ELCN100
Electronic Lab.
Instruments and Measurements
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Lecture 02:
Measurements Errors

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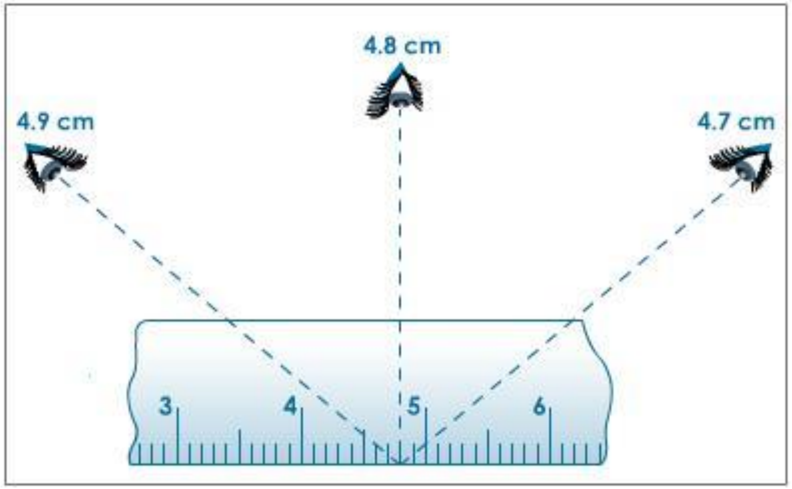
Measurement Errors

- No electronic component or instrument is perfectly accurate
- Sometimes, errors might completely cancel each other out, the worst case combination of errors must always be assumed

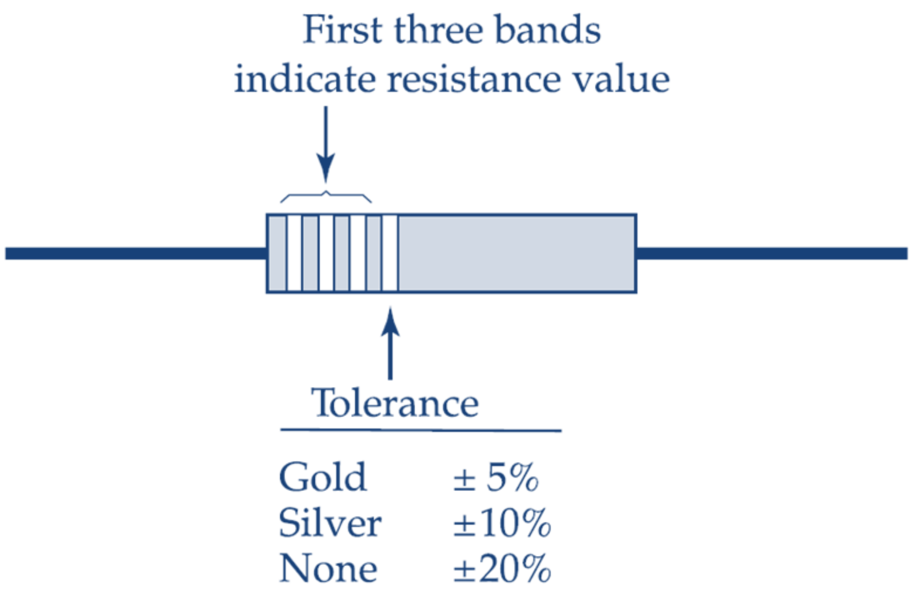


Sources of Errors

- Errors due to equipment imperfection
- Errors of unexplainable origin (classified as random errors)
- Gross errors which are essentially human errors that are the result of carelessness
  - such as misreading of an instrument
- Errors occur because the instrument is not calibrated before usage
- Errors occur because the measurement system affects the measured quantity
  - such as the loading effect
- Errors due to environmental conditions such as temperature and humidity

Absolute Errors and Relative Errors

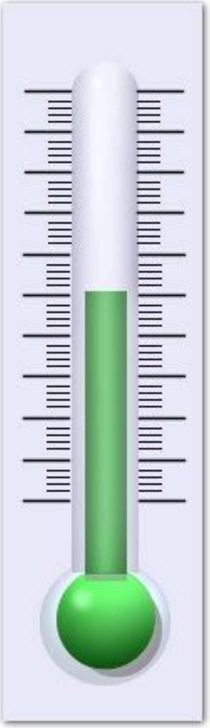
- If a resistor is known to have a resistance of 500Ω with a possible error of ±50Ω, the ±50Ω is an absolute error
- When the error is expressed as a percentage or as a fraction of the total resistance, it becomes a relative error. For example, 500Ω ± 10%.



The relative error in a measured or specified quantity is expressed as a percentage of the quantity. The absolute error is determined by converting the relative error into an absolute quantity.

Absolute Errors and Relative Errors

- ppm = part per million
- Example
  - The temperature coefficient of 1MΩ resistor =100ppm/^{o}C,
  - 100ppm/^{o}C means 100 parts per million per degree Celsius.
  - One millionth of 1MΩ is 1Ω, consequently, 100ppm of 1MΩ = 100Ω.
  - So, the 1^{o}C change in temperature might cause the 1MΩ resistance to increase or decrease by 100Ω.



Accuracy, Precision, Resolution, and Range

Accuracy

- is defined as how much can the value obtained from measurement differ from the actual value.
  - For example, a Voltmeter with an error of ±1% reads exactly 100V, the true level of the measured voltage is somewhere between 99V and 101V.

Precision

- is defined as the degree to which the instrument produces similar results for repeated measurements of the same quantity.

- Difference between Accuracy and Precision

(a) Digital voltmeter display with a 1 mV precision

Accuracy, Precision, Resolution, and Range

Resolution

- is defined as the smallest change in the measured quantity that can be detected.
- This definition is very close to the measurement precision.

Range

- is defined as the limits of the measured quantity values between which the instrument operates correctly.

Measurement Error Combination

❑ Sum of Quantities

- Let $y = u+v$ and assume u has an absolute error of Δu and v has an absolute error of Δv
- $y+\Delta y = (u+\Delta u)+(v+\Delta v)=[u+v]+[\Delta u+\Delta v]$ -->

$$\underline{\Delta y = \Delta u + \Delta v}$$

- In terms of the relative error,
- $\Delta y/y = \Delta u/y + \Delta v/y$ -->

$$\underline{\Delta y/y = (\Delta u/u)*(u/y)+ (\Delta v/v)*(v/y)}$$

Measurement Error Combination

- **Example**
  - Two resistors with values 100 Ω ± 1% and 80 Ω ± 5% are connected in series. What is the relative error in the total resistance?

- **Solution**
  - Rt = R1 + R2 = 100 + 80 = 180Ω
  - ΔR1 = 0.01*100 = 1Ω
  - ΔR2 = 0.05*80 = 4Ω
  - ΔRt = ΔR1+ΔR2 = 1 + 4 = 5Ω
  - Relative error of Rt = ΔRt/Rt = (5/180)*100 = 2.8%
  - So, the relative error in the total resistance = ±2.8%

- The absolute error in the total resistance is still larger than the absolute error in any of them

Measurement Error Combination

Difference of Quantities

- Let $y = u-v$ and assume u has an absolute error of Δu and v has an absolute error of Δv
- $y+\Delta y = (u+\Delta u)-(v+\Delta v)=[u-v]+[\Delta u+\Delta v]$ -->

$$\underline{\Delta y = \Delta u + \Delta v}$$

- In terms of the relative error,
- $\Delta y/y = \Delta u/y + \Delta v/y$ -->

$$\underline{\Delta y/y = (\Delta u/u)*(u/y)+ (\Delta v/v)*(v/y)}$$

Measurement Error Combination

Example

- Calculate the maximum percentage error in the sum and the difference of the following two measured voltages when V1= 100 V ± 1% and V2= 80 V ± 5%.

Solution

- Let S = V1 + V2 = 180V and D = V1 - V2=20V.
- $\Delta V1 = 0.01*100 = 1V$
- $\Delta V2 = 0.05*80 = 4V$
- $\Delta S = \Delta V1 + \Delta V2 = 5V$
- $\Delta S/S = (5/180)*100 = 2.8\%$
- $\Delta D = \Delta V1 + \Delta V2 = 5V$
- $\Delta D/D = (5/20)*100 = 25\%$
- The difference between two quantities should be avoided during measurements if possible due to its large percentage error

Measurement Error Combination

- **Product of Quantities**
  - **Let $y = u*v$ and assume u has an absolute error of Δu and v has an absolute error of Δv**
  - **$y+\Delta y = (u+\Delta u)*(v+\Delta v)=u*v + u*\Delta v + v*\Delta u + \Delta u*\Delta v$**
  - **Assuming $\Delta u*\Delta v$ is very small**
  - **$y+\Delta y \approx u*v + u*\Delta v + v*\Delta u$ --> $\underline{\Delta y = u*\Delta v + v*\Delta u}$**
  - **In terms of the relative error,**
  - **$\Delta y/y = \Delta y/(u*v) = (u*\Delta v + v*\Delta u)/(u*v) = \underline{\Delta u/u + \Delta v/v}$**

- The absolute error of the sum (or difference) of two quantities equals the sum of the absolute errors of these two quantities

- The percentage relative error of the product (or quotient) of two quantities equals the sum of the percentage relative errors of these two quantities.

Measurement Error Combination

Example

- A 820 Ω resistor with an accuracy of ±10% carries a current of 10mA. The current is measured by an analog ammeter on a 25mA range with an accuracy of ±2% of the full scale. Calculate the voltage across this resistor and power dissipated in it and determine the accuracy of your results.

Solution

- V = I*R = 10mA * 820 = 8200 mV = 8.2V
- P = I^2*R = $(10mA)^2$*820Ω = 82 mW
- % error in R = ±10%
- Absolute error in I = ±2% of 25mA (full scale) = ±0.5mA
- % error in I =±(0.5/10)*100= ±5%
- % error in V = I*R = (% error in I)+ (% error in R) = ± (5%+10%) = ±15%

Measurement Error Combination

- **Solution**
  - % error in I^2 = (% error in I)+ (% error in I)

    = ± (5%+5%) = ±10%
  - % error in P = $I^2 * R$ = (% error in I^2)+ (% error in R)

    = ± (10%+10%) = ±20%

Measurement Error Combination

Example

- The current flowing in a resistance is measured with a relative limiting error of ± 1.5%, and power fed is measured with relative limiting error of ±1%. Find the relative limiting error in the calculated value of the resistance.

Solution

- $R = P/I^2$
- % error in R = % error in P + % error in I^2

  = % error in P + % error in I + % error in I

  =± (1%+1.5%+1.5%) = ± 4%.