Radio Resource Management for LTE-A Relay-Enhanced Cells with Spatial Reuse and Max-Min Fairness[*]

Omar A. Elgendy[1], Mahmoud H. Ismail[2,3] and Khaled Elsayed[3]

[1] Dept. of Engineering Mathematics and Physics, Faculty of Engineering,
Cairo University, Giza 12613, Egypt. Email: omar.abdallah@eng.cu.edu.eg.
[2] Dept. of Electrical Engineering, American University of Sharjah,
PO Box 26666, Sharjah, UAE. Email: mhibrahim@aus.edu. Tel: +971-6-515-4937
[3] Dept. of Electronics and Communications Engineering, Faculty of Engineering,
Cairo University, Giza 12613, Egypt. Email: {mhismail, khaled}@ieee.org.

***Abstract*— One of the major issues in LTE-Advanced (LTE-A) systems is the poor capacity at the cell edge. This is mainly due to the interference experienced by the users as a result of the aggressive frequency reuse usually implemented. Relaying offers an attractive solution for this problem by offering better links than those with the eNodeB (eNB) for the terminals suffering from high path loss or high interference. However, adding relays complicates the resource allocation problem at the eNB and therefore the need for more efficient schemes arises. This is also aggravated by the reuse of resource blocks (RBs) by the relays to fully exploit the scarce spectrum, which, in turn, leads to intra-cell interference. In this paper, we study the joint power and resource allocation problem in LTE-A relay-enhanced cells that exploit spatial reuse. To guarantee fairness among users, a max-min fair optimization objective is used. This complex problem is solved using coordinate ascent and the difference of two convex functions (DC) programming techniques and the proposed scheme indeed converges to a local-optimum quickly. This is shown to be a satisfactory solution according to the simulation results that indicate an almost sevenfold increase in the 10th percentile capacity when compared to previously proposed solutions.**

Keywords*- LTE-A, Relaying;Resource Allocation; Power allocation; Max-Min Fairness; DC Programming.

I. INTRODUCTION

LTE-Advanced (LTE-A) is a 4th Generation mobile communications standard under development by the Third Generation Partnership Project (3GPP) for meeting the requirements set by the International Telecommunication Union IMT-Advanced vision for future mobile networks [1]. LTE-

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[*] This work is part of the 4G++ project supported by the National Telecom Regulatory Authority of Egypt.

[*] This work has been presented in part at the 2014 IEEE Wireless Communications and Networking Conference (WCNC), Istanbul, Turkey.

A uses a frequency reuse that approaches unity among neighboring cells in order to achieve the expected higher system capacity. As a consequence, due to their close proximity to the adjacent cells, the cell-edge (CE) user equipment (UEs) suffer from large inter-cell interference. This problem is even aggravated by the already weak connections maintained by those users due to their remote locations away from their home eNodeB (eNB). The poor signal levels combined with the large interference values experienced by a CE UE, lead to low signal to interference-plus-noise ratio (SINR) consequently resulting in small achievable rates.

One promising technique for solving the above-mentioned problem is via the use of relay stations (RSs). In this scenario, the weak eNB-UE link is divided via the use of an intermediate node called the relay station (RS) into two better connections; one from the eNB to the RS (eNB-RS) and the other from the RS to the UEs (RS-UE). Moreover, relaying can be used to improve the network capacity through spatial reuse. In other words, multiple simultaneous transmissions can occur over the same channel within the same cell [2]. This is possible as a result of the limited power budget of the RSs along with the high pathloss usually associated with urban environments, which limits intra-cell interference [3]. However, to make use of the benefits offered by the RSs, efficient resource allocation (RA) algorithms, which consider the inter- and intra-cell interference constraints, should be sought. Unfortunately, this kind of problem is quite challenging since the placement of relays close to the CE UE of one cell actually increases the interference levels experienced by the CE UEs of the neighboring ones [4].

Trying to reach an optimum solution for the RA problem, lots of works in the literature proposed various approaches. For example, efficient allocation of subcarriers in the orthogonal frequency division multiple access (OFDMA) context was tackled in [5] and [6], efficient power control mechanisms were studied in [7], optimal RSs assignment was addressed in [5] and [8], and optimal RSs placement was investigated in [9]. Joint techniques have also been proposed in the literature. For example, the authors in [10] tackled the joint resource block (RB) assignment and RS selection problem in a multi-cell scenario with proportional fair considerations.

The work at hand addresses the problem of joint resource block assignment (RBA) and power allocation (PA) with a max-min fairness objective. Most of the reported works in the literature, addressing similar problems, usually consider maximizing the sum capacity as an objective. Although this eventually leads to maximizing the whole system capacity, this comes at the expense of the fairness among the CE UEs. Specifically, maximizing the sum capacity will tend to assign low rates for the CE UEs because of their unfavourable channel conditions thus treating these users unfairly most of the time. That is why works like [11], [12], [13] and [14] add a fairness constraint to the problem to guarantee a minimum achievable rate by all UEs or by a subset of delay-sensitive UEs [15]. In this paper, however, we target maximizing the fairness among the UEs by using a max-min fairness criterion instead of the sum capacity criterion.

As for the optimization framework used in the literature, many of the works in the literature also use the Lagrangian dual decomposition strategy such as in [11], [12], [13], [14], [15], and [16] to solve similar RA problems. This strategy is reported to be slowly convergent [17]. Furthermore, the dual problem is not recommended to be solved when the original problem is non-convex with mixed-type variables (i.e., both continuous and binary) due to the non-zero duality gap [18]. Other works, such as [19], use game theoretic approaches for optimization.

In this paper, on the other hand, we solve the joint RBA and PA problem using the coordinate ascent (CA) strategy. In particular, we split the joint problem into two sub-problems where the solution will be iterating between them. The first sub-problem will tackle PA and will be solved using the difference of convex functions (DC) programming strategy, which has been used for power control in [20], [21] and [22]. It is important to stress here that these works only solved the PA problem, whereas our work tackles the more complicated joint RBA and PA problem. Our results indicate that this technique indeed converges to an optimal solution, which is not necessarily a global one. Nevertheless, it will be shown in the results that this local optimal solution is satisfactory. The second sub-problem will deal with RB allocation as a binary assignment problem, which is a well-studied problem in literature that has many efficient solutions.

The rest of the paper is organized as follows: Section II introduces the system model while the formulation of the problem is given in Section III. The proposed solution approach is presented in Section IV along with other earlier approaches that will be used for comparison and benchmarking. Section V presents the performance evaluation results before the paper is finally concluded in Section VI.

II. SYSTEM MODEL

The studied system in this work is depicted in Fig. 1. A single cell is considered where M in-band decode-and-forward (DF) RSs are uniformly distributed on a circle with radius R_{RS} from the eNB and K UEs are uniformly distributed across the cell. The RSs and UEs are respectively indexed by the

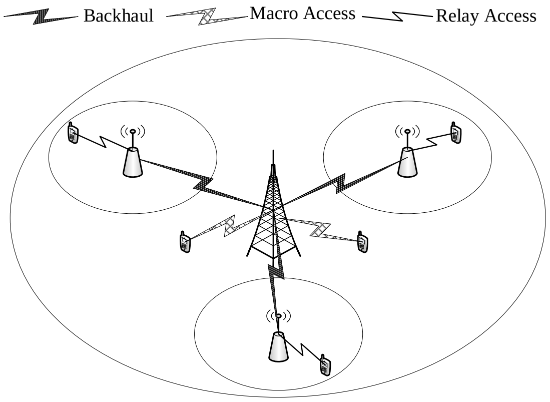

Figure 1: System model.

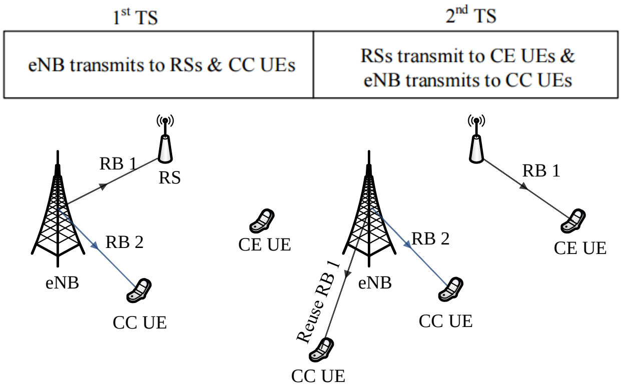

Figure 2: Time resource allocation.

sets $\mathbb{M} = \{1,2,..,m,..,M\}$ and $\mathbb{K} = \{1,2,..,k,..,K\}$. Since only one cell is considered in this work, the inter-cell interference is not taken into account. Also, the different users within the cell are categorized as being either cell-center (CC) UEs (indexed by the set $\mathbb{K}^{CC}$) or CE UEs (indexed by the set $\mathbb{K}^{CE}$) where $\mathbb{K}^{CC} \cup \mathbb{K}^{CE} = \mathbb{K}$. This classification is based on a specific average channel gain (h_{th}), which is proportional to the received signal received power (RSRP) parameter defined in the LTE-A standard. Also, according to LTE-A, UEs will be assigned to a RS based on the strongest downlink received reference signal strength indicator [23]. The CE UEs that are assigned to the k^{th} RS are called the k^{th} RS legacy users. The total bandwidth of the system is divided into N RBs indexed by the set $\mathbb{N} = \{1,2,..n,..,N\}$ and the RBs are assumed to be reused within the same cell. In particular, the RBs assigned for the RS-UE links may be reused by the eNB to transmit to CC UEs, which implies the presence of intra-cell interference.

The studied system operates over two time slots (TSs) as depicted in Fig. 2. During the 1st, data is transmitted from the eNB to the CC UEs as well as to the RSs on non-interfering RBs. During the 2nd, the eNB proceeds with its transmission to the CC UEs and the RSs decode the messages received in the 1st TS and re-transmit them to the UEs using the same RBs of the 1st TS. Note that it is possible that some RS-UE RBs can be reused by the eNB to communicate with the CC UEs in order to increase the cell capacity.

III. POWER AND RESOURCE BLOCK ALLOCATION OPTIMIZATION PROBLEM

In this section, the joint power and resource allocation optimization problem is formulated and possible simplifications will be proposed. The used optimization variables are first summarized in

Table 1. We also assume that the power budget of the eNB over the n^{th} RB during the 1st and 2nd TSs is respectively set to $P_1^{eNB}(n)$ and $P_2^{eNB}(n)$ and the transmitted power used by the m^{th} RS to transmit over the n^{th} RB in the 2nd TS is $P_m^{RS}(n)$. Based on the above, the achievable rate of the k^{th} UE relayed by the m^{th}RS can be written as [24]:

$$R_{m,k}^{CE} = \sum_{n=1}^{N} \rho_{m,k}(n) \times R_{m,k}^{CE}(n) \tag{1}$$

where

$$R_{m,k}^{CE}(n) = \frac{1}{2}\min\left\{C\left(SNR_{eNB-RS_m}(n)\right), C\left(SINR_{RS_m-UE_k}(n)\right)\right\}, \tag{2}$$

$$SNR_{eNB-RS_m}(n) = \frac{P_1^{eNB}(n)h_m(n)}{N_o W}, \tag{3}$$

$$SINR_{RS_m-UE_k}(n) = \frac{P_m^{RS}(n)h_{m,k}(n)}{N_o W + P_2^{eNB}(n)h_k(n)}, \tag{4}$$

and $C(x) \triangleq \log_2(1+x)$ is Shannon's capacity. Eqs. (3) and (4) represent the signal-to-noise ratio (SNR) and SINR at RS_m and UE_k in the 1st and 2nd TSs, respectively. In (3) and (4), $h_x(n)$ represents the gain of the channel between the eNB and the x^{th} node (RS_m or UE_k) over the n^{th} RB, and $h_{m,k}(n)$ denotes the channel gain of the $RS_m - UE_k$ link over the n^{th} RB. These channel gains can generically include both large-scale and small-scale fading channel variations.

TABLE 1: SUMMARY OF THE OPTIMIZATION VARIABLES USED IN THIS WORK.

Optimization Variable	Type	Usage
$\rho_{m,k}(n)$	binary assignment variable (BAV)	Indicates whether the n^{th} RB is assigned to the $eNB - RS_m$ link during the 1st TS and the $RS_m - UE_k$ link during the 2nd one.
$\gamma_k(n)$	BAV	Indicates whether the n^{th} RB is assigned to the $eNB - UE_k$ link during the 1st TS.
$\beta_k(n)$	BAV	Indicates whether the n^{th} RB is assigned to the $eNB - UE_k$ link during the 2nd TS.

The achievable rate by the k^{th} CC UE (served solely by the eNB) can also be written as

$$R_k^{CC} = \frac{1}{2}\sum_{n=1}^{N} \gamma_k(n) \times R_k^{CC,1}(n) + \frac{1}{2}\sum_{n=1}^{N} \beta_k(n) \times R_k^{CC,2}(n) \tag{5}$$

where

$$R_k^{CC,1}(n) = C\left(SNR_{eNB-UE_k,1}(n)\right), \tag{6}$$

and

$$SNR_{eNB-UE_k,1}(n) = \frac{P_1^{eNB}(n)h_k(n)}{N_o W}, \tag{7}$$

respectively denote the capacity and the received SNR at the k^{th} CC UE when using the n^{th} RB during the 1st TS. Similarly,

$$R_k^{CC,2}(n) = C\left(SINR_{eNB-UE_k,2}(n)\right), \tag{8}$$

and

$$SINR_{eNB-UE_k,2}(n) = \frac{P_2^{eNB}(n)h_k(n)}{N_o W + \sum_{m\in M} P_m^{RS}(n)h_{m,k}(n)}, \tag{9}$$

respectively denote the capacity and the received SNR at the k^{th} CC UE when using the n^{th} RB during the 2nd TS.

Now, the joint RBA and PA can be formulated as an optimization problem for the worst-case UE capacity (referred to as the bottleneck capacity in the sequel) as

$$\max_{\boldsymbol{\rho},\boldsymbol{\gamma},\boldsymbol{\beta},\boldsymbol{p}} \min\left\{\min_{k\in\mathbb{K}^{CE}} R_{m,k}^{CE}, \min_{k\in\mathbb{K}^{CC}} R_k^{CC}\right\} \tag{10a}$$

s.t.

$$\sum_{k\in\mathbb{K}^{CE}}\sum_{m=1}^{M} \rho_{m,k}(n) + \sum_{k\in\mathbb{K}^{CC}} \gamma_k(n) \leq 1, \qquad \forall\, n \in \mathbb{N}, \tag{10b}$$

$$\sum_{k\in\mathbb{K}^{CC}} \beta_k(n) \leq 1, \qquad \forall\, n \in \mathbb{N}, \tag{10c}$$

$$RB_{CE}^{min} \leq \sum_{n=1}^{N} \rho_{m,k}(n) \leq RB_{CE}^{max}, \qquad \forall k \in \mathbb{K}^{CE}, \tag{10d}$$

$$RB_{CC}^{min} \leq \sum_{n=1}^{N} \beta_k(n) \leq RB_{CC}^{max}, \qquad \forall k \in \mathbb{K}^{CC} \tag{10e}$$

$$\sum_{n=1}^{N} P_t^{eNB}(n) \leq P_{max}^{eNB}, \qquad \forall\, t \in \{1,2\}, \tag{10f}$$

$$\sum_{n=1}^{N} P_{m,k}^{RS}(n) \leq P_{max}^{RS}, \qquad \forall\, m \in \mathbb{M}, \tag{10g}$$

$$P_1^{eNB}(n) \leq \left(\sum_{k \in \mathbb{K}^{CE}} \sum_{m=1}^{M} \rho_{m,k}(n) + \sum_{k \in \mathbb{K}^{CC}} \gamma_k(n) \right) P_1^{eNB}(n) \tag{10h}$$

$$P_2^{eNB}(n) \leq \left(\sum_{k \in \mathbb{K}^{CC}} \beta_k(n) \right) P_2^{eNB}(n) \tag{10i}$$

$$P_m^{RS}(n) \leq \left(\sum_{k \in \mathbb{K}^{CE}} \rho_{m,k}(n) \right) P_m^{RS}(n) \tag{10j}$$

$$P_t^{eNB}(n),\ P_m^{RS}(n) \geq 0, \qquad t \in \{1,2\}, m \in \mathbb{M}, \tag{10k}$$

$$\rho_{m,k}(n), \gamma_k(n), \beta_k(n) \in \{0,1\}. \tag{10l}$$

In the above optimization problem, constraint (10b) ensures that every RB is assigned to either a single relayed link (eNB- RS_m- UE_k) during the two TSs or to a direct link to the CC UE during the 1st TS. Constraint (10c) ensures no co-channel interference between different direct links during the 2nd TS. Constraints (10d) and (10e) are needed to ensure that no UEs are assigned a large number of or no RBs at all. Constraints (10f) and (10g) make sure that the eNB and RSs respectively use powers that are within their budget. Constraints (10h) - (10j) ensure that no power will be allocated to unused RBs. Last but not least, constraints (10k) and (10l) are respectively the non-negativity and binary constraints imposed on the power and BAVs. In what follows, we will re-formulate the problem in order to put it in a more tractable form.

We first start by simplifying the problem via defining R_{min}^{CE}, R_{min}^{CC} and R_{min}. These correspond to the bottleneck CE UE capacity, the bottleneck CC UE capacity, and the bottleneck UE capacity, respectively. Hence, the new RA problem can be formulated as:

$$\max_{\boldsymbol{\rho},\boldsymbol{\gamma},\boldsymbol{\beta},\boldsymbol{p},R_{min}^{CE},R_{min}^{CC},R_{min}} R_{min} \tag{11a}$$

s.t.

$$\text{Constraints (10b)– (10l)} \tag{11b}$$

$$R_{m,k}^{CE} \geq R_{min}^{CE}, \qquad m \in \mathbb{M}, k \in \mathbb{K}^{CE} \tag{11c}$$

$$R_{k}^{CC} \geq R_{min}^{CC}, \qquad k \in \mathbb{K}^{CC} \tag{11d}$$

$$0 \leq R_{min} \leq R_{min}^{CE}, R_{min}^{CC} \tag{11e}$$

Substituting from (2) and (5) into (11), the following new formulation is obtained:

$$\max_{\boldsymbol{\rho},\boldsymbol{\gamma},\boldsymbol{\beta},\boldsymbol{p},R_{min}^{CE},R_{min}^{CC},R_{min}} R_{min} \tag{12a}$$

s.t.

$$\text{Constraints (10b)– (10l)} \tag{12b}$$

$$\frac{1}{2}\sum_{n=1}^{N} \rho_{m,k}(n) \times C\left(SNR_{eNB-RS_m}(n)\right) \geq R_{min}^{CE} \tag{12c}$$

$$\frac{1}{2}\sum_{n=1}^{N} \rho_{m,k}(n) \times C\left(SINR_{RS_m-UE_k}(n)\right) \geq R_{min}^{CE} \tag{12d}$$

$$\begin{aligned} &\frac{1}{2}\sum_{n=1}^{N} \gamma_{k}(n) \times C\left(SNR_{eNB-UE_{k},1}(n)\right) \\ &+\frac{1}{2}\sum_{n=1}^{N} \beta_{k}(n) \times C\left(SINR_{eNB-UE_{k},2}(n)\right) \geq R_{min}^{CC} \end{aligned} \tag{12e}$$

$$0 \leq R_{min} \leq R_{min}^{CE}, R_{min}^{CC} \tag{12f}$$

The following section discusses in details our proposed approach for solving the problem in (12) as well as other solutions that we use for performance comparison.

IV. THE PROPOSED SOLUTION APPROACH

The CA approach [7] is a commonly-used solution for complex resource and power allocation problems like the one discussed above. The CA is an iterative algorithm where each iteration is composed of 2 stages: the first stage optimizes the RB assignment vectors and the second one optimizes the power vector. Equal power allocation is typically chosen as an initial point. This is then followed by an optimization step on the RB assignment vectors, based on which, the 2nd stage will then optimize on the power vector using the DC programming method. Multiple iterations are executed till a specific stopping criterion is achieved, e.g., the difference between the resulting R_{min} vectors in two successive iterations becomes lower than some prescribed error value ϵ. Although there is no guarantee on the convergence of this algorithm to a global optimum, convergence to a local optimum is guaranteed [7]. In the following two subsections, we provide details on these two stages.

A. *The resource block assignment sub-problem*

In this 1st stage, the power vector is assumed constant, hence, $R_{m,k}^{CE}(n)$ will also be a constant with respect to the RB assignment vectors. Consequently, the problem can be formulated as a mixed integer linear program (MILP) optimization as follows:

$$\max_{\boldsymbol{\rho},\boldsymbol{\gamma},\boldsymbol{\beta},R_{min}^{CE},R_{min}^{CC},R_{min}} \quad R_{min} \tag{13a}$$

s.t.

$$\text{Constraints (10b)–(10c), (10l), (12c)–(12f).} \tag{13b}$$

The solution of this problem can be obtained using any available MILP software package such as MOSEK, TOMLAB, or CPLEX.

B. *The power allocation sub-problem*

In the 2nd stage, the RB assignment variables are fixed and the optimization problem becomes

$$\max_{\boldsymbol{p},R_{min}^{CE},R_{min}^{CC},R_{min}} \quad R_{min} \tag{14a}$$

s.t.

$$\text{Constraints (10f)–(10i), (12c)–(12f).} \tag{14b}$$

Unfortunately, (14) is a non-convex problem due to the interference terms in $SINR_{RS_m-UE_k}$ and $SINR_{eNB-UE_k,2}$ in constraints (12d) and (12e) , respectively. However, reformulating these constraints in the form of a difference between two concave functions, one gets:

$$C\left(SINR_{eNB-UE_k,2}(n)\right) = \log_2\left(1+\frac{P^{eNB}(n)h_k(n)}{N_oW+P_{m^*}^{RS}(n)h_{m^*,k}(n)}\right) \tag{15}$$

$$= u_{m^*,k,n}(\boldsymbol{p}) - v_{m^*,k,n}(\boldsymbol{p})$$

where

$$\begin{gathered} u_{m^*,k,n}(\boldsymbol{p}) = \log_2\left(N_o W + P_{m^*}^{RS}(n) h_{m^*,k}(n) + P^{eNB}(n) h_k(n)\right) \\ v_{m^*,k,n}(\boldsymbol{p}) = \log_2\left(N_o W + P_{m^*}^{RS}(n) h_{m^*,k}(n)\right) \end{gathered} \tag{16}$$

are now concave in the power allocation vector. In the above equations, m^* denotes the RS that reuses the n^{th} RB, where m^* can be obtained from the RB assignment vector. Similarly,

$$\begin{gathered} C\left(SINR_{RS_m-UE_k}(n)\right) = \log_2\left(1 + \frac{P_m^{RS}(n) h_{m,k}(n)}{N_o W + P^{eNB}(n) h_k(n)}\right) \\ = f_{m,k,n}(\boldsymbol{p}) - g_{k,n}(\boldsymbol{p}) \end{gathered} \tag{17}$$

where

$$\begin{gathered} f_{m,k,n}(\boldsymbol{p}) = \log_2\left(N_o W + P^{eNB}(n) h_k(n) + P_m^{RS}(n) h_{m,k}(n)\right) \\ g_{k,n}(\boldsymbol{p}) = \log_2\left(N_o W + P^{eNB}(n) h_k(n)\right) \end{gathered} \tag{18}$$

are also concave in the power vector. We can get rid of the non-concavity in (15) and (17) by linearization using 1st order Taylor expansion around $\boldsymbol{p}^{(i)}$, as follows:

$$C\left(SINR_{eNB-UE_{k,2}}\right) \approx u_{m^*,k,n}(\boldsymbol{p}) - v_{m^*,k,n}\left(\boldsymbol{p}^{(i)}\right) - \boldsymbol{\nabla} v_{m^*,k,n}^T\big|_{\boldsymbol{p}^{(i)}} \times \left(\boldsymbol{p} - \boldsymbol{p}^{(i)}\right) \tag{19}$$

$$C\left(SINR_{RS_m-UE_k}(n)\right) \approx f_{m,k,n}(\boldsymbol{p}) - g_{k,n}\left(\boldsymbol{p}^{(i)}\right) - \boldsymbol{\nabla} g_{k,n}^T\big|_{\boldsymbol{p}^{(i)}} \times \left(\boldsymbol{p} - \boldsymbol{p}^{(i)}\right) \tag{20}$$

where $\boldsymbol{p}^{(i)}$ is the power vector of the ith iteration. The problem is now an affine maximization with a group of concave and affine constraints. Consequently, it can be solved using regular convex optimization solvers. In the sequel, we will refer to the overall algorithm detailed above as Optimized PA + Optimized RBA (OPA+ORBA).

C. *Complexity analysis of the proposed algorithm*

As mentioned above, our algorithm uses the CA optimization approach. It solves the overall problem in an iterative way. Each iteration comprises two sub-iterations:

1. PA sub-iteration: This step involves a convex optimization problem, which can be solved by the interior point algorithm in polynomial time complexity. Clearly, for a network with m relays, the dimension of the optimization variable is $M = m + 5$ and the computational

complexity of SeDuMi interior point solver is estimated to be in the order of $\mathcal{O}(N^2M^{2.5} + M^{3.5})$ [25], where N is the number of inequality constraints.

2. RBA sub-iteration: This step involves an MILP as indicated earlier. For a network with m relays and k UEs, the dimension of the optimization variable is $2k + mk + 3$. For solving such problem, software like CPLEX applies the branch and cut algorithm, which, unfortunately, has a worst-case exponential complexity. However, from a more practical point of view and as will be shown in the next section, the proposed algorithm actually converges after approximately 8 iterations (16 sub-iterations).

D. *Alternative solution approaches based on existing algorithms*

Since the proposed optimization objective has not been previously tackled in the literature, we develop herein an alternative solution technique in order to use it for comparison purposes. This alternative solution is also based on structuring the problem into two sub-problems as follows.

1) *Proposed low-complexity heuristic for the RBA problem*

In [6], a low-complexity scheme for allocation of sub-channels to relays was proposed assuming equal power allocation across all sub-channels. We modify this scheme in order to make it more suitable for our problem and system model. Assuming power is initially allocated equally among the RB's and neglecting the intra-cell interference, we can allocate RBs to UEs using the channel gains averaged on all RBs. The pseudo-code of the proposed modified scheme is depicted in Algorithm 1. The scheme works as follows: the CE UEs are first arranged based on their average channel gains and then RBs are allocated to them in that order. This is done to make sure that all CE UEs get some RBs. All UEs are then arranged based on their average channel gains, and the remaining RBs are assigned to them (during the 1st and 2nd TSs for CE UEs relayed transmissions and the 1st TS for CC UEs) until all RBs are consumed. Using the above steps, some CC UEs might not be assigned any RBs because their channel is relatively poor compared to other CC UEs. Therefore, these CC UEs are arranged again as before and RBs are assigned based on the new arrangement.

2) *Iterative multi-level water filling (IMLWF) algorithm for the PA problem*

Recall that the main objective here is to find the transmit powers of all the nodes with the maximization of the worst-case capacity in mind as in (12a). It is clear that system under investigation involves $M + 2$ transmitting nodes. The eNB, which transmits to the RSs and the CC UEs during the 1st TS and transmits to CC UEs during the 2nd TS, effectively constitutes 2 nodes. The remaining nodes are the M RSs that transmit to their legacy CE UEs during the 2^{nd} TS. In order to achieve the objective mentioned above, we propose to find the required transmit power of each of the $M + 2$ nodes such that the bottleneck capacity among its legacy users is maximized. Using this approximation, the main PA problem is effectively divided into a number of PA sub-problems each of which has the form:

$$\max_{\boldsymbol{P_{k,i}}} R_{min} \tag{21a}$$

s.t.

$$\sum_{i=1}^{L_k} \log_2\left(1 + P_{k,i}\lambda_{k,i}\right) \geq R_{min} \;\forall k, i \tag{21b}$$

$$\sum_{k=1}^{K}\sum_{i=1}^{L_k} P_{k,i} \leq P_{max} \tag{21c}$$

$$R_{min} \geq 0\;, P_{k,i} \geq 0, \forall k \in \{1, \dots, K\}, i \in \{1, \dots, L_k\}. \tag{21d}$$

In each sub-problem, the k^{th} user is assigned L_k RBs, which are controlled by $\rho_{m,k}(n), \gamma_k(n)$ and $\beta_k(n)$ found according to the heuristic detailed in Section IV.C.1. In (21b), $P_{k,i}$ is the power assigned to the k^{th} user on the i^{th} assigned RB and $\lambda_{k,i}$ is the channel-to-interference-plus-noise ratio (CINR) similar to SINR except that the signal power in the numerator is omitted. It is worth mentioning that

Algorithm 1: RB Assignment Heuristic

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Step 1: Initialization
Set $\rho_{m,k}(n), \gamma_k(n), \beta_k(n) = 0 \; \forall \; m, k, n$. Let $\mathcal{A} = \{1,2,\dots,N\}$ be the set of unassigned RBs.
Step 2: Sort CE UEs ascendingly based on the average channel gains to their assigned relays
$\overline{h_{m,k}} = \sum_n h_{m,k}(n)/N \,, \forall k \in \mathbb{K}^{CE}$. Let $\mathbb{K}_S^{CE}$ be the set of indices of the sorted CE UEs.
for $k \in \mathbb{K}_S^{CE}$
    Choose the best RB for the $k^{th}$ CE UE:
    $n^* = \arg\max_n h_{m,k}(n)\,,$
    $\rho_{m,k}(n^*) = 1,$
    $\mathcal{A} = \mathcal{A}/\{n^*\}.$
end for
Step 3: Sort CE and CC UEs ascendingly according to average channel gains to their transmitting nodes
$\overline{h_{m,k}} \; \forall \; k \in \mathcal{K}^{CE}$ and $\overline{h_k} = \sum_n h_k(n)/N \,, \forall k \in \mathbb{K}^{CC}$. Let $\mathbb{K}_S$ be the set of indices of the sorted CE UEs.
for $k \in \mathbb{K}_S$
    Choose the best RB for the $k^{th}$ UE:
    if $k \in \mathbb{K}^{CE}$
        $n^* = \arg\max_n h_{m,k}(n)\,,$
        $\rho_{m,k}(n^*) = 1,$
    else
        $n^* = \arg\max_n h_k(n)\,,$
        $\gamma_k(n^*) = 1,$
    end if
    $\mathcal{A} = \mathcal{A}/\{n^*\}.$
    if $\mathcal{A} = \phi$
        break;
    end if
end for
Step 4: Find CC UEs with no RBs during 1st TS, let $\mathcal{J}^{CC}$ be the set of their indices. Sort these CC UEs ascendingly based on the average channel gains to eNB $\overline{h_k} \; \forall k \in \mathcal{J}^{CC}$. Let $\mathcal{J}_S^{CC}$ be the set of indices of the sorted CC UEs, $\mathcal{A} = \{1,2,\dots,N\}$ be the set of unassigned RBs.
for $k \in \mathcal{J}_S^{CC}$
    Choose the best RB for the $k^{th}$ CC UE:
    $n^* = \arg\max_n h_k(n)\,,$
    $\beta(n^*) = 1,$
    $\mathcal{A} = \mathcal{A}/\{n^*\}.$
end for
```

the interference power in CINR is considered constant and is determined from the last iteration. The Lagarangian of (21) can now be written as:

$$\mathcal{L} = R_{min} + \sum_{k=1}^{K} \alpha_k \left(R_{min} - \sum_{i=1}^{L_k} log_2\left(1 + P_{k,i}\lambda_{k,i}\right) \right) + \eta \left(\sum_{k=1}^{K} \sum_{i=1}^{L_k} P_{k,i} - P_{max} \right) \qquad (22)$$

Differentiating with respect to $P_{k,i}$, equating to zero, and after straightforward manipulations, one gets

$$P_{k,i} = \left(\mu_k - {\lambda_{k,i}}^{-1}\right)^{+} \qquad (23)$$

where $\mu_k = \frac{\alpha_k}{\eta \ln 2}$, $(x)^{+} \triangleq \max\{x, 0\}$. The result in (23) can be thought of as a multi-level water filling (MLWF) problem with the k^{th} user having the water level (μ_k). In [26], an efficient scheme for calculating μ_k $\forall k \in \{1, \dots, K\}$ was proposed. This algorithm aims at strictly equalizing the rates of all users thus realizing the long-term behaviour of max-min fair RA problems. In our scenario, the MLWF algorithm is applied to the eNB and separately to each RS in the 2nd TS to perform power allocation to their users. The regular water filling solution is applied to the eNB in the 1st TS to assign power to CC UEs and RSs. In order to account for the intra-cell interference in the 2nd TS, the iterative water-filling (IWF) algorithm proposed in [27] is used with each node assuming that interference resulting from other nodes as noise. The interference level is then adjusted and the further iterations of the algorithm are performed until a Nash equilibrium point is reached. In the rest of the paper, we refer to this algorithm as the IMLWF+Heuristic RBA (IMLWF+HRBA) scheme.

V. PERFORMANCE EVALAUTION AND SIMULATION RESULTS

In this section, we evaluate the performance of the proposed optimization framework (Optimized PA + Optimized RBA (OPA+ORBA)) and compare it with the IMLWF+HRBA algorithm via MATLAB simulations. Ten different random snapshots were used, where each snapshot includes ten transmission time intervals (TTIs). All the links are simulated using the Winner II channel model [28]. Moreover, the eNB-UE, RS-UE and eNB-RS links are modeled as typical urban macro-cell (C2), typical urban micro-cell (B1), and feeder link (B5f), respectively. The value of h_{th} is chosen empirically such that 30 – 40% of the users are CE UEs. All the other simulation parameters are provided in Table 2. We also use CPLEX 12.5 [29] to solve the RB assignment problem. As for the PA optimization problem, it is modelled as a DC problem and solved iteratively using the MOSEK package within CVX [30] modelling framework.

TABLE 2: SYSTEM PARAMETERS

Parameter	**Value**
Cell radius (R)	500 m
Number of RBs (N)	50
Number of UEs (K)	60
Number of RSs (M)	3
Distance of RSs from the center (R_{RS})	$0.7R$
Threshold average channel gain (h_{th})	0.5×10^{-12}
Maximum eNB power (P_{max}^{eNB})	46 dBm
Maximum RS power (P_{max}^{RS})	30 dBm
Noise PSD (N_0)	-174 dBm/Hz

Fig. 3 shows the convergence speed of the proposed scheme for an arbitrary simulation run vs. the number of iterations. Since the optimization algorithm alternates between the PA and RB problems as discussed earlier starting with the PA one (assuming equal power allocation), all the even-numbered iterations represent a PA optimization step while the odd-numbered ones represent an RB optimization step. In this specific scenario, the number of CC and CE UEs were chosen to be 39 and 21, respectively. The convergence behavior indicates that the scheme succeeds in progressively increasing the bottleneck capacity (CE or CC) until a plateau effect is reached. Accordingly, the iterations can be stopped after a reasonable number by either carefully choosing a tolerance value ϵ or after a fixed number. In our simulations, we found out that 10 overall iterations are adequate for noticeable improvement in the bottleneck capacity and fairness as will be shown in the reported results. In addition, we notice that the algorithm maintains the value of the worst-case CC capacity always above the value of bottleneck CE capacity. This is due to the ease of increasing the rate of CC UEs compared to CE UEs since an eNB is capable of transmitting higher power compared to a RS. From a complexity point of view, however, the complexity of the IMLWF+HRBA algorithm is lower because it involves a single RB and PA assignment steps with the latter employing IMLWF. For comparison purposes, we propose another simplified approach based on the OPA+ORBA algorithm. In this approach, the optimal PA solution is replaced with equal power allocation (EPA), hence the name EPA+ORBA.

The CDF of the bottleneck capacity as well as all users' capacities (CC UEs and CE UEs) for the 3 algorithms are depicted in Figs. 4 and 5. In Fig. 4, the 10th percentile capacities for the OPA+ORBA, EPA+ORBA and IMLWF+HRBA are 1.35 bps/Hz, 0.16 bps/Hz, and 0.15 bps/Hz, respectively. In Fig. 5, the 10th percentile capacities for the three methods are 1.8 bps/Hz, 0.9 bps/Hz, and 0.5 bps/Hz, respectively. We notice that OPA+ORBA exhibits the best performance among all algorithms in terms of the bottleneck as well as all users' capacities. Specifically, the 10th percentile capacity for OPA+ORBA, EPA+ORBA, and IMLWF+HRBA are 1.86 bps/Hz, 0.77 bps/Hz and0.28 bps/Hz, respectively. This translates to an OPA+ORBA capacity gain of 2.4 and 6.6 times when compared to EPA+ORBA and IMLWF+HRBA algorithms, respectively.

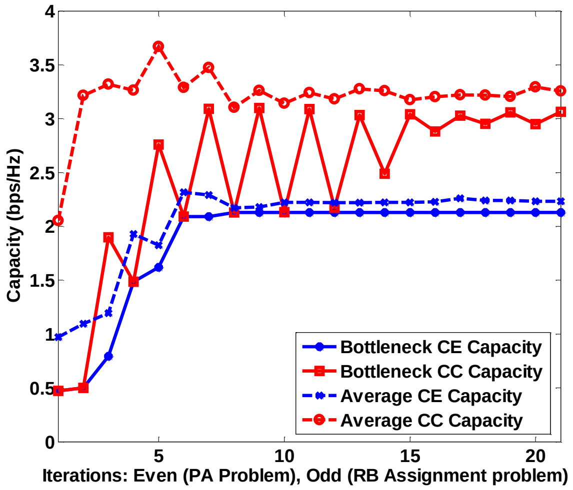


Figure 3: Convergence behavior of the OPA+ORBA algorithm.

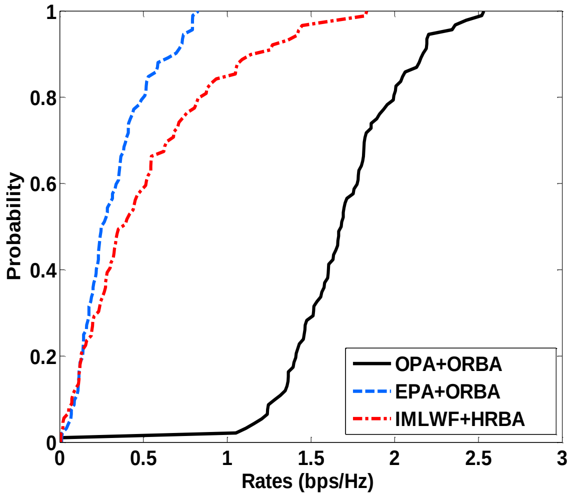


Figure 4: CDF of worst-case capacity.

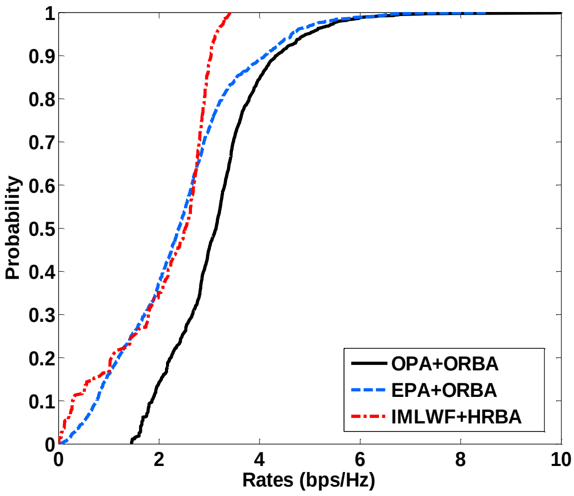


Figure 5: CDF of all users' capacities.

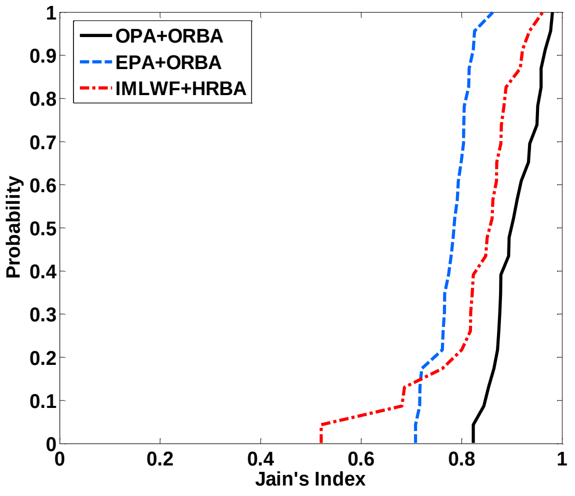
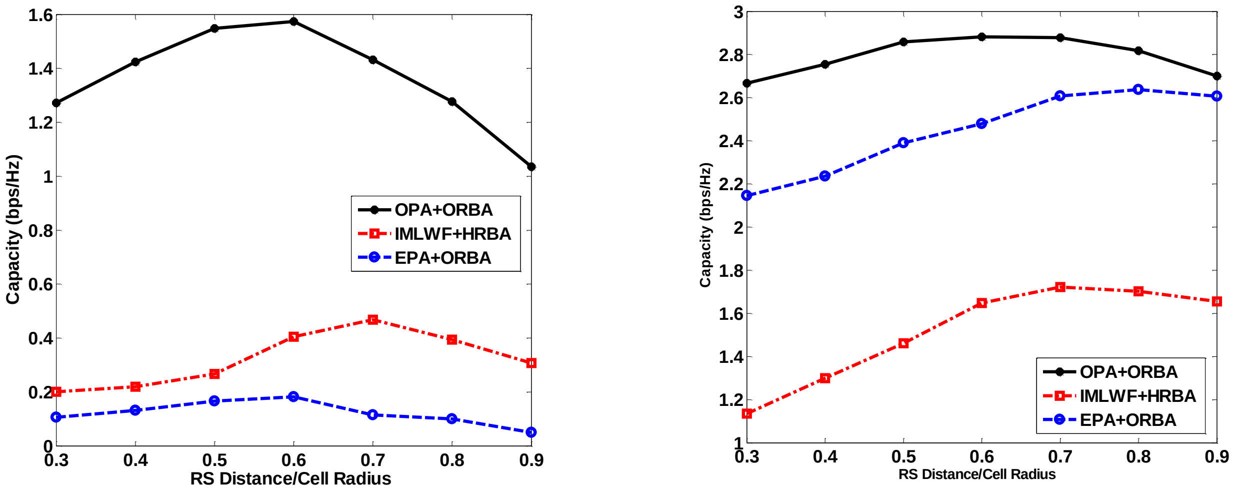


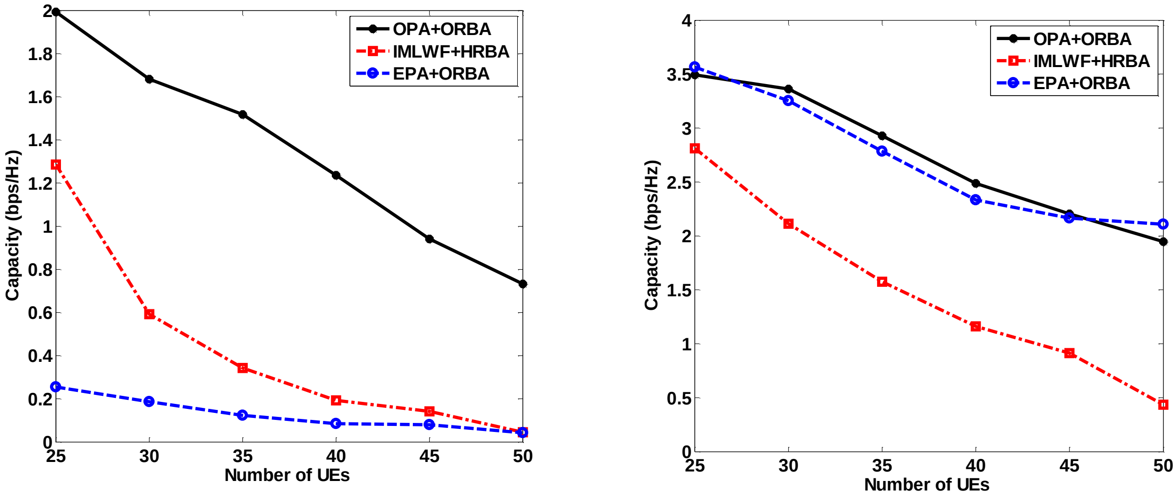
Figure 6: CDF of Jain's fairness index.



(a)

(b)

Figure 7: (a) Bottleneck and (b) Average capacity vs. relay distance.



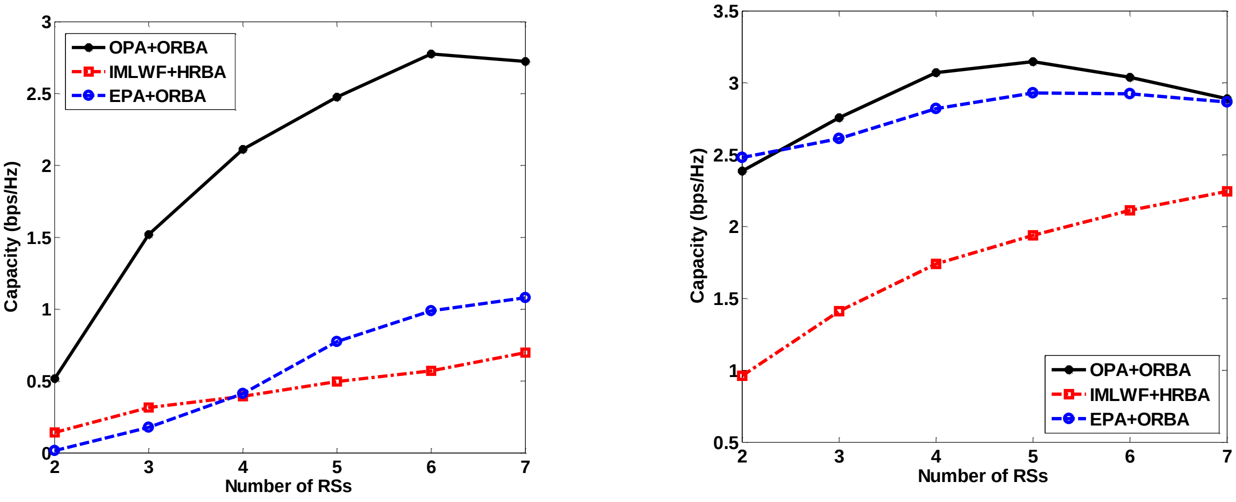
(a)

(b)

Figure 8: (a) Bottleneck and (b) Average capacity vs. number of UEs.

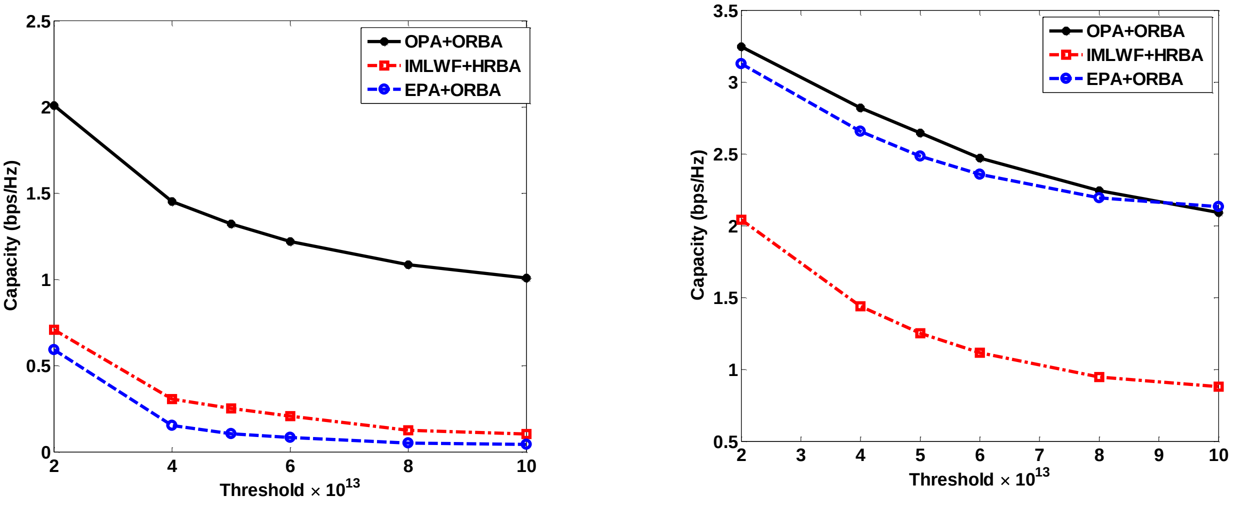
To assess the fairness performance, we use the well-known Jain's fairness index [5]. We report the CDF of Jain's fairness index for the three algorithms in Fig. 6. As expected, OPA+ORBA achieves the highest fairness due to the utilization of a max-min fair allocation criterion that ensures fairness among users. We also observe that EPA+ORBA exhibits the worst fairness because EPA is indeed an unfair way of allocating power to the UEs as it does not cater well for the UEs that have bad channel conditions.

We also investigate the effect of the RSs-eNB distance on the bottleneck and average capacities. Intuitively, there should be an optimal distance that is not too large or too small to guarantee good RS backhaul links and good relay-access links as well. Fig. 7 shows the results of this experiment, averaged over 80 random snapshots, and 5 TSs/snapshot. The results show that an optimal distance of $0.6R$ maximizes the bottleneck and average capacities. We also notice that the bottleneck capacity of EPA+ORBA algorithm has the lowest values due to the fact that EPA is not fair in allocating resources.



(a) (b)

Figure 9: (a) Bottleneck and (b) Average capacity vs. number of RSs.



(a) (b)

Figure 10: (a) Bottleneck and (b) Average capacity vs. threshold (h_{th}).

As the number of UEs increases, the bottleneck and average capacities should decrease because of the increased competition on resources according to spatial reuse. Fig. 8 shows the results of this experiment on bottleneck and average capacities by averaging over 110 random snapshots, and 5 TSs/snapshot. As expected, the bottleneck and average capacities clearly decrease as the number of UEs increases, and our algorithm has the best performance in terms of the bottleneck capacity and a comparable performance with EPA+ORBA in terms of the average capacity.

An important question that comes into mind is what effect increasing the number of RSs has on the bottleneck and average capacities. Intuitively, the bottleneck capacity should benefit from the increased number of RSs because of the resulting diversity of RSs. However, the number of RSs cannot be increased arbitrarily because of spatial reuse. Fig. 9 shows the bottleneck and average capacities (averaged over 80 random snapshots, and 5 TSs/snapshot) for this experiment where the bottleneck capacity results agree with our intuition. We notice that the average capacity increases at first because of the increased capacity of CE UEs, then it decreases when the number of RSs exceeds 5 because of the increased intra-cell interference caused by the RSs due to spatial reuse.

One final important factor that needs to be assessed is the effect of the channel gain threshold defining CE users. Essentially, increasing this threshold will increase the number of UEs that are classified as CE, which leads to more competition on relays resources. Accordingly, the bottleneck capacity of this increased number of CE UEs decreases. This is indeed depicted in Fig. 10, which shows the deterioration of both bottleneck and average capacities when increasing this threshold.

VI. CONCLUSIONS

A novel algorithm for allocating resources in OFDMA relay-enhanced cells with spatial reuse has been proposed. The proposed algorithm allocates power and RBs in a max-min fair fashion, which is of paramount importance in multiuser systems to attain fair resource allocation. Performance comparison with other techniques shows the superiority of our proposed algorithm in terms of capacity and fairness; the 10th percentile capacity has improved by a factor of 6.6 compared to previously proposed solution techniques. However, this comes at the expense of increased complexity.

VII. CONFLICT OF INTEREST

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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