Electron Dynamics in Presence of Static Helical Magnet Inside Circular Waveguide

B. F. Mohamed, A. M. Gouda, and L. Z. Ismail

***Abstract*—The dynamics of an electron in the fields associated with a TE electromagnetic wave propagating inside a circular waveguide is analytically studied. The motion of this electron along the axis of the waveguide is investigated in the existence of a helical magnet (in which the field is perpendicular to the axis of the waveguide and rotating as a function of position along the magnet). It is shown that it can be accelerated due to its interaction with polarized fields of microwave radiation propagating along the waveguide. The fields for the lowest order TE_{11} modes and the deflection angle of electron trajectory, due to these fields, are obtained. Also, an expression of the acceleration gradient of the electron and its energy gain are evaluated for different intensities and frequencies of the microwave.**

***Index Terms*—Electron acceleration, polarized microwave, waveguide.**

I. Introduction

In RECENT years, a number of studies have been dedicated to the dynamics of electrons in electromagnetic fields depending on the basis of the Newton equation with the Lorentz force [1], [2]. This subject is of great interest due to its diverse applications to particle acceleration in the field of nuclear physics, thermonuclear fusion research, and high-energy particle physics.

Also, another means for coupling electromagnetic energy to a particle has been discussed as a possible origin of cosmic rays [3]. The same mechanism has been proposed for most of the investigations, including the direct acceleration scheme which makes use of short-pulse high-intensity lasers [4], [5]. However, some of the researchers have made theoretical, as well as experimental, attempts for particle acceleration by using microwave radiation [6]–[9]. The description of the electron dynamics in a high-frequency field is complicated because of the large number of oscillations. The problem becomes more intricate analytically with the addition of an extra electric or magnetic field.

In 1982, Pantell and Smith [7] analyzed the interaction of a slow microwave signal and a laser light beam and calculated the energy gradient for an electron as a function of fields of both the electromagnetic waves.

Manuscript received May 21, 2010; revised September 11, 2010, October 20, 2010, and November 28, 2010; accepted December 29, 2010. Date of publication February 17, 2011; date of current version March 9, 2011.

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Digital Object Identifier 10.1109/TPS.2011.2104369

The energy gain by an electron during its motion in the field of a fundamental TE_{10} mode excited by a microwave in a rectangular waveguide was analyzed by Malik [10]. He has also studied particle acceleration due to two fundamental TE_{10} modes which interfere obliquely [11] and obtained the expressions for the energy gain and the acceleration gradient. Moreover, the microwave breakdown threshold in a circular waveguide excited in the lowest order TE_{11} mode has been investigated by Tomala *et al.* [12]. Pioneer experiments on inverse free-electron laser (IFEL) acceleration depending on a microwave accelerating structure which consists of a TE_{11} rotating waveguide mode and an axial magnetic field have been reported by Yoder *et al.* [9].

In the present analysis, we study the dynamics of an electron in the fields associated with a TE electromagnetic wave propagating inside a circular waveguide. We investigate the possibility of acceleration for this electron inside the waveguide when it is injected along the direction of propagation of the TE_{11} mode excited by microwave radiation. It is also considered the motion of this electron along the axis of a helical magnet (in which the field is perpendicular to the axis of the waveguide and rotating as a function of position along the magnet).

II. Electron Trajectory and Energy Gain

A. Electron Dynamics

A high-intensity microwave of frequency ω is used to excite the lowest order TE_{11} mode (with $E_z = 0$) in an evacuated circular waveguide of radius a. Therefore, one can obtain the field components of this mode from the following Maxwell equations [13] (with time dependence as $e^{-i\omega t}$):

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\vec{E} = 0 \tag{1}$$

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\vec{H} = 0 \tag{2}$$

so that, inside a waveguide of ideally conducting material, the fields of the excited TE_{11} mode can be obtained under the condition that the tangential component of the electric field must vanish on the boundary as follows:

$$E_r = \left(\frac{i\omega H_o}{rk_c^2}\right) J_1(k_c r)\sin\phi e^{-i\beta_g z}$$

$$E_\varphi = \left(\frac{i\omega H_o}{k_c}\right) J_1'(k_c r)\cos\phi e^{-i\beta_g z}$$

$$H_r = \left(\frac{-ik_g H_o}{k_c}\right) J_1'(k_c r)\cos\phi e^{-i\beta_g z}$$

$$H_\varphi = \left(\frac{ik_g H_o}{rk_c^2}\right) J_1(k_c r)\sin\phi e^{-i\beta_g z}$$
$$H_z = H_o J_1(k_c r)\cos\phi e^{-i\beta_g z}. \tag{3}$$

Here, H_o is the amplitude of the magnetic field intensity which is related to the intensity of the microwave radiation, k_c is the wavenumber corresponding to the cutoff condition (i.e., $J_1'(k_c a) = 0$, where J_1 is the first-order cylindrical Bessel function), and β_g is the propagation constant with $k_c^2 = \omega^2\mu\varepsilon - \beta_g^2$. It is noticed that the strength of the fields depends on both radius r and azimuthal angle φ such that its magnitude is largest in the center of the waveguide and decreases radially outward.

Now, we discuss the dynamics of an electron in the fields of this mode when the electron is injected along the propagation mode inside the waveguide. The electron is deflected from the z-axis due to the force of the field $E_\perp$ and also the force due to the field $H_\perp$ which also acts on it at the same time. Moreover, we consider an external magnetic field that is always perpendicular to the mean particle motion but whose direction rotates about that axis as a function of position along that axis. This field can be produced by a magnet referred to as "helical magnet" and given by $B = B_o \exp(2\pi iz/\Lambda)$, where Λ is the length of one turn of the helix and B_o is the helical magnetic field induction [6].

Under the force of all these fields, the deflection angle η of the electron during its motion along the z-axis can be calculated from the following electron equation of motion:

$$\frac{d\vec{p}}{dt} = -e\left[\vec{E} + \vec{v}\times(\mu_o\vec{H} + \vec{B})\right]. \tag{4}$$

The rate of change for transverse momentum $P_\perp$ of an electron injected with momentum P_z in the z-direction is calculated from the following:

$$\frac{dP_\perp}{dz} = -\frac{e}{v_z}\left[E_\perp + v_z \times (\mu_o H_\varphi + B)\right] \tag{5}$$

where

$$E_\perp = \frac{i\omega}{k_c} H_o e^{-i\beta_g z}\left[\left(\frac{1}{rk_c}\right)^2 J_1^2 \sin^2\phi + J_1'^2\cos^2\phi\right]^{\frac{1}{2}}.$$

Equation (5) has been obtained under $|P_\perp| \ll P_z$ (which means that the propagation of the electron is solely specified in the z-direction); then, one can put the values of the field components and integrate them to find $P_\perp$ at $\varphi = \pi/2$. Therefore, the angle of deflection can be evaluated as

$$\tan\eta = \frac{P_\perp}{P_z} = \left(\frac{e\mu_o H_o}{m_e v_z^2 k_c^2 \beta_g r}\right) J_1(k_c r)(\omega - \beta_g v_z)\cos(\beta_g z) + \frac{eB_o\Lambda}{2\pi m_e v_z}\sin\left(\frac{2\pi}{\Lambda}z\right). \tag{6}$$

B. Electron Acceleration

Now, we analyze the electron acceleration during its motion in the field of the TE_{11} mode. The acceleration gradient and energy gain by the electron can be obtained by considering the following momentum equation and energy equation:

$$\frac{d}{dt}(m_e\gamma\vec{v}) = -e\left[\vec{E} + \vec{v}\times(\mu_o\vec{H} + \vec{B})\right] \tag{7}$$

$$\frac{d}{dt}(m_e\gamma c^2) = -e\vec{v}\cdot\vec{E}. \tag{8}$$

We transform the coordinates into $\xi = v_g t - z$ in (7) and (8), and then, they are simplified under the aforementioned condition ($|P_\perp| \ll P_z$) to obtain the following relations:

$$\frac{d(\gamma v_r)}{d\xi} = \frac{-e}{m_e(v_g - v_z)}\left[E_r - v_z(\mu_o H_\phi + B_0)\right] \tag{9}$$

$$v_r = \frac{-m_e(v_g - v_z)}{e(\mu_o H_\phi + B_o)}\frac{d(\gamma v_z)}{d\xi} \tag{10}$$

$$\frac{d\gamma}{d\xi} = \frac{-eE_r}{m_e c^2(v_g - v_z)} v_r. \tag{11}$$

Equations (10) and (11) give the following relation:

$$\frac{d^2\gamma}{d\xi^2} = \frac{E_r}{c^2(\mu_o H_\varphi + B_0)}\frac{d^2(\gamma v_z)}{d\xi^2}. \tag{12}$$

Therefore, the radial component of the electron momentum equation (9) with (10) gives

$$(v_g - v_z)\gamma\frac{d^2(\gamma v_z)}{d\xi^2} + v_g\left(\frac{d\gamma}{d\xi}\right)\left(\frac{d(\gamma v_z)}{d\xi}\right) - \left[\frac{d(\gamma v_z)}{d\xi}\right]^2 = \frac{e^2(\mu_o H_\varphi + B_0)}{m_e^2(v_g - v_z)}\left[E_r - v_z(\mu_o H_\varphi + B_0)\right]. \tag{13}$$

Now, from (12) and (13) and for the sake of simplicity, γ and v_z (in the case of the microwave [11]) can be considered as slowly varying. Therefore, the first term in (13) is dominated over the other two terms. Then, (13) can be integrated considering ξ as the dummy variable [9], and putting the field components in it, we have

$$\left(\frac{d\gamma}{d\xi}\right) = \sqrt{2R}\left[\ln\frac{\gamma}{\gamma_o}\right]^{\frac{1}{2}} \tag{14}$$

where

$$R = \frac{e^2\omega\mu_o^2 H_o^2}{m_e^2 c^2(v_g - v_z)^2 r^2 k_c^4} J_1^2(k_c r)\sin^2\phi \times \Bigg\{\left[\omega - \beta_g v_z\right]\sin^2(\beta_g z) - \frac{r v_z k_c^2 B_o}{\mu_o H_o J_1(k_c r)\sin\phi}\sin\left((\beta_g - 2\pi/\Lambda)z\right)\Bigg\}.$$

Multiplying (14) with $(m_e c^2/e)$, one can obtain the energy ($U = m_e\gamma c^2$) gradient (in eV/m) achieved by the electron as

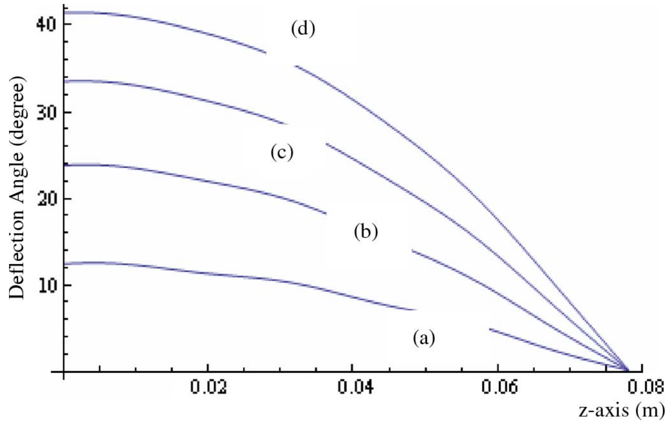

Fig. 1. Variation of the deflection angle along the z-axis for different microwave intensities of (a) 10^4, (b) 2×10^4, (c) 3×10^4, and (d) 4×10^4 W/m^2 with microwave frequency $f = 2$ GHz, helical magnetic field of 0.1 T, and electron kinetic energy of 140 keV.

follows:

$$\left(\frac{dU}{dz}\right)_{\frac{\mathrm{eV}}{\mathrm{m}}} = \frac{\sqrt{2}c\omega\mu_o H_o}{rk_c^2(v_g - v_z)} J_1(k_c r)\sin(\beta_g z)\left[\ln\frac{\gamma}{\gamma_o}\right]^{\frac{1}{2}} \times \left\{1 - \frac{\beta_g v_z}{\omega} - \frac{r v_z k_c^2 B_o}{\omega\mu_o H_o J_1(k_c r)}\left[\frac{\sin\left((\beta_g - \frac{2\pi}{\Lambda})z\right)}{\sin(\beta_g z)}\right]\right\}^{\frac{1}{2}}.$$

It is worth mentioning that, for deducing (6) and (15), we considered that the electron is injected at angle $\varphi = \pi/2$. This expression shows that the acceleration gradient depends on the parameters of the waveguide and the properties of the microwave.

III. Numerical Results

The motion of the injected electron is investigated by studying the expressions of the deflection angle and energy gradient. It can be seen from (6) that the deflection angle depends on the amplitude of the magnetic field H_o (which is related to the intensity of the incident microwave). It also changes sinusoidally along the z-axis and is maximum at the entrance of the waveguide. In addition, (15) illustrates that the energy gradient of the electron could be increased rapidly along the z-axis depending on different parameters.

In this section, numerical evaluation of the formulas of the deflection angle and the energy gradient derived in the previous sections is employed to further explore the behavior of the motion of the electron.

A numerical example taken to investigate the propagation of the dominant TE_{11} mode in a circular waveguide of radius $r = 5$ cm is considered. It can be found that the microwave frequency f should be higher than 1.75×10^9 Hz, which corresponds to cutoff wavenumber $k_c = 36.82$ m^{-1}. The other used parameters, in our case, the initial energy of the injected electron (related to the initial velocity) and the helical magnetic field, are taken to be 140 keV and 0.1 T, respectively.

Fig. 1 shows the variation of the deflection angle of the electron during its motion along the z-axis for different microwave intensities. It is noted that the deflection angle is directly proportional to the intensity of the microwave and that the angle is maximum at the entrance of the waveguide, decreases along its axis, and reaches zero at $z = \lambda_g/4$.

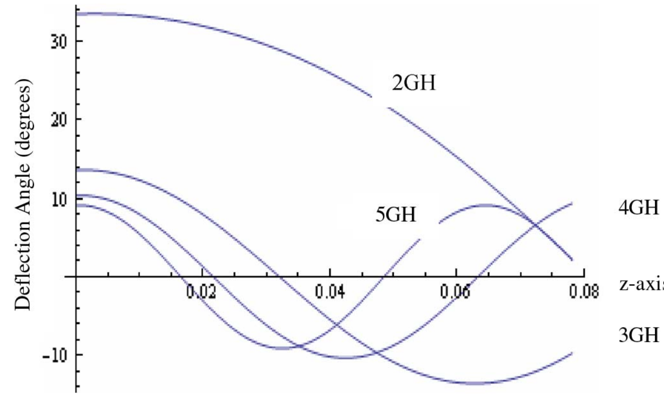

Fig. 2. Variation of the deflection angle along the z-axis for different microwave frequencies with a microwave intensity of 3×10^4 W/m^2. The other parameters are the same as in Fig. 1.

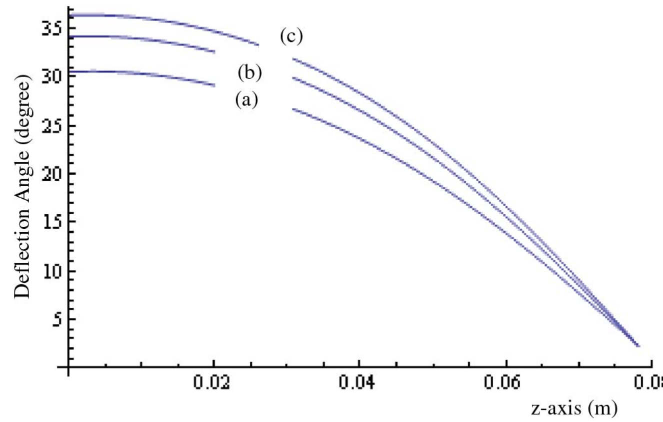

Fig. 3. Deflection angle along the z-axis at different points of injections of (a) $3a/4$, (b) $a/2$, and (c) $a/4$ with a microwave intensity of 3×10^4 W/m^2. The other parameters are the same as in Fig. 1.

The dependence of the deflection angle on the frequency of the microwave along the z-axis is also investigated in Fig. 2 for a microwave intensity of 3×10^4 W/m^2. It can be noted that the angle of deflection is decreasing with the increase of the microwave frequency and it is approaching to zero at smaller distance with higher frequency along the z-axis. This is due to that λ_g (the guide wavelength) depends on the microwave frequency.

Fig. 3 shows that the dynamics of the electron and the deflection of its motion strongly depends on the point of injection for this electron at the entrance of the waveguide. It shows that the angle of deflection is decreasing when the electron is injected at longer distance from the center of the entrance of the waveguide. One can note from Fig. 4 that the deflection angle changes inversely proportionally with the kinetic energy of the injected electron in the front of the waveguide along its axis. Moreover, the effect of the helical magnetic field on the deflection angle of the electron can also be investigated in Fig. 5. It shows that the deflection is increased with the strong magnetic field.

On the other hand, the acceleration gradient of the electron during its motion along the axis of the waveguide has also

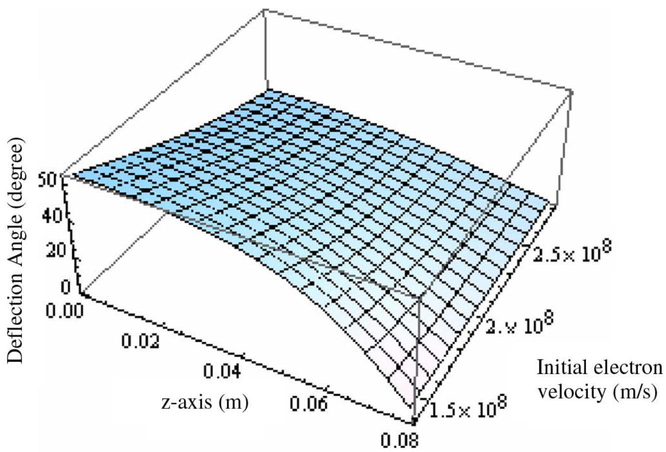

Fig. 4. Effect of the initial kinetic energy of the electron on the deflection angle along the z-axis for a microwave intensity of 3×10^4 W/m^2. The other parameters are the same as in Fig. 1.

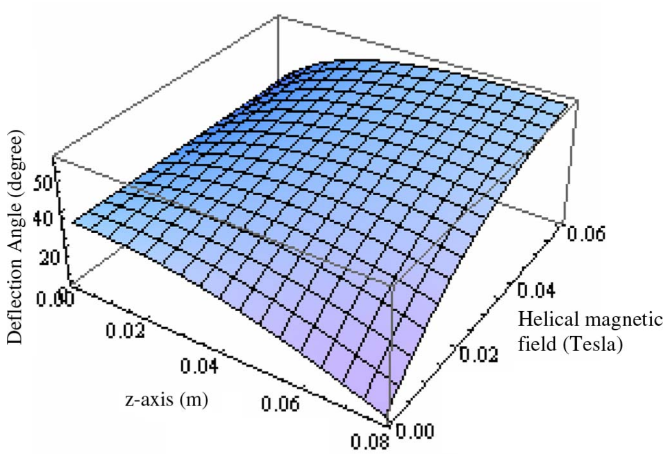

Fig. 5. Effect of the helical magnetic field on the deflection angle along the z-axis with a microwave intensity of 3×10^4 W/m^2. The other parameters are the same as in Fig. 1.

been studied with different parameters. First, Fig. 6 shows the effect of different intensities of the microwave on the energy gradient along the z-axis for microwave frequency $f = 2$ GHz and electron kinetic energy of 140 keV. It can be seen that the energy gradient is directly proportional to the microwave intensity. Also, the gradient started from a point closer to the front of the waveguide with higher intensity.

The effect of microwave frequency on the maximum acceleration gradient is also shown in Fig. 7. It is clear that the acceleration gradient increases with microwave frequency and makes to decrease the acceleration length (the distance on which the energy gradients become maximum).

Here, it can be seen that the values of λ_g and v_g decrease as the microwave frequency is increased. These results on the variation of β_g (or λ_g) are similar to the ones obtained by Shenggang *et al.* [14] and Xiao *et al.* [15] in a circular waveguide for the similar types of the modes.

The plot in Fig. 8 illustrates the dependence of the energy gradient on the electron initial energy of the electron. The injected electron with higher kinetic energy also increases the energy gradient.

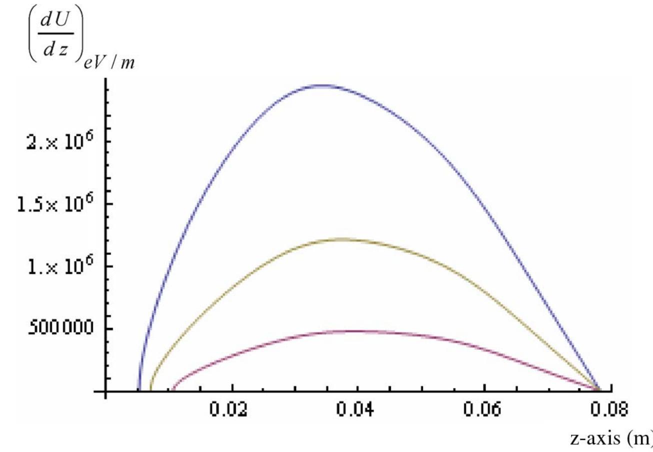

Fig. 6. Variation of the energy gradient along the z-axis for different intensities of the microwave of (a) 10^4, (b) 2×10^4, and (c) 4×10^4 W/m^2. The other parameters are the same as in Fig. 1.

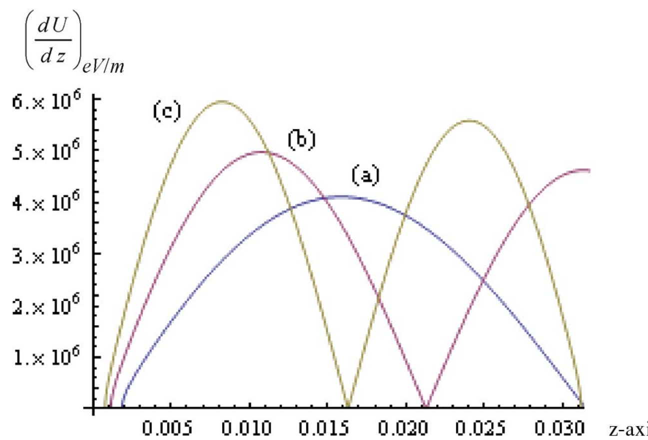

Fig. 7. Variation of the energy gradient along the z-axis for different microwave frequencies of (a) 3, (b) 4, and (c) 5 GHz, with a microwave intensity of 5×10^4 W/m^2. The other parameters are the same as in Fig. 1.

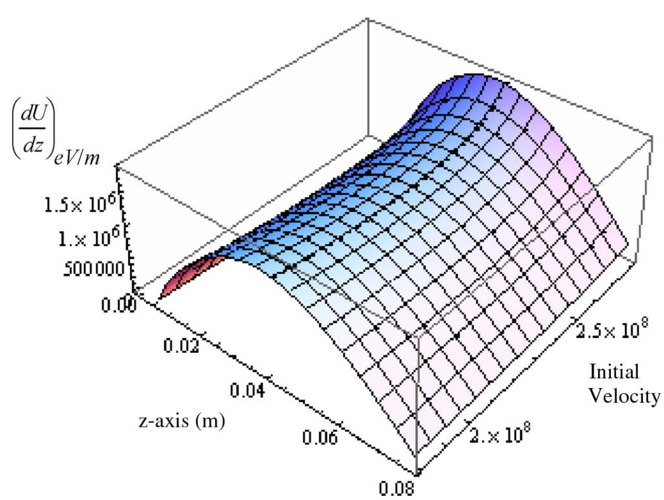

Fig. 8. Effect of the initial energy of the electron on the energy gradient along the z-axis with a microwave intensity of 3×10^4 W/m^2. The other parameters are the same as in Fig. 1.

In summary, it can be deduced that the acceleration gradient and, hence, the energy gain attained by the electron could be enhanced by optimizing the parameters of the microwave and the initial energy. Therefore, it must be taken into account

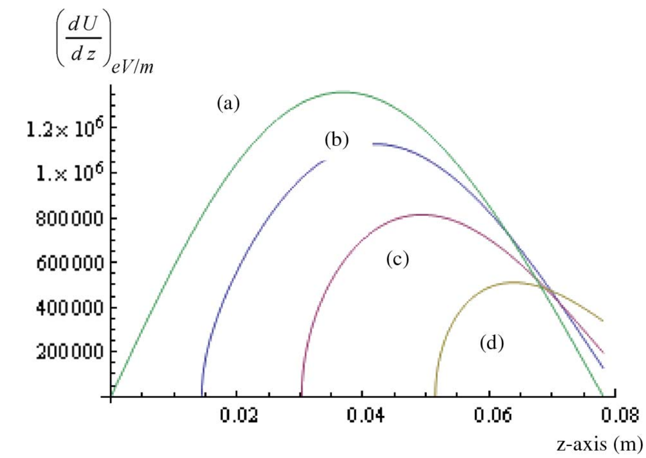

Fig. 9. Variation of the energy gradient along the z-axis for different helical magnetic fields B_0 of (a) 0, (b) 0.1, (c) 0.2, and (d) 0.3 T, with a microwave intensity of 3×10^4 W/m^2. The other parameters are the same as in Fig. 1.

that, when we increase the intensity of the microwave, it must increase the microwave frequency to keep the amplitude of oscillation (through the deflection angle) less than the radius of the waveguide.

Finally, the effect of the helical magnetic field on the energy gradient is shown in Fig. 9. The energy gradient has the highest value in the case of without a magnetic field ($B_0 = 0$). It is noted that the magnetic field makes to quench the gradient and away the starting point of acceleration from the entrance of the waveguide. The effect of the length of one turn of the helix on the energy gradient is examined which shows that the gradient has weak effect with the change of length.

Here, for example, it can be seen from Figs. 6 and 7 (as we have mentioned) that, when the microwave frequency is 2 GHz, the acceleration gradient corresponding to a microwave intensity of 4×10^4 W/m^2 is 2.6 MeV/m, but at the same frequency, there is no acceleration gradient corresponding to the intensity of 5×10^4 W/m^2. However, by increasing the microwave frequency to 3 GHz or more with the microwave intensity still 5×10^4 W/m^2, it is possible to obtain acceleration gradient more than 4.2 MeV/m. Therefore, for realizing a larger acceleration gradient, the microwave frequency should be higher with higher microwave intensity. Also, it is clear that the energy gradient should be enhanced with the increase of the microwave frequency. These results are in agreement with that of Malik [10], [11] in a rectangular waveguide for the TE$_{10}$ mode. Moreover, the parameters of the experiment by Yoder *et al.* [9], on IFEL acceleration with a circular waveguide of radius 3.14 cm, are applied and analyzed in the present case. It is found that there is a close agreement between the theoretical and experimental results (under the same conditions), where the theory predicts an output energy of 6.47 MeV compared to 6 MeV in the experiment of Yoder *et al.* [9].

IV. Conclusion

We have presented exact analytic solutions for the dynamics of a single electron injected along the propagation direction of a microwave radiation inside a circular waveguide. It has been investigated that the electron is deflected due to the field components of the TE$_{11}$ mode of this microwave, and at the same time, it is accelerated by these fields.

The effect of the different parameters of the microwave radiation (frequency and intensity) and the initial energy of the injected electron on the angle of deflection and the acceleration gradient has been studied. It can be noted that the deflection angle of the electron is maximum at the entrance of the waveguide (at $z = 0$), and it gets lower with the increasing energy (or v_z) of the electron along the z-axis. Also, when the electron travels and completes a distance $\lambda_g/4$, it still deflects with a small angle equal to

$$\tan^{-1}\left[\frac{eB_0\Lambda}{2\pi m_e v_z}\sin\left(\frac{\pi\lambda_g}{2\Lambda}\right)\right].$$

Finally, it is also found that a large amount of energy gain and the larger acceleration gradient for the injected electron along the waveguide can be achieved by using the present mechanism when the high intensity and frequency of the microwave are used with high initial kinetic energy. Since large gradients are achieved when the electron is injected with higher energy, it is possible to use the proposed mechanism in multiple stages to obtain a large energy gain for the electron by optimizing all the other parameters.

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