

# A numerical technique for an accurate determination of formation resistivity factor using $F_R$ - $R_O$ overlays method

Walid M. Mabrouk · Khaled S. Soliman

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**Abstract** The exactness of water saturation value for given reservoir conditions depends on the accuracy of Archie parameters  $a$ ,  $m$ , and  $n$ . The terms of Archie relationship have been subjected to many laboratory investigations and even more speculation. There are many factors that affect porosity exponent  $m$ , saturation exponent  $n$ , and tortuosity factor  $a$ . Usually, assumptions are made to approximate  $a$  and  $m$ ; often  $m$  is 2, while  $a$  may be 0.81 or 1.0 depending on the type of lithology. But it is very difficult to fix Archie parameters regardless of reservoir characteristics; rock wettability, formation water salinity, permeability, porosity, and fluids distribution. This work illustrates a simple numerical method to calculate  $a$  and  $m$  which depends on  $F_R$ - $R_O$  overlays method which is used as hydrocarbon indicator. The method is tested using synthetic and real data to ensure its ability in determining formation factor parameters  $a$  and  $m$ .

**Keywords** Formation resistivity factor ·  $a$  and  $m$  · Tortuosity and porosity exponent

## Nomenclature

$F_R$	Formation resistivity factor
$R_O$	Resistivity of fully saturated S.S. ( $\Omega$ m)
$A$	Tortuosity factor
$R_t$	True resistivity ( $\Omega$ m)
$S_w$	Water saturation
$R_{cal}$	Calculate resistivity ( $\Omega$ m)
R-Calc (0.52–1.6)	Calculated resistivity (a-m)
$\phi$	Porosity

$R_w$	Water resistivity ( $\Omega$ m)
$M$	Cementation exponent
$R_{xo}$	Flushed zone resistivity
$\phi_{Syn.}$	Synthetic porosity

## Introduction

The need for greater accuracy for the determination of in situ fluid saturations has become more important in recent years because of efforts to develop new technology to enhance oil recovery from old oil fields.

This accurate assessment of the existing reservoir oil saturation is required to project the overall economic feasibility of the new or improved oil-recovery process. This need for greater accuracy and more data from in situ testing has led to the development of new well-logging tools (the repeat formation tester, the 64-electrode microresistivity tool, and the Phasor induction tool).

The formation resistivity factor of a porous medium is a valuable concept in the area of formation evaluation. It has been defined as the ratio of the resistivity of the medium when completely saturated with a conducting fluid to the resistivity of the saturating fluid as follows:

$$F_R = R_O/R_w \quad (1)$$

Next, Archie cross-plotted porosity versus formation factor on log-log scales, and noted a linear trend that is equivalent to the following:

$$F_R = \frac{1}{\phi^m} \quad (2)$$

W. M. Mabrouk (✉) · K. S. Soliman  
Geophysics Department, Faculty of Science, Cairo University, Giza,  
Egypt  
e-mail: walid\_mabrouk@yahoo.com

**Table 1** Coefficients and exponents used to calculate formation factor ( $F$ ) as a function of porosity

$F = a / \phi^m$	General relationship (Archie 1942) where: $a$ = tortuosity factor $m$ = cementation exponent $\phi$ = porosity
$F = 1 / \phi^2$	General equation for Chalky Rocks
$F = 0.81 / \phi^2$	For consolidated sandstones
$F = 0.62 / \phi^{2.15}$	Humble formula for unconsolidated sands (Winsauer et al. 1952; Wyllie and Gregory 1953)
Phillips equation	For average sands, 793 sandstone data points gathered by Carothers (1968)
$F = 1.45 / \phi^{1.54}$	
$F = 1.65 / \phi^{1.33}$	For shaly-sands (after Carothers 1968)
$F = 1.45 / \phi^{1.70}$	For calcareous sands (after Carothers 1968)
$F = 0.85 / \phi^{2.14}$	For carbonates (after Carothers 1968)
$F = 1 / \phi^{2.04}$	For 188 carbonate data points (Carothers 1968)
$F = 1 / \phi^{2.2 \text{ to } 2.5}$	Recommended for compacted carbonate rocks (Schlumberger 1979; Worthington 1985)
Shell equation	Recommended for low-porosity, nonfractured carbonates. (Schlumberger 1979; Worthington 1985)
$F = 1.0 / \phi^{1.87+0.019/\phi}$	
$F = 2.45 / \phi^{1.08}$	For Pliocene sands, Southern California; 1575 data points was gathered from 11 offshore wells. (After Porter and Carothers 1970)
$F = 1.97 / \phi^{1.29}$	For Miocene sands, Texas Louisiana, Gulf Coast; 720 data points was collected from 4 offshore wells. (After Porter and Carothers 1970)
$F = 1.0 / \phi^{(2.05-\phi)}$	For clean granular formations (after Sethi 1979)
Chevron Formula	Timur et al. (1972) gathered an extensive collection of $F$ - $\phi$ data. The data consists of 1,833 sandstone samples.
$F = 1.13 / \phi^{1.73}$	

Winsauer et al. 1952 established experimentally a more generalized formation resistivity factor equation, expressed as follows:

$$F_R = a\phi^{-m} \tag{3}$$

Where  $F$  is the formation factor (dimensionless),  $\phi$  is the porosity (dimensionless),  $a$  is the cementation factor

**Table 2** Factors affecting the formation factor parameters ( $a$  and  $m$ ) (adopted after R. C. Ransom 1974)

Formation factor “ $a$ ”	Cementation exponent “ $m$ ”
1. Surface conductance and ionic mobility occurring in water films adsorbed to solid surface.	For the rock-water interface: 1. Pore geometry: i. Surface area to volume ratio of the rock particle, angularity, and sphericity.
i. The cation exchange capacity of particular solid materials	ii. Cementation
ii. The quantity of water adsorbed to clay particles in the rock framework or within the interstices	iii. Compaction
2. Salinity of formation water,	iv. Uniformity of mineral mixture
3. Wettability relations between particular solid surface and hydrocarbons, as it influences cation exchange capacity	2. Anisotropy
4. The presence and distribution of electrically conductive solid minerals	3. Degree of electrical isolation by cementation
	4. The occurrence of an open fracture

(dimensionless), and  $m$  is the cementation exponent (dimensionless).

Several different values, depending mostly on the types of formations selected for the experiments, have been proposed for “ $a$ ” and “ $m$ ,” but the curves showing  $F$  versus  $\phi$  for these various values of “ $a$ ” and “ $m$ ” do not differ much from one another.

Table 1 represents a general equations for computing formation factor as a function of porosity with references to the values of “ $a$ ” and “ $m$ ” used in such calculation, and illustrates how lithology affects the formation factor. Also, the complexities of the interplaying factors affecting such parameters (Table 2) have always been preventing a precise estimation of  $F$ , especially when indirect methods are used.

**Table 3** Intervals used for  $a$  (0.5 to 1.5) and  $m$  (1.5 to 2.5)

$a$	$m$
0.5	1.5, 1.6, 1.7, ..... 2.5
0.51	1.5, 1.6, 1.7, ..... 2.5
.	.
.	.
.	.
1.5	1.5, 1.6, 1.7, ..... 2.5

**Table 4** Arrangements of the well-logging data

Row #	Column #1	Column #2	Column #3
1	Depth (ft)	Rt (ohm m)	Porosity (p.u.)

This paper aims to apply a numerical method to enhance the evaluation of “*a*” and “*m*” with the aim of enhancing water and hence hydrocarbon saturations.

**Formulation of the problem**

It is important to point out which intervals need careful formation evaluation, only intervals with hydrocarbons need wellsite evaluation. With several thousand feet of hole to evaluate quickly, methods were needed to remove the non-hydrocarbon-bearing “wet” intervals and quickly locate possible hydrocarbon-bearing intervals. The five prime techniques used (Best et al. 1978; Bigelow and abt 1972; Fertl 1978) as hydrocarbon indicators are as follows:

1. Apparent water resistivity ( $R_w$ )
2.  $F_R$ - $R_o$  overlays
3. Movable-oil plot (MOP)
4. The  $R_{xo}/R_t$  indicator

5. The neutron-density gas overlay
6. BVW method (Mabrouk 2005)

Most of these older techniques are used in current wellsite computer interpretation. However, the present paper depends on  $F_R$ - $R_o$  overlays method where Archie (1942) introduces the following water saturation equation:

$$S_w = \frac{F_R R_w}{R_t} \tag{4}$$

Rewrite equation (4), we get the following:

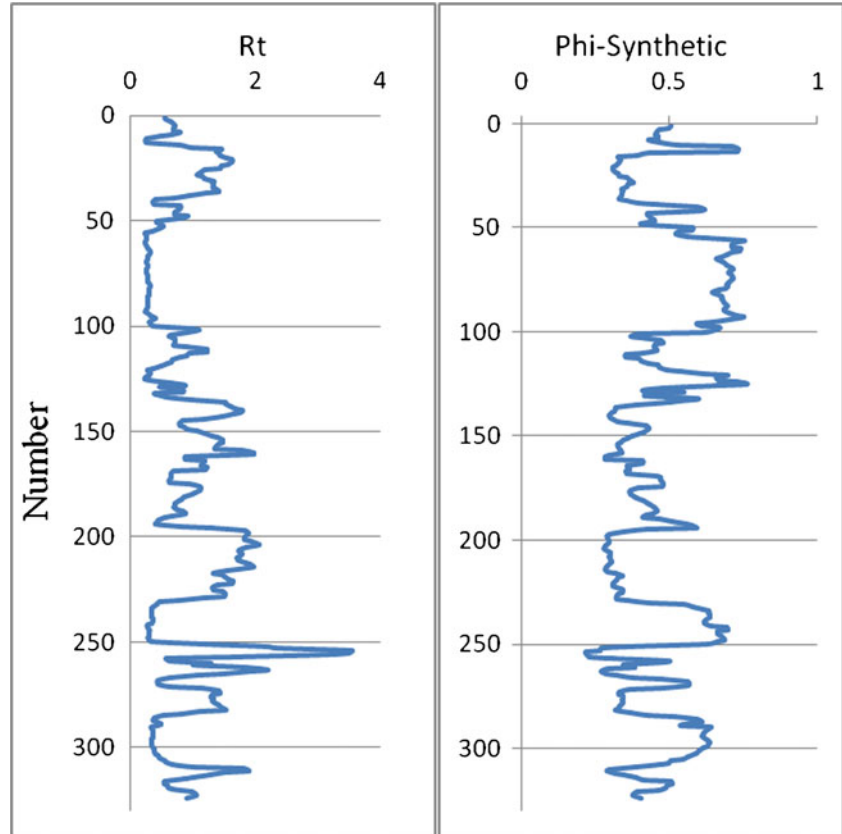
$$R_t S_w = F_R R_w \tag{5}$$

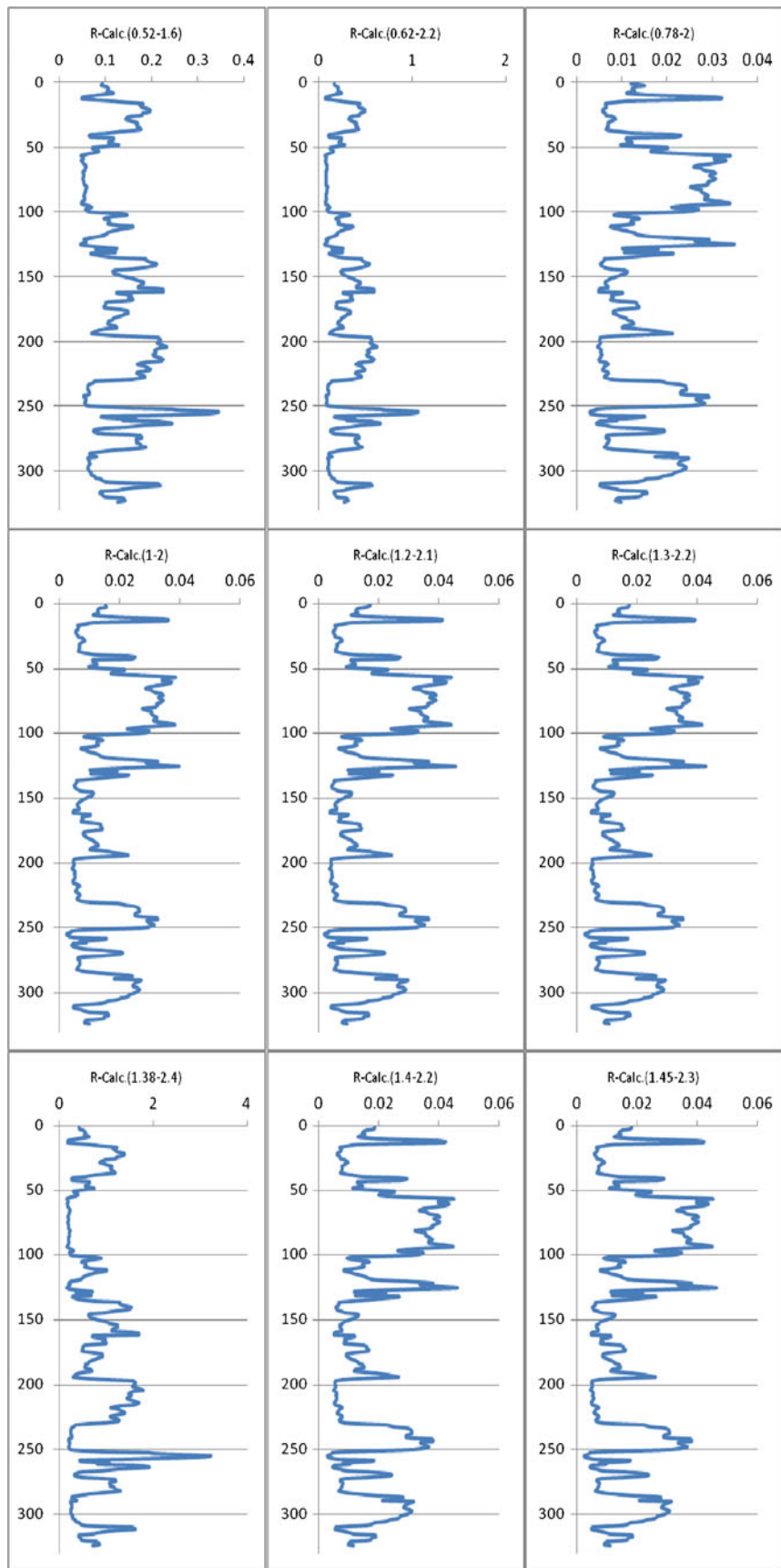
In water-bearing zones, where  $S_w=100\%$ , Eq. (5) can be represented by Eq. (1) by plotting  $R_o$  and  $F_R$  vertically with depth; by knowing  $R_w$ , we can easily differentiate between water and hydrocarbon bearing zones where:

1. In water-bearing zones,  $R_o$  is very close to  $F_R$ .
2. In hydrocarbon-bearing zones, there is a separation between them.

However, in water-bearing zones,  $R_o$  must be to equal  $F_R$  and not close to it, but the use of wrong values for *a* and *m* in

**Fig. 1** Synthetic values for  $R_t$  and the corresponding  $\phi$  from Eq. (7)





**Fig. 2** A set of 10 graphs selected randomly from 1,100 graphs of  $R_{\text{calculated}}$  from Eq. (8) using different values of  $\alpha$  and  $m$  including the known values

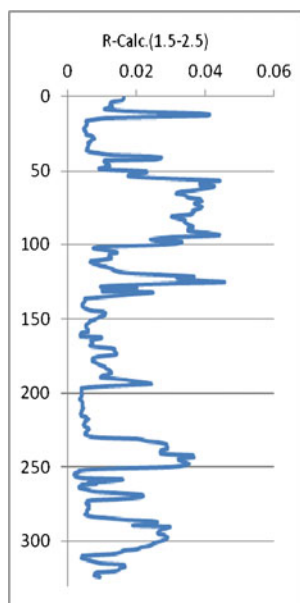


Fig. 2 (continued)

determining  $F_R$  is the only responsible for that; however, this paper introduce a numerical technique to minimize this error and get more accurate values for  $a$  and  $m$  based on the following steps:

1. Equation (1) after substituting with Eq. (3) can be written as follows:

$$R_o = a\phi^{-m}R_w \tag{6}$$

2. Based on Eq. (6), a set of graphs is constructed for  $R_o$  with knowing  $\phi$  and  $R_w$  and by taking  $a=0.5$  and  $m$  from 1.5 to 2.5, 0.1 step and changing  $a$  by 0.01 till 1.5, as listed in Table 3.
3. From step 2, we get 1,100 graphs of  $R_o$  and  $\phi$  versus depth.
4. By comparing  $R_o$  and  $a\phi^{-m}$  (or  $F_R$ ), the closest interval depending on standard deviation and root mean square error will be a water-bearing zones with the accurate values of  $a$  and  $m$ .
5. The selected  $a$  and  $m$  can be used for the entire well to determine the saturation in the other intervals.
6. Since 1,100 graphs of  $R_o$  and  $\phi$  and comparing their result to find the standard deviation and root mean square error for all is very hard, so a program is designed to serve the authors' objective in order to save time and get the required values of  $a$  and  $m$  easily.

**Program description**

The program is designed by C++ language in order to read the input data which include the  $R_w$  value and the different values

of  $R_t$  and  $\phi$  for all intervals. There are three steps to run the program.

1. Data gathering: Log data have to be digitized and put into the format shown in Table 4.
2. Entering  $R_w$  value and the range used for  $a$  and  $m$  from Table 3
3. The output will be in two files, the first file includes the  $R$  calculated for different values of  $a$  and  $m$  with the observed standard deviation and root mean square errors, while the second file contains the final values of  $a$  and  $m$ , which belong to the closest value of the standard deviation observed between the input and output resistivities and the minimum RMS error between them.

**Proposed technique testing and application**

The following is mainly concerning with the running of the proposed technique for the following:

1. Synthetic data to explain clearly how to use it
2. Some core samples selected randomly from different wells from Gulf of Suez Basin, Egypt

**Test number 1: synthetic data**

The authors used synthetic values for  $R_t$  to represent water-bearing zones and use it to determine  $\phi$  (Fig. 1) using  $a=0.62$ ,  $m=2.2$ , and  $R_w=0.06 \Omega m$ , through Eq. (6) after rearrangement as follows:

$$\phi = \left( \frac{0.62 * 0.06}{R_t} \right)^{1/2.2} \tag{7}$$

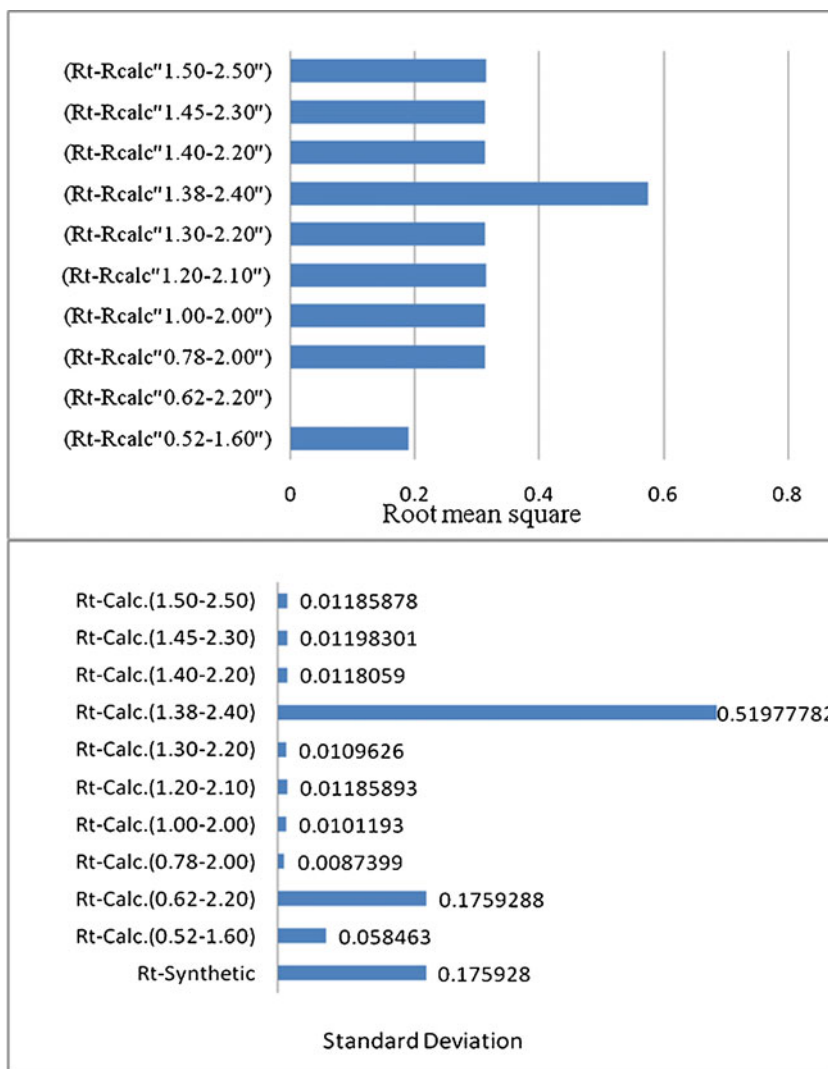
Through that, the authors know the actual values for  $a$  and  $m$ .

The program uses the following Eq. (8) to find the different  $R_{calculated}$  at different values of  $a$  and  $m$ .

$$R_{calculated} = 0.06a\phi_{Synthetic}^{-m} \tag{8}$$

By using different values of  $a$  and  $m$  based on Table 3, and by comparing the synthetic values of  $R_t$  with  $R_{calculated}$  from Eq. (8), Fig. 2 represent a set of 10 graphs selected randomly from 1,100 graphs using different values of  $a$  and  $m$  including the known values which are used in the synthetic data; from Fig. 2, no vision difference can be illustrated, but Fig. 3 can give us the accurate  $a$  and  $m$  after comparing between all

**Fig. 3** RMS error between the  $R_t$  values and  $R_t$  calculated with the observed standard deviation



$R_{calculated}$  data using root mean square error and the standard deviation where the lowest RMS and the closest standard deviation to the actual  $R_t$  belong to  $a=0.62$  and  $m=2.2$ , which is the used values in the present test.

**Test number 2: core samples, Gulf of Suez Basin, Egypt**

Ten core samples selected from different wells at different interval from Gulf of Suez basin, Egypt (compiled by

**Table 5** Measured (Mabrouk 2013) and calculated values for ten core samples, Gulf of Suez Basin, Egypt

Core #	Rw ohm m	Rt ohm m	Phi %	a	m	a-calculated	m-calculated
1	0.26	3.1	18.84	0.68	1.71	0.68	1.72
2	0.23	2.67	19.3	0.72	1.69	0.71	1.70
3	0.37	4.3	19.9	0.59	1.84	0.60	1.84
4	0.089	1.8	18.76	0.89	1.88	0.90	1.89
5	0.25	2.58	25.2	1.40	1.45	1.43	1.44
6	0.021	0.45	23.4	1.20	1.92	1.20	1.87
7	0.06	0.88	21.9	1.10	1.70	1.10	1.68
8	0.068	1.7	15.9	1.01	1.79	1.02	1.80
9	0.038	1.82	18.6	1.30	2.20	1.30	2.20
10	0.25	7	17.8	1.60	1.65	1.61	1.66



Mabrouk et. al. 2013) are used to serve the authors objectives to find  $a$  and  $m$  with the suggested numerical technique where as follows:

1.  $R_w$  calculated from chemical composition of uncontaminated connate water produced from these cores.
2.  $R_t$  calculation is done based on the following general resistance formula:

$$R = (R_t \times L)/A \quad (9)$$

Where  $R$ =resistance ( $\Omega$ ),  $R_t$ =core resistivity ( $\Omega$  m),  $L$ =length of conducting path (m), and  $A$ =cross-sectional area of the path (m).

3. The porosity is calculated from the summation method of the fluid.
4. The resistivity of each water-saturated core is graphed on a log-log graph with a porosity to determine  $a$  and  $m$ .

Using the numerical technique to get  $a$  and  $m$  depending on Table 3, and by using the suggested program, one can easily calculate  $a$  and  $m$  for each sample. The results of calculated values for each sample get extremely close to the measured one with the neglected amount of error (Table 5).

## Conclusions

The use of wrong values of  $a$  and  $m$  in Archie's water saturation equation can yield to overlooking producible zones. This paper introduce a simple numerical technique to calculate  $a$  and  $m$  accurately which depends on  $F_R$ - $R_O$  overlays method for hydrocarbon indicator. The method is tested using synthetic and real data to represent its ability in determining formation factor parameters  $a$  and  $m$  with very high degree of accuracy where the amount of error can be neglected.

The paper also introduced a simple program to help apply the technique easily where 1,100 graphs must

be held in order to get the final correct values of  $a$  and  $m$ .

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## References

- Archie GE (1942) The electrical resistivity log as an aid in determining some reservoir characteristics. *Trans AIME* 146:54–62
- Best DL, Gardner JS, Dumanoir JL (1978) A computer-processed wellsite log computation: 19th Ann. Soc. Prof. Well Log Analysts Logging Symposium, El Paso, Paper Z
- Bigelow EL, abt (1972) The "F" overlay technique and construction of the M.O.P
- Carothers JE (1968) A statistical study of the formation factor relation to porosity. *Log Anal* 9:38–52
- Fertl WH (1978) Rwa method: fast formation evaluation. *Oil Gas J* 76(37):73–76, Advanced Interpretation of Wireline Logs
- Mabrouk WM (2005) BVW As An indicator for hydrocarbon and reservoir homogeneity. *J Pet Sci Eng* 49(1-2):57–62
- Mabrouk WM, Soliman KS, Anas SS (2013) *J Pet Sci Eng* 104(2013): 49–52
- Porter CR, Carothers JE, (1970) Formation factor-porosity relation from well log data: Soc. Professional Well log Analysts, 11th Ann. Logging Symp., Trans., Paper D
- Ransom RC (1974) The bulk volume water concept of resistivity well-log interpretation. A theory based on a new reservoir rock resistivity model. *The Log Analyst*, Vol. 14. No. 1
- Schlumberger (1979) Log Interpretation charts, Houston, Texas
- Sethi DK (1979) Some considerations about the formation resistivity factor-porosity relationships: Soc., Professional Well Log Analysts, 20th Ann. Logging Symp. Trans., Paper L
- Timur A, Hempkins WB, Worthington AE (1972) Porosity and pressure dependence of formation resistivity factor for sandstones. *Trans., Cdn. Well Logging Soc.*, 4, paper D
- Winsauer WO, Shearin Jr., HM, Masson PH, Williams M, (1952) Resistivity of brine-saturated sands in relation to pore geometry, *Bull. AAPG*, Vol. 36 (2)
- Worthington PF, (1985) The evaluation of shaly-sand concepts in reservoir evaluation. *The Log Analyst*, 24, No.1
- Wyllie MRJ, Gregory AR (1953) Formation factors of unconsolidated porous medium: Influence of practical shale and effect of cementation: *Trans. AIME*