A comparison study using particle swarm optimization inversion algorithm for gravity anomaly interpretation due to a 2D vertical fault structure

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**A B S T R A C T**

A new approach to the inversion of gravity data utilizing the Particle Swarm Optimization (PSO) algorithm is used to model 2D vertical faults. The PSO algorithm is stochastic in nature; its development was motivated by the communal in-flight performance of birds looking for food. The birds are represented by particles (or models). Individual particles have a location and a velocity vector. The location vectors represent the parameter value. PSO is adjusted with random particles (models) and searches for targets by updating generations.

Herein, the PSO algorithm is applied to three synthetic data sets (residual only with and without noise, residual plus regional, residual plus anomaly generated by a buried cylinder structure) and two field gravity data sets acquired across known faults in Egypt. Assessment of the synthetic data demonstrates that the PSO algorithm generates superior results if a first horizontal gradient (FHG) filter is applied first. The robustness of the PSO inversion algorithm is demonstrated for both synthetic and field gravity data.

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1. Introduction

The gravity method has an extensive variety of applications, for examples, sedimentary basin delineation (Singh and Singh, 2017), hydrocarbon exploration (Assaad, 2009; Eppelbaum and Khesin, 2012), mineral exploration (Essa, 2007; Lelièvre et al., 2012), archaeology (Linford, 2006; Panisova and Pasteka, 2009), hydrogeology (Murty and Raghavan, 2002; Al-Garni, 2005; Arafia et al., 2015), fault investigation (Essa, 2013; Abdelrahman and Essa, 2015) and cavity detection (Camacho et al., 1994; Essa, 2011). The conventional inversion of gravity data is subject to limitations including ill-posedness and non-uniqueness and requires a priori information about density contrasts (Tarantola, 2005; Essa, 2014; Mehanee, 2014; Mehanee and Essa, 2015). To over-come some of these limitations, various alternate inversion methods have been developed (e.g., Nettleton, 1962; Paul et al., 1966; Green, 1976; Jain, 1976; Telford et al., 1976; Kilty, 1983; Gupta and Pokhriyal, 1990; Abdelrahman et al., 2003; Abdelrahman et al., 2006; Abdelrahman et al., 2013; Biswas, 2015).

For example, metaheuristic techniques have been used as alternatives to the conventional inversion techniques and are designed to solve hard optimization problems with the objective of finding a more precise solution in limited time (Sen and Stoffa, 2013). These metaheuristic techniques include several different approaches including genetic algorithm (Tiamo et al., 2004; Amjadi and Najj, 2013; Kaftan, 2017), particle swarm optimization (Toushmali, 2013; Singh and Biswas, 2016), differential evolution (Wu et al., 2014; Balkaya et al., 2017), simulated annealing (Biswas et al., 2014; Biswas et al., 2017), ant colony optimization (Liu et al., 2014; Alavand and Asil, 2018) and hybrid-genetic-price algorithm (Di Maio et al., 2016).

For the research presented herein, the particle swarm optimization (PSO) approach was used to invert gravity datasets for a 2D vertical fault structure in an effort to calculate fault parameters (depth (z), amplitude factor (K), and the origin of the fault trace (xo)). The PSO algorithm has been applied to three synthetic data sets and two field data sets. Some of the synthetic and field Bouguer gravity data contain both residual and regional anomalies. In these cases, accuracy of the output of the PSO inversion depends on the algorithms ability to differentiate the regional and residual anomalies using the first horizontal gradient (FHG) method for several window lengths (s-value).

The PSO approach has been applied to three synthetic models. The first model represents Bouguer gravity data across a 2D vertical fault with a 1st order polynomial regional. The second model includes random noise on a pure residual gravity anomaly. The third model was designed to demonstrate the impact of an interfered structure. The PSO approach method is also applied to two real field data sets acquired...
across known faults in Egypt in an attempt to assess the robustness and applicability of the PSO approach when applied to real gravity data.

2. Methodology

The measured Bouguer gravity anomaly is comprised of the residual anomaly generated by the faulted structure and the undesired regional anomaly as follows:

$$\Delta g(x_i) = R(x_i, z) + Z(x_i)$$

where $\Delta g(x_i)$ is the Bouguer gravity anomaly (mGal), $R(x_i, z)$ is the residual gravity anomaly (mGal) and $Z(x_i)$ is the regional gravity anomaly (mGal). The objectives are to use the first horizontal gradient method to isolate the residual gravity anomaly and the PSO approach to invert it.

2.1. The 2D vertical fault forward modelling

The gravity anomaly of a 2D one-sided vertical fault or semi-infinite thin sheet can be expressed as (Abdelrahman and Essa, 2013; Hinze et al., 2013) (Fig. 1):

$$R(x_i, z) = K \left( \frac{1}{z} + \frac{1}{\pi} \tan^{-1}\left( \frac{x_i - x_0}{z} \right) \right), \quad i = 1, 2, 3, 4, \ldots \ldots \ldots \ldots \ldots N$$

Fig. 1. Top panel represents synthetic 2D vertical fault model ($K = 80$ mGal, $z = 6$ m, $x_0 = 0$ m and profile length = 120 m) and a 1st-order polynomial for the regional anomaly. Lower panel represents a schematic figure showing the cross-sections and parameters.
where \( x_i \) is the horizontal location, \( z \) is the depth, \( x_0 \) is the origin and \( K \) is the amplitude factor \( (=2\pi G\Delta\sigma) \), \( G \) is the gravitational constant, \( \Delta\sigma \) is the density contrast, and \( t \) is the thickness or throw of the fault.

2.2. The PSO inversion algorithm

The PSO algorithm was developed by Eberhart and Kennedy (1995). The PSO progression is stochastic in nature; its development was motivated by the communal flight performance of birds looking for food. The birds are represented by particles (or models). Individual particles have a location and a velocity vector. The location vectors represent the parameter value. PSO is adjusted with random particles (models) and searches for targets by updating generations. In each iteration, every particle updates its velocity and location using Eqs. (3) and (4) (Essa and Elhussein, 2018). The best position \( (\text{Tbest}) \) reached by particle is kept in the memory of the particle while \( \text{Jbest} \) model represent the best area reached by any particle. The following formulas represent the update of particle’s velocity and particle’s location respectively.

\[
V_{i}^{k+1} = c_3 V_i^k + c_1 \text{rand}() \left( \text{Tbest} - P_i^k \right) + c_2 \text{rand}() \left( \text{Jbest} - P_i^k \right) \tag{3}
\]

\[
P_{i}^{k+1} = P_i^k + V_i^{k+1} \tag{4}
\]

where \( V_i^k \) is the \( i \)th particle velocity at the \( k \)th iteration, \( P_i^k \) is the current \( i \)th particle position at the \( k \)th iteration, \( \text{rand}() \) is a random number between 0 and 1, \( c_1 \) and \( c_2 \) are cognitive and social coefficients and equal 2 (Parsopoulos and Vrahatis, 2002; Sweilam et al., 2007), \( c_3 \) is the inertial coefficient that governs the particle velocity and its value less than 1.

The PSO algorithm was applied to several different data sets. For those examples where a regional anomaly was present, the first horizontal gradient method was used to remove the regional background using the following approach based on Eq. (1), using two observation points \( (x_i - s, x_i + s) \) along anomaly profile. The filtered first horizontal gradient (FHG) of the gravity anomaly is assumed by the subsequent form:

\[
\text{FHG}(x_i, z, s) = \frac{\Delta g(x_i + s) - \Delta g(x_i - s)}{2s} \tag{5}
\]

where \( s = 1, 2, ..., M \) spacing units which is called graticule spacing, \( \Delta g \) is the Bouguer gravity anomaly.

The PSO algorithm was then applied to each FHG anomaly profile to calculate the fault parameters \( z, K \) and \( x_0 \).

For the anomalies where only residual gravity data were present, the PSO algorithm was applied directly to the Bouguer gravity data.

### Table 1

{| Parameters | Used ranges | Using the PSO-inversion for the Bouguer gravity data | Using the PSO-inversion for the pure residual gravity data |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( s = 3 ) m</td>
<td>( s = 4 ) m</td>
<td>( s = 5 ) m</td>
</tr>
<tr>
<td>( K ) (mGal)</td>
<td>10–300</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>( z ) (m)</td>
<td>1–10</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>( x_0 ) (m)</td>
<td>(-10 - 10)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Fig. 3. Synthetic 2D vertical fault model \( K = 100 \) mGal, \( z = 7 \) m, \( x_0 = 3 \) m and profile length = 120 m) without and with a 10\% random noise.

Fig. 4. FHG (first horizontal gradient) anomalies for Fig. 3 in case of 0\% noise.
2.3. The inverse modelling

Inverse modelling of gravity data across a fault is an attempt to determine the best-fit fault parameters for the Bouguer gravity data (either synthetic or real). In most cases, initial parameters must be assumed (Tarantola, 2005; Mehanee et al., 2011). A good initial model is generally developed based on available information from geology, drilling or other geophysical techniques (Zhdanov, 2002; Mehanee and Essa, 2015). The initial model is progressively refined at each iterative step until a best-fit between the measured and the predicted data is achieved. In every iterative step, the fault parameters are changed to get the best values by mimicking the next objective function ($\phi_{obj}$), where:

$$\phi_{obj} = \frac{2 \sum_{i=1}^{N} |g_i^o - g_i^p|}{\sum_{i=1}^{N} |g_i^o| + \sum_{i=1}^{N} |g_i^o + g_i^p|}$$  \hspace{1cm} (6)

N is the number of observed points, $g_i^o$ is the observed gravity anomaly and $g_i^p$ is the predicted gravity anomaly at the point $x_i$.

After estimating the fault parameters ($z, K, x_o$) of the 2D buried vertical fault, the whole error (RMSE) between the measured and calculated fields is estimated using the following formula:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} (g_i^o(x_i) - g_i^p(x_i))^2}{N}}$$  \hspace{1cm} (7)

3. Application of PSO approach to synthetic examples

The PSO inversion approach was applied to three synthetic models. The first synthetic model is the residual gravity anomaly profile (Fig. 3 and Eq. 9) which is consisting of a 2D vertical fault model ($K = 100$ mGal, $z = 7$ m and profile length = 120 m) without and with 10% random noise.

Table 2
Numerical results of the PSO-inversion algorithm for the residual gravity anomaly profile (Fig. 3 and Eq. 9) which is consisting of a 2D vertical fault model ($K = 100$ mGal, $z = 7$ m, $x_o = 3$ m and profile length = 120 m) without and with 10% random noise.

<table>
<thead>
<tr>
<th>Parameters Used ranges</th>
<th>Using the PSO-inversion for the Bouguer gravity data</th>
<th>Using the PSO-inversion for the pure residual gravity data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without using random noise</td>
<td>with using 10% random noise</td>
<td></td>
</tr>
<tr>
<td>$s = 3$ m</td>
<td>$s = 4$ m</td>
<td>$s = 5$ m</td>
</tr>
<tr>
<td>$K$ (mGal) 10–300</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$z$ (m) 1–10</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>$x_o$ (m)</td>
<td>−10 - 10</td>
<td>3</td>
</tr>
</tbody>
</table>

Fig. 5. FHG (first horizontal gradient) anomalies for Fig. 3 in case of 10% noise.
**Fig. 6.** Synthetic 2D vertical fault model (\(K = 150 \text{ mGal}, z = 5 \text{ m}, x_0 = -3 \text{ m} \) and profile length = 120 m) and an interfered structure of a horizontal cylinder model (\(K = 200 \text{ mGal} \times \text{m}, z = 10 \text{ m}, x_0 = 30 \text{ m} \) and profile length = 120 m).

**Fig. 7.** FHG (first horizontal gradient) anomalies for Fig. 6.
without and with 10% noise. The third model is the residual anomaly of 2D vertical fault superposed on the gravity anomaly generated by a proximal buried cylindrical structure the effect of interfered structure with the target source.

3.1. 1st synthetic model

The first model consists of a 2D vertical fault with $K = 80 \text{ mGal}$, $z = 6 \text{ m}$, $x_0 = 0 \text{ m}$, profile length = 120 m plus a 1st order polynomial (regional background) (Fig. 1) as:

$$\Delta g(x, z) = 80 \left[ \frac{1}{2} \pi + \frac{1}{\pi} \tan^{-1}\left(\frac{x}{6}\right) \right] + (2x - 10) \quad (8)$$

The FHG method was used to minimize the regional anomaly at various window lengths ($s = 3, 4, 5$ and 6 m) (Fig. 2). The PSO inversion algorithm was then used to calculate the fault parameters ($z, K, x_0$) for every $s$-value (Table 1). Table 1 is a summary of the assessed results and the ranges of every parameter. The assessed results for each parameter ($z, K, x_0$) are in close agreement with the known input model parameters.

The combination anomaly (regional and residual) was also treated as a pure residual anomaly and inverted using the PSO approach. The results are summarized in Table 1. The errors in model parameters ($z, K$) are 33.3% and 25%, respectively, and the RMSE is 76.6 mGal. These results indicate that the PSO inversion algorithm is not effective in the presence of a significant regional anomaly.

Table 3

Numerical results of the PSO-inversion algorithm for the Bouguer gravity anomaly profile (Fig. 6 and Eq. 10) which is consisting of a 2D vertical fault model ($K = 150 \text{ mGal}$, $z = 5 \text{ m}$, $x_0 = -3 \text{ m}$ and profile length = 120 m) and an interfered structure of a horizontal cylinder model ($K = 200 \text{ mGal} \cdot \text{m}, z = 10 \text{ m}, x_0 = 30 \text{ m}$ and profile length = 120 m).

<table>
<thead>
<tr>
<th>Parameters Used ranges</th>
<th>Using the PSO-inversion for the Bouguer gravity data</th>
<th>Using the PSO-inversion for the pure residual gravity data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Results</td>
<td>Results</td>
</tr>
<tr>
<td>$s = 3 \text{ m}$</td>
<td>$s = 4 \text{ m}$</td>
<td>$s = 5 \text{ m}$</td>
</tr>
<tr>
<td>$K (\text{mGal})$</td>
<td>10–300</td>
<td>141.4</td>
</tr>
<tr>
<td>$z (\text{m})$</td>
<td>1–10</td>
<td>5.1</td>
</tr>
<tr>
<td>$x_0 (\text{m})$</td>
<td>$-10$</td>
<td>$-2.9$</td>
</tr>
</tbody>
</table>

The FHG method was used to minimize the regional anomaly at various window lengths ($s = 3, 4, 5$ and 6 m) (Fig. 2). The PSO inversion algorithm was then used to calculate the fault parameters ($z, K, x_0$) for every $s$-value (Table 1). Table 1 is a summary of the assessed results and the ranges of every parameter. The assessed results for each parameter ($z, K, x_0$) are in close agreement with the known input model parameters.

The combination anomaly (regional and residual) was also treated as a pure residual anomaly and inverted using the PSO approach. The results are summarized in Table 1. The errors in model parameters ($z, K$) are 33.3% and 25%, respectively, and the RMSE is 76.6 mGal. These results indicate that the PSO inversion algorithm is not effective in the presence of a significant regional anomaly.

Fig. 8. Geological map of South Aswan area, showing the location of the Gazelle fault (modified after Woodward-Clyde Consultants., 1985; Abdelrahman et al., 2013).
3.2. 2nd synthetic model

The second model consists of a pure residual gravity anomaly for a 2D vertical fault with \( K = 100 \) mGal, \( z = 7 \) m, \( x_0 = 3 \) m, profile length = 120 m (Fig. 3) and can be described as:

\[
\Delta g_{x_i,z}(x) = \frac{100}{2} \left( \frac{x_i - 3}{7} \right) \tan^{-1}\left( \frac{x_i - 3}{7} \right) + \frac{2000}{\left| x_i - 30 \right|^2 + 100} \tag{9}
\]

The FHG method was used to simulate the minimization of the non-existent regional anomaly at various window lengths (\( s = 3, 4, 5 \) and 6 m) (Fig. 4). The PSO inversion algorithm was then used to calculate the fault parameters (\( z, K, x_0 \)) for every \( s \)-value (Table 2). Table 2 indicates that the output parameters are similar to the actual model parameters. The errors in all parameters and the RMSE are zero.

In order to study the effect of background noise, 10% random noise was added to the residual gravity anomaly (Eq. (9)) (Fig. 3). The process described above was used to calculate the fault parameters (\( z, K, x_0 \)) for a 2D vertical fault model. The FHG anomalies are represented in Fig. 5 for the same \( s \)-value (\( s = 3, 4, 5 \) and 6 m). The estimated fault parameters are tabulated in Table 2. As noted, the PSO inversion of noisy data after the application of the FHG algorithm is superior to simply processing data without applying FHG because the RMSE = 5.42 mGal (in the 1st case) and is less than the RMSE (10.9 mGal) in the 2nd case.

3.3. 3rd synthetic model

The third model consists of a 2D vertical fault with \( K = 150 \) mGal, \( z = 5 \) m, \( x_0 = -3 \) m, profile length = 120 m and a proximal buried horizontal cylinder with \( K = 200 \) mGal×m, \( z = 10 \) m, and \( x_0 = 30 \) m (Fig. 6) described by the following formula:

\[
\Delta g_{x_i,z}(x) = \frac{150}{2} \left( \frac{x_i - 3}{5} \right) \tan^{-1}\left( \frac{x_i + 3}{5} \right) + \frac{2000}{\left| x_i - 30 \right|^2 + 100} \tag{10}
\]

The FHG algorithm and PSO inversion were applied to the interfered gravity model. The FHG gravity anomalies for numerous \( s \)-values (\( s = 3, 4, 5 \) and 6 m) are depicted in Fig. 7. The output fault parameters estimated are presented in Table 3. Table 3 shows that the RMSE (4.4 mGal) for the Bouguer gravity data is less than the RMSE (11.1 mGal) for the gravity data when used directly, implying it is best to apply the FHG method first to the gravity data to remove the unwanted anomalies.
4. Application of PSO approach to field examples

Two field examples from Egypt were inverted to demonstrate the robustness and efficiency of the PSO inversion algorithm. The importance of this study is to explore various subsurface issues related to fault or lineament analysis.

4.1. Gazelle fault example, Egypt

The Gazelle fault is located south of Aswan and trends N-S, has a length of 35 km and an inferred left-slip sense of displacement with no active features or ground cracks observed along the fault trace. The fault plane is nearly vertically (Issawi, 1969). The composite fault plane description indicates a nearly strike-slip fault with a normal-fault component (Fat-Helbary and Tealeb, 2002; Sawires et al., 2015). This fault is situated wholly through rocks latest Cretaceous sandstones and shale of Nubian Formation (Woodward-Clyde Consultants, 1985; Abdelrahman et al., 2013) (Fig. 8).

Table 4
Numerical results for the Gazelle fault, Egypt using the first technique.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Used ranges</th>
<th>Results</th>
<th>Average RMSE (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$ (m)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K$ (mGal)</td>
<td>1–100</td>
<td>16.27</td>
<td>13.25 ± 2.42</td>
</tr>
<tr>
<td>$z$ (m)</td>
<td>150–400</td>
<td>321.88</td>
<td>314.39 ± 19.12</td>
</tr>
<tr>
<td>$x_0$ (m)</td>
<td>−6–6</td>
<td>−0.91</td>
<td>−0.79 ± 0.96</td>
</tr>
</tbody>
</table>

This fault is situated wholly through rocks latest Cretaceous sandstones and shale of Nubian Formation (Woodward-Clyde Consultants, 1985; Abdelrahman et al., 2013) (Fig. 8). Fig. 9 shows a Bouguer gravity profile of length 5000 m and was digitized at an interval of 62.5 m. First case, the PSO inversion algorithm was applied to the FHG anomalies using Eq. (5) and different $s$-value ($s = 125, 187.5, 250, 312.5, 375, 437.5, 500$ m).

Table 5
Numerical results for the Gazelle fault, Egypt using the second technique.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Used ranges</th>
<th>Results</th>
<th>RMSE (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$ (mGal)</td>
<td>1–100</td>
<td>10.61</td>
<td>2.12</td>
</tr>
<tr>
<td>$z$ (m)</td>
<td>150–400</td>
<td>319.79</td>
<td>318.13</td>
</tr>
<tr>
<td>$x_0$ (m)</td>
<td>−6–6</td>
<td>−0.93</td>
<td>−0.99 ± 0.15</td>
</tr>
</tbody>
</table>
The estimated results are $K = 13.25 \pm 2.42$ mGal, $z = 308.49 \pm 19.12$ m and $x_0 = -0.99 \pm 0.15$ m with the RMSE = 1.24 mGal (Table 4). Second case, the PSO inversion algorithm was also applied to the Bouguer gravity data considering these data as the pure residual gravity anomaly. The predicted result are: $K = 10.61$ mGal, $z = 319.79$ m and $x_0 = -0.93$ m with a RMSE = 2.12 mGal (Table 5). The results by using the PSO method convolved with the FHG method have a reasonable agreement with the results attained from drilling information ($z = 300$ m) (Evans et al., 1991; Abdelrahman et al., 2013; Essa, 2013; Abdelrahman and Essa, 2015).

4.2. Mersa Matruh fault example, Egypt

The Mersa Matruh fault example is from the Mersa Matruh basin in the Northwestern Desert of Egypt. The fault zone trends NE-SW as determined from stratigraphy of the boreholes MM (Mersa Matruh) and S (Siqueifa) in the study area (Fig. 11). According to Said (1962) and Barakat and Darwish (1984), the faulting is Lower Cretaceous in age. The throw of the fault is nearly 610 m and the depth extent of the fault is more than 4000 m. A Bouguer gravity anomaly profile of length 43,200 m (Fig. 11) was digitized at an interval of 450 m. First case, the digitized profile was subjected to FHG filtering using different $s$-value ($s = 900, 1350, 1800, 2250, 2700, 3150$ and $3600$ m) (Fig. 12). The PSO inversion algorithm was applied to the output FHG anomalies to obtain the fault parameters (Table 6). The calculated parameters are consistent with borehole information. Second case, the PSO inversion algorithm was also applied to the Bouguer gravity data considering these data as the pure residual gravity anomaly, the predicted results were shown in Table 7.
5. Conclusions

The robustness of the PSO-inversion algorithm was demonstrated for both synthetic and real gravity data and for residual gravity data (only) and residual plus regional gravity data. The gravity data processing was a two-step process. In step 1, the FHG algorithm is applied to minimize regional trends and background noise. In step 2, the PSO inversion algorithm is applied to determine fault parameters. The assessment of results confirmed that the FHG method is more stable than the conventional direct interpretation of gravity data. The PSO applications to the interpretation of the fault models is superior because it does not need a priori model information, provides for quick convergence and is robust with respect to the computation of the model parameters. In subsequent studies, this approach will be extended to the analyses of magnetic and self-potential anomalies in support of mineral exploration.

Data availability

The data will be available upon request.

Authors’ contributions

Anderson N. L., Khalid S. Essa and Mahmoud Elhussein wrote the text, Anderson N. L. revised the text. Khalid S. Essa and Mahmoud Elhussein interpreted synthetic examples, Khalid S. Essa, Mahmoud Elhussein and Anderson N. L. interpreted field examples. Mahmoud Elhussein produced Figures and Khalid S. Essa produced Tables. Mahmoud Elhussein, Khalid S. Essa and Anderson N. L., made the replies to the reviewers’ comments.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
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References


Essa, K.S., Ellhussine, M., 2018. PSO (Particle Swarm Optimization) for Interpretation of Magnetic Anomalies caused by simple geometrical structures. Pure Appl. Geophys. 175, 3593–3595.
