Mu, \tag{9}$$

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial y} = f_1(\theta)\frac{\partial^2 w}{\partial y^2} + \frac{\partial f_1(\theta)}{\partial y}\frac{\partial w}{\partial y} - \frac{Ha^2}{1+m^2}(w - mu) - Mw, \tag{10}$$

$$t = 0 : u = w = 0, \tag{11a}$$

$$t > 0 : y = -1, \quad u = w = 0, \tag{11b}$$

$$t > 0 : y = 1, \quad u = 1, \quad w = 0, \tag{11c}$$

$$\frac{\partial \theta}{\partial t} + S\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}f_2(\theta)\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{\Pr}\frac{\partial f_2(\theta)}{\partial y}\frac{\partial \theta}{\partial y} + Ecf_1(\theta)\left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right) + \frac{EcHa^2}{1+m^2}(u^2 + w^2), \tag{12}$$

$$t = 0 : \theta = 0, \tag{13a}$$

$$t > 0 : \theta = 0, \quad y = -1, \tag{13b}$$

$$t > 0 : \theta = 1, \quad y = 1. \tag{13c}$$

Equations (9), (10) and (12) represent a system of coupled non-linear partial differential equations which are solved numerically under initial and boundary conditions given by Eqs. (11a, 11b, 11c) and (13a, 13b, 13c) using the method of finite differences. A linearization technique is first applied to replace the nonlinear terms at a linear stage with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank–Nicolson implicit method is used at two successive time levels [28]. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas-algorithm [28]. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the y-direction. The diffusion terms are replaced by the average of central differences at two successive time-levels. The computational domain is divided into meshes each of dimension Δt and Δy in time and space, respectively. We define the variables $A = \partial u/\partial y$, $B = \partial w/\partial y$ and $H = \partial \theta/\partial y$ to reduce the second order differential Eqs. (9), (10) and (12) to first order differential equations, and an iterative scheme is used at every time step to solve the linearized system of difference equations. All calculations are carried out for the non-dimensional variables and parameters given by, $G = 5$, $\alpha = 1$, $Pr = 1$, and $Ec = 0.2$ where G and α are related to the externally applied pressure gradient and where the chosen given values for Pr and Ec

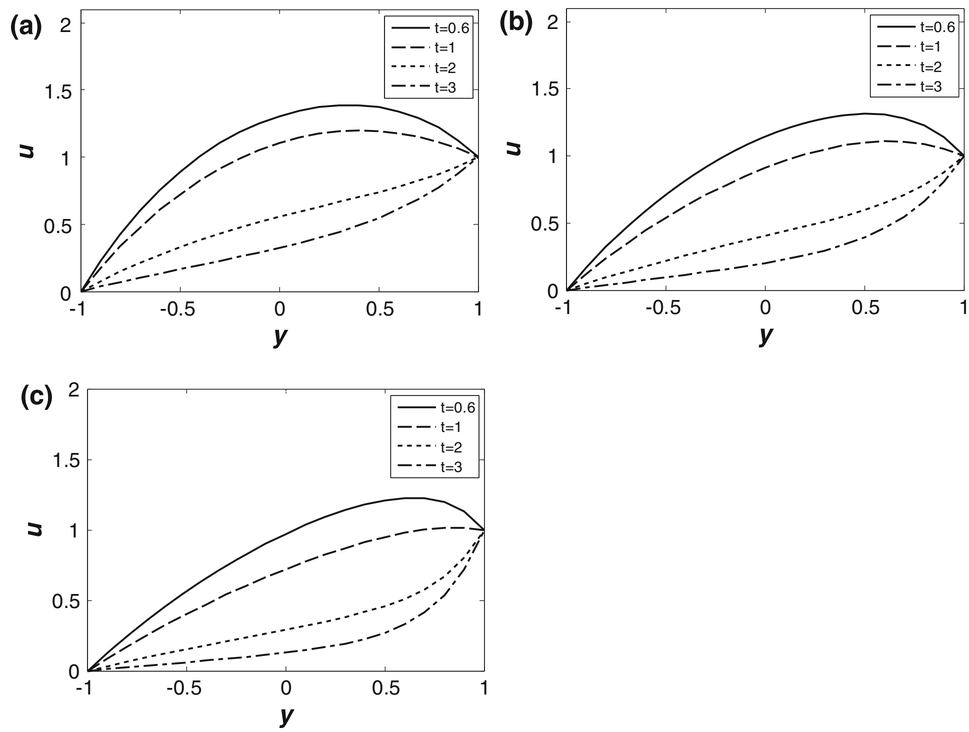

Fig. 1 Evolution of the profile of u: (**a**) $S = 0$; (**b**) $S = 1$ and (**c**) $S = 2$ ($Ha = 1,\ m = 1,\ M = 1,\ a = 0.5,\ b = 0.5$)

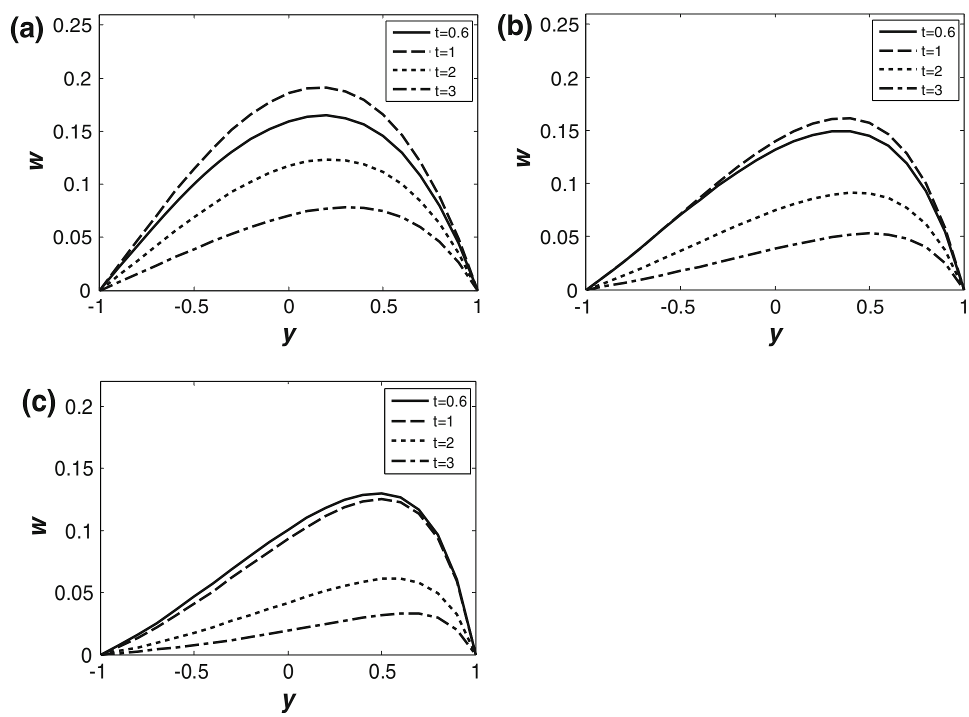

Fig. 2 Evolution of the profile of w: (**a**) $S = 0$; (**b**) $S = 1$ and (**c**) $S = 2$ ($Ha = 1,\ m = 1,\ M = 1,\ a = 0.5,\ b = 0.5$)

are suitable for steam or water vapor. Grid-independence studies show that the computational domain $0 < t < \infty$ and $-1 < y < 1$ is divided into intervals with step sizes $\Delta t = 0.0001$ and $\Delta y = 0.005$ for time and space respectively. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when all of the unknowns u, w, A, B, θ and H for the last two approximations differ from unity by less than 10^{-6} for all values of y in $-1 < y < 1$ at every time step. Less than 7 approximations are required to satisfy this convergence criteria for all ranges of the parameters studied here.

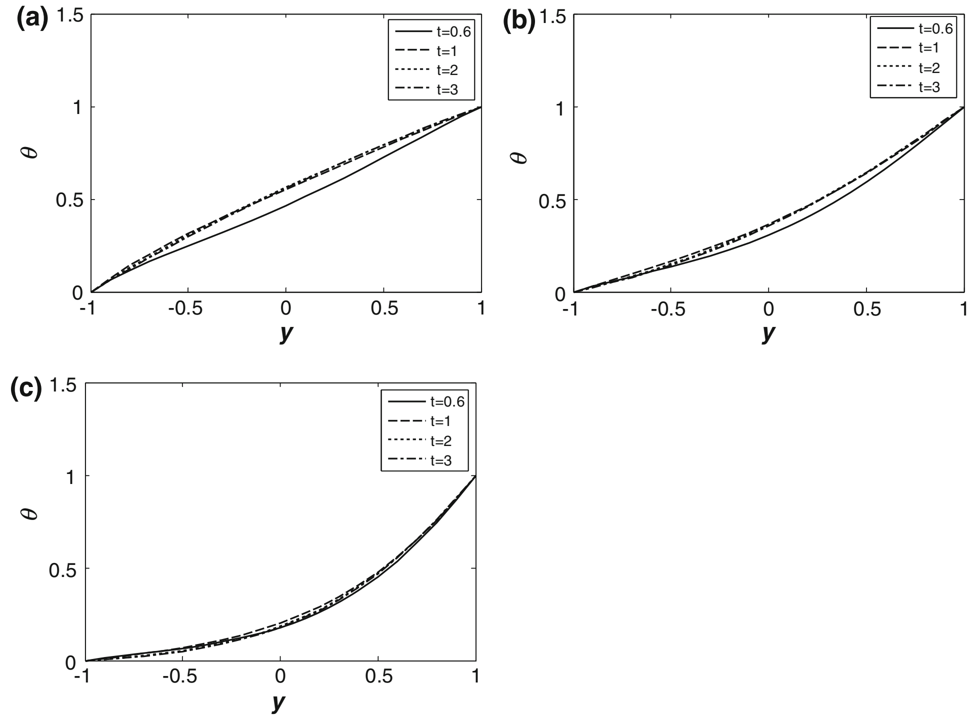

Fig. 3 Evolution of the profile of θ: (**a**) $S = 0$; (**b**) $S = 1$ and (**c**) $S = 2$ ($Ha = 1,\ m = 1,\ M = 1,\ a = 0.5,\ b = 0.5$)

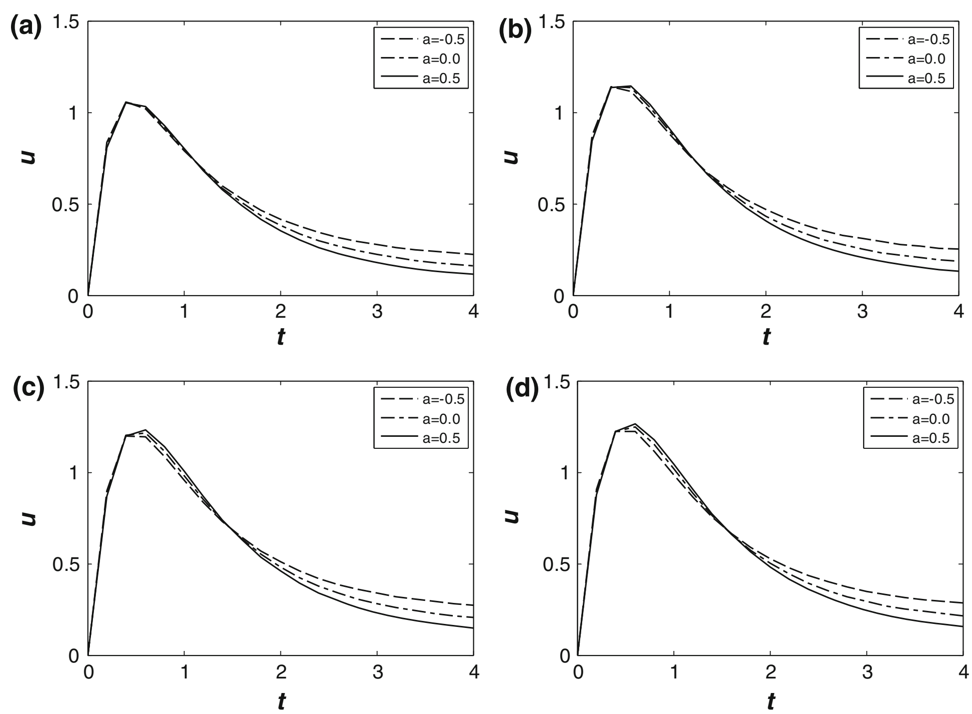

Fig. 4 Evolution of u at $y = 0$ for various values of a, and m: (**a**) $m = 0$; (**b**) $m = 1$; (**c**) $m = 2$ and (**d**) $m = 3$ ($Ha = 1,\ M = 1,\ S = 1,\ b = 0$)

3. Results and discussion

Figures 1, 2 and 3 show of the profiles of velocity components u and w and temperature respectively for various values of the suction parameter S and for $Ha = 1$, $m = 1$, $M = 1$, $a = 0.5$ and $b = 0.5$. Figures 1 and 3 show that velocity component u and the temperature distributions reach steady state monotonically whereas the velocity component w do not reach the steady state monotonically as shown in Fig. 2. The velocity component w increases with time till a maximum value and then decrease up to steady state under the effect of decaying pressure gradient.

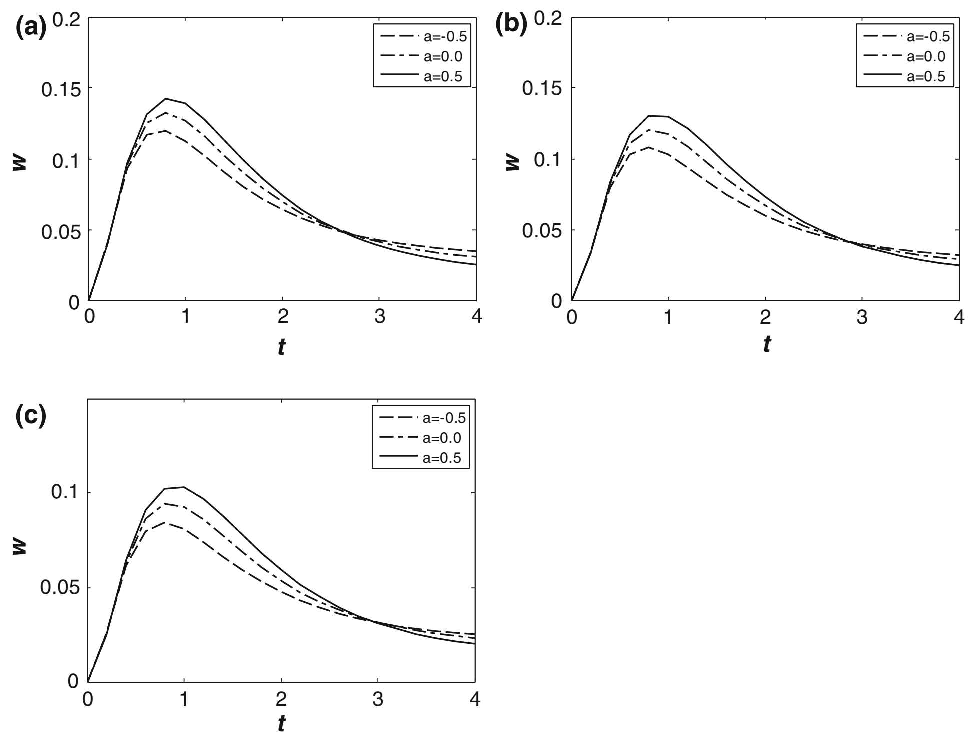

Fig. 5 Evolution of w at $y = 0$ for various values of a, and m: (**a**) $m = 1$; (**b**) $m = 2$ and (**c**) $m = 3$ ($Ha = 1$, $M = 1$, $S = 1$, $b = 0$)

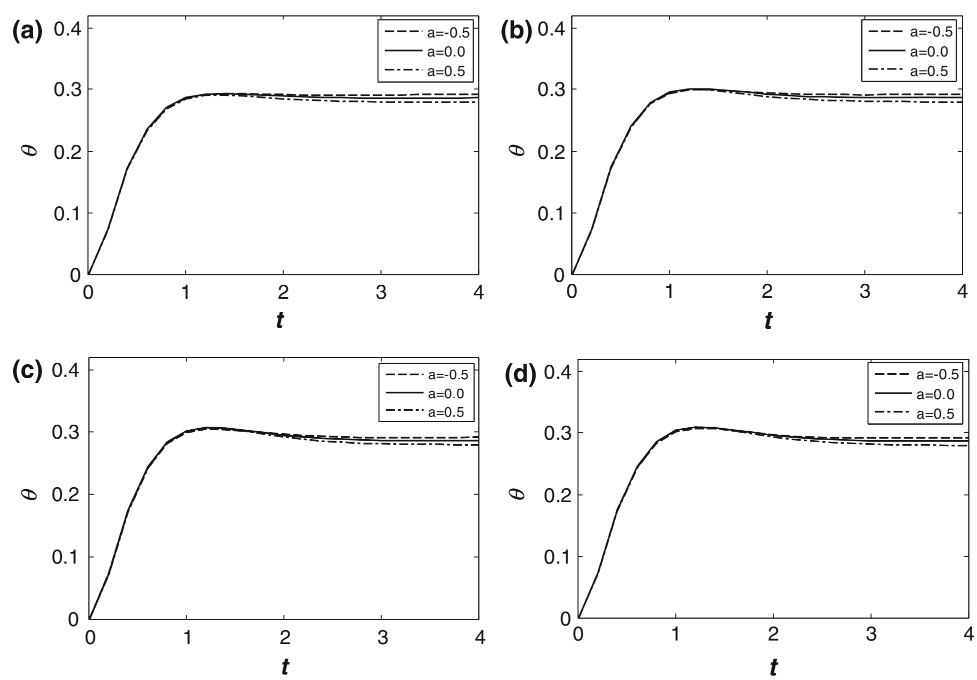

Fig. 6 Evolution of θ at $y = 0$ for various values of a, and m: (**a**) $m = 0$; (**b**) $m = 1$; (**c**) $m = 2$ and (**d**) $m = 3$ ($Ha = 1$, $M = 1$, $S = 1$, $b = 0$)

Figure 2 indicates influence of the suction velocity in controlling the overshooting occurs in velocity component w with time progression. The velocity component u reaches steady state faster than w which, in turn, reaches steady state faster than θ. This is expected as u is the source of w, while both u and w are sources of θ. The suction parameter has a great effect on the steady state time for these profiles.

Figures 4, 5 and 6 present the evolution of velocity components u and w and temperature, respectively, at the

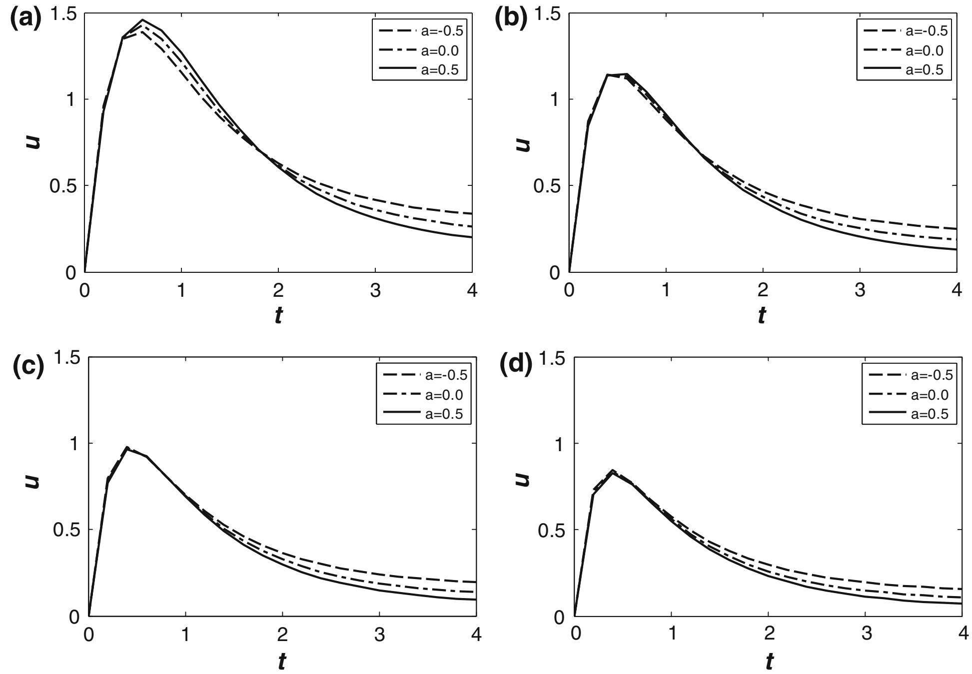

Fig. 7 Evolution of u at $y = 0$ for various values of a, and M: (**a**) $M = 0$; (**b**) $M = 1$; (**c**) $M = 2$ and (**d**) $M = 3$ ($Ha = 1,\ m = 1,\ S = 1,\ b = 0$)

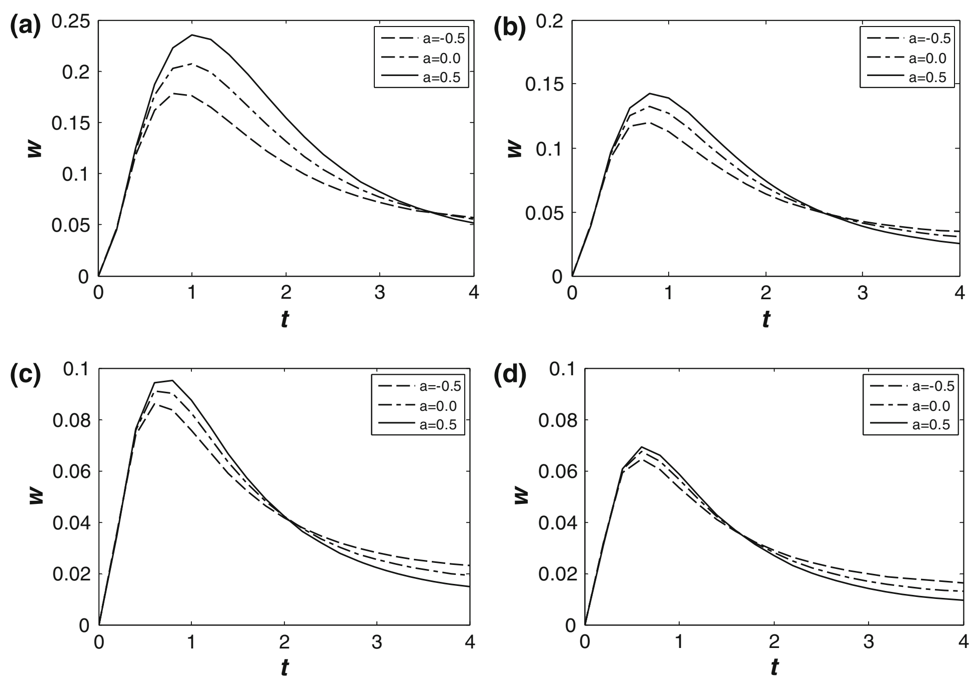

Fig. 8 Evolution of w at $y = 0$ for various values of a, and M: (**a**) $M = 0$; (**b**) $M = 1$; (**c**) $M = 2$ and (**d**) $M = 3$ ($Ha = 1,\ m = 1,\ S = 1,\ b = 0$)

centre of the channel ($y = 0$) for different values of m and a and for $b = 0$, $S = 1$, $Ha = 1$ and $M = 1$. Figure 4 shows that u increases with m for all values of a which can be attributed the fact that an increase in m decreases the effective conductivity ($\sigma/(1 + m^2)$) and hence the magnetic damping. Figure 4 also, indicates that the variation of u with the viscosity parameter a depends on time t which results in a crossover in the u–t profiles. Figure 5 shows w decreases with increasing m for all values of a as a result of the damping effect. This can be understood by studying the term $(-(w - mu)/(1 + m^2))$ in Eq. (10), which is the source term of w. For some time w is very small and this

Table 1 Variation of the steady state Nusselt number at the lower plate Nu_L for various values of a and b ($Ha = 1,\ m = 0,\ S = 0,\ M = 0$)

Nu_l	$a = -0.5$	$a = -0.1$	$a = 0.0$	$a = 0.1$	$a = 0.5$
$b = -0.5$	0.5317	0.5210	0.5179	0.5149	0.5051
$b = -0.1$	0.5085	0.4975	0.4945	0.4916	0.4816
$b = 0.0$	0.5110	0.4998	0.4968	0.4939	0.4839
$b = 0.1$	0.5151	0.5037	0.5007	0.4977	0.4878
$b = 0.5$	0.5362	0.5243	0.5213	0.5185	0.5086

Table 2 Variation of the steady state Nusselt number at the upper plate Nu_U for various values of a and b ($Ha = 1,\ m = 0,\ S = 0,\ M = 0$)

Nu_u	$a = -0.5$	$a = -0.1$	$a = 0.0$	$a = 0.1$	$a = 0.5$
$b = -0.5$	0.6766	0.6736	0.6767	0.6806	0.7003
$b = -0.1$	0.4134	0.4156	0.4178	0.4202	0.4318
$b = 0.0$	0.3717	0.3744	0.3764	0.3787	0.3893
$b = 0.1$	0.3358	0.3388	0.3407	0.3428	0.3526
$b = 0.5$	0.2294	0.2329	0.2345	0.2363	0.2438

term may be approximated to $(mu/(1 + m^2))$, which decreases with increasing m. Figure 5 indicates also, that variation of w with the viscosity parameter a depends on time t which results in a crossover in the w–t profiles.

Figure 6 shows the variation of θ with a depends on time t. For small t, increasing a decreases θ while for large t increasing a increases θ. It is observed that the effect of m on θ depends on t. When $m > 1$, increasing m decreases θ slightly at small times but increases θ at large times. This is because when t is small, u and w are small and an increase in m results in an increase in u but a decrease in w, so the Joule dissipation which is proportional also to $(1/(1 + m^2))$ decreases. When t is large, u and w increase and so do the Joule and viscous dissipations. It is difficult to predict the effect of a on θ, because while increasing a increases the velocities and the velocity gradients, it decreases the function f_1. All the same, Fig. 6 indicates that increasing a increases θ and its effect on the steady state time for θ is negligible.

Figures 7, 8 and 9 present the evolution of the velocity components u and w and temperature θ, respectively, at the centre of the channel ($y = 0$) for different values of M and a and for $b = 0, S = 1, Ha = 1$ and $M = 1$. As depicted in Figs. 7 and 8, increasing M decreases u and w for all values a due to increase in resisting force. Increasing M decreases θ as a result of reducing the viscous dissipation.

Tables 1 and 2 show the influence of the parameters a and b on the Nusselt numbers at both walls Nu_L and Nu_U, respectively, for $Ha = 1$, $m = 0$, $S = 0$ and $M = 0$. Increasing b decreases both Nu_L and Nu_U. On the other hand, increasing a increases Nu_L but decreases Nu_U. The influence of b on Nu_U is more pronounced than its effect on Nu_L.

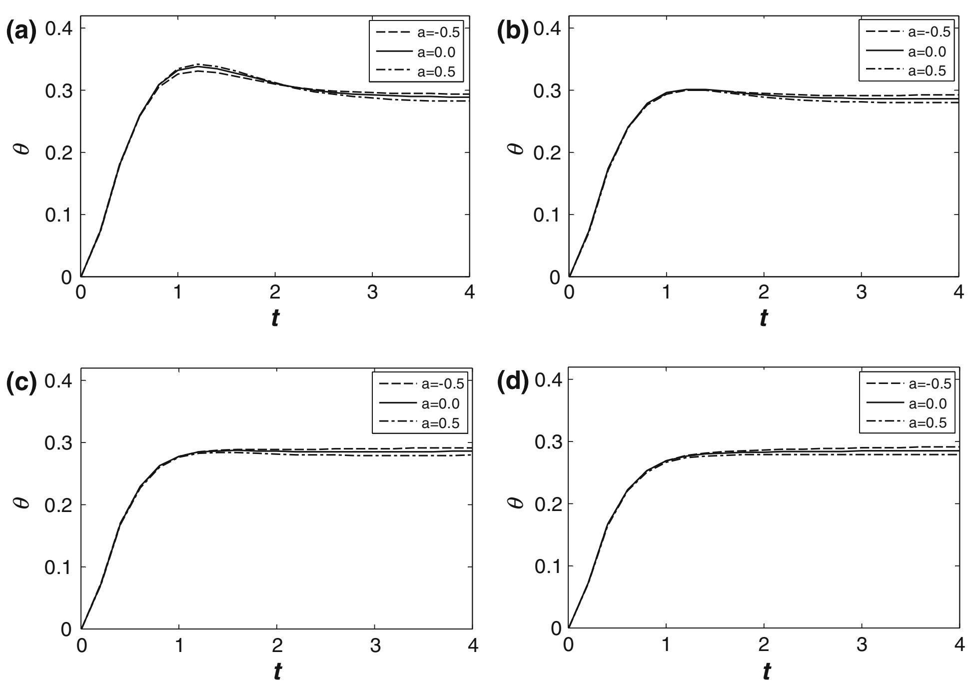

Fig. 9 Evolution of θ at $y = 0$ for various values of a, and M: (**a**) $M = 0$; (**b**) $M = 1$; (**c**) $M = 2$ and (**d**) $M = 3$ ($Ha = 1,\ m = 1,\ S = 1,\ b = 0$)

4. Conclusions

The transient generalized Couette MHD flow with heat transfer through a porous medium between two parallel plates was investigated considering the Hall current and variable properties. The viscosity and thermal conductivity of the fluid are temperature dependent. Effect of porosity parameter M, the Hartmann number Ha, the Hall parameter m, the viscosity variation parameter a and the thermal conductivity variation parameter b on velocity and temperature fields at the centre of the channel are discussed. Introducing the Hall term gives rise to a velocity component w in the z-direction and affects the main velocity u in the x-direction. It is found that parameter a has a marked effect on the velocity components u and w for all values of M. The porosity parameter M has a marked effect on the velocity and temperature distributions, however, its effect on the velocity and its steady state time is more pronounced than that for the temperature.

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