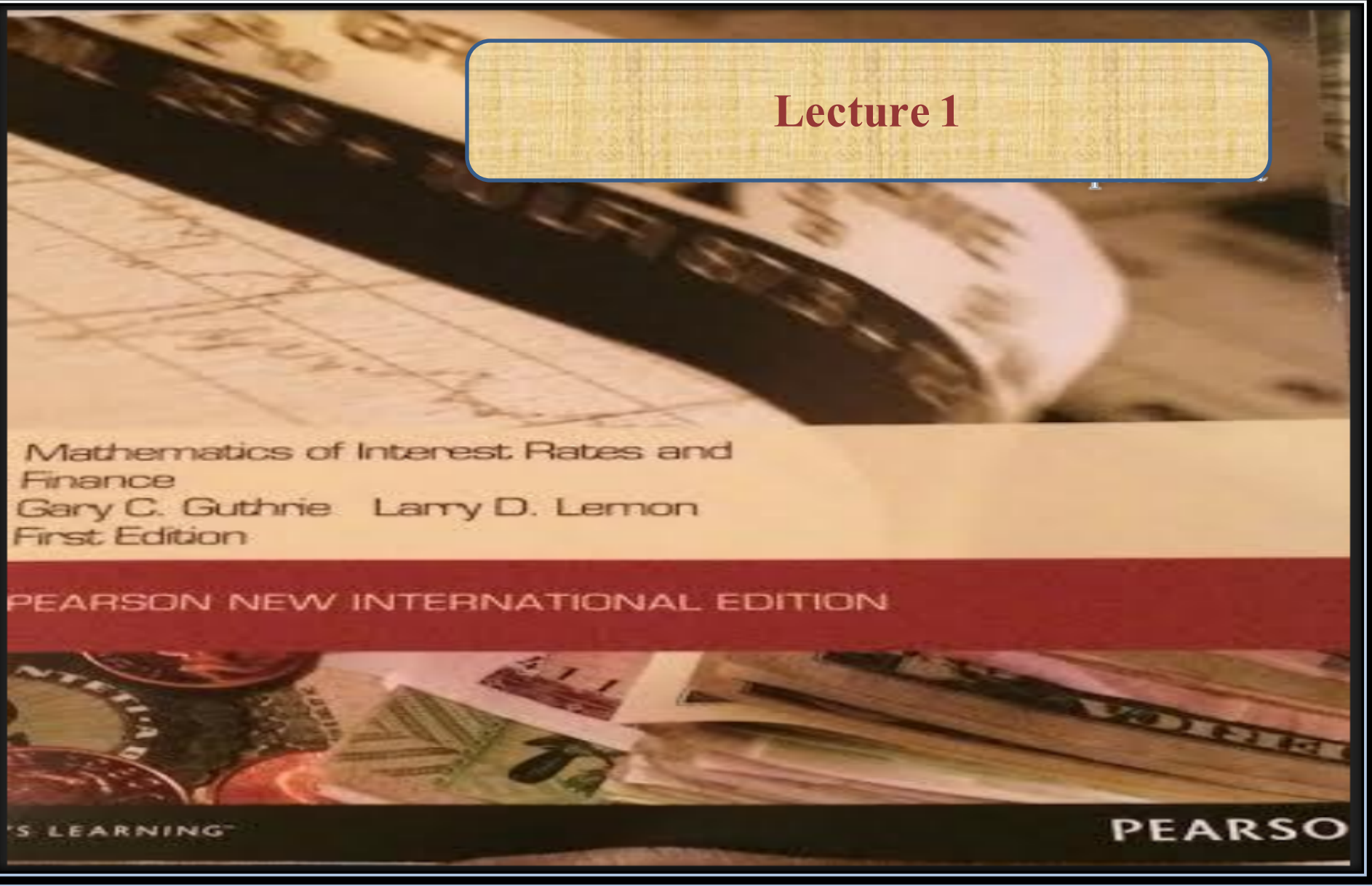
Lecture 1


Mathematics of Interest Rates and Finance
Gary C. Guthrie Larry D. Lemon
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Text Book:

Mathematics of Interest Rates and Finance. Gary C. Guthrie Larry D. Lemon.

Required Chapters and its Sections:

Chapter number and name	Exercises
Chapter 1 **Simple Interest** 1.2 The Importance of Interest and Basic Formula. 1.3 Types of Time and Interest. 1.4 Future Value at Simple Interest. 1.5 Present Value at Simple Interest. 1.6 Simple Interest Debt Instruments. 1.7 Equations of Value. 1.8 Investments- Net Present Value and Internal Rate of Return. 1.9 Partial Payments. 1.10 Equivalent Time.	All Exercises at the end of each required section, Review Test Questions at the end of the chapter, and Sample Test Questions at the end of the chapter.

Chapter 2
Discount Interest

2.1 Discount Interest Basic Formulas and Applications.

2.2 Comparing Simple and Discount Interest.

2.3 Discounting Negotiable Instruments-Notes.

All Exercises at the end of each required section, Review Test Questions at the end of the chapter, and Sample Test Questions at the end of the chapter.

Chapter 3
Compound Interest

3.1 Compound Interest-Future Value Formula.

3.2 The Present Value Formula and Discounting.

3.4 nominal Rates and Effective Interest (Note that; For compound interest only not compound discount).

3.5 Finding the Compound Rate.

3.6 Finding the Time for an Investment to Grow.

3.7 Equations of Value to Find the Unknown.

All Exercises at the end of each required section, Review Test Questions at the end of the chapter, and Sample Test Questions at the end of the chapter.

*** Mid Term Exam Will Cover:**

Chapter 1 (1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, and 1.10).
Chapter 2 (2.1, 2.2, and 2.3).
Chapter 3 (3.1, 3.2, 3.4, 3.5, 3.6, and 3.7).

*** Mid Term Exam questions will be**

in between 20 to 25 questions. (weight of each questions will be in between 0.2 to 0.16).

Chapter 1 Simple Interest

1.2 The Importance of Interest and the Basic Formula :

Summary of Basic Definitions

Principal: amount of money invested or borrowed (present value).
Interest :cost or charge for the use of borrowed money.
Interest Rate: percent of the principal that is the basis for the interest.
Term: length of the loan in time units corresponding to the rate.

The Basic Simple Interest Formula:

$I = Pit$ (1.1)

I = interest P = principal i = rate per year t = time in years

Example 2.1 page 14

A simple interest loan for $800 at 7% per annum is paid off after 6 months. What are the interest charges?

Solution

- Since the interest is needed, use the formula I = Pit and substitute the values. Note that; 6 months will be (6÷12) of a year or 0.5.
- Remember to change the rate to a decimal, so 7%= 0.07.

  I= (800) (0.07) (0.5) = $28

Example 2.2 page 14

If the interest charges on a $950 loan for 9 months amount to $42.75, what is the interest rate for the loan?

Solution

- **Since the rate is needed, solve the formula for *i* either before or after substituting the known information.**

Nine months have to be by years unit. So;

$$\mathbf{t=}\frac{9}{12}=\frac{3}{4}\mathbf{= 0.75\ year.}$$

$$\boldsymbol{i}=\frac{I}{Pt}=\frac{42.75}{(950)(0.75)}\mathbf{= 0.06 = 6\%\ yearly\ or\ per\ year.}$$

Example 2.3 page 14

How long will it take $4000 invested at 7% per annum to earn $350?

Solution

- **The time is needed, so solve the formula for t either before or after substituting the known information.**

$$t = \frac{I}{Pi} = \frac{350}{(4000)(0.07)} = 1.25 \text{ year or } 15 \text{ months}$$

(1 year=12months + 0.25 years means 0.25 ×12 =3 months, so the total period is 12+3=15 months).

A promissory note is a document on which one party (the maker) writes his or her promise to pay another party (the payee) the principal and interest for a loan due at some date in the future. It often looks like a fancy check, but it can be as simple as a handwritten piece of paper. Certain information must appear on the note: the principal borrowed, the interest rate, the length of the term or **maturity date**, the signing date, and the borrower's signature. Sometimes a note will have an additional signature of a witness to the borrower's signature. Often this witness is a notary public. The amount of money the borrower will pay back is the sum of the principal and interest, and it is called the **maturity value**, the **future value**, or the **amount**.

The letter **S** represents the **maturity value**, so that **S = P + I .**

Example 2.4 page 15

A promissory note for $5000 at 8% per annum is due in 18 months. What is the maturity value of the note?

Solution

- The interest charges are calculated first and then added to the principal. This total is the maturity value (or future value) of the note. Remember that 18 months is (18÷12=1.5) of a year

$$I = Pit = 5000(0.08)(1.5) = \$600$$

Maturity value: $S = P + 1 = \$5000 + \$600 = \$5600$

Exercise 17 page 15

On the second of November 2002 Mark invests $1000 at 5%. Find the interest and the future value of the investment on April /2/04.

Solution

By getting a year from 2004 and means 12 months, and by adding these 12 months to 4 , it means it will be 16 months that is substitute from 11, what makes the result 5 months. After got a year from 2004, it became 2003, and by substituting it from 2002, the result will be a year.

	Month	Day	Year
Due date or maturity date or future date	04	02	2004
Investment date	11	02	2002
Substitute result to calculate the time of investment	05	0	01

t= a year + 5 months = 12 + 5= 17 months = 17 ÷12 year

$I=Pit = (1000)(0.05)\left(\frac{17}{12}\right) = \$70.83$

S= P + I = 1000 + 70.83 = $1070.83

Exercise 19 page 16

On September 5/03 Penny takes out a loan for $5000 from her parents who are charging 8%, when Penny graduates on May 5/07, how much interest does she owe, and what is the amount she will repay her parents ?

Solution

	Month	Day	Year
Due date or maturity date or future date	05	05	2007
Investment date	09	05	2003
Substitute result to calculate the time of investment	08	0	03

$t = 3 \text{ years} + 8 \text{ months} = 12 \times 3 + 8 = 44 \text{ months} = 44$

$\div 12 \text{ year}$

$I = Pit = (5000)(0.08)\left(\frac{44}{12}\right) = \$1466.67$

$S = P + I = 5000 + 1466.67 = \$6466.67$

Chapter 1 Simple Interest

3 Types of Time and Interest

- **Types of time:**

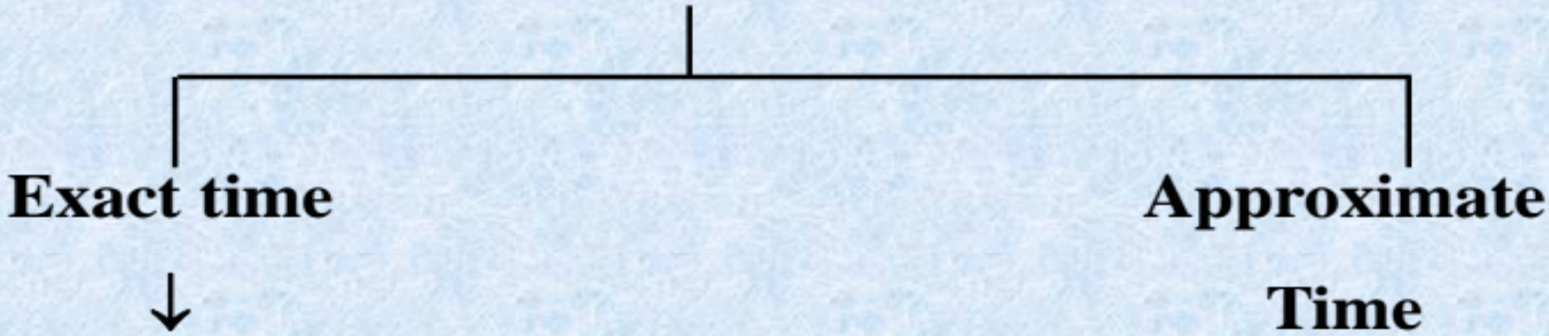
If the time is days :

$$t = \frac{no.of\ days}{the length of\ a\ year}$$

← Types of time

← Types of interest

To calculate the No. of days or types of time, there are :



Use the exact days in each month, except the start date it will be the total days in this month minus the given date

Consider each month has 30 days with exact time for any portion of a month

❑ Example 3.1 page 17

Find the exact time and approximate time for a loan taken out on April 14 and due on October 20.

Solution

Exact time will be as follows:

April May June July August September October

30-14 + 31 + 30 + 31 + 31 + 30 + 20

= 189 days

To compute the approximate time , April is the 4^{th} month and October is the 10^{th} month, so there are 6 months of 30 days each from April 14 to October 14. The exact time from October 14 to October 20 is 6 days. The math gives us

$$6\times30+6=180+6=186 \text{ days.}$$

Example 3.2 page 17

A loan is taken out on March 15 and due on September 19.

Find

a) The exact time for this loan

b) The approximate time for this loan

Solution

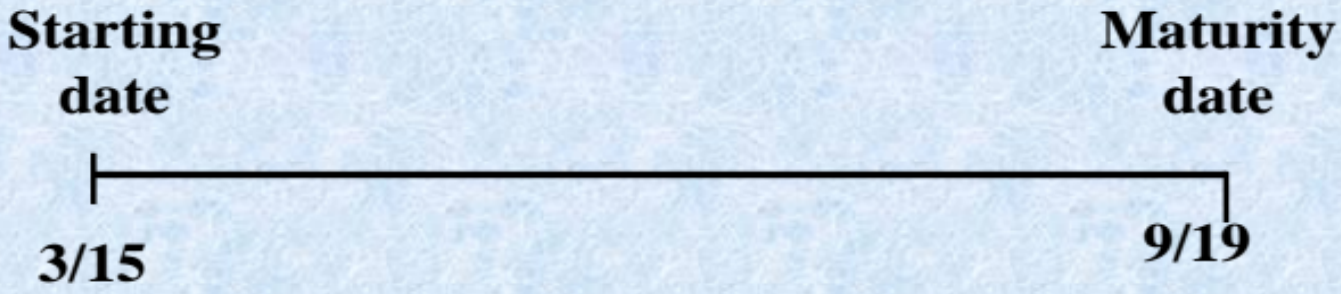
(a) Exact time for this loan

March April May June July August September

31-15 + 30 + 31 + 30 + 31 + 31 + 19

= 188 days

b) Approximate time



- March is the third month and September is the ninth month, so there are 6 months of 30 days each from March 15 to September 15 and the Exact time from Sep. 15 to Sep. 19 is 4 days.
- So → The Approximate time = $6 \times 30 + 4 = 184$

❑ Example3.3 page 17

Find the exact time for a promissory note signed on February 11, 2004, with a maturity date of July31,2004.

Solution

Because 2004 is a leap year, so February is 29 days,
Exact time for this promissory note will be:

February	March	April	May	June	July
29-11	+ 31	+ 30	+ 31	+ 30	+ 31

= 171 days

❑ Example3.4 page 18

Find the exact time for a note signed on December 1, 2005, with a maturity date of March 15,2006.

Solution

(31-1+31+28+15=104)

Because 2006 is not a leap year, so February is 28 days,
Exact time for this note will be:

December	January	February	March
31-1	+ 31	+ 28	+ 15

= 104 days

❑Types of Interest:

Exact Interest

The length of the year is 365 days and 366 days in the leap year.

Ordinary Interest

The length of the year is assumed to be 360 days.

Example3.5 page 18

Calculate the simple interest on a $12,000 loan for 65 days at 7% per annum by using

a. Ordinary interest, and

b. Exact interest.

a. **Substitute the given parameters into the formula using ordinary interest. Use the original form since we are looking for interest. I= pit=12,000(0.07)($\frac{65}{360}$) = $151.67**

b. **Using exact interest, we get**

$$I = Pit = 12{,}000 \times (0.07)\left(\frac{65}{365}\right) = \$149.59$$

Example3.6 page 18

Calculate the simple interest on a $2500 loan signed June 30, 2005, with a maturity date of November 2, 2005, and bearing interest at 9.5% per annum. Use exact time and ordinary interest.

Solution

Exact time will be:

June July August September October November

30-30 +31 + 31 + 30 + 31 + 2

= 125 days

Ordinary Interest is 360 days

$$I = pit = 2500(0.095)\left(\frac{125}{360}\right) = \$82.47$$

Example3.7 page 18 and 19

Anthony has $5680 in a Money Market account paying 4.25%; interest is paid every quarter. Any money withdrawn between interest dates will earn no interest for the entire quarter. If Anthony has a pressing $4550 financial need 45 days before interest accrues, should he withdraw the money or take a credit card loan that will cost him 18%?

Solution

- If he withdraws $4550 he will lose the following interest:

  $I = pit = 4550(0.0425)(3 \div 12) = \$48.34.$

- The credit card loan will cost the following:

  $I = pit = 4550(0.18)(45 \div 360) = \$102.38.$

- In fact, the amount of money he withdraws from the account is immaterial since

$0.425(0.25) < (0.18(45 \div 360).$

He should take the money from savings .

Because bankers lend money on a regular basis, the use of exact time and ordinary interest has been called Banker's Rule. In this text Banker's Rule will be used unless the exercise says otherwise.

Notes:

- Banker Rule: means, use Exact time to compute the number of days, and use ordinary interest to compute the length of the year.
- We will use banker's rule unless the exercise says otherwise.
- In the case of Exact interest, consider that the length of the given year is 365 unless the exercise determine that it is a leap year.

Thank You