Circuits I

Lecture 3-5: Methods of DC Solutions

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Modified from Dr. Mohamed Fathy Slides

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Methods of solution of DC circuits

- **<u>Definition :</u>**

The problem of solution of DC circuits is finding the response (voltage, current, power) in any circuit element due to input excitation (voltage and/or current sources).

Methods of solution of DC circuits

- **Method 1:Branch Method :** KCL & KVL

- Circuit Topology

N Nodes **L** Loops **B=N+L-1** Branches

  - **Write N-1 equations at (N-1 nodes) using KCL**
  - **Write L equations at (L independent Loops) using KVL**
  - **Solve the N+L-1 equations to find all branch currents.**

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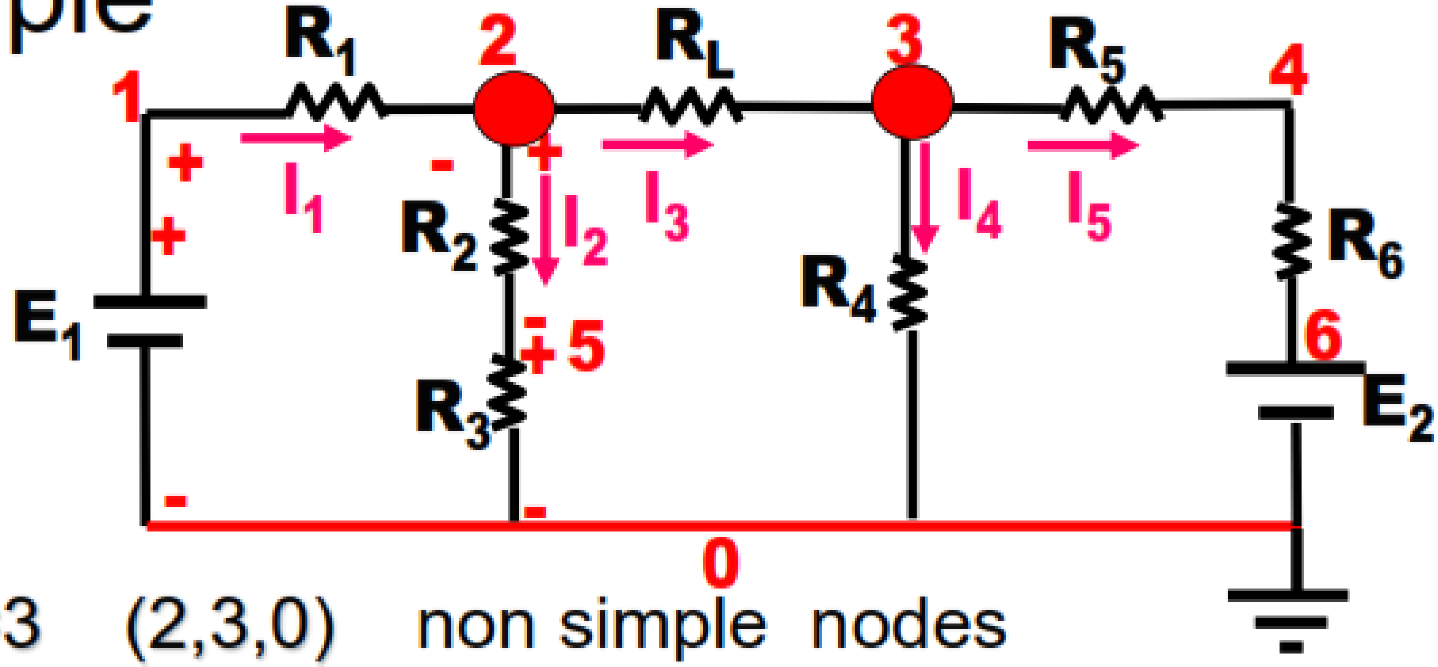
Example



- Nodes: N=3 (2,3,0) non simple nodes
- Loops: L=3 independent loops 12501, 23052, 34603
- Branches B=5 012, 250, 23, 30, 3460
- B=N+L-1

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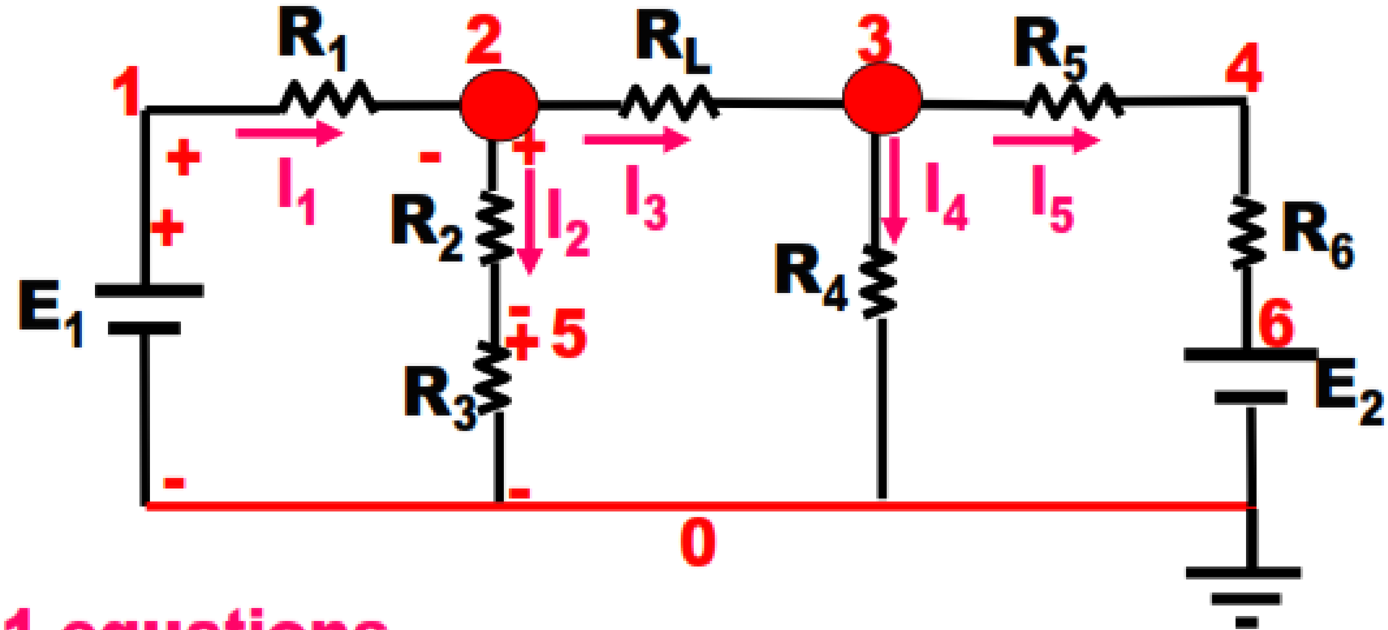

✓ **Write N-1 equations using KCL (usually node 0 is excluded)**

Node 2

Node 3

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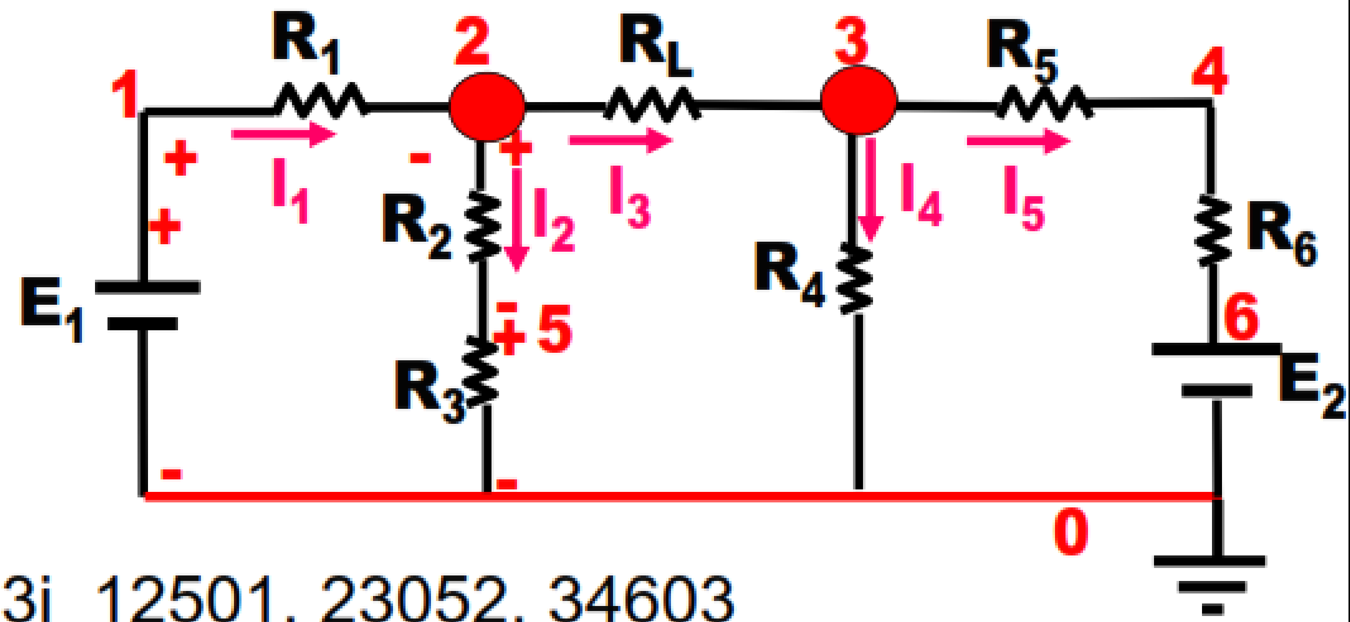

- Loops: L=3i 12501, 23052, 34603

✓ **Write L equations using KVL**

Loop 12501

$$V_{12} + V_{25} + V_{50} + V_{01} = 0$$

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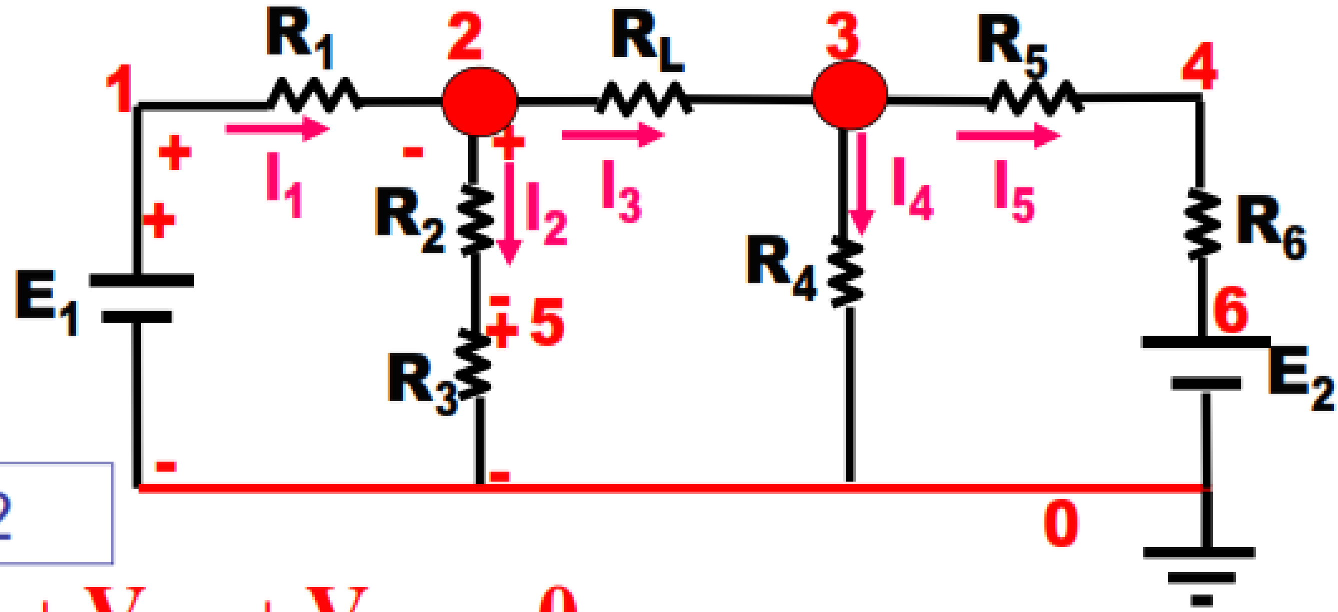
✓ Write L equations using KVL



Loop 23052

$$V_{23}+V_{30}+V_{05}+V_{52}=0$$

Loop 34603

$$V_{34}+V_{46}+V_{60}+V_{03}=0$$

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Example

$$I_1 - I_2 - I_3 = 0 \qquad (1)$$

$$I_3 - I_4 - I_5 = 0 \qquad (2)$$

$$I_1R_1 + I_2R_2 + I_2R_3 - E_1 = 0 \qquad (3)$$

$$I_3R_L + I_4R_4 - I_2R_3 - I_2R_2 = 0 \qquad (4)$$

$$I_5R_5 + I_5R_6 + E_2 - I_4R_4 = 0 \qquad (5)$$

How to put Equations in a matrix form?

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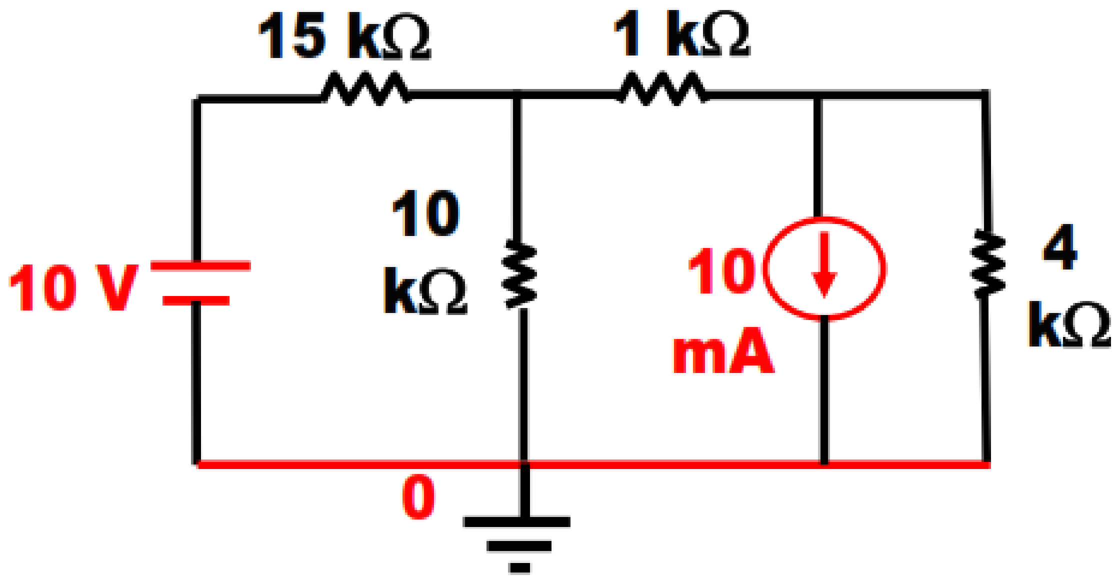
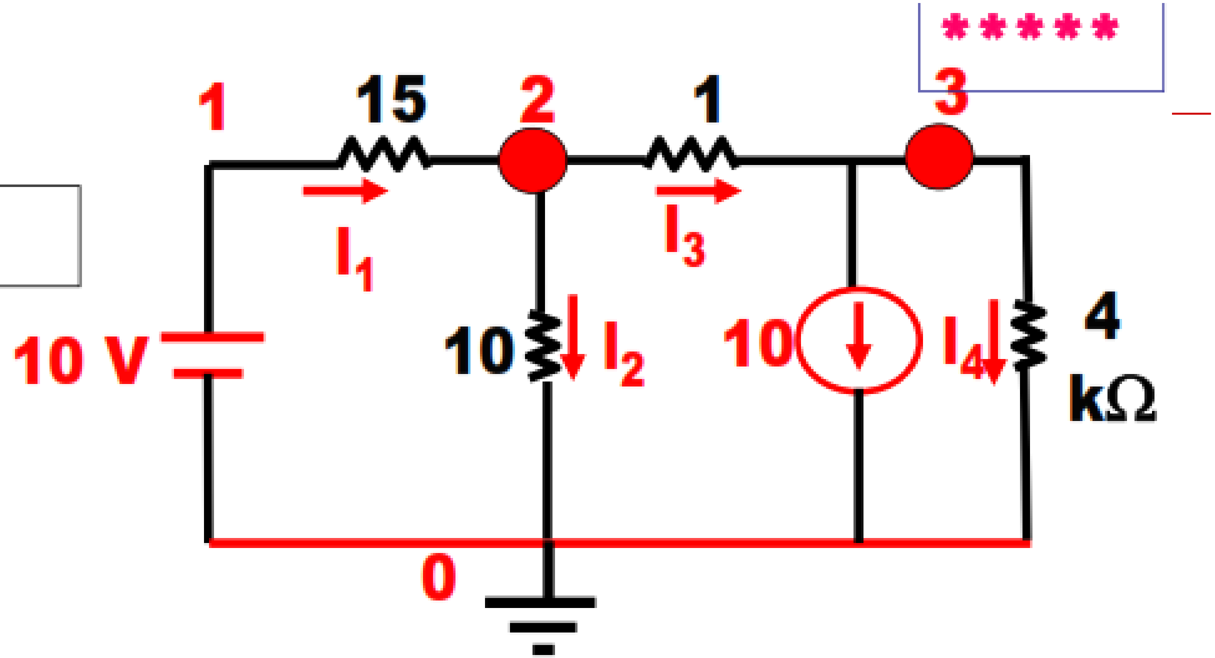
For the shown circuit, find all branch currents and check the power balance



N=

L=

B=



KCL Node 2

KCL Node 3

KVL loop 1201

KVL loop 2302

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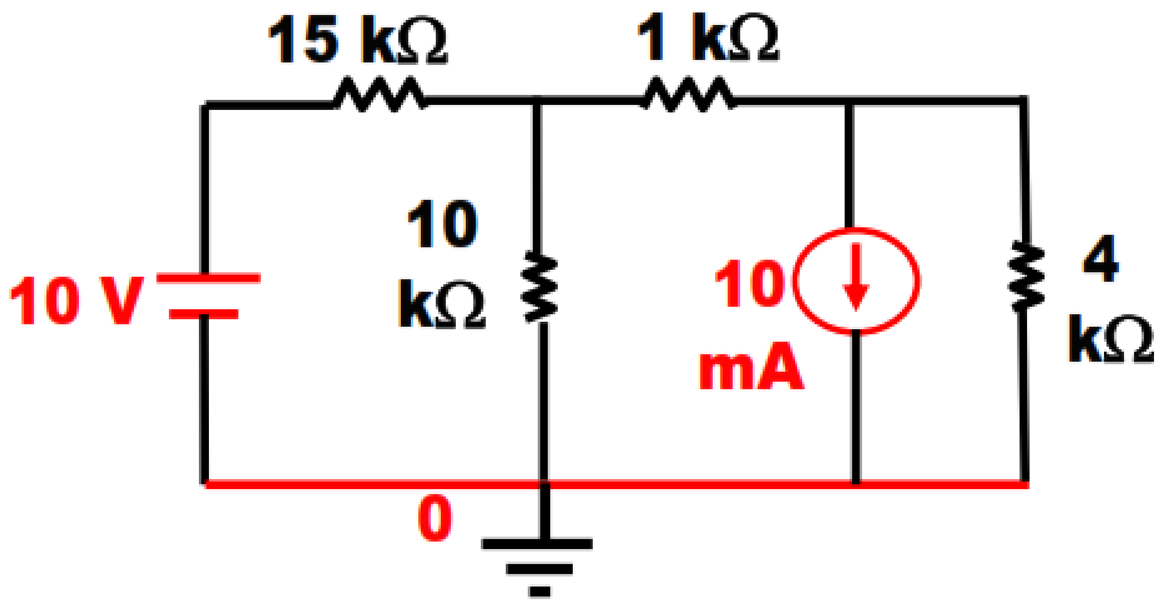
For the shown circuit, find all branch currents and check the power balance

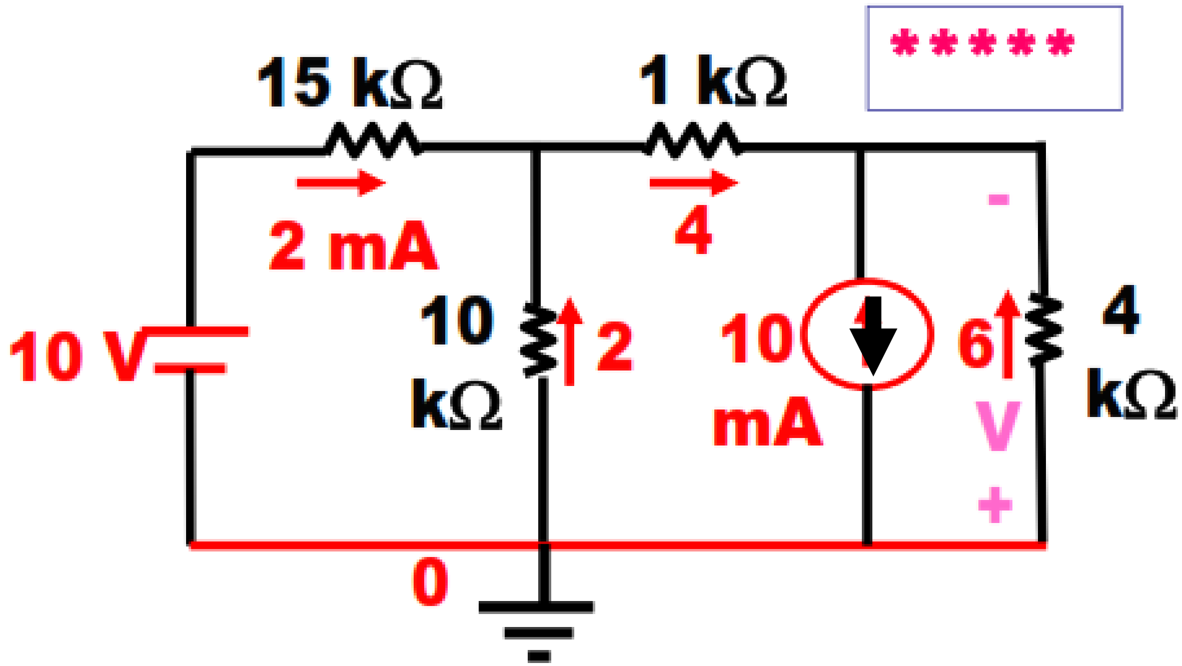


Solve equations 1-4 to get:

$I_1=2 \quad I_2=-2 \quad I_3=4 \quad I_4=-6$

Power balance

R kΩ	Power (mw) I^2R
15	
10	
1	
4	
Total	



Source	Power (mw)
10 V	
10 mA	
Total	

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BEAT QUESTION

Two random students are selected and keep asking questions till one beats the other…

Fourth row, second student from the left wall

third row, fourth student from the right wall

Punishment: Read the following:

سبع لحاليح استلحلحناهم من عند المستلحلحين تقدر يا ملحلح يا مستلحلح تستلحلحلنا سبع لحاليح زى ما استلحلحناهم من عند المستلحلحين

Methods of solution of DC circuits

- <u>Method 2</u> Step by Step Simplification

1. Transforming current sources into equivalent voltage sources or visa versa

2. Combination of active elements

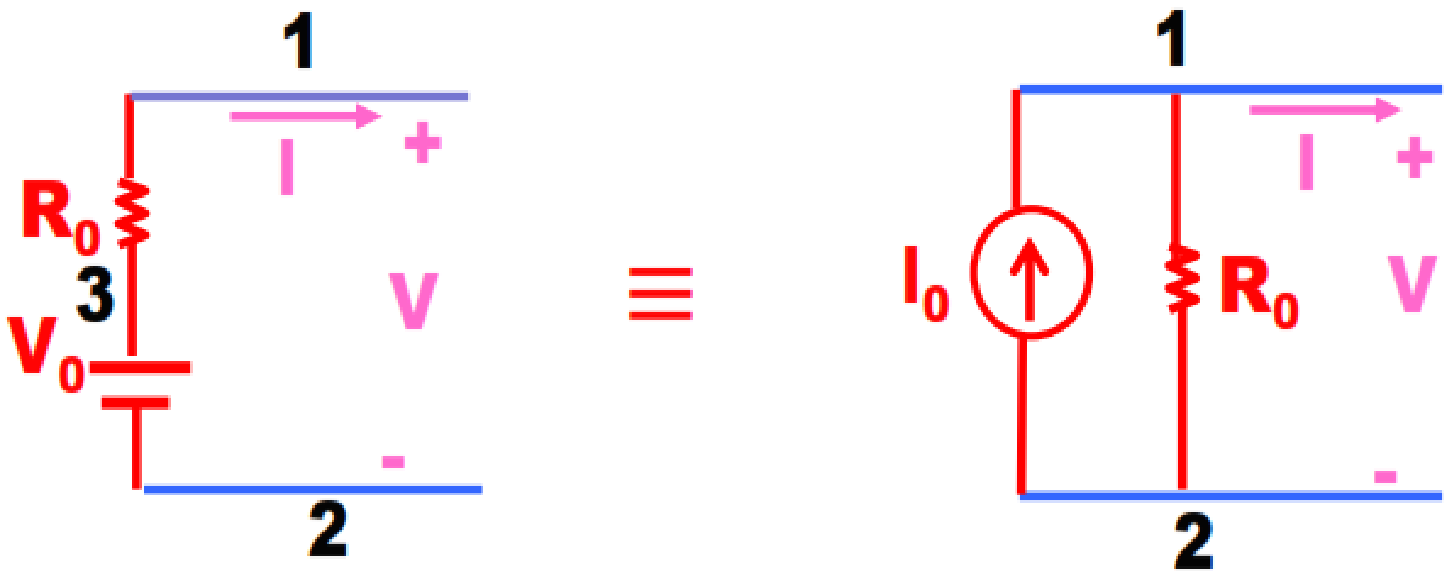
3. Replacing series and parallel resistors by its equivalent (All resistors are assumed linear)

4. Star-Delta or Delta-Star transformation

Method 2
Step by Step Simplification

- Source Equivalence
- Combination of active elements

Solution



$$V = V_{12} = V_{13} + V_{32} = -IR_0 + V_0 \qquad I_0 = I + \frac{V}{R_0}$$

$$V = V_0 - I\,R_0$$

$$V = I_0\,R_0 - I\,R_0$$

$$V_0 = I_0 R_0$$

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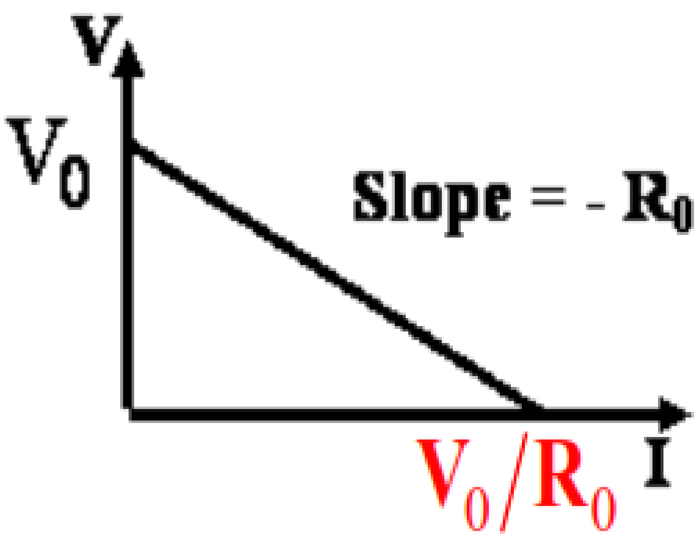
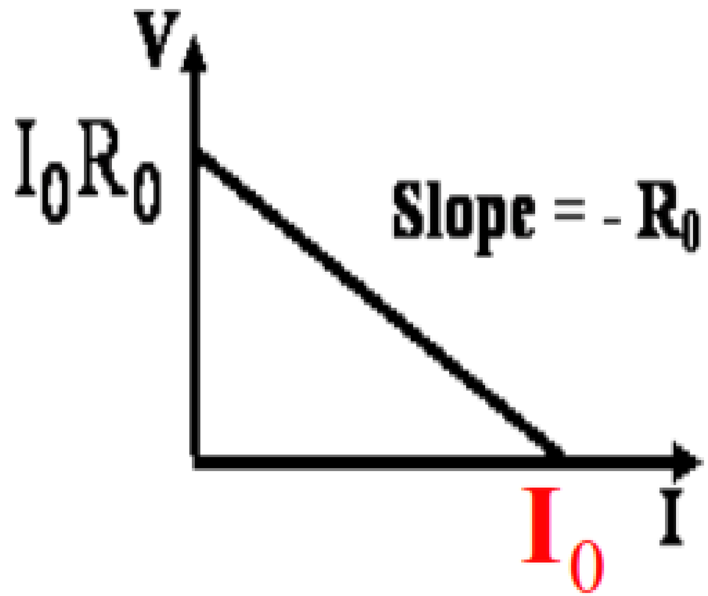
$$V = V_0 - IR_0$$

$$V = I_0R_0 - IR_0$$

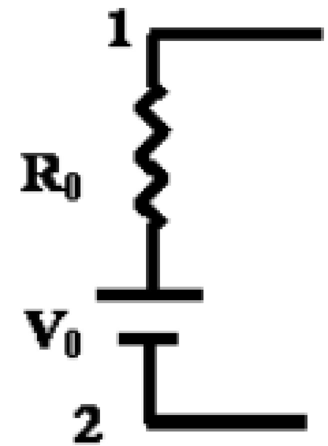




$$V_0 = I_0R_0$$

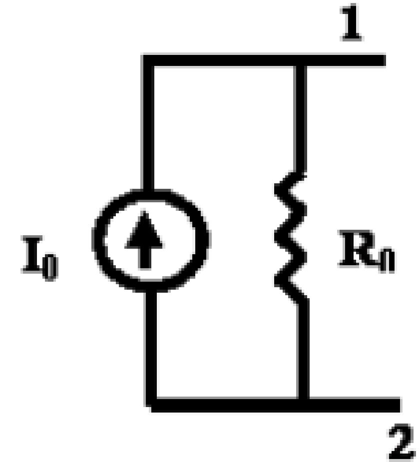
Example

- Convert the shown voltage source $\left(V_0 = 3v \,\&\, R_0 = 100\Omega\right)$ into its equivalent current source.



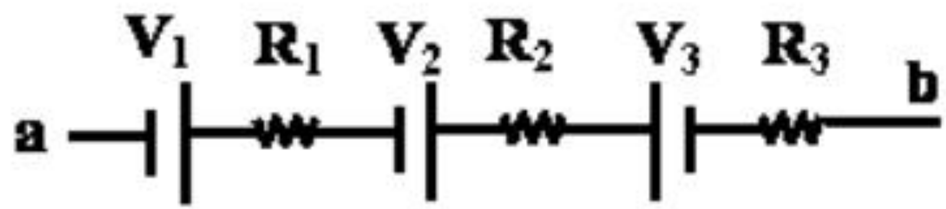
Solution

$$I_0 = \frac{V_0}{R_0} = \frac{3}{0.1} = 30\text{mA}$$



Combination of active elements

i) *Series Connection*



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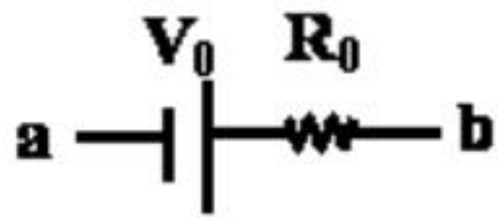
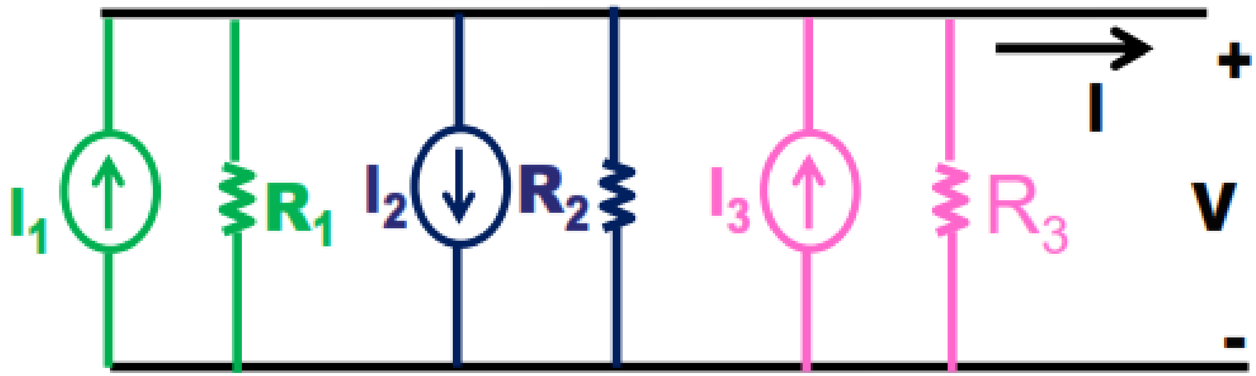
$$V_0 = V_1 + V_2 - V_3$$

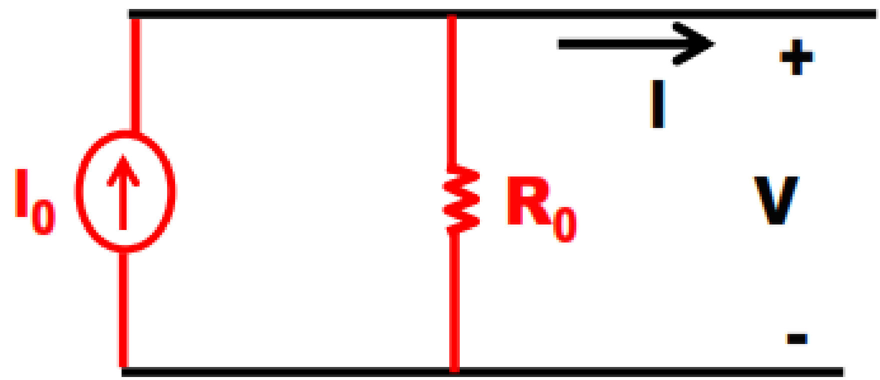
$$R_0 = R_1 + R_2 + R_3$$



Combination of active elements

ii) Parallel Connection



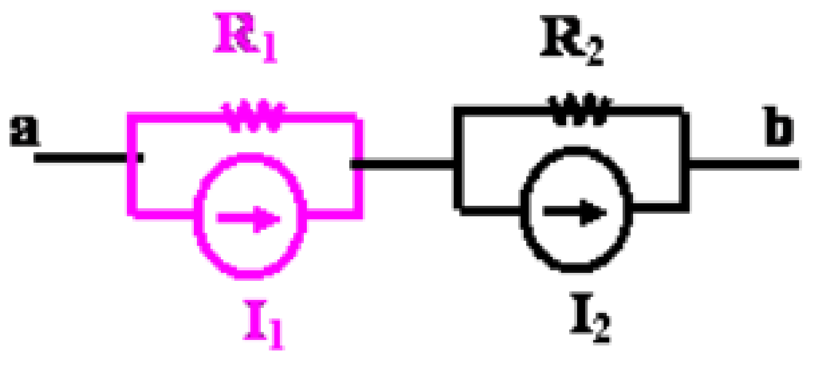


$$I_0 =$$

$$\frac{1}{R_{eq}} =$$

Example

- Convert the shown current sources
$\left(I_1 = 100\ mA, R_1 = 50\ \Omega, I_2 = 200\ mA, \textit{and}\ R_2 = 50\ \Omega\right)$
into one equivalent current source.



Solution

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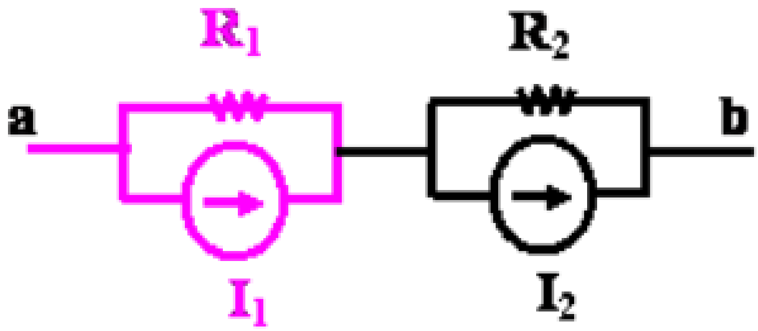
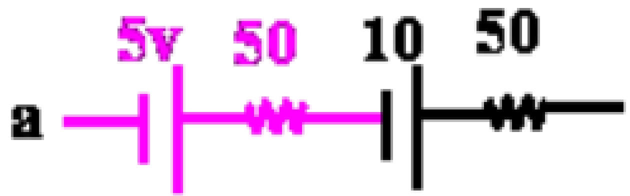
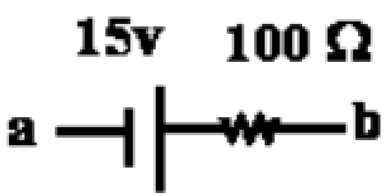
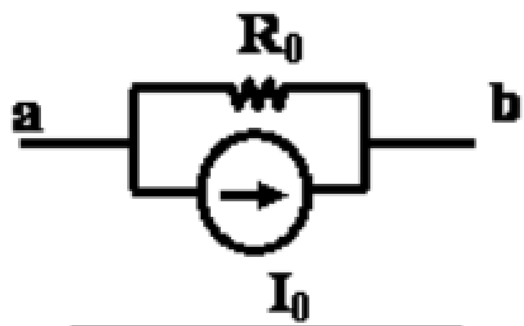
$\left(I_1 = 100 \ \text{mA}, R_1 = 50\,\Omega, I_2 = 200\,\text{mA}, \ \text{and}\ R_2 = 50\,\Omega\right)$







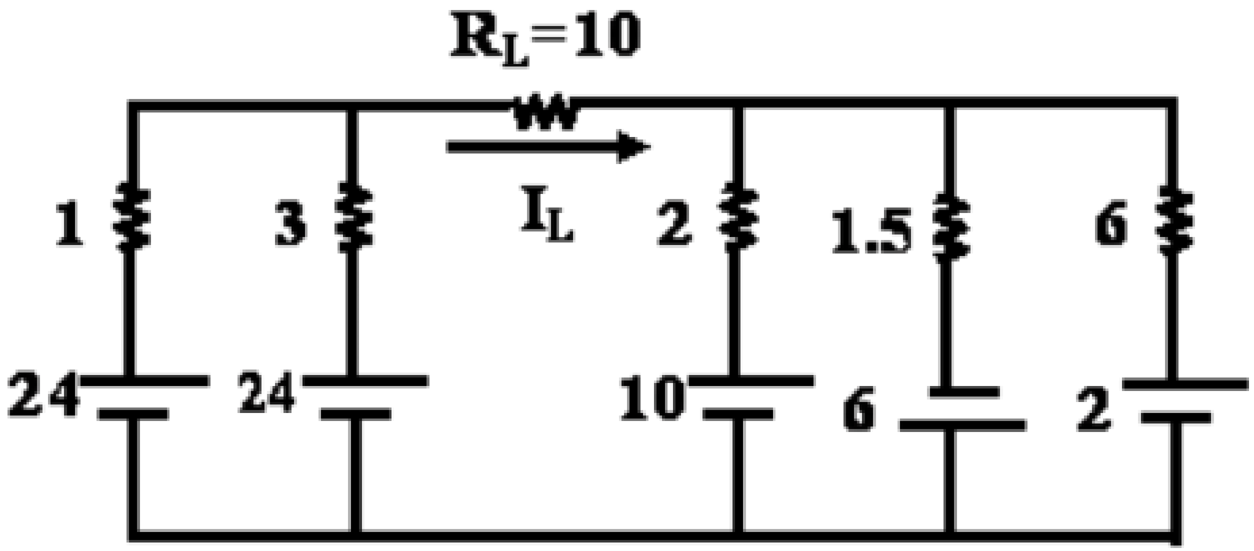


$I_0 =$

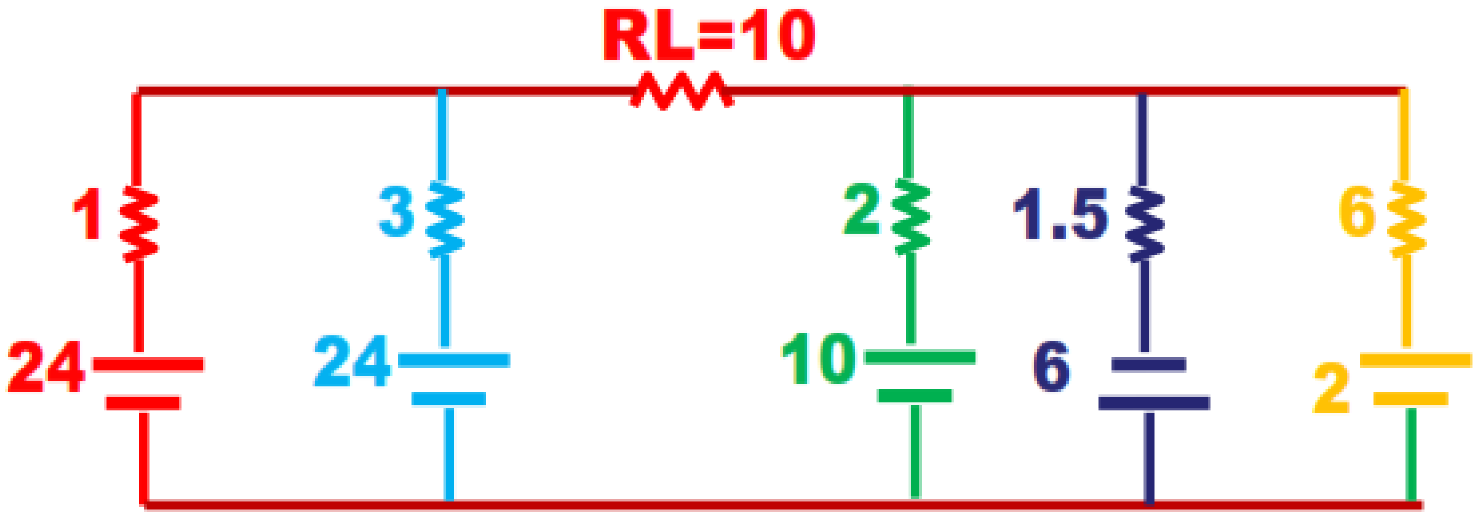
$R_0 =$

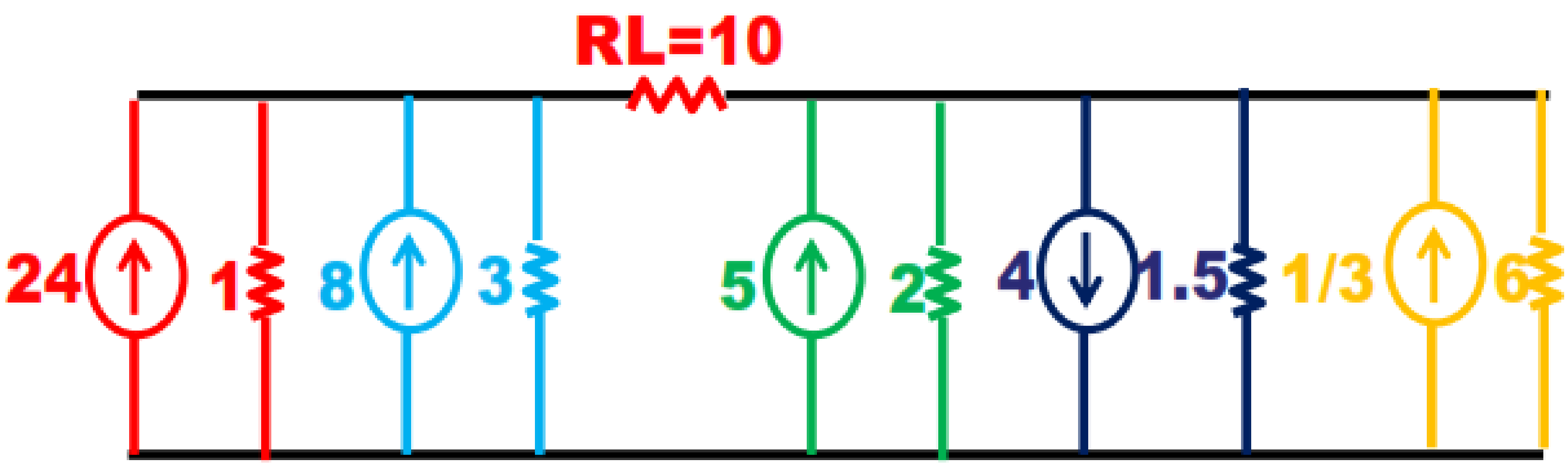
Example

- *For the* shown current, use simplification method to find the load current I_L. (All voltages are in Volts & all resistors are in k Ω).



Solution





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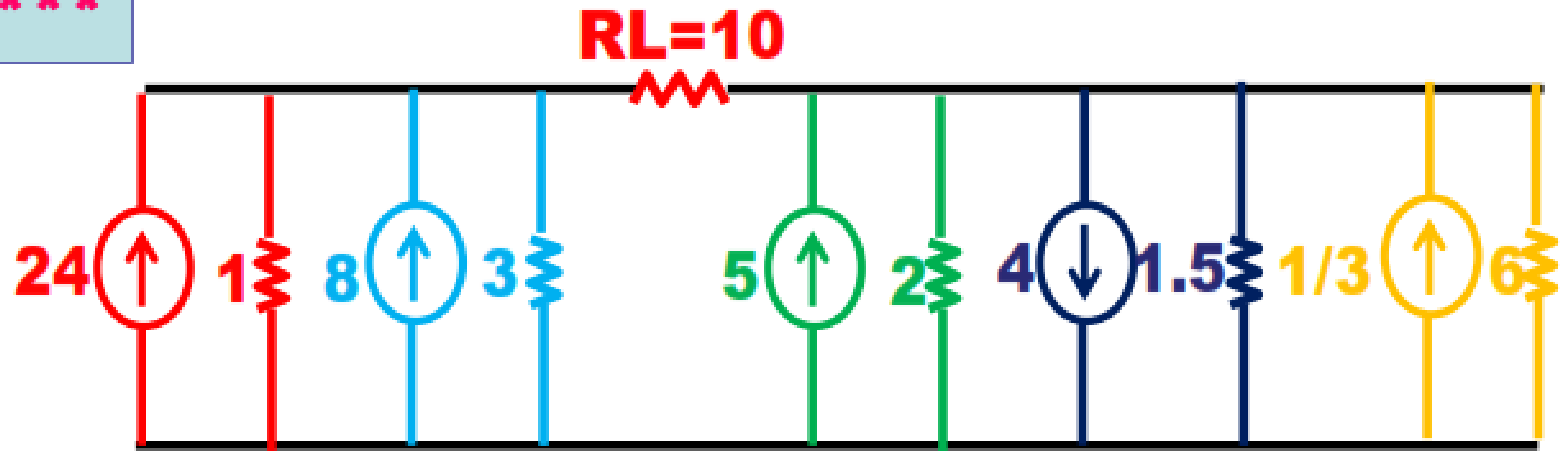
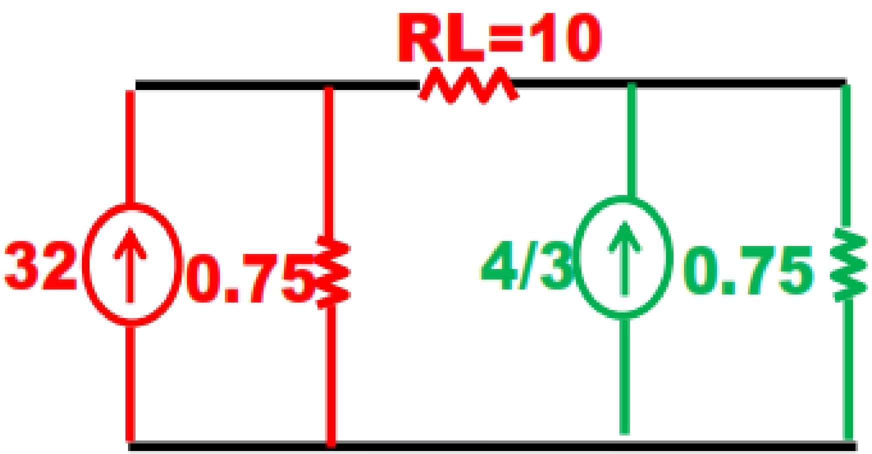
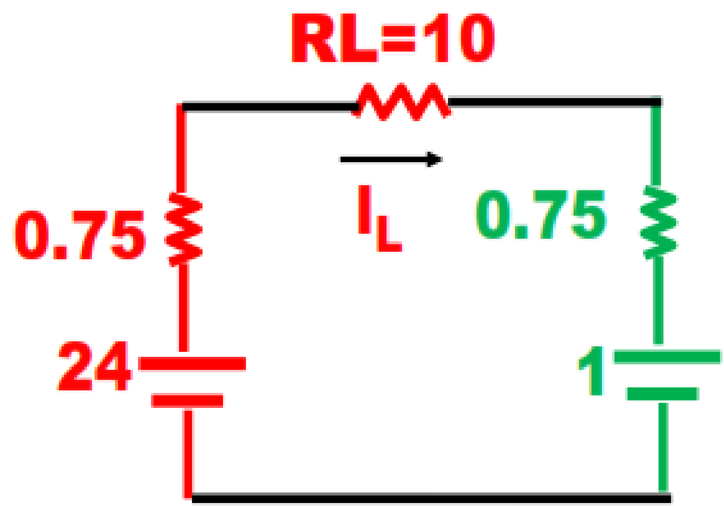
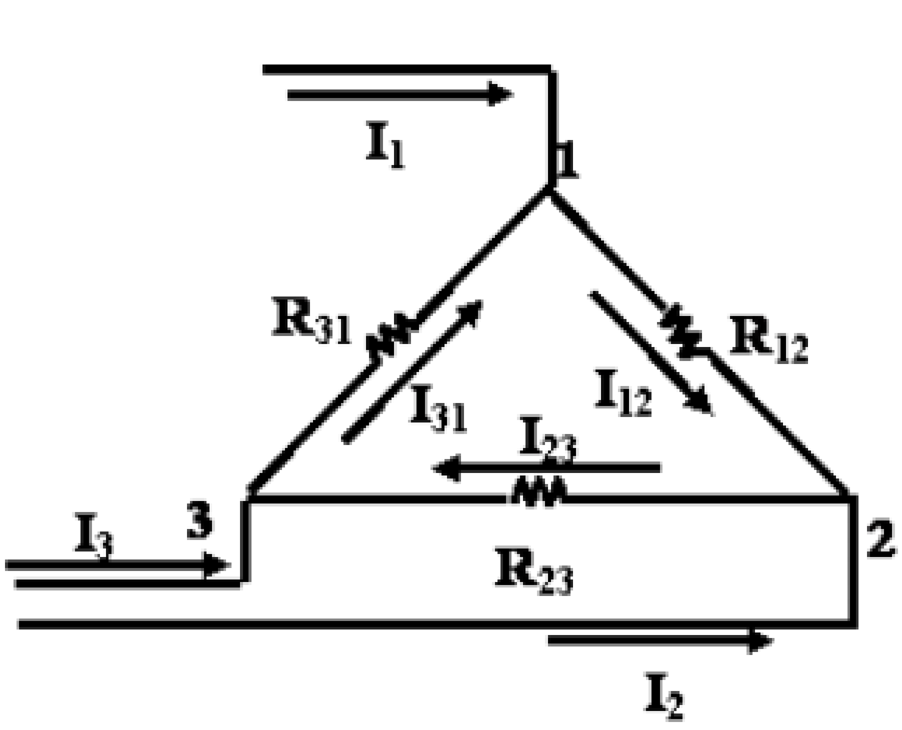




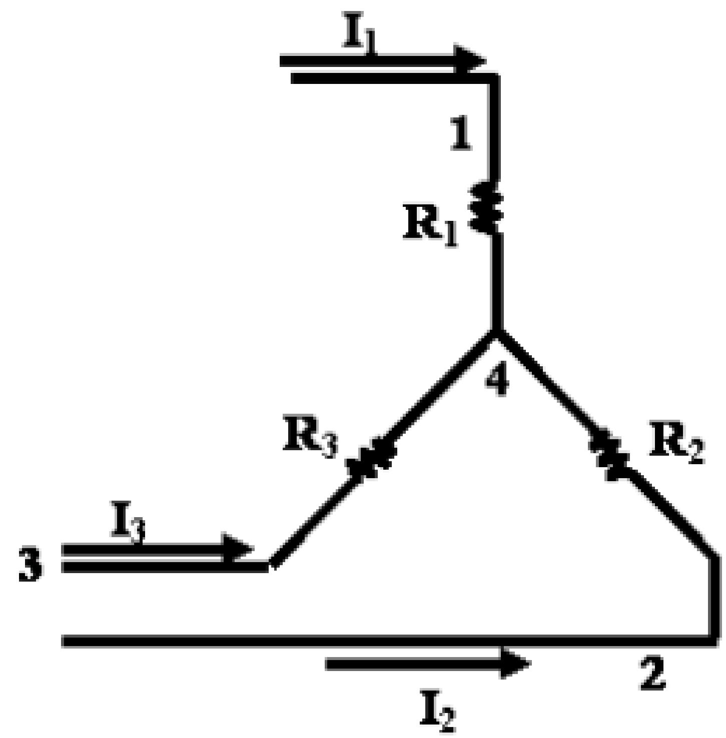


$$I_L =$$

Star - Delta Transformation

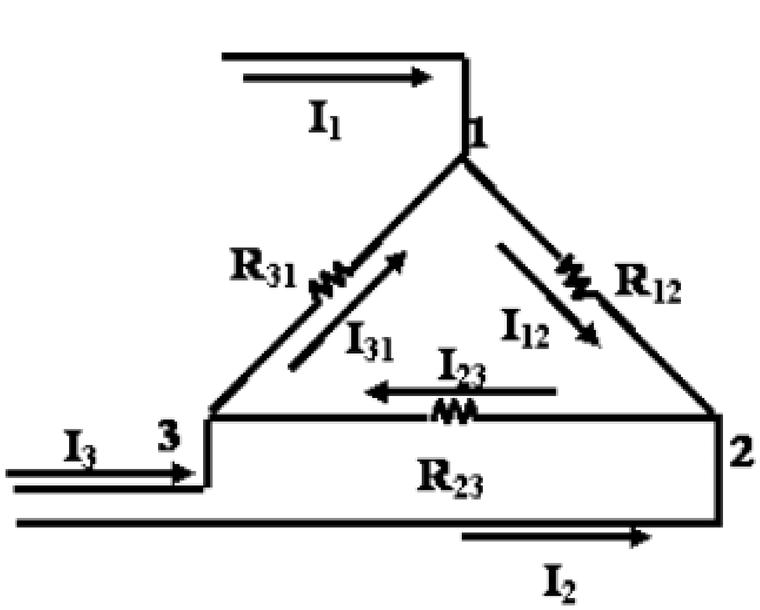


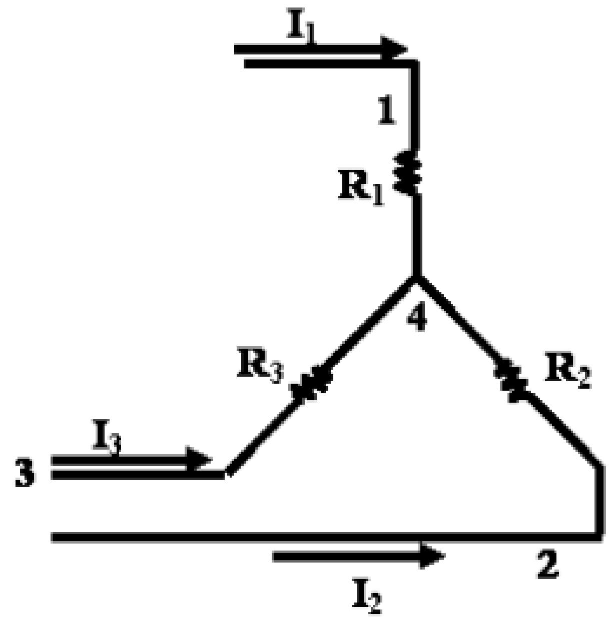
Delta (Pi or π)



Star (Y or T)

Δ to Y Transformation





$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}}$$

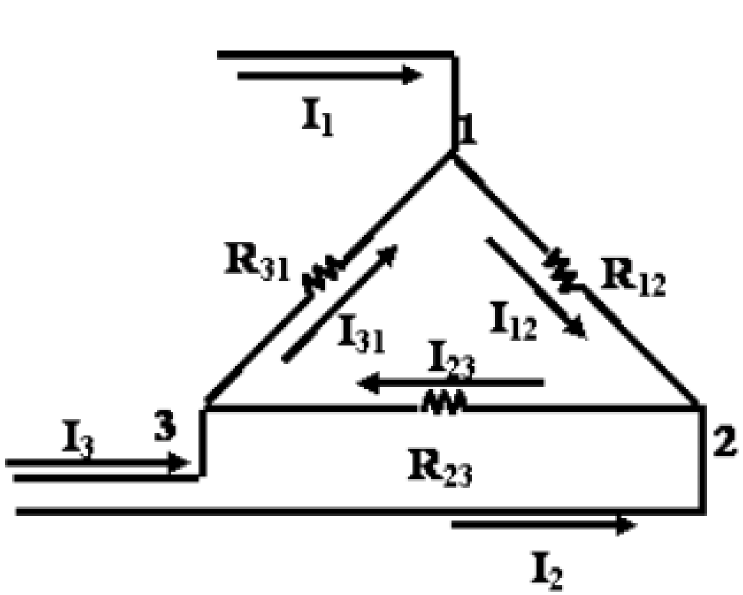
$$R_3 = \frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$$

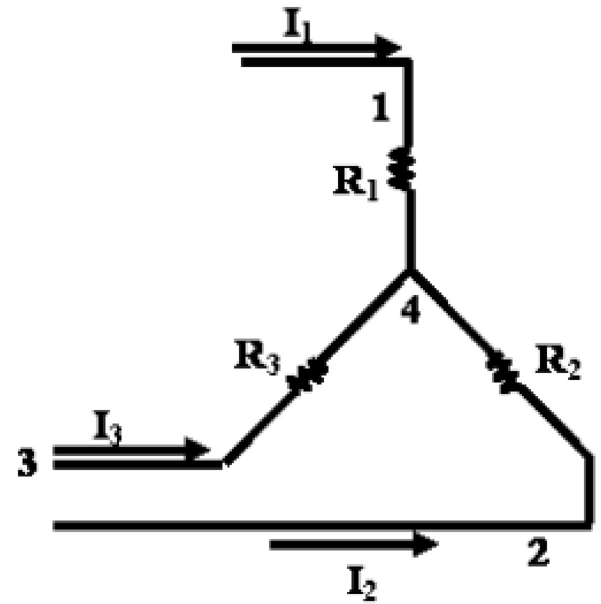
Special case

$$R_{12}=R_{23}=R_{31}=R$$

$$R_1 = R_2 = R_3 = \frac{R}{3}$$

Y - Δ Transformation





$$R_{12}=R_1+R_2+\frac{R_1R_2}{R_3}$$

$$R_{23}=R_2+R_3+\frac{R_2R3}{R_1}$$

$$R_{31}=R_3+R_1+\frac{R_3R_1}{R_2}$$

Special case

$$R_1=R_2=R_3=R$$

$$R_{12}=R_{23}=R_{31}=3R$$

Assignment 1

(Deadline Oct. 27, 2019)

You are required to prove the previous relationships between Delta-Y transformations and Y-Delta transformations
Submit a hard copy assignment showing the proof

You are required to write a online program that takes on the values of the Delta branch resistors and produce the values of the Y branch resistors And vice versa. You can use whatever language but the program should be online

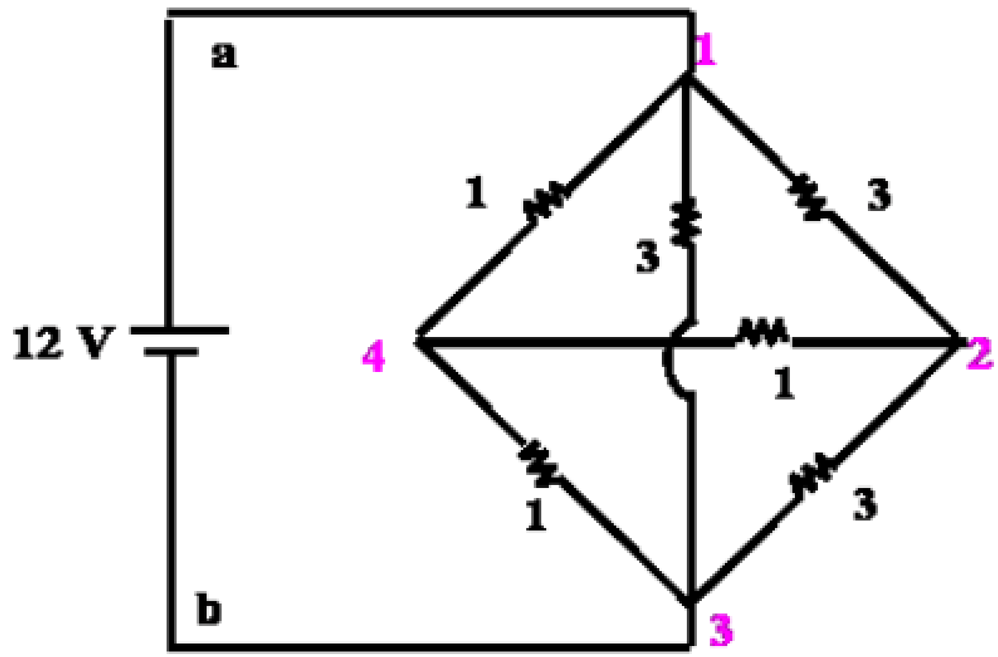
You should make an acceptable GUI for the program
Send your program link to
hmostafa@staff.cu.edu.eg with subject:
CMP_Assignment1_DeltaY_Your_Full_Name

Example

For the shown circuit , Find:

a) The input resistance between the two terminals 1 and 3

b) V_{23}



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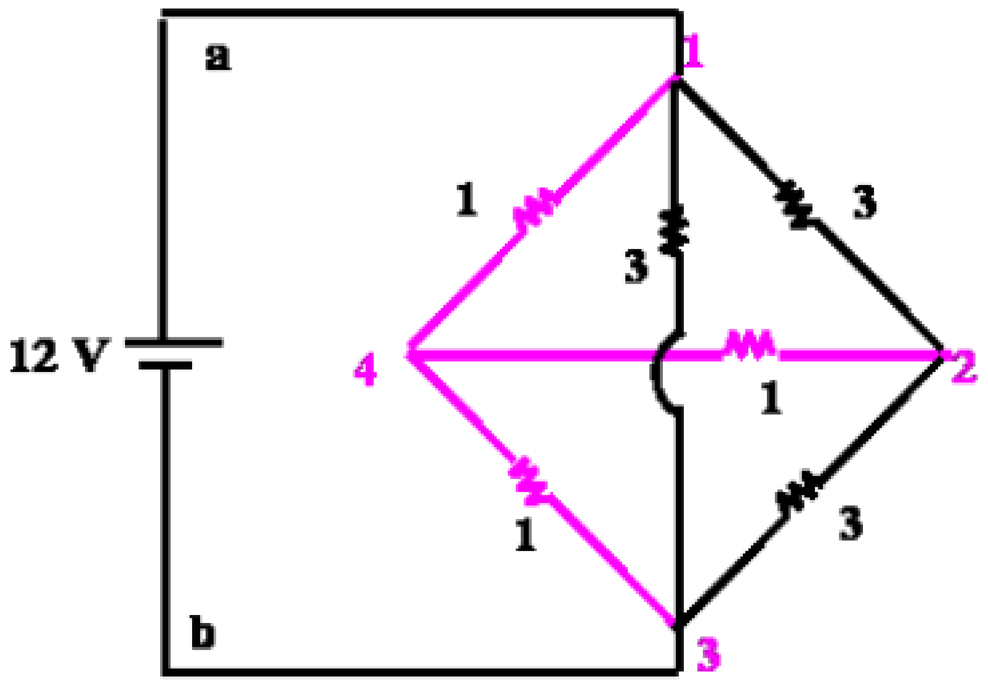
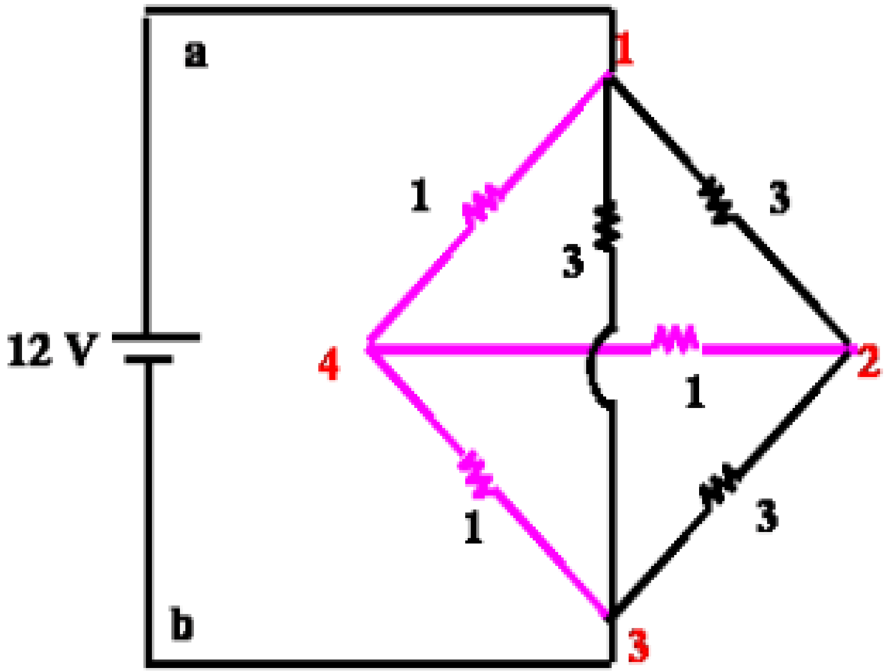
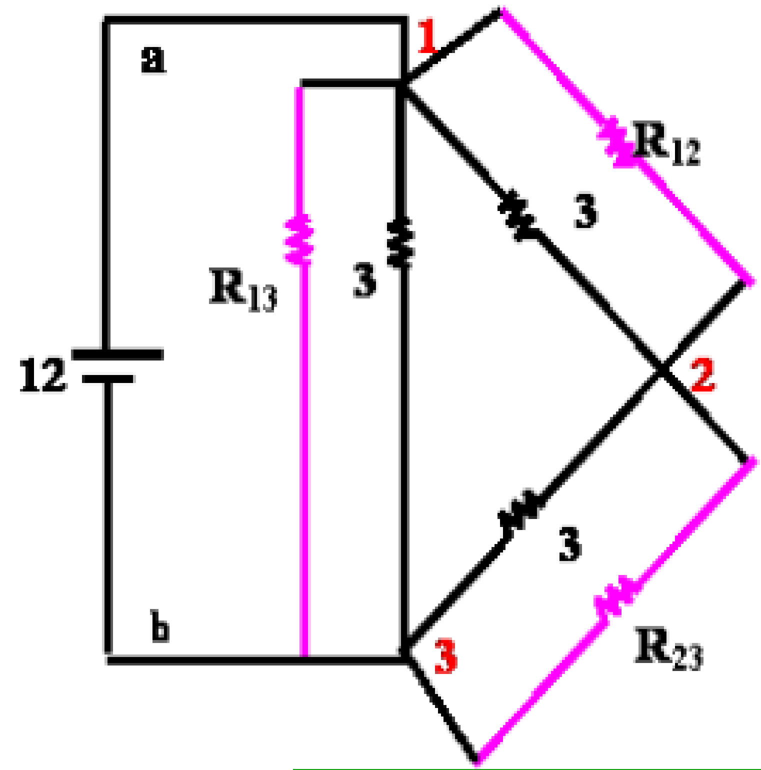
Solution 1

By converting Y 123-4 ⇒ Δ 123







Special case

$R_1 = R_2 = R_3 = R$

$R_{12} = R_{23} = R_{31} = 3R$

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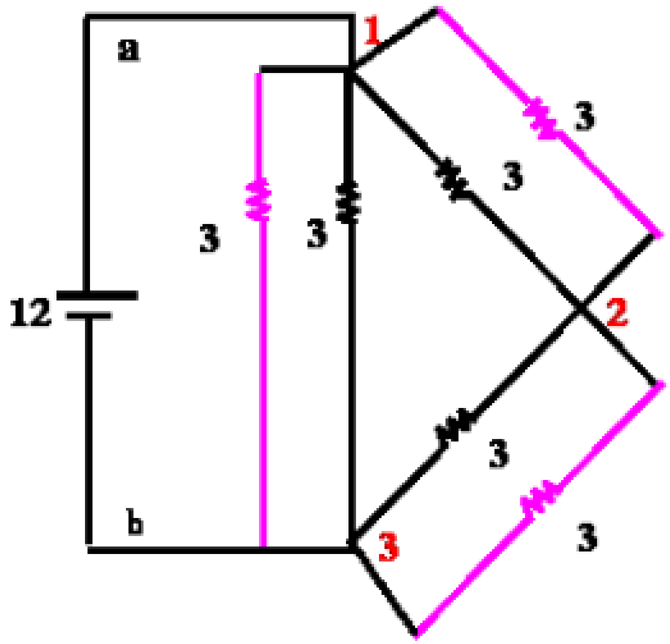

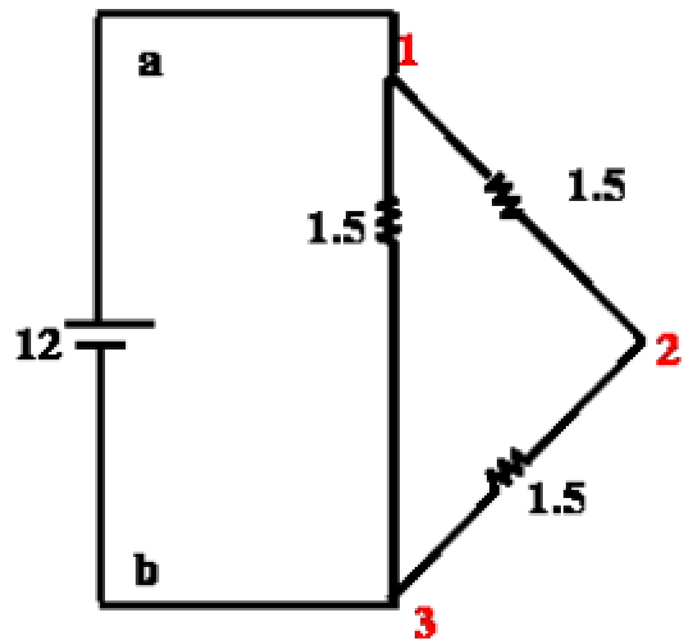

$R_{13} =$

$V_{23} =$

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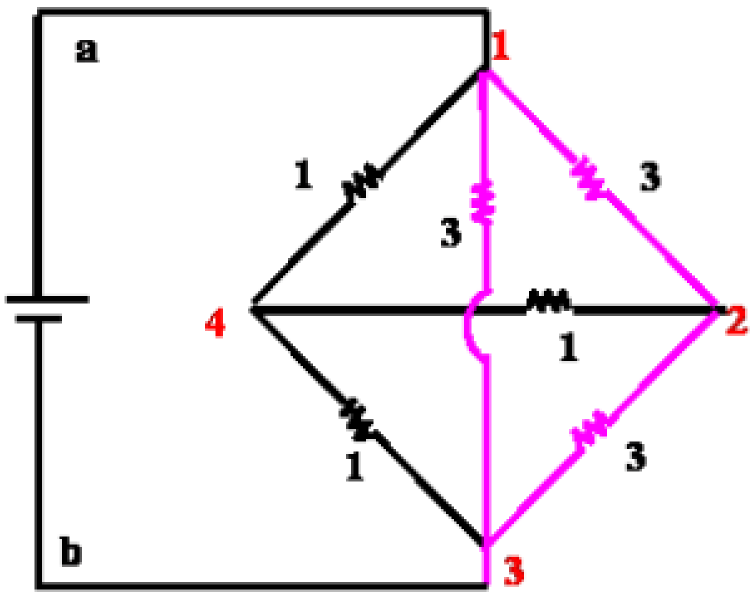
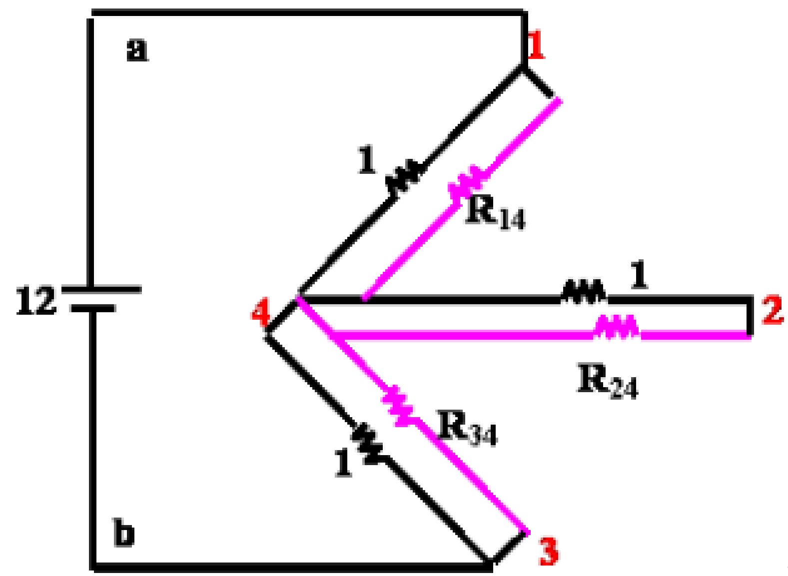
Solution 2

By converting Δ 123 ⇒ Y 123-4





Special case

$R_{12}=R_{23}=R_{31}=R$

$$R_1 = R_2 = R_3 = \frac{R}{3}$$

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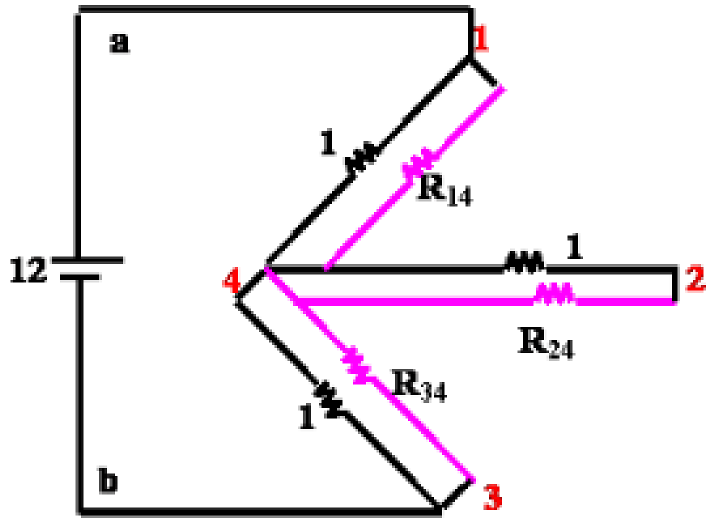
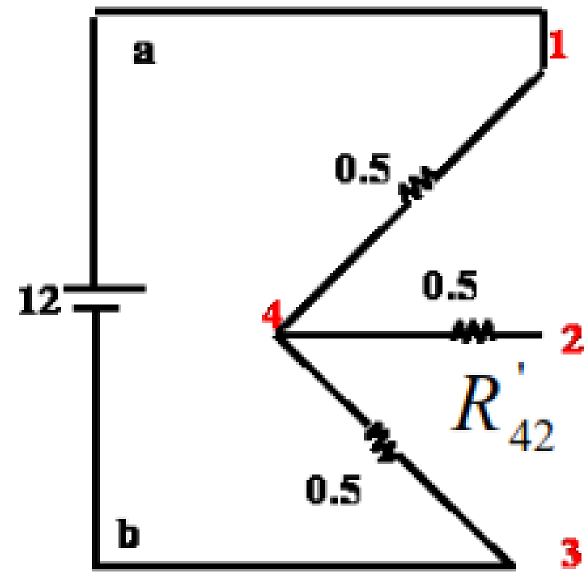
Solution 2

By converting Δ 123 ⇒ Y 123-4





Note that R'_{42} has no effect

$R_{13} =$

$V_{23} =$

IQ

Find five consecutive numbers in the list below that total 21.

58236394721658342594 23

IQ

Find five consecutive numbers in the list below that total 21.

5823639472165834259423

Answer: 72165

Methods of solution of DC circuits (Cont.)

<u>Method 3:</u> Loop Analysis

Methods of solution of DC circuits

- **<u>Method 3</u>** Loop Current Method (Loop Analysis)

1-Convert independent current sources into equivalent voltage sources

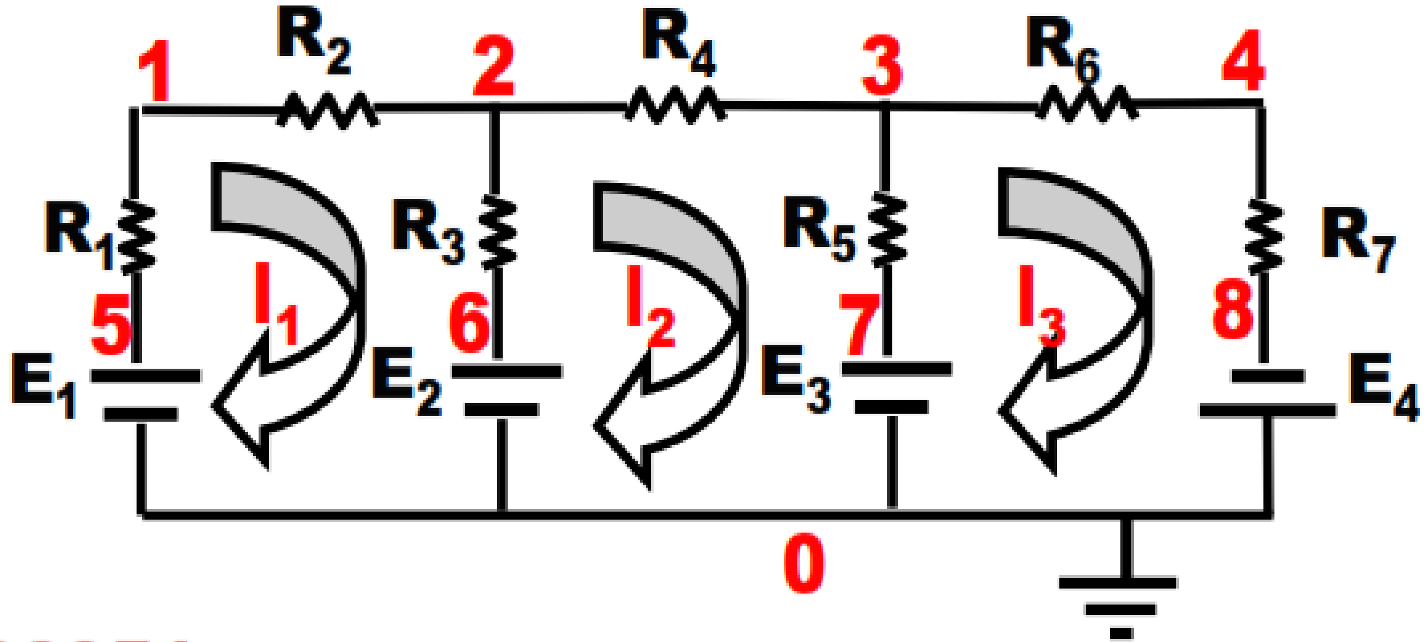
2- Identify the number of independent loop (*L*) on the circuit

3- Label a loop current on each loop (Clock wise direction).

4- Write an expression for the KVL around each loop.

5- Solve the resultant system of algebraic equations to find the *L* loop currents.

$L=3;$

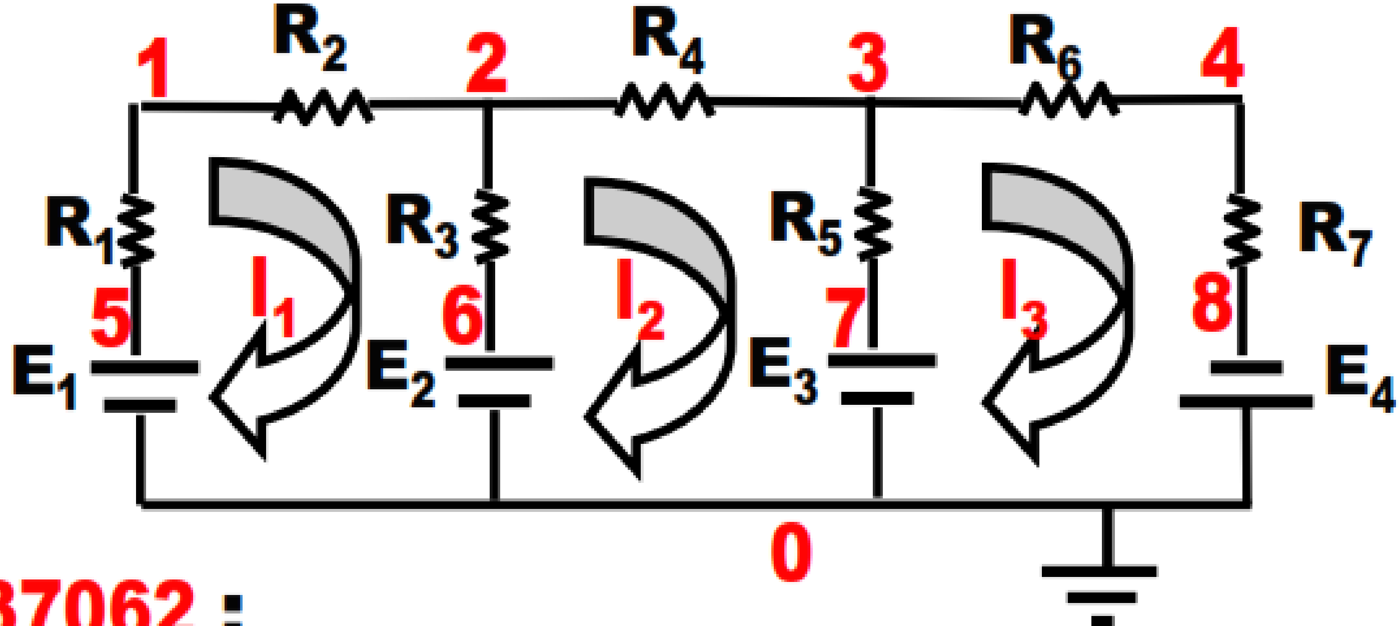


- **Loop 126051 :**

$$V_{12}+V_{26}+V_{60}+V_{05}+V_{51}=0$$

$$I_1R_2+(I_1-I_2)R_3+E_2-E_1+I_1R_1=0$$

$$I_1(R_1+R_2+R_3)-I_2R_3-I_3(0)=E_1-E_2 \qquad (1)$$

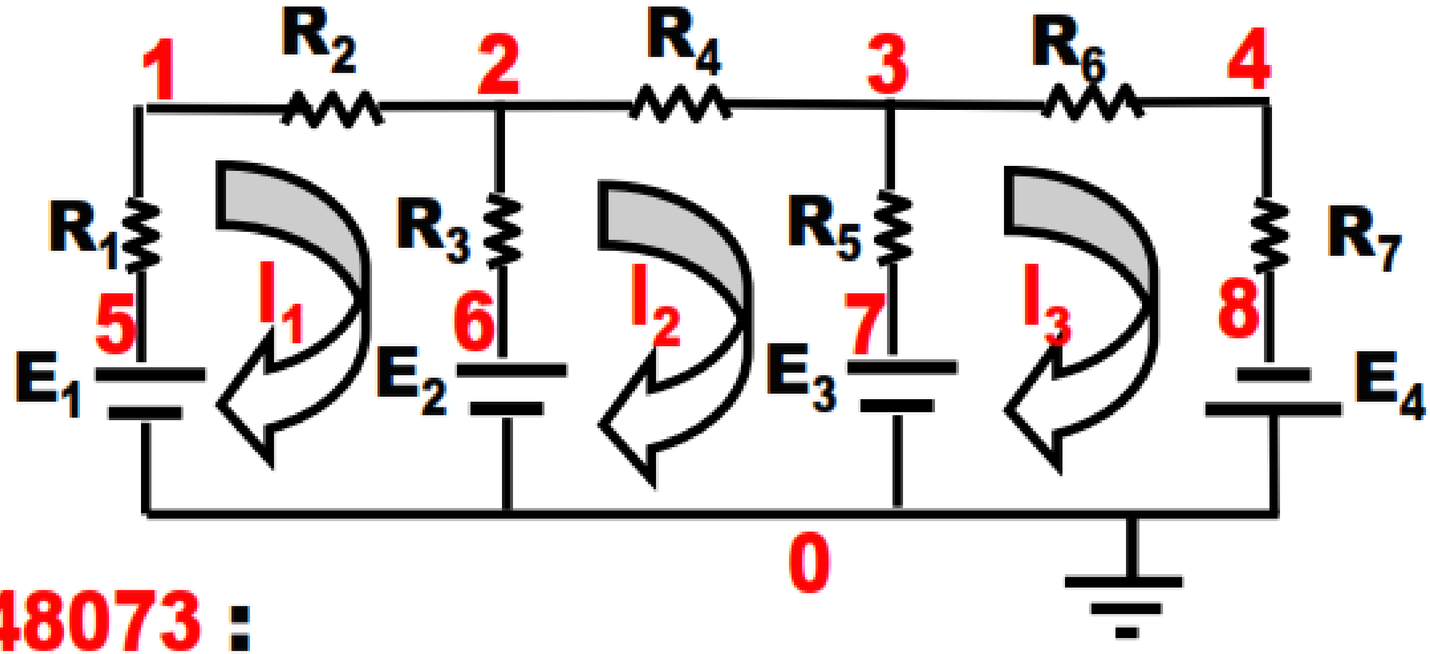


- **Loop 237062 :**

$$V_{23}+V_{37}+V_{70}+V_{06}+V_{62}=0$$

$$I_2R_4+\left(I_2-I_3\right)R_5+E_3-E_2+\left(I_2-I_1\right)R_3=0$$

$$-I_1R_3+I_2\left(R_3+R_4+R_5\right)-I_3R_5=E_2-E_3 \qquad (2)$$

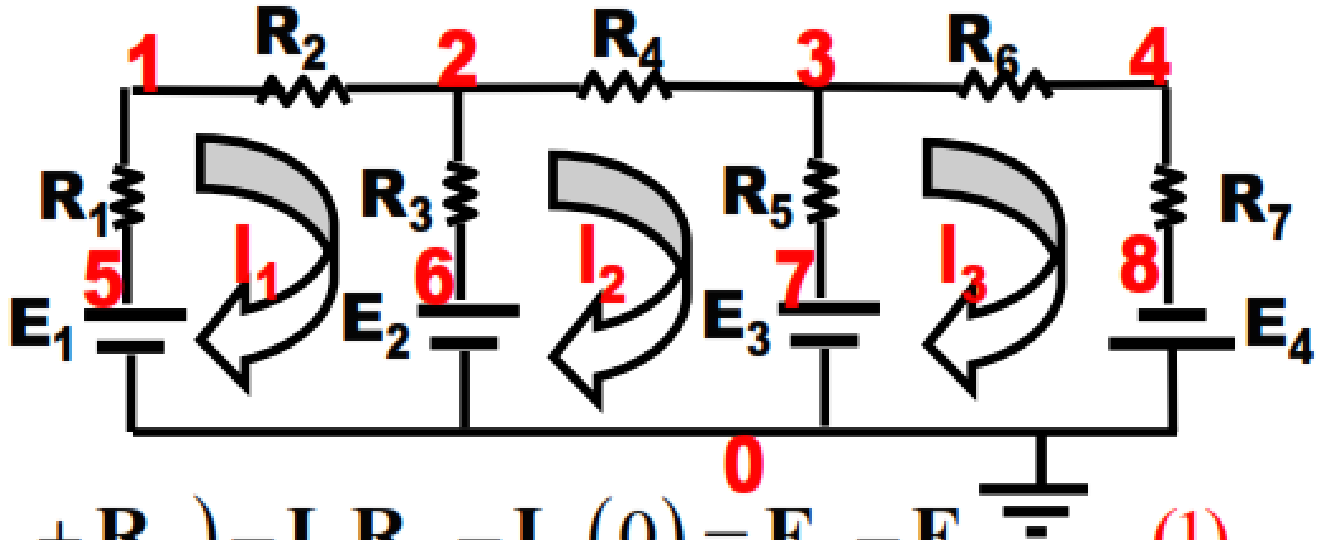


- **Loop 348073 :**

$$V_{34}+V_{48}+V_{80}+V_{07}+V_{73}=0$$

$$I_3R_6+I_3R_7-E_4-E_3+(I_3-I_2)R_5=0$$

$$I_1(0)-I_2R_5+I_3(R_5+R_6+R_7)=E_3+E_4 \quad (3)$$



$$I_1\left(R_1+R_2+R_3\right)-I_2R_3-I_3\left(0\right)=E_1-E_2 \qquad (1)$$

$$-I_1R_3+I_2\left(R_3+R_4+R_5\right)-I_3R_5=E_2-E_3 \qquad (2)$$

$$-I_1\left(0\right)-I_2R_5+I_3\left(R_5+R_6+R_7\right)=E_3+E_4 \qquad (3)$$

$$\begin{pmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}=\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

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$$\begin{pmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

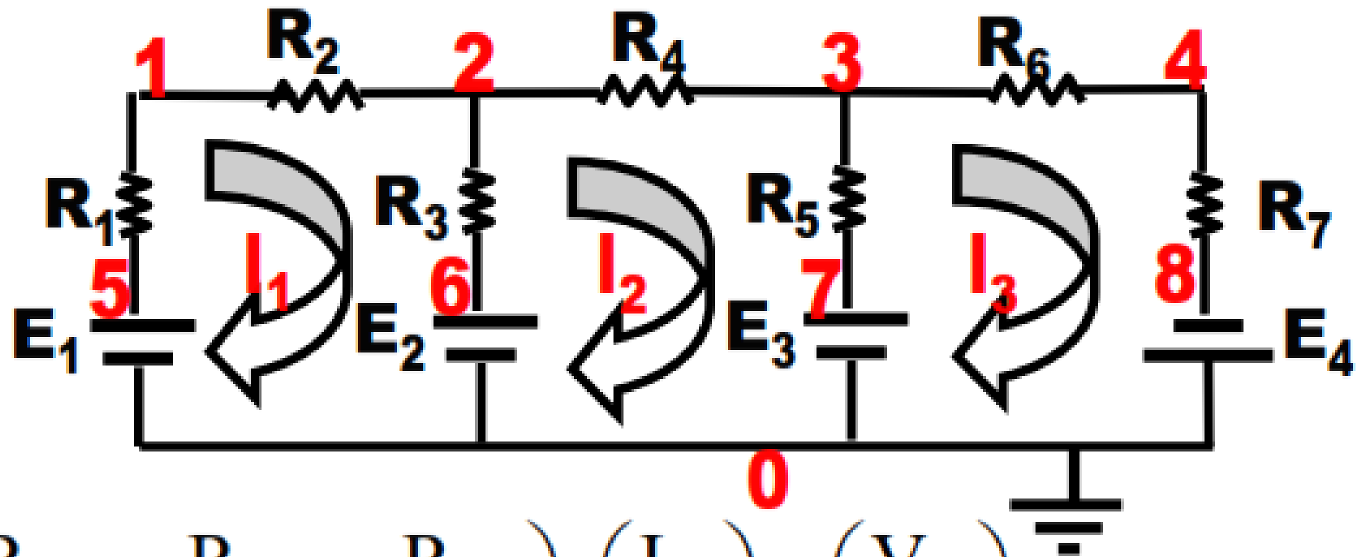
$$\begin{pmatrix} \mathbf{R} \end{pmatrix} \begin{pmatrix} \mathbf{I} \end{pmatrix} = \begin{pmatrix} \mathbf{V} \end{pmatrix}$$

$$\mathbf{R_{ii}} = \sum \textbf{Resistance in loop i} \equiv \textbf{Self resistance of loop i}$$

$$\mathbf{R_{ij}} = \mathbf{R_{ji}} = \sum \textbf{Common Resistance between loops i\& j}$$

$$\mathbf{V_i} = \sum \textbf{voltage sources in loop i}$$

- A voltage source is assumed +ve if it pushes current in the loop direction and –ve otherwise. 98



$$\begin{pmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$R_{11}=(R_1+R_2+R_3)$ $\quad R_{12}=R_3$ $\quad R_{13}=0$ $\quad V_1=E_1-E_2$

$R_{21}=R_3$ $\quad R_{22}=(R_3+R_4+R_5)$ $\quad R_{23}=R_5$ $\quad V_2=E_2-E_3$

$R_{31}=0$ $\quad R_{32}=R_5$ $\quad R_{33}=R_5+R_6+R_7$ $\quad V_3=E_3+E_4$

$$\begin{pmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta} \quad \text{where,} \quad \Delta = \begin{vmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{12} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{vmatrix}$$

and

$$\Delta_1 = \begin{vmatrix} V_1 & -R_{12} & -R_{13} \\ V_2 & R_{22} & -R_{23} \\ V_3 & -R_{32} & R_{33} \end{vmatrix}$$

$$\begin{pmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{12} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

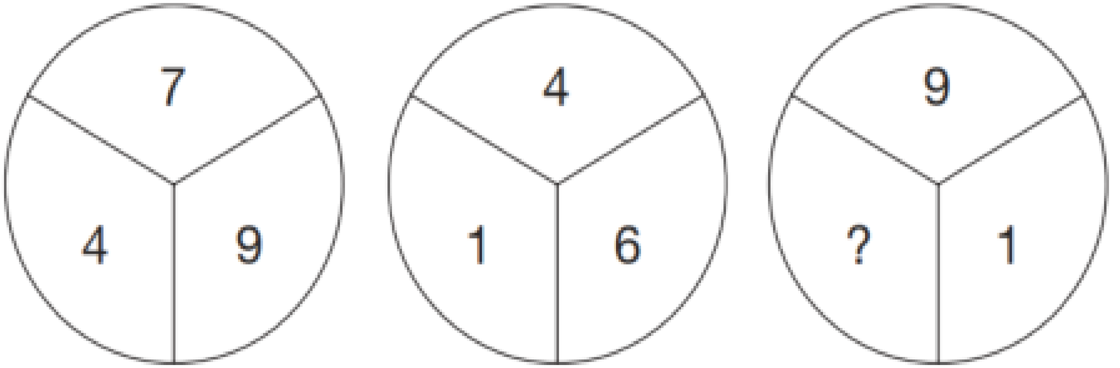
$$\Delta_2 = \begin{vmatrix} \mathbf{R}_{11} & \mathbf{V}_1 & -\mathbf{R}_{13} \\ -\mathbf{R}_{12} & \mathbf{V}_2 & -\mathbf{R}_{23} \\ -\mathbf{R}_{31} & \mathbf{V}_3 & \mathbf{R}_{33} \end{vmatrix}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_3 = \begin{vmatrix} \mathbf{R}_{11} & -\mathbf{R}_{12} & \mathbf{V}_1 \\ -\mathbf{R}_{12} & \mathbf{R}_{22} & \mathbf{V}_2 \\ -\mathbf{R}_{31} & -\mathbf{R}_{32} & \mathbf{V}_3 \end{vmatrix}$$

$$\mathbf{I}_3 = \frac{\Delta_3}{\Delta}$$

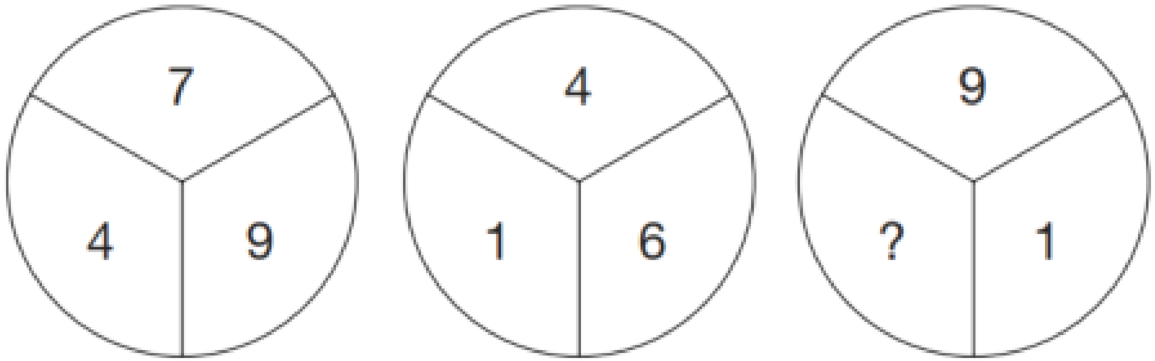
Lecture Pause



What number should replace the question mark?

Answer:

Lecture Pause



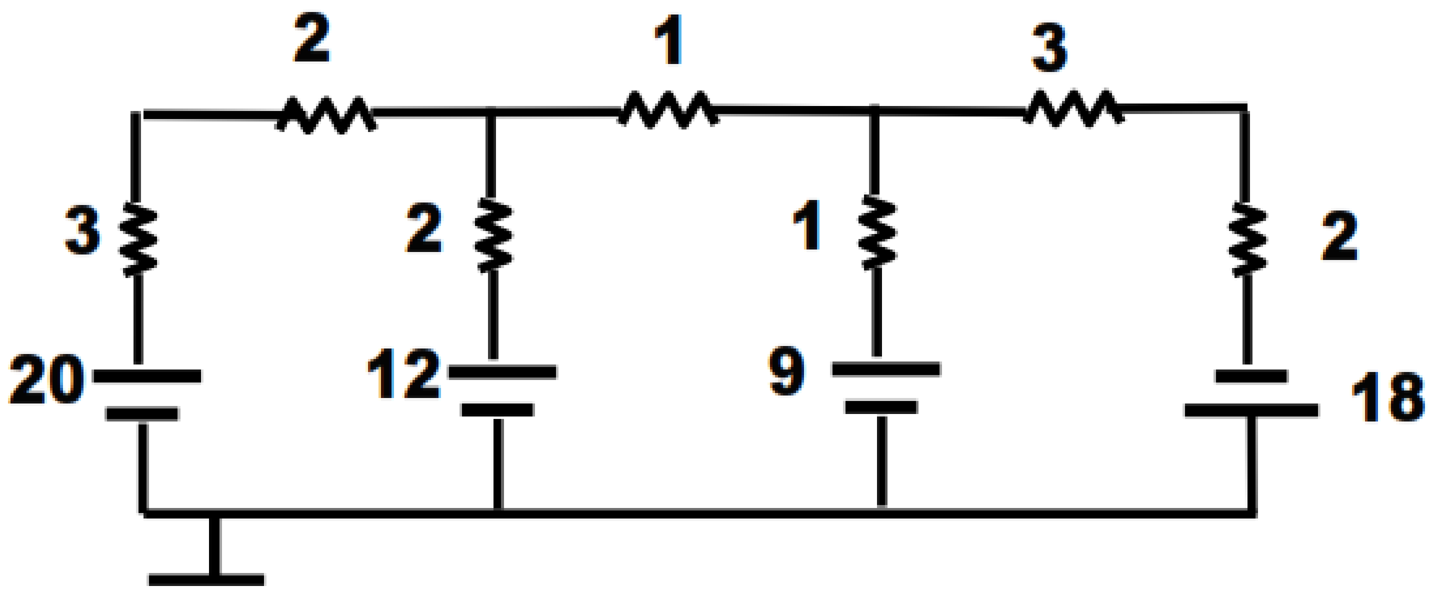
What number should replace the question mark?

Answer: 8

Example

For the shown circuit, **use loop analysis to:**

a) Find the branch currents
b) Check the power balance of the circuit.



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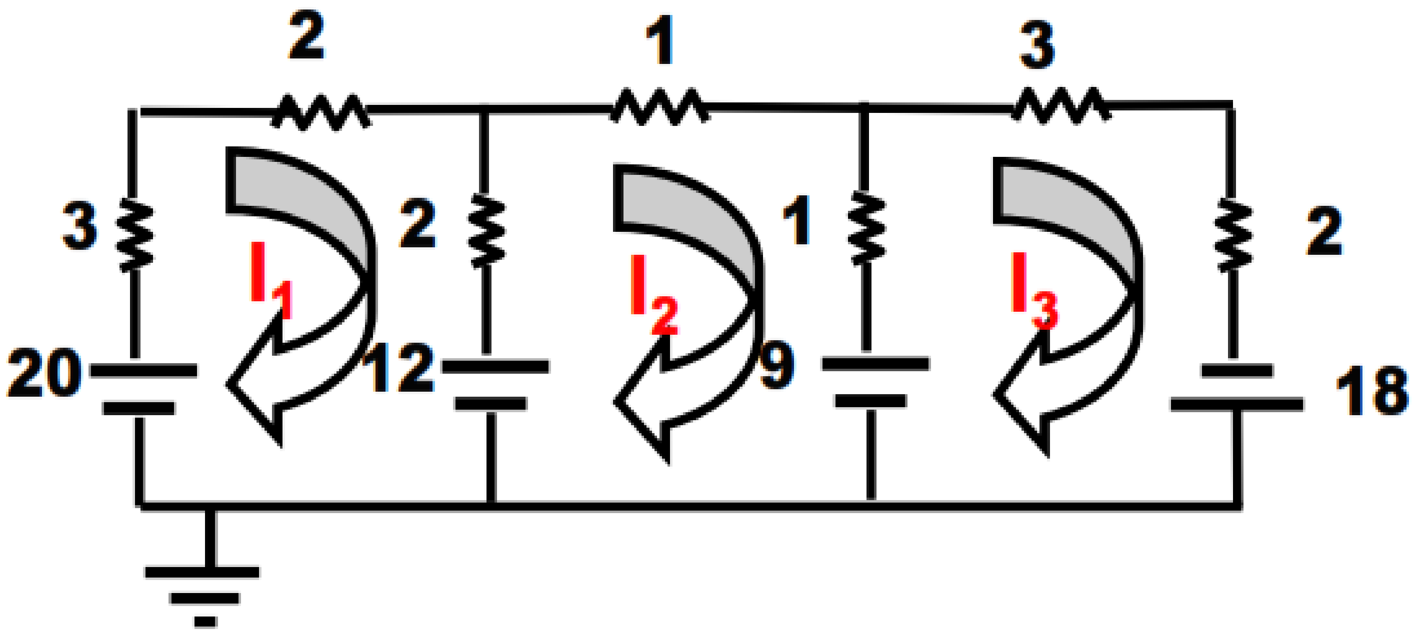
Solution



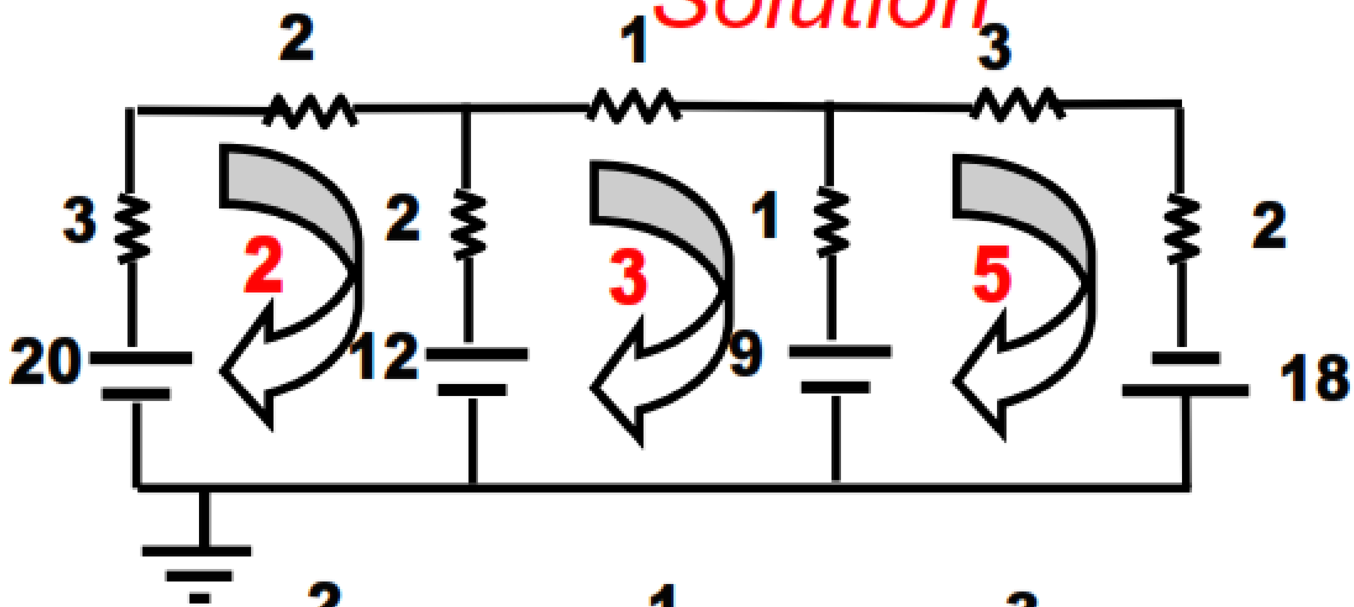
$$\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

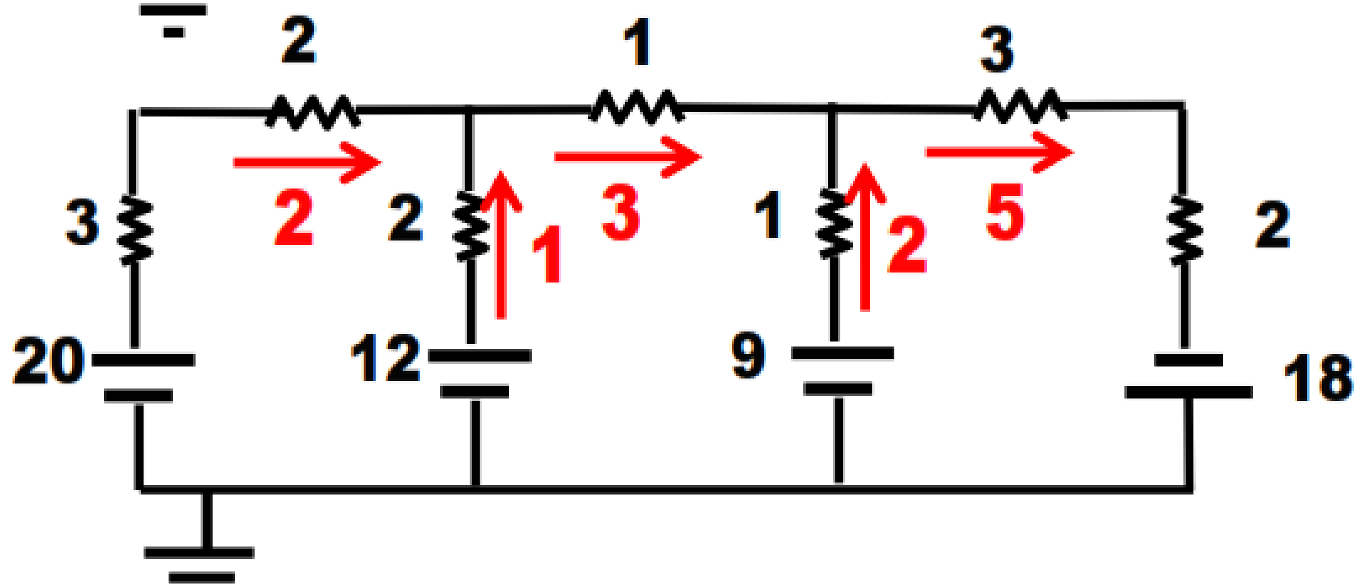
$I_1 = 2\ mA$

$I_2 = 3\ mA$

$I_3 = 5\ mA$

Solution





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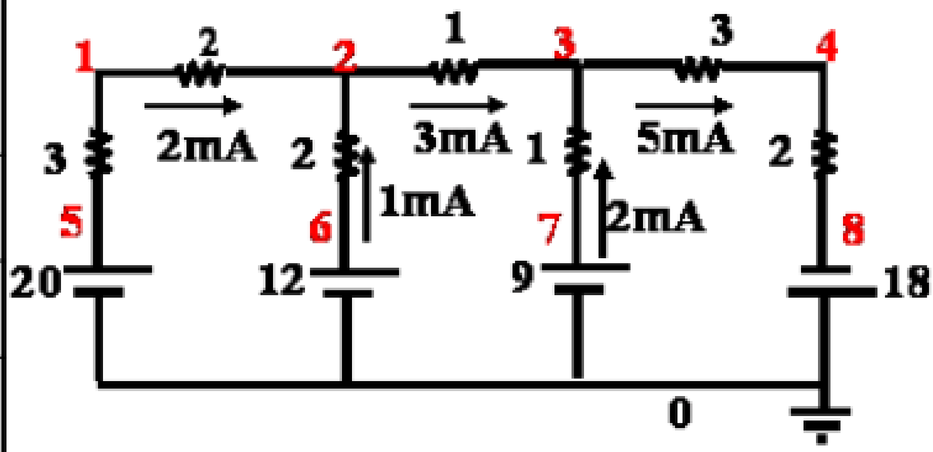
power dissipated by resistors

Resistance	Power (mw) I^2R
R15	
R12	
R26	
R23	
R37	
R34	
R48	
Total	160



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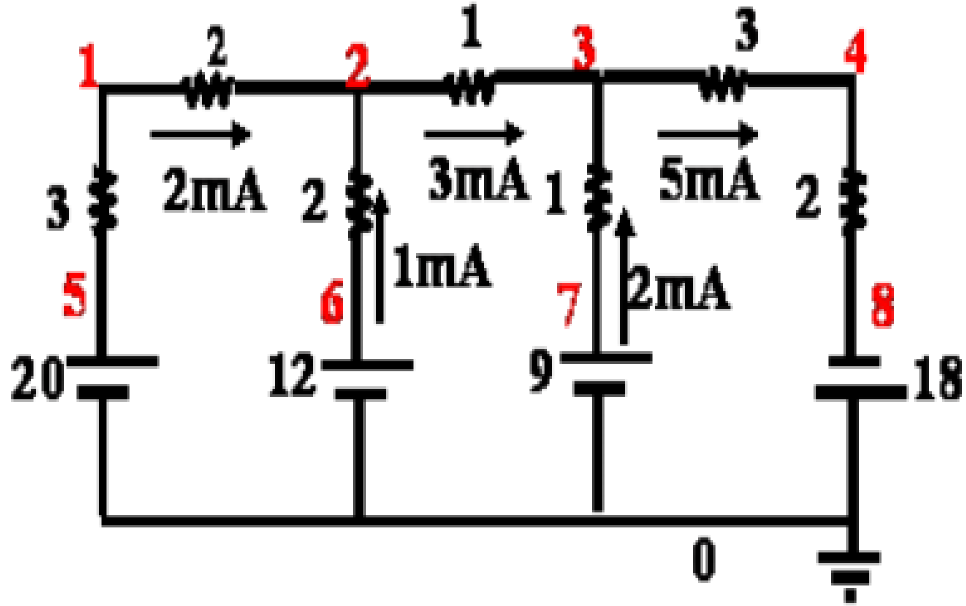
power delivered by sources

Source	Power (mw) E*I
E20	
E12	
E9	
E18	
Total	**160**



Power balance

Σ Power delivered by sources

= Σ Power dissipated by resistors = 160 mw

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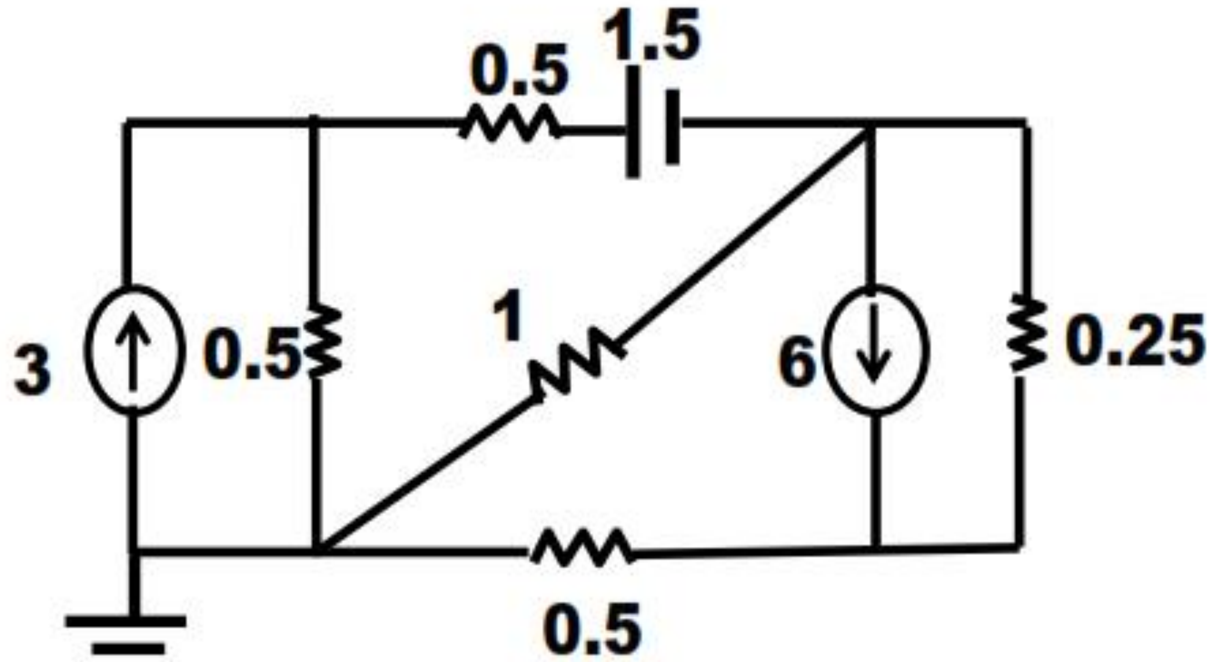
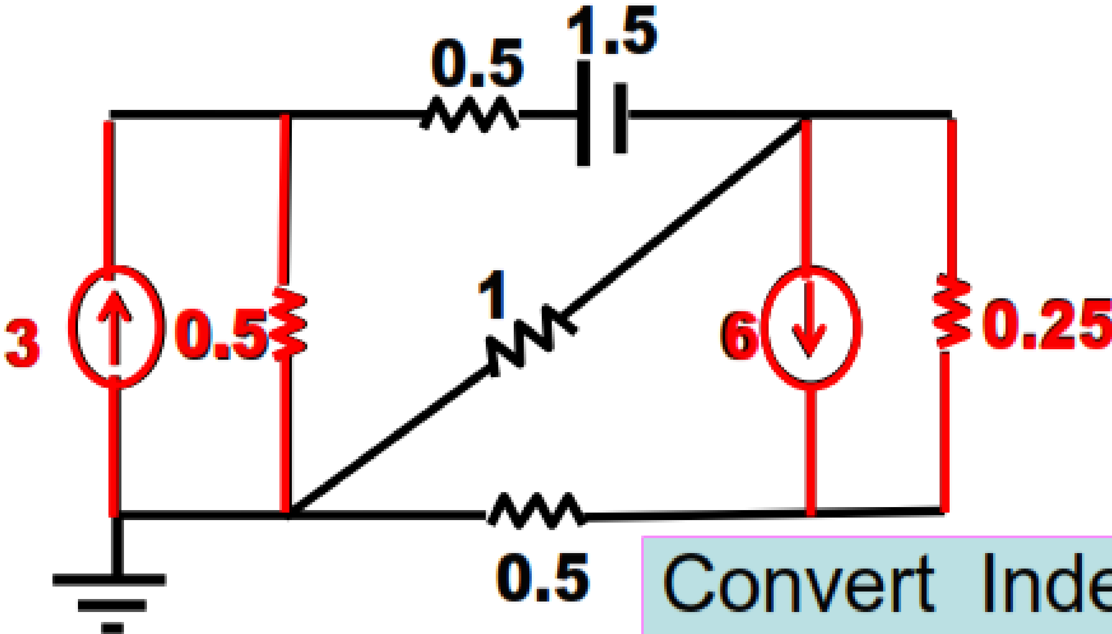
Example

For the circuit shown, **use loop analysis** to find:

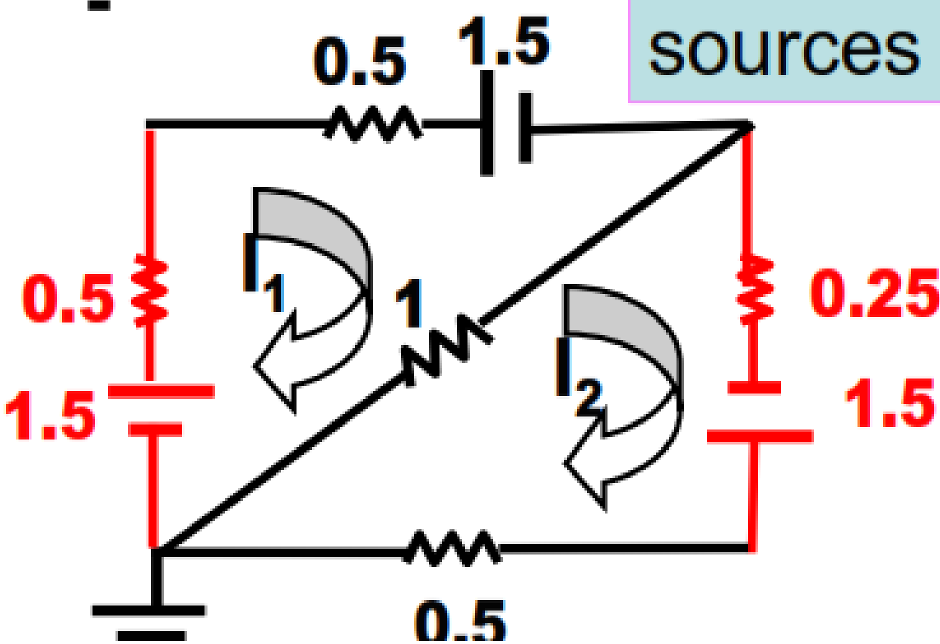
a) The branch currents

b) Check the power balance





Convert Independent Current sources into voltage sources



$$\begin{pmatrix} & & \\ & R & \\ & & \end{pmatrix} \begin{pmatrix} \\ I \\ \\ \end{pmatrix} = \begin{pmatrix} \\ V \\ \\ \end{pmatrix}$$

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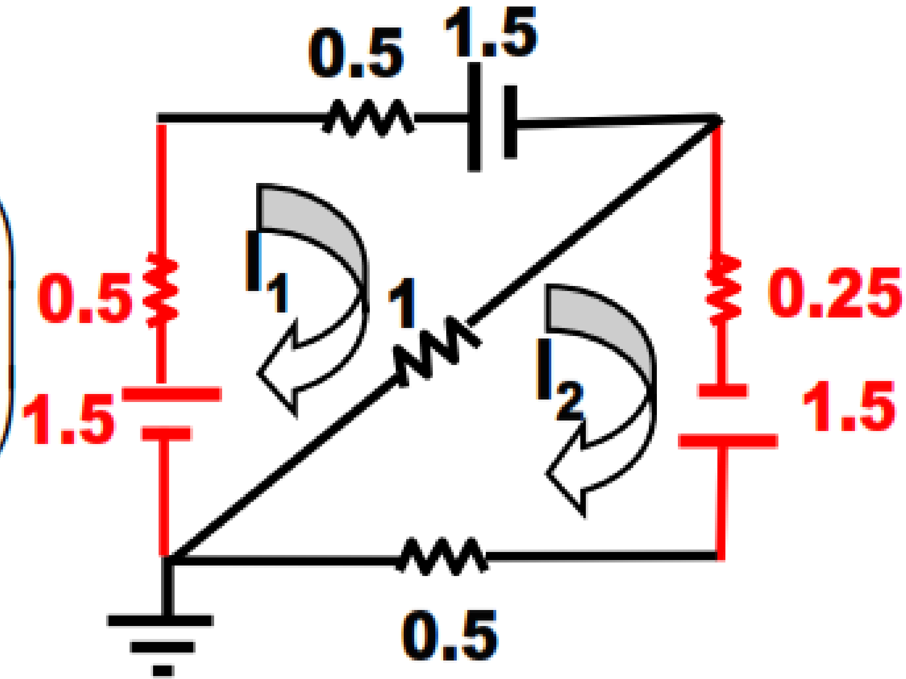
*****

$$\begin{pmatrix} R_{11} & -R_{12} \\ -R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$



$$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$$

$I_1 = 0.6\ \text{mA}$

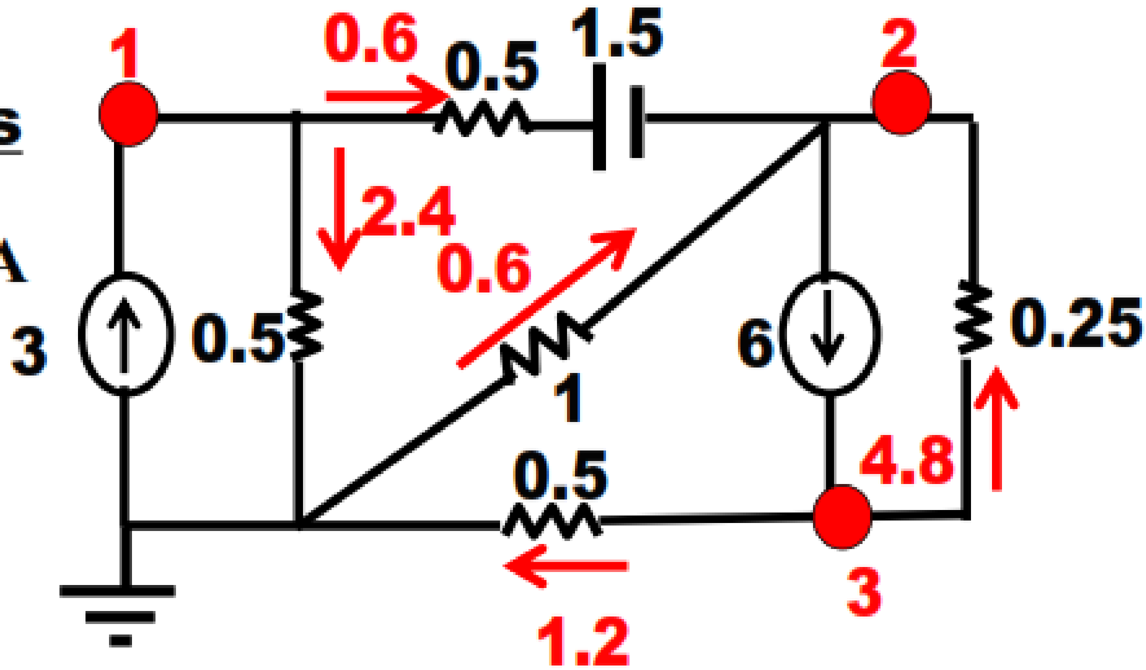
$I_2 = 1.2\ \text{mA}$

*****

a) Branch Currents

$I_1 = 0.6$ mA $I_2 = 1.2$ mA

b) Nodal Voltages



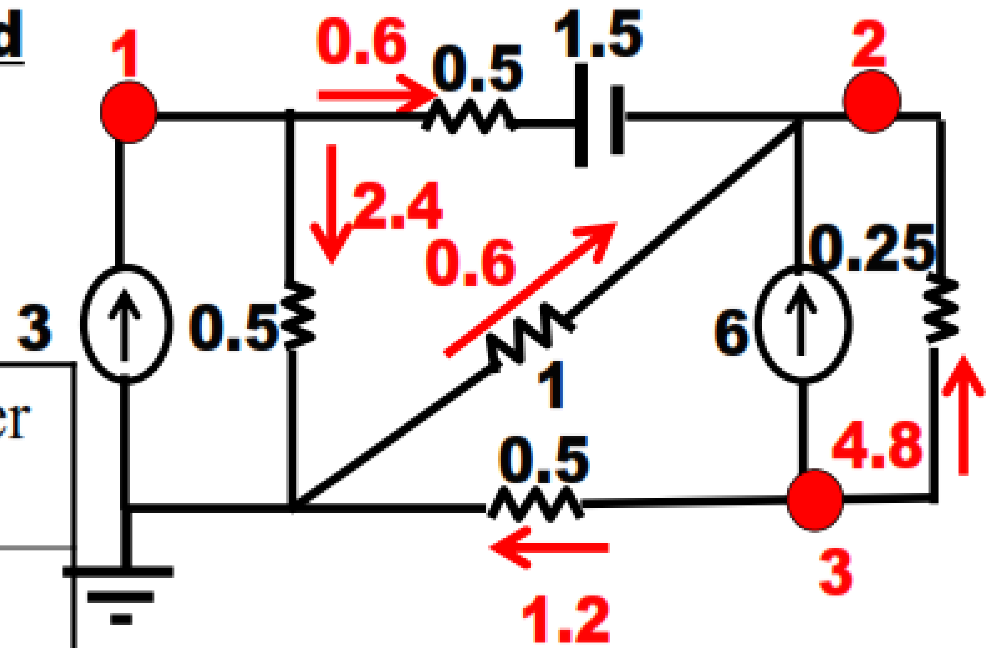
$V_1 =$

$V_2 =$

$V_3 =$

*****

Power dissipated by resistors



R	Dissipated power (mw)
R_{10}	
R_{32}	
R_{12}	
R_{30}	
R_{02}	
Total	9.9

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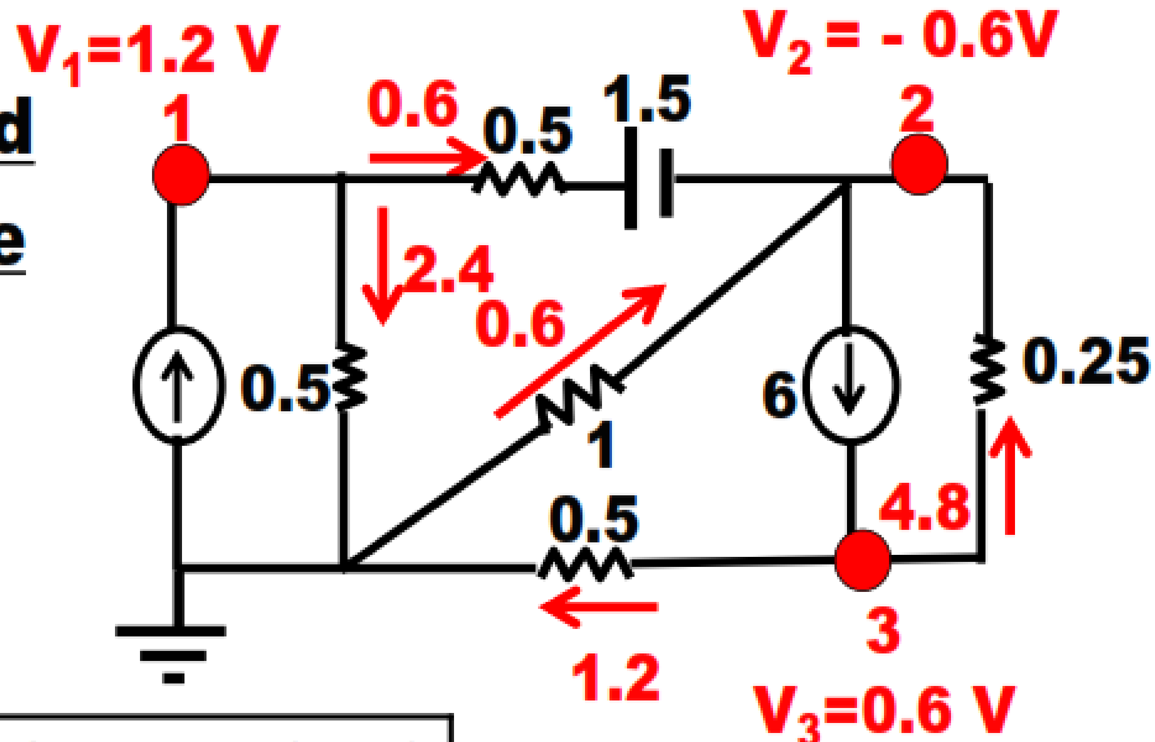
*****

Power supplied by each source



Source	Delivered power (mw)
1.5 V	
3 mA	
6 mA	
Total	9.9 mw

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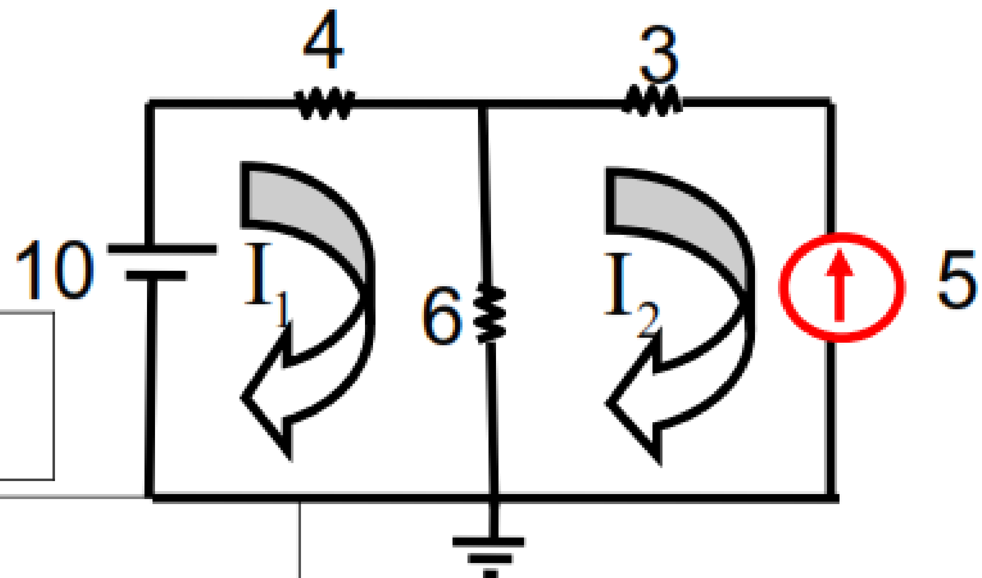
*****

Loop Analysis With an Ideal Current Source

<u>CASE 1:</u> When an ideal current source exists in one loop

Write loop equation

for loop 1



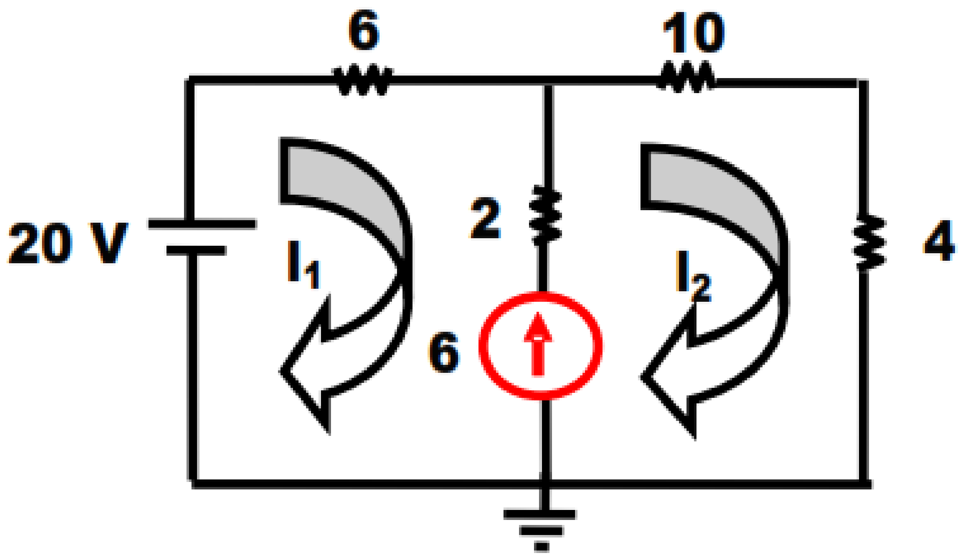
- For loop 2, $I_2 =$

$\therefore I_1 =$

*****

Loop Analysis With Current Source

CASE 2: **When an ideal current source exists between two loops**



(1)

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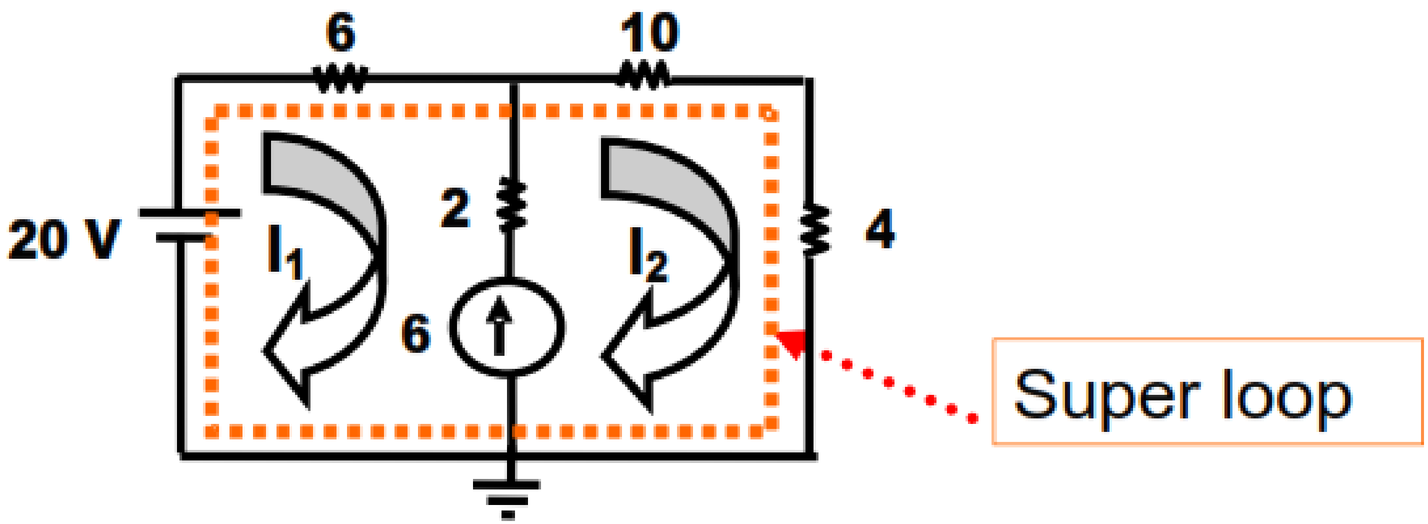
*****

Loop Analysis With Current Source

Super loop: A loop consisting of 2 or more loops.



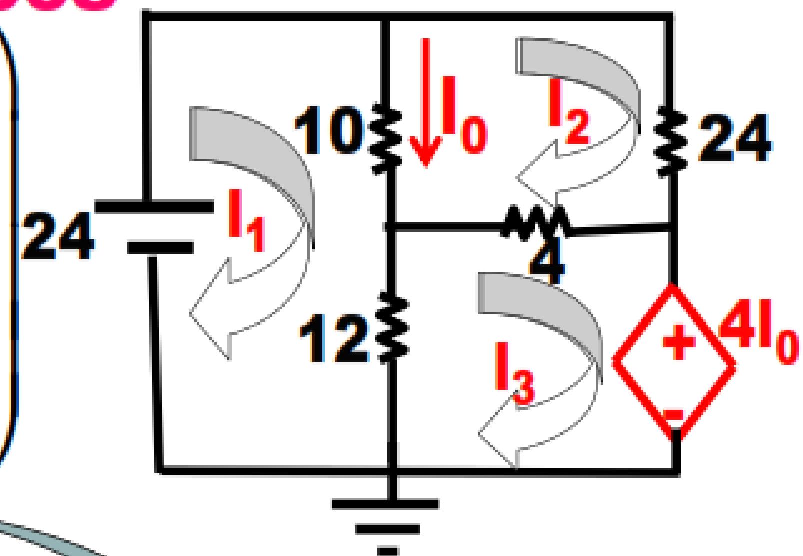
(2)

From (1), (2), $\quad I_1 = -3.2\, m \qquad I_2 = 2.8\, mA$

*****

Loop Analysis With Dependent Sources

$$\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$



Loop 3

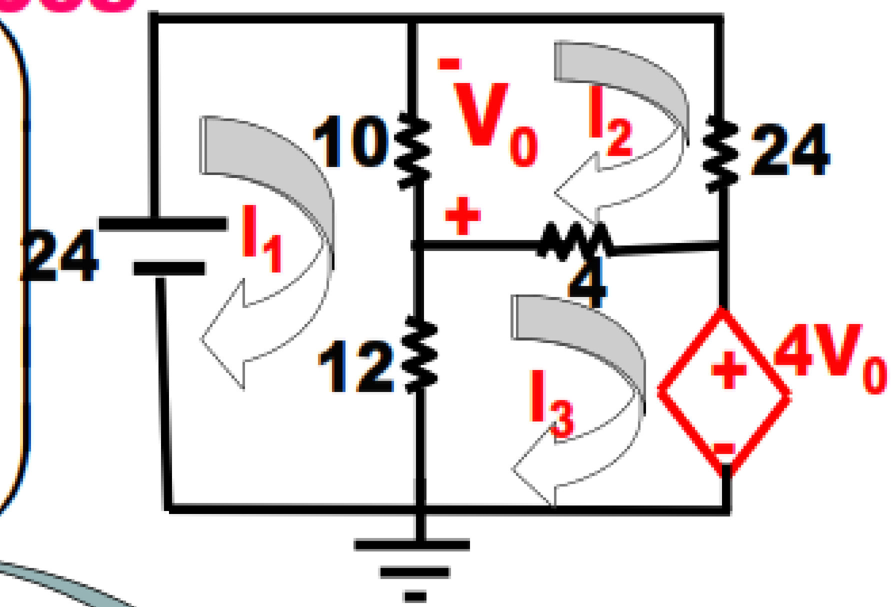
$$I_0 = I_1 - I_2$$

$$I_1 = 2.25 \text{ mA} \quad I_2 = 0.75 \text{ mA} \quad I_3 = 1.5 \text{ mA}$$

*****

Loop Analysis With Dependent Sources

$$\begin{pmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} \quad \\ \quad \\ \quad \end{pmatrix}$$



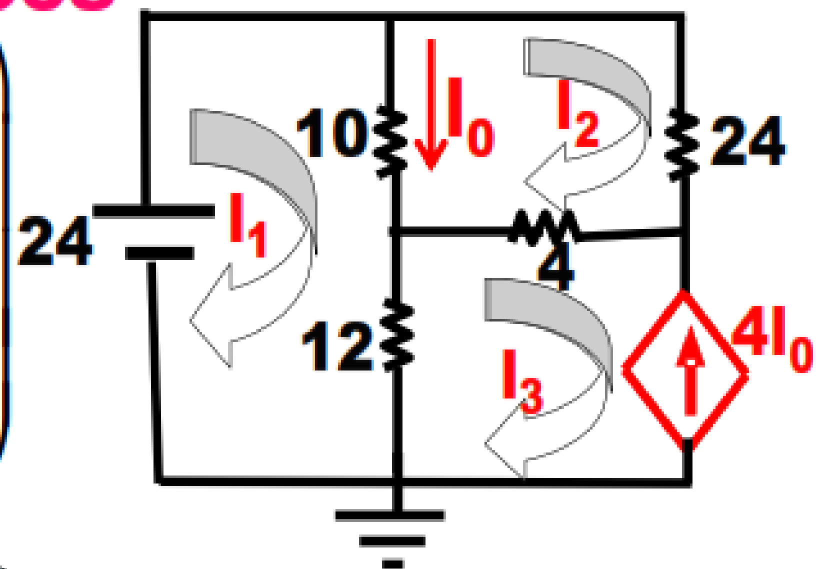
Loop 3

$$V_0 = 10(I_2 - I_1)$$

*****

Loop Analysis With Dependent Sources

$$\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$



Loop 3

$I_0 = I_1 - I_2$

Important Applications Using Loop Analysis

Balanced bridge

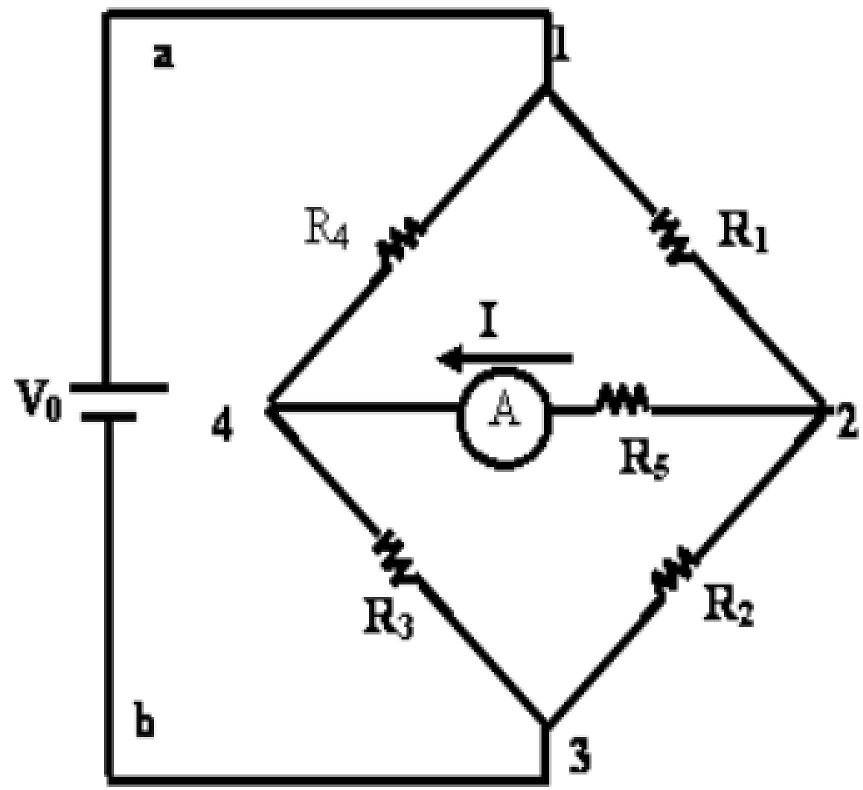
- For the bridge to be balanced $I = 0$
- $V_2 = V_4$ Hence,
- $V_{12} = V_{14}$

$I_{12} R_1 = I_{14} R_4$ (1)

- $V_{23} = V_{43}$

$I_{23} R_2 = I_{43} R_3$ (2)

- Since $I = 0$, $I_{12} = I_{23}$ & $I_{14} = I_{43}$



$$\frac{(1)}{(2)} \Rightarrow \frac{R_1}{R_2} = \frac{R_4}{R_3} \quad \text{Hence,} \quad R_1 R_3 = R_2 R_4$$

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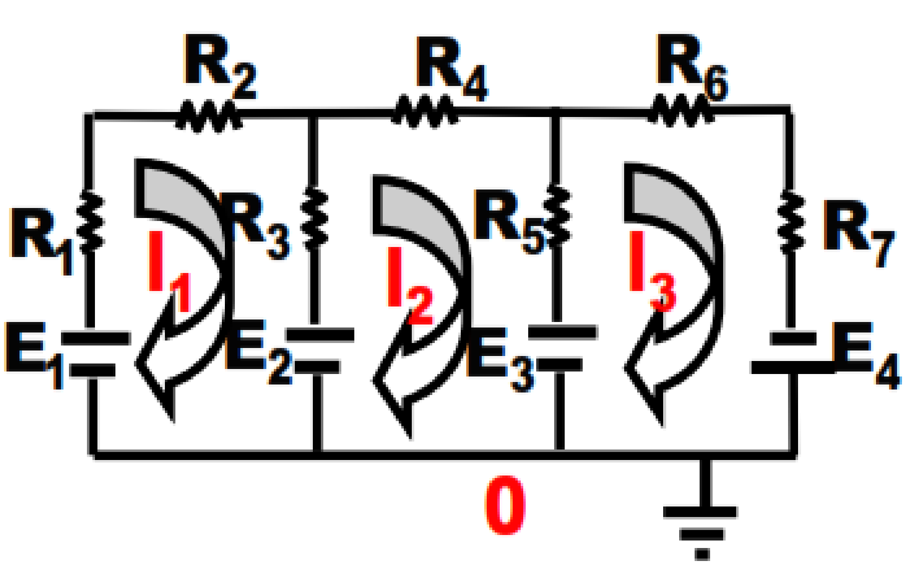
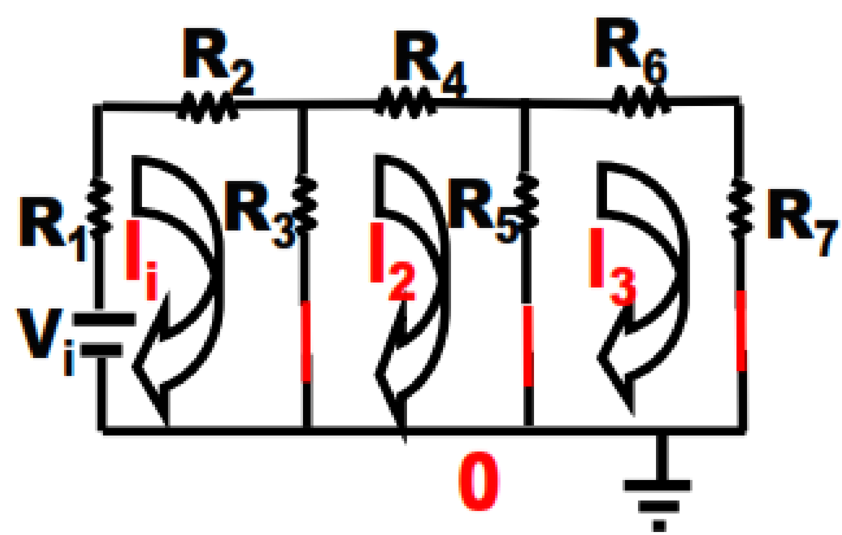
Input Resistance

$$R_i = \frac{V_i}{I_i}\bigg|_{\text{all circuit sources are cancelled}}$$

- Voltage sources are SC & current sources are OC

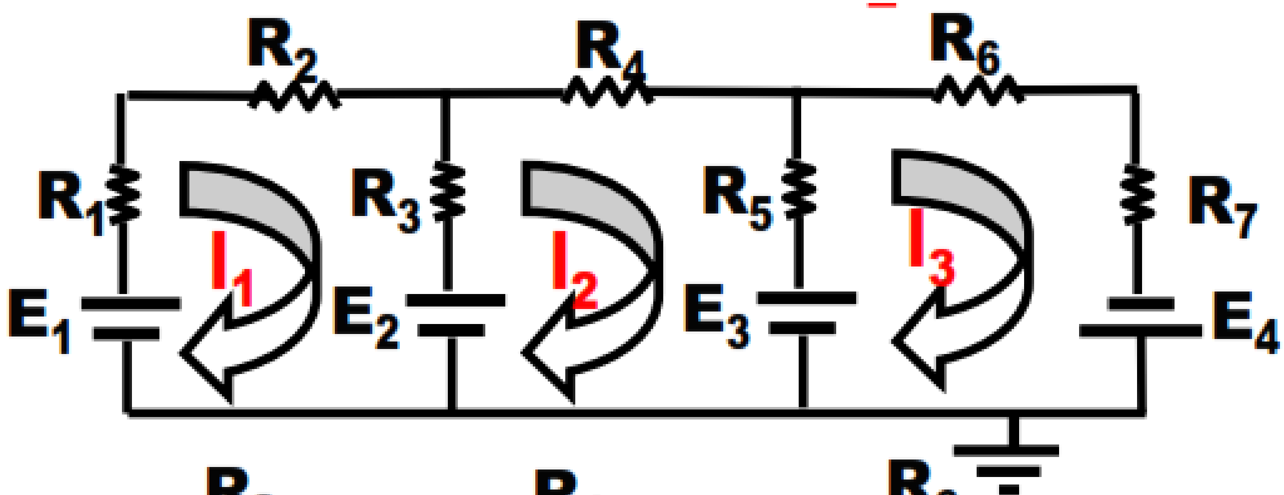
1- Resistance Seen by a source in one Loop (e.g. E_1)

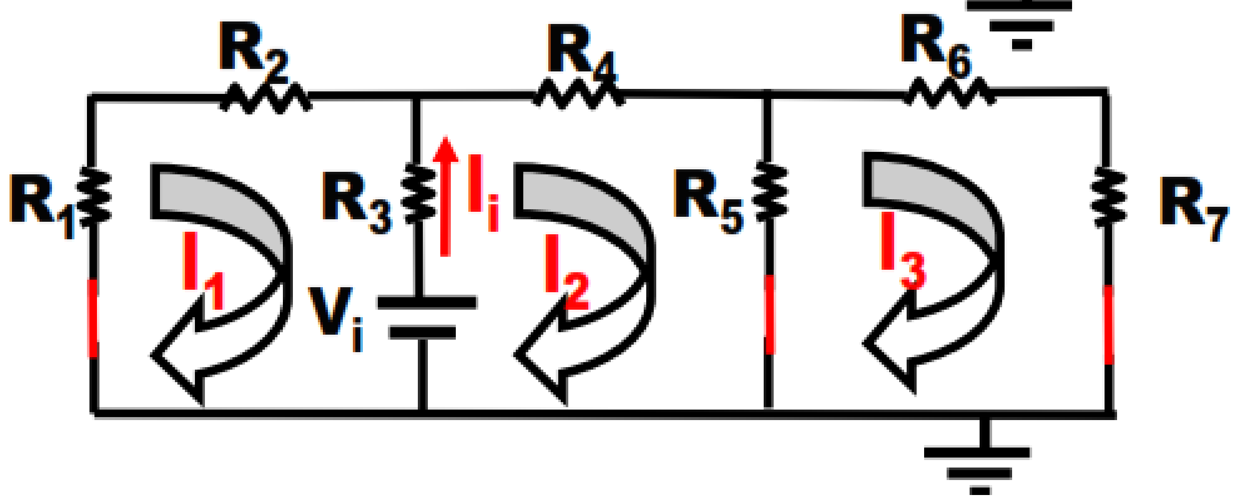




Input Resistance

2- Resistance Seen by a source common between two Loops (e.g. E_2)





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4. Nodal Analysis

Method 4 Nodal Analysis

- Steps of solution:

1- Convert independent voltage sources into equivalent current sources

2- Identify the number of nodes (*N*) of the circuit

3- Write an expression for the KCL at *N-1* nodes (Exclude the ground node).

4- Solve the resultant system of algebraic equations to find the node voltages.

100, 1, 91.5, 9.5, 83, 18, 74.5, 26.5, ?, ?

What two numbers come next?

Answer:

100, 1, 91.5, 9.5, 83, 18, 74.5, 26.5, ?, ?

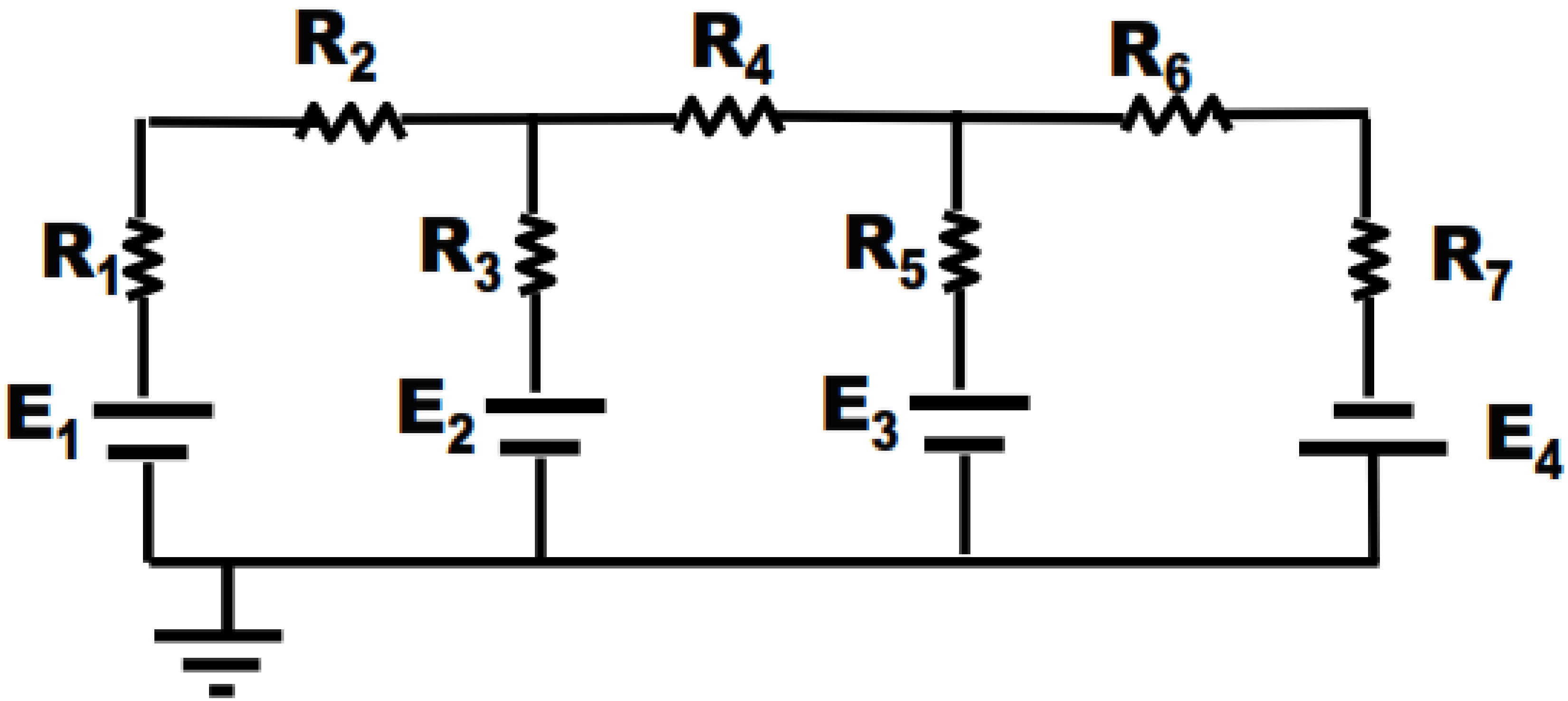
What two numbers come next?

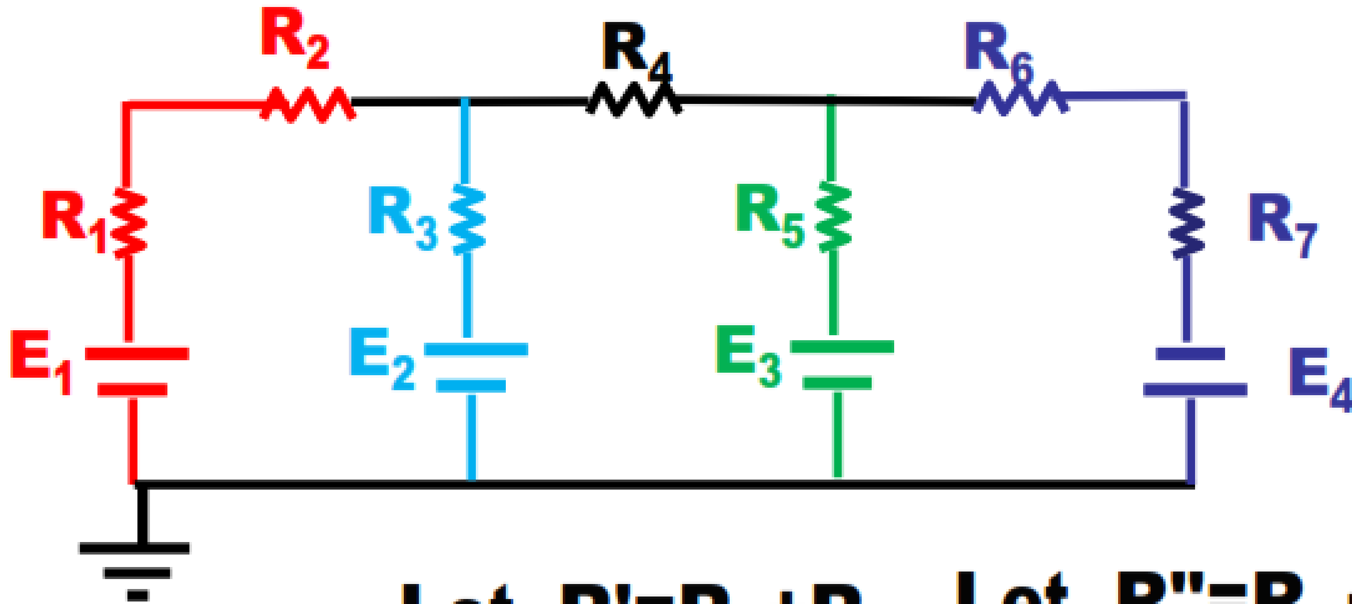
Answer: 66, 35

Explanation:

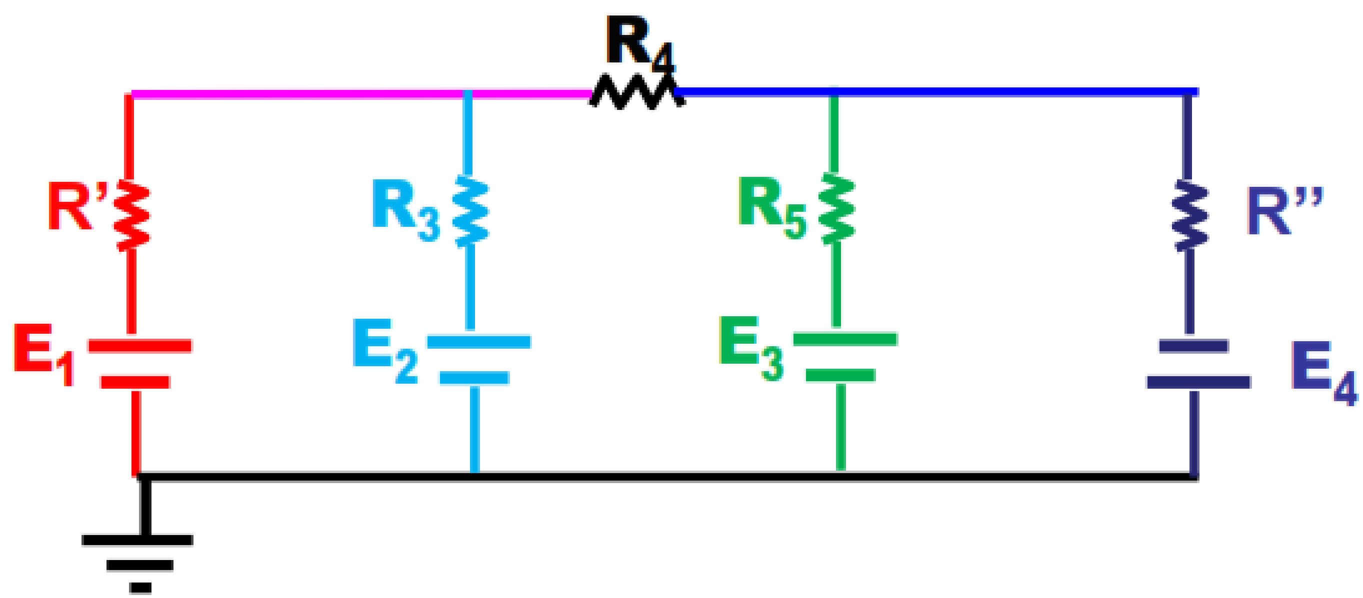
There are two interwoven sequences. Start at 100 and deduct 8.5. Start at 1 and add 8.5.

Example





Let $R' = R_1 + R_2$ Let $R'' = R_6 + R_7$



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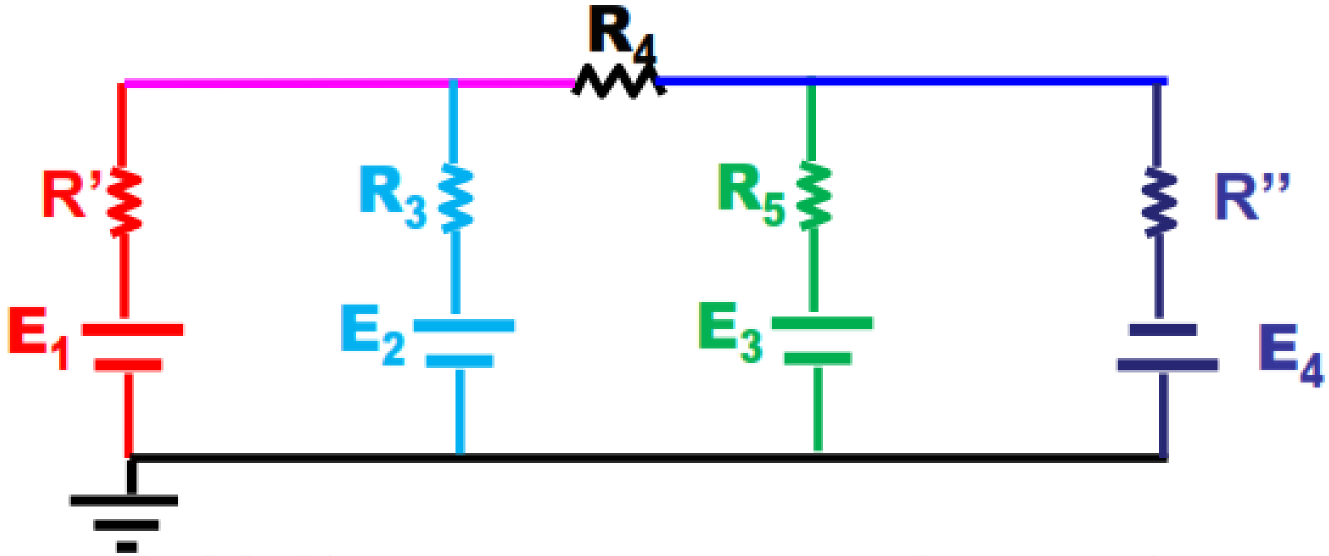
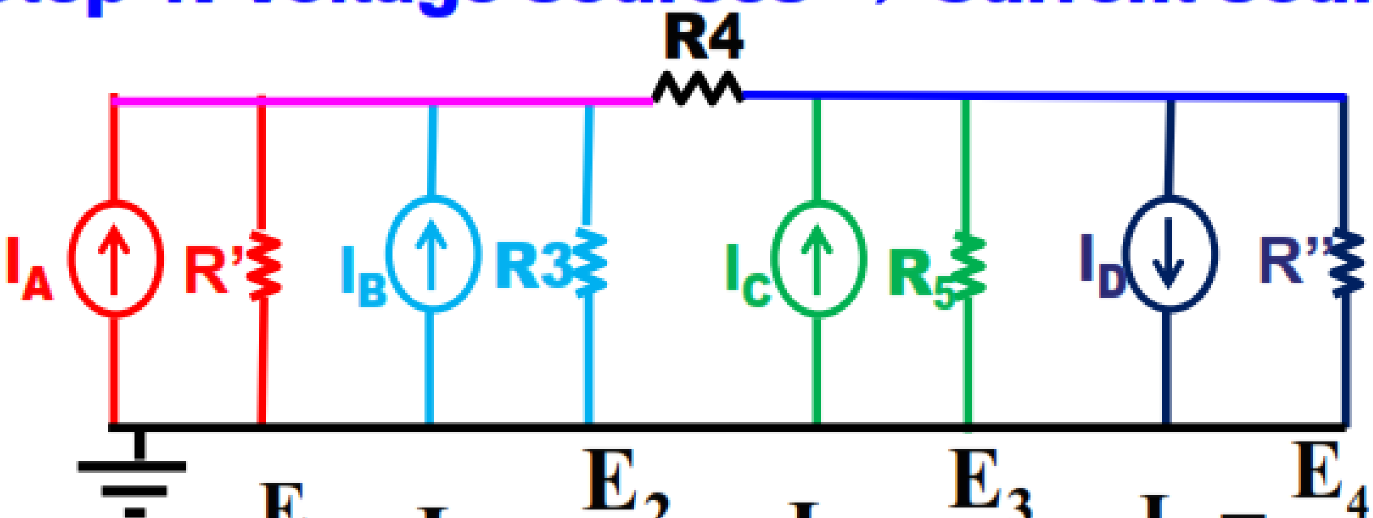
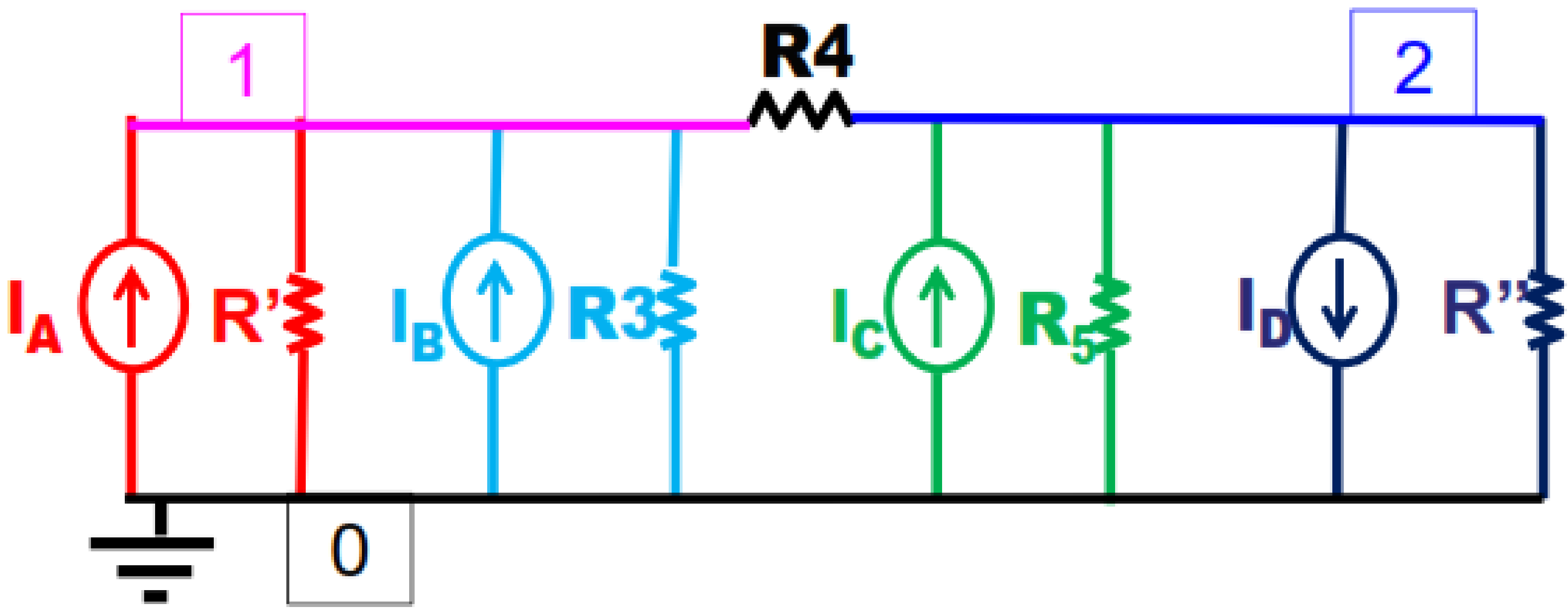


Step 1: Voltage sources ⇒ Current sources



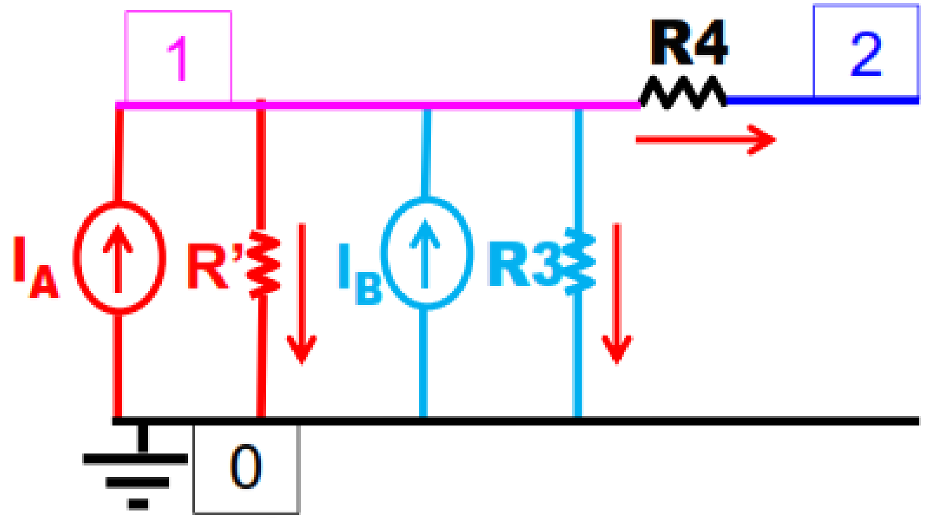
$$I_A = \frac{E_1}{R'} \quad I_B = \frac{E_2}{R_3} \quad I_C = \frac{E_3}{R_5} \quad I_D = \frac{E_4}{R''}$$

- **Step 2 : N=3**
- **Step 3: KCL at each node** $\sum I_{out} = 0$



Node 1

$$\sum I_{out} = 0$$



$$I_{R'} + I_{R3} + I_{R4} - I_A - I_B = 0$$

$$\frac{V_1}{R'} + \frac{V_1}{R_3} + \frac{V_1 - V_2}{R_4} - I_A - I_B = 0$$

$$G' V_1 + G_3 V_1 + G_4 (V_1 - V_2) - I_A - I_B = 0$$

$$(G' + G_3 + G_4) V_1 - G_4 V_2 = I_A + I_B \qquad (1)$$

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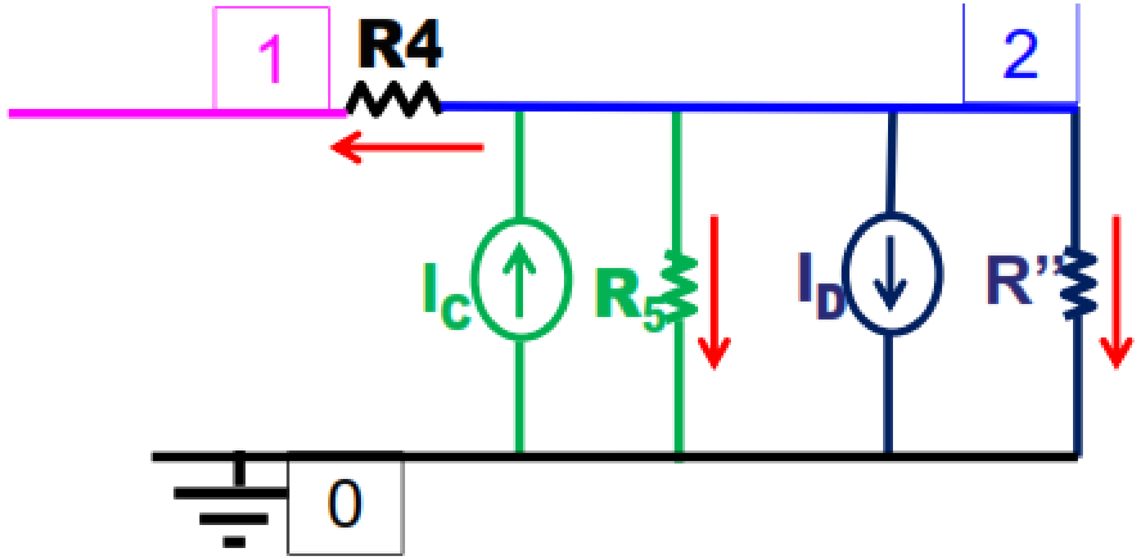
Node 2

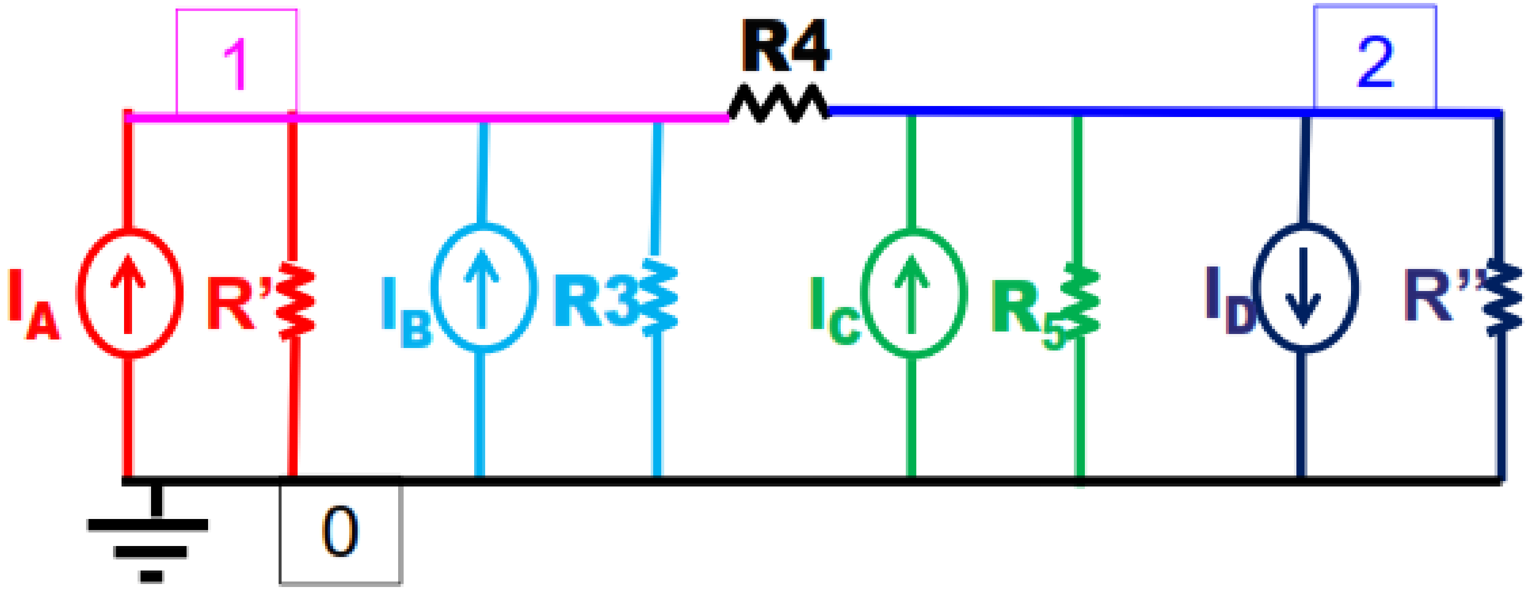


$$\sum I_{out} = 0$$

$$I_{R4} + I_{R5} + I_{R''} - I_C + I_D = 0$$

$$G_4 \left(V_2 - V_1 \right) + G_5 V_2 + G'' V_2 = I_C - I_D$$

$$-G_4 V_1 + \left(G_4 + G_5 + G'' \right) V_2 = I_C - I_D \qquad (2)$$



- **Equations**

$$\left(G' + G_3 + G_4\right)V_1 - G_4 V_2 = I_A + I_B \qquad (1)$$

$$-G_4 V_1 + \left(G_4 + G_5 + G''\right)V_2 = I_C - I_D \qquad (2)$$

$$\begin{pmatrix} G_{11} & -G_{12} \\ -G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$\mathbf{G_{ii}} = \sum \textbf{Conduc}\tan \textbf{ce of node i} \equiv \textbf{ Self conductance of node i}$$

$$\mathbf{G_{ij}} = \mathbf{G_{ji}} = \sum \textbf{Common cnduc}\tan \textbf{ces between nodes i\& j}$$

$$\mathbf{I_i} = \sum \mathit{Curents\ sources}\ \textbf{at node i}$$

A current source is assumed +ve if it pushes current toward the node direction and –ve otherwise.

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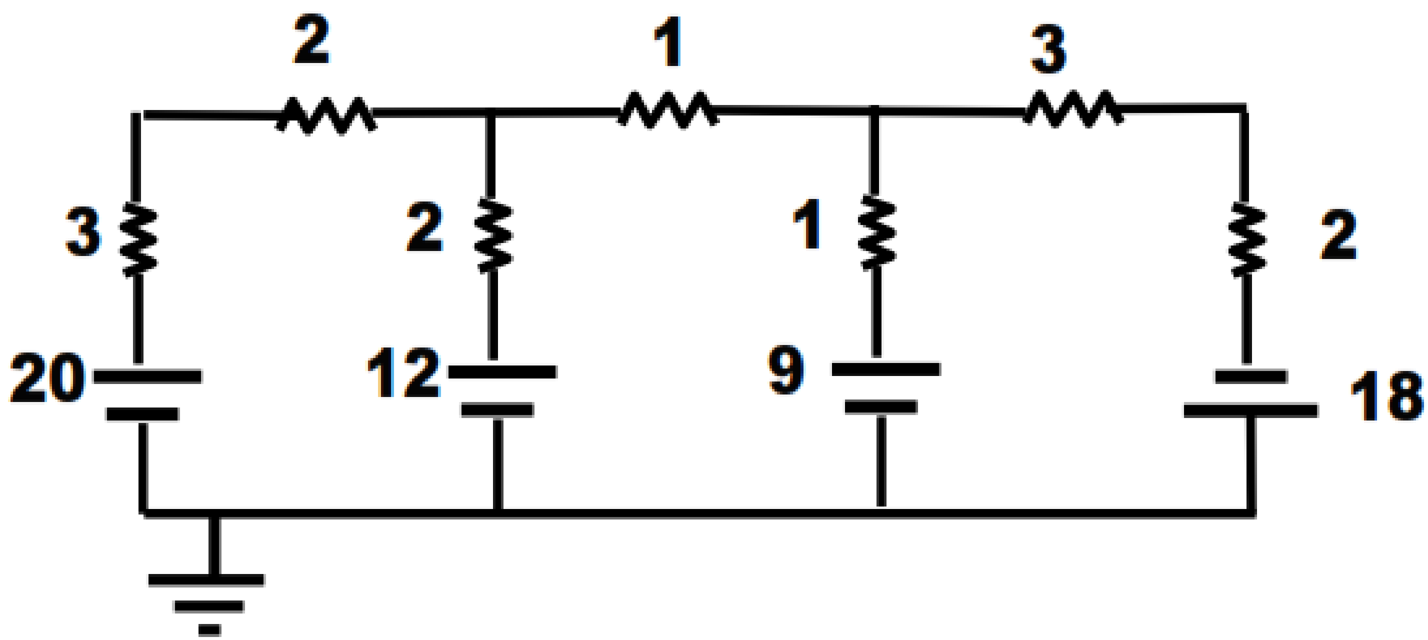
$$\begin{pmatrix} & & \\ & G & \\ & & \end{pmatrix} \begin{pmatrix} \\ V \\ \\ \end{pmatrix} = \begin{pmatrix} \\ I \\ \\ \end{pmatrix}$$

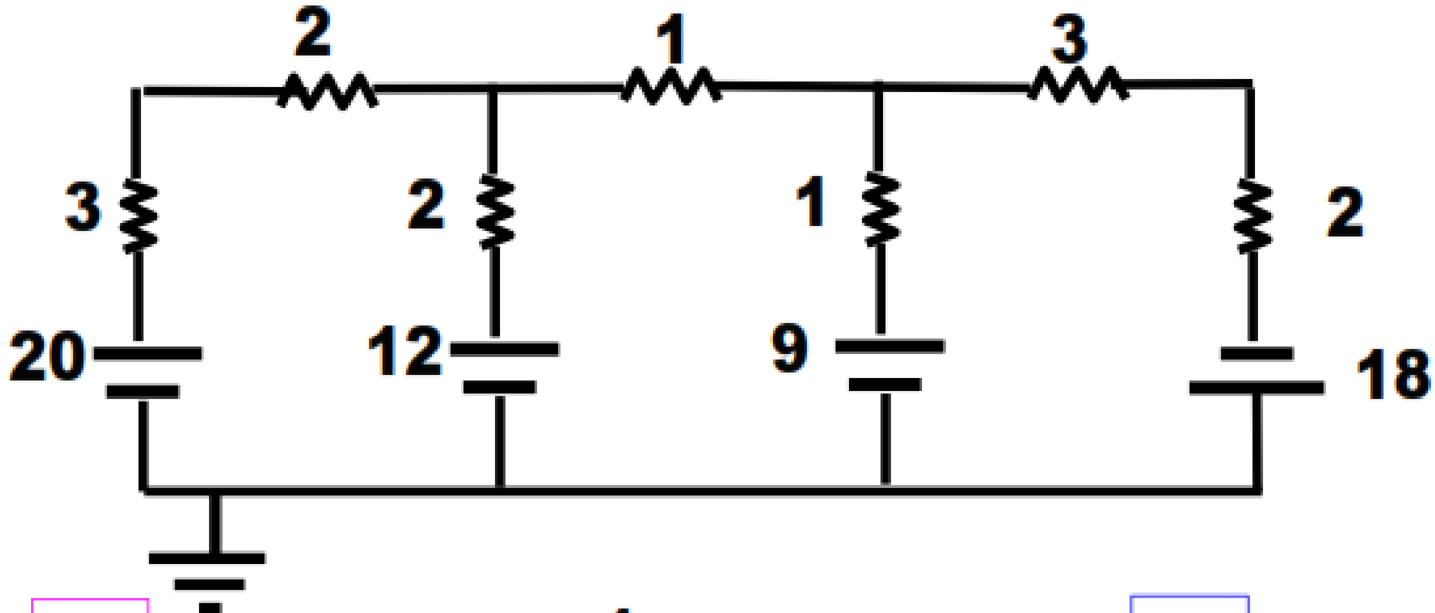
- $G \equiv$ Conductance Matrix
- $V \equiv$ Unknown nodal voltages
- $I \equiv$ Current Vector

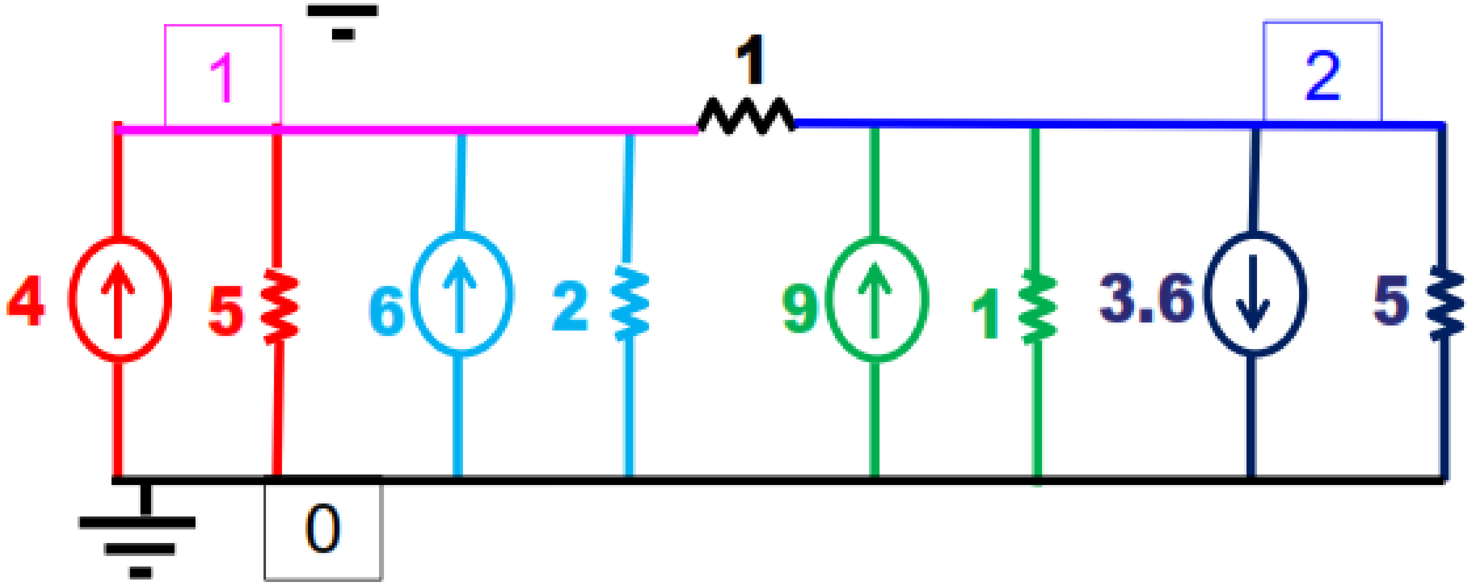
Example

For the shown circuit, **Use node method** to find the nodal voltages & the branch currents



Solution





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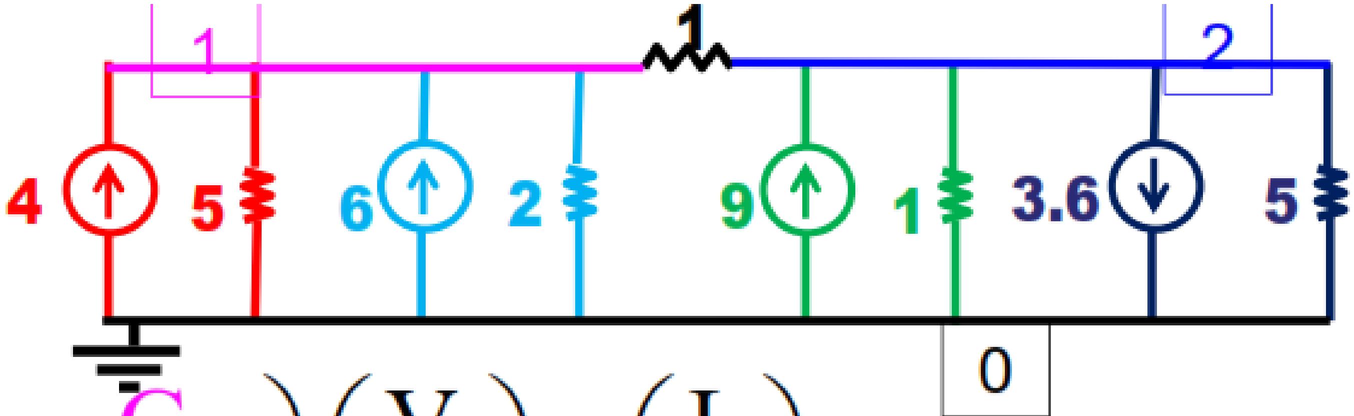

$$\begin{pmatrix} G_{11} & -G_{12} \\ -G_{21} & G_{22} \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}=\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$\begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}=\begin{pmatrix} \\ \end{pmatrix}$$

$$\begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}=\begin{pmatrix} \\ \end{pmatrix}$$

$V_1 = 10\ V$

$V_2 = 7\ V$

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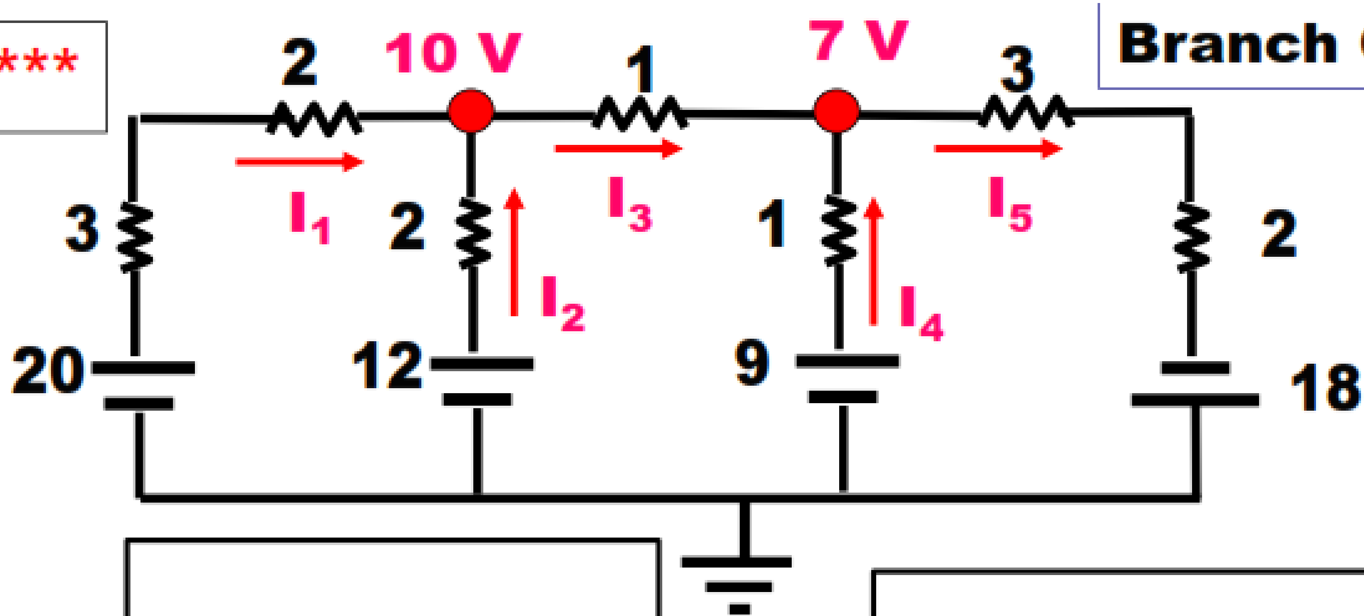
****

Branch Currents



$I_1 =$

$I_2 =$

$I_3 =$

Check KCL at node 1

$I_4 =$

$I_5 =$

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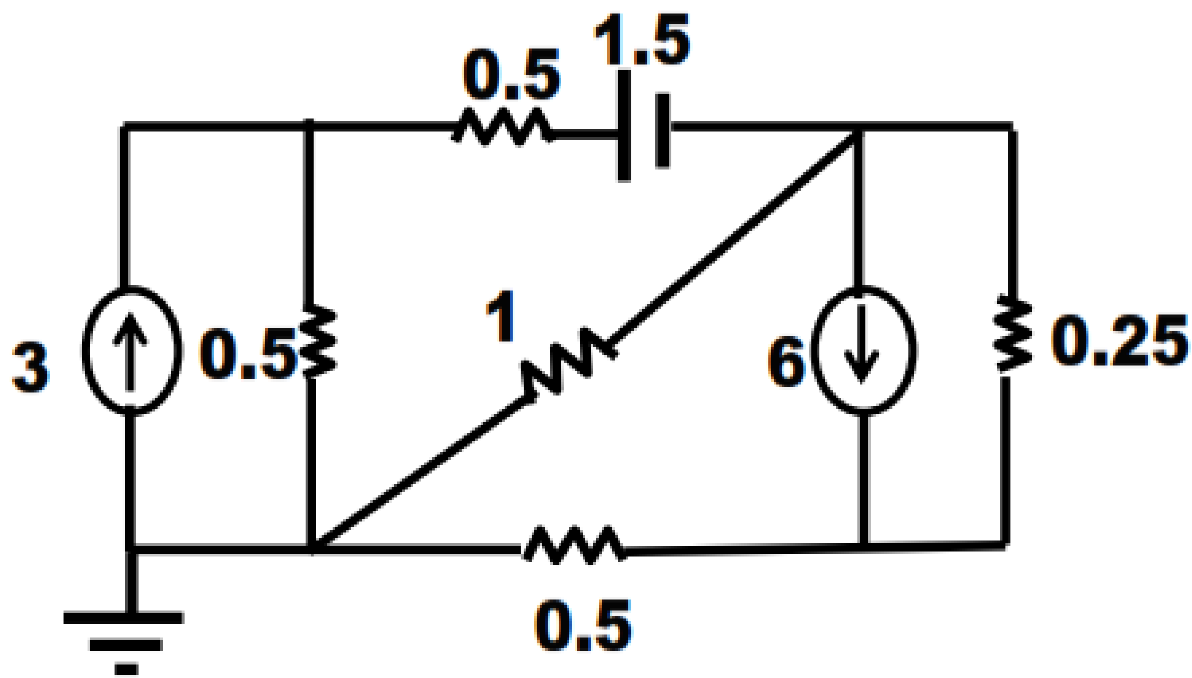
Check KCL at node 2

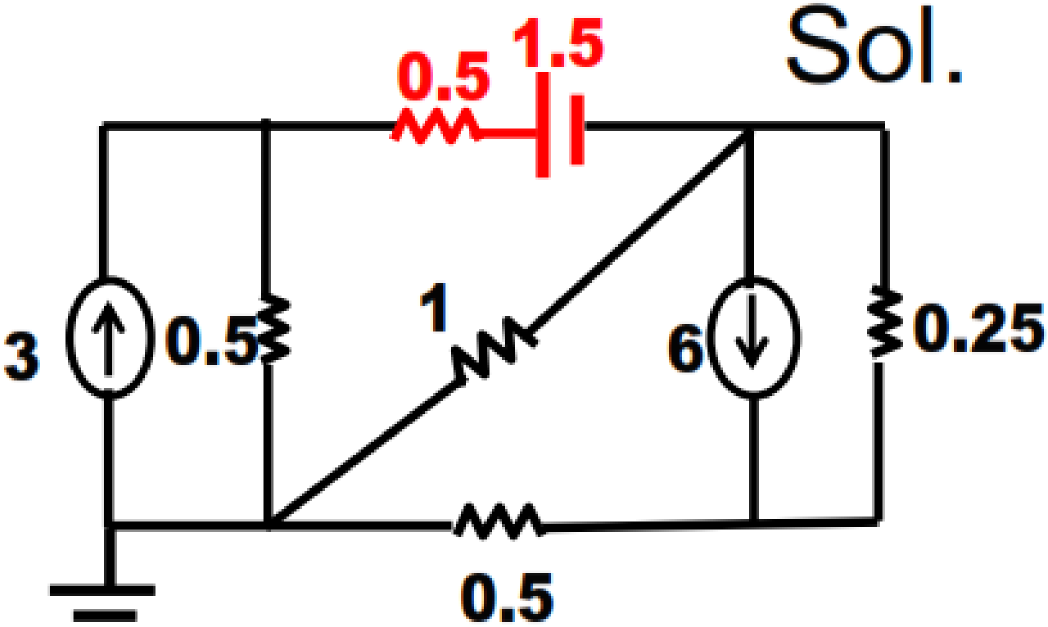
Example

- For the circuit shown, use node analysis to find:

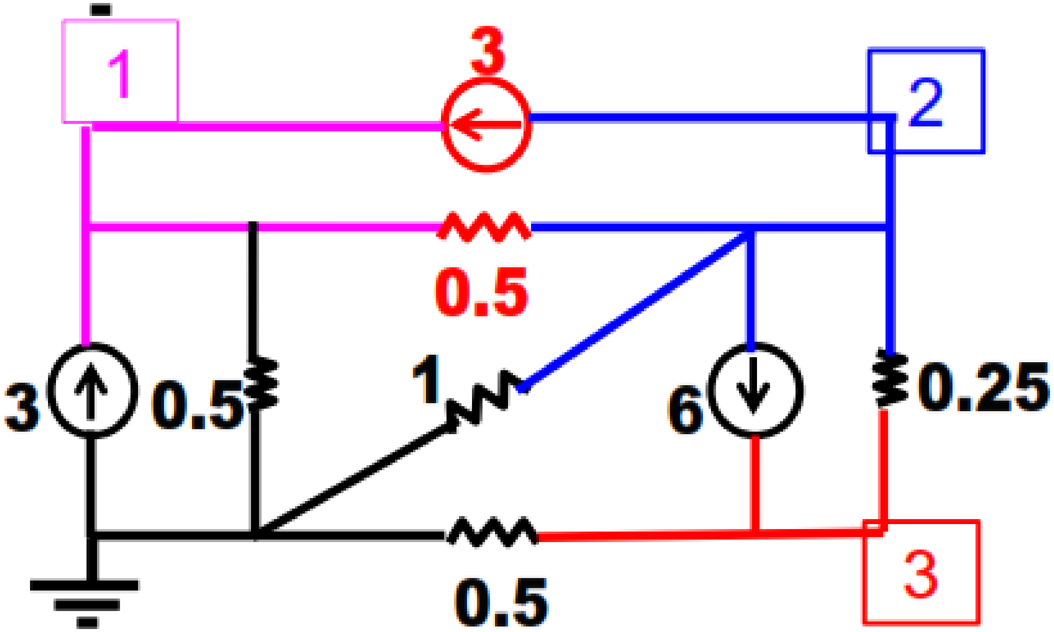


a) The nodal voltages and the branch currents
b) Check the power balance

Sol.



Independent
VS ⇒ CS



N=4

$$\begin{pmatrix} & & \\ & G & \\ & & \end{pmatrix} \begin{pmatrix} \\ V \\ \\ \end{pmatrix} = \begin{pmatrix} \\ I \\ \\ \end{pmatrix}$$

153

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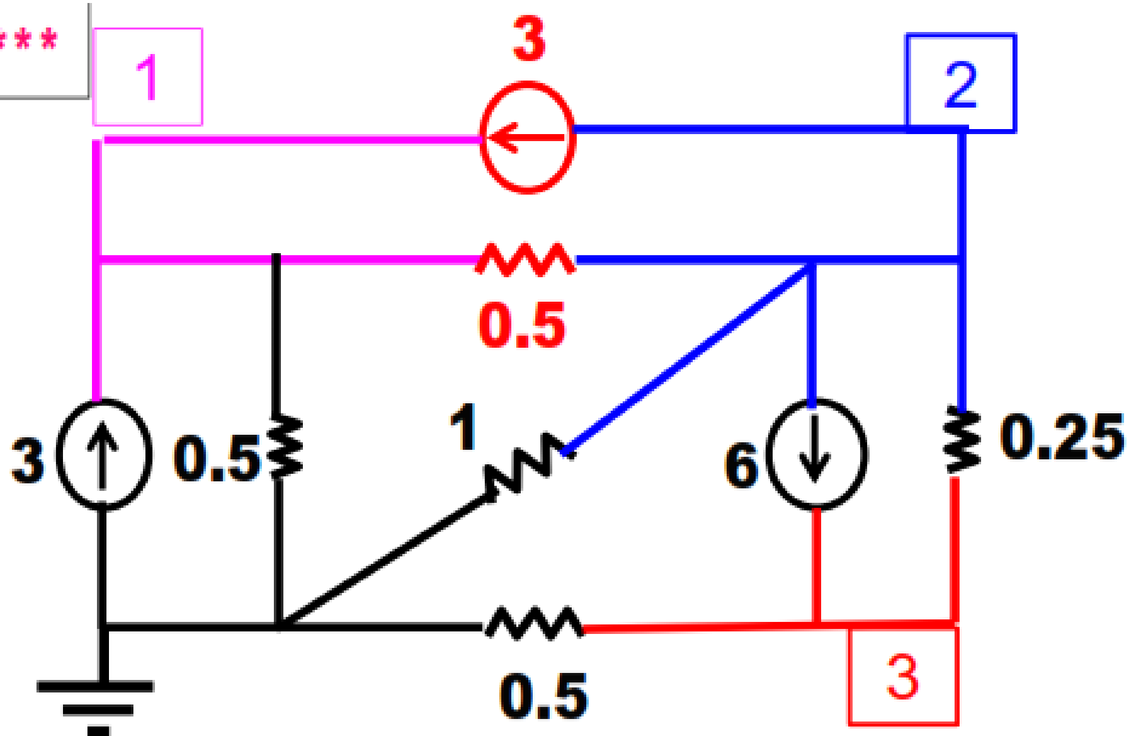

$V_1 = 1.2\ V$

$V_2 = -0.6\ V$

$V_3 = 0.6\ V$

$$\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

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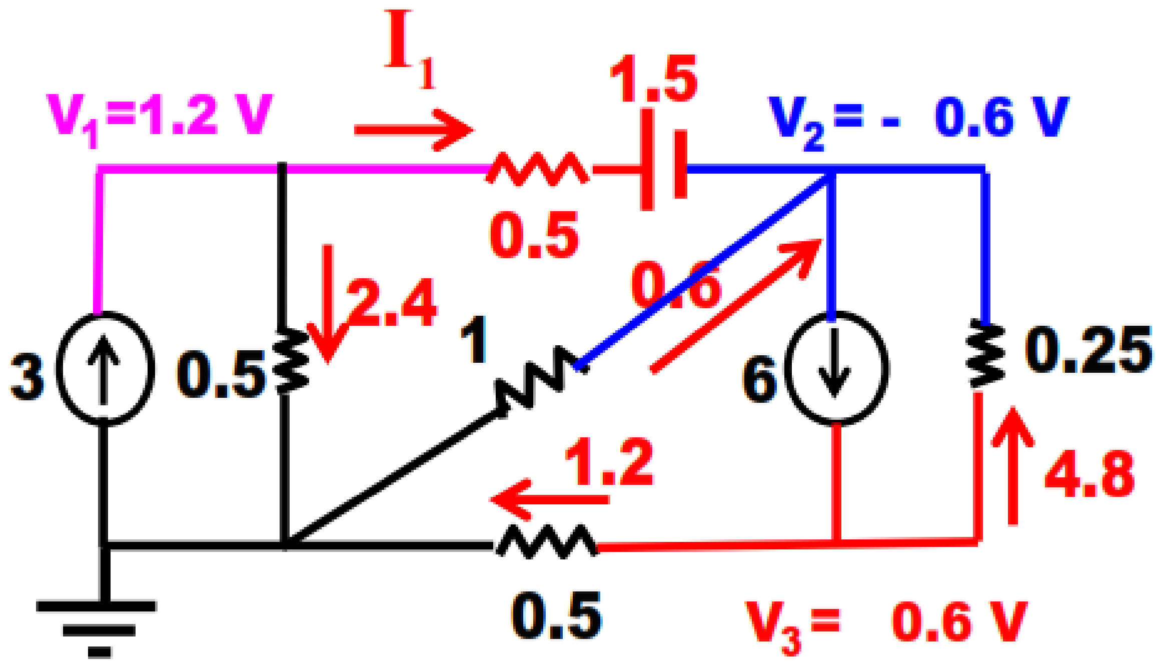
*****

Branch Currents



$V_{12} =$ [ ]

$I_1 =$ [ ]   Check **KCL** t node 1

Check **KCL** ıt node 2

Check **KCL** at node 3

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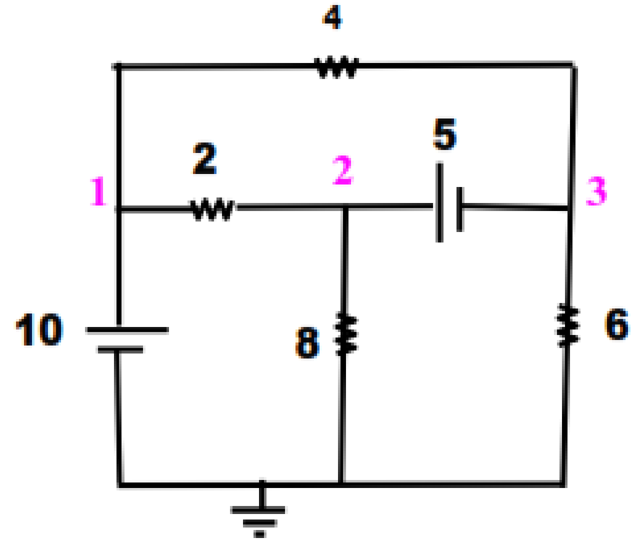
Node Analysis With Voltage Source

<u>CASE **1:**</u> When a voltage source exists between one node and the reference node

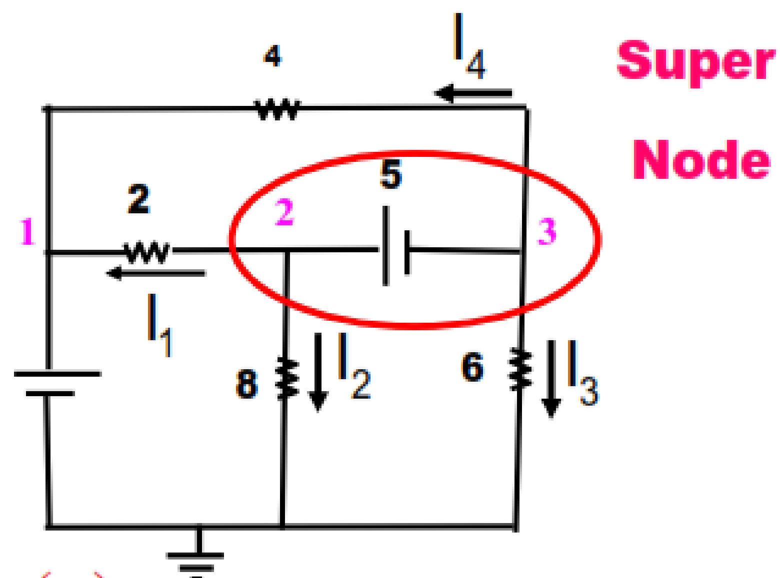
$$V_1 = 10\,V \qquad (1)$$



Node Analysis With Voltage Source

<u>CASE **2:**</u> When a voltage source exists between two non-reference nodes

A super node is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements in parallel with it.



KVL $\quad V_2 - V_3 = 5\text{ V} \quad (2)$

KCL $\quad I_1+I_2+I_3+I_4=0$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{8} + \frac{V_3}{6} + \frac{V_3 - V_1}{4} = 0 \quad (3)$$

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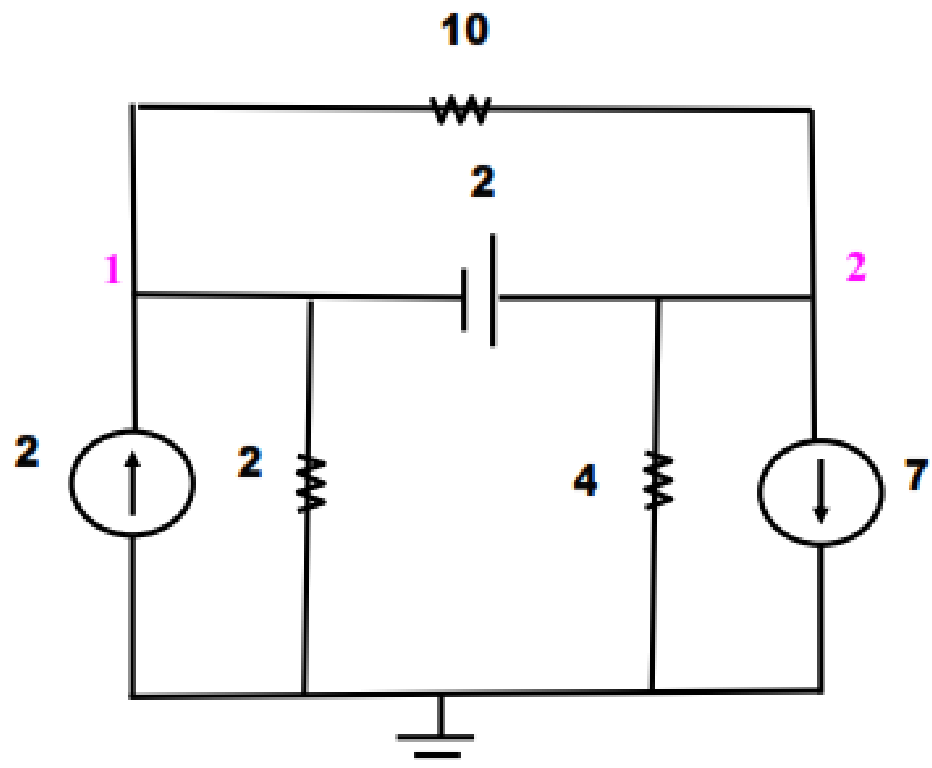
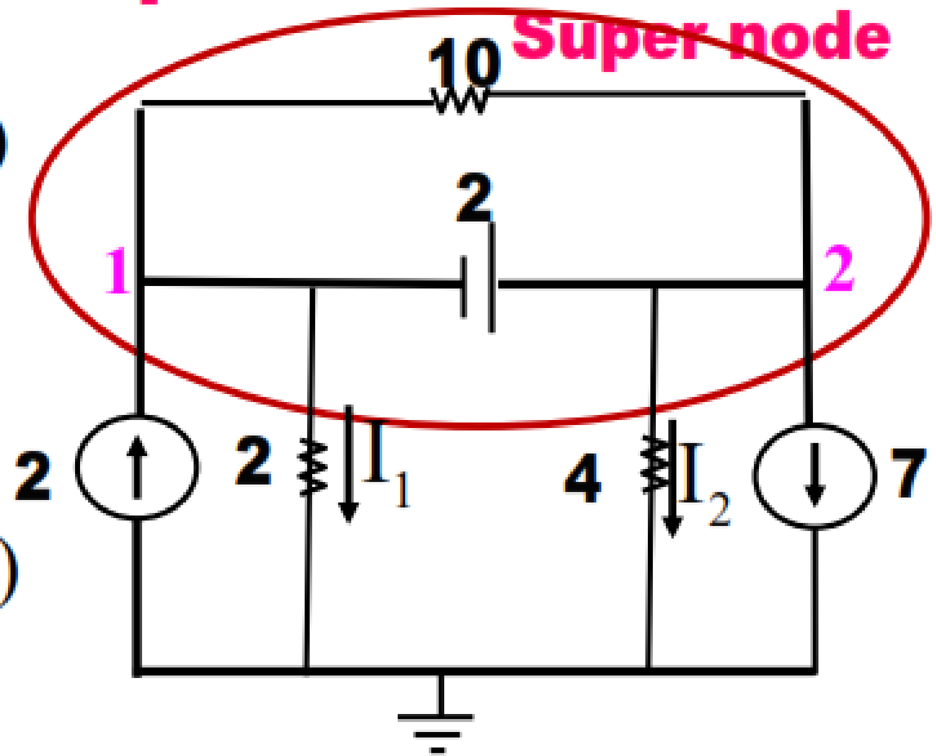
Example

Find the node voltages for the circuit shown.



Example



KVL $V_2 - V_1 = 2\text{ V} \quad (1)$

KCL $I_1 + I_2 + 7 - 2 = 0$

$$\frac{V_1}{2} + \frac{V_2}{4} = -5 \quad (2)$$

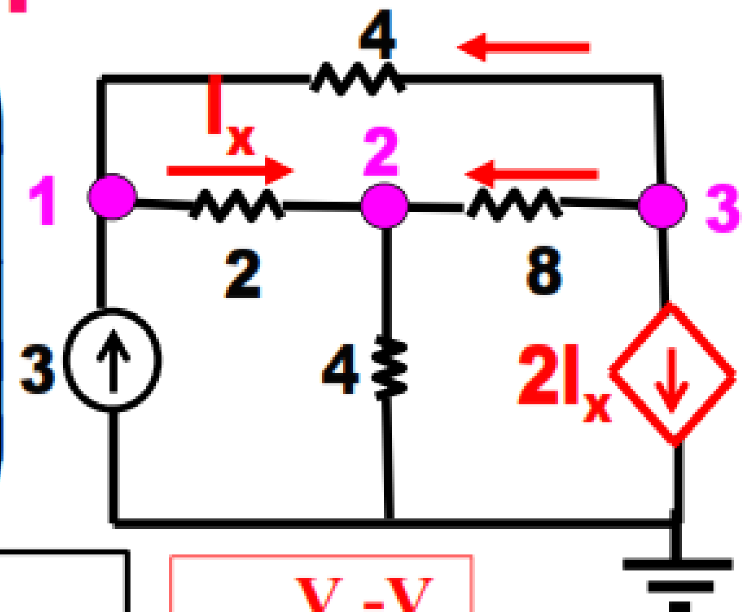
From 1, 2

$$V_1 = -\frac{22}{3}\text{ V} \quad V_2 = -\frac{16}{3}\text{ V}$$

*****

Node Analysis With Dependent Sources

$$\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$



Node 3:

$$I_x = \frac{V_1 - V_2}{2}$$

$$\sum I_{out} = 0$$

$$V_1 = 4.8V \quad V_2 = 2.4V \quad V_3 = -2.4V$$

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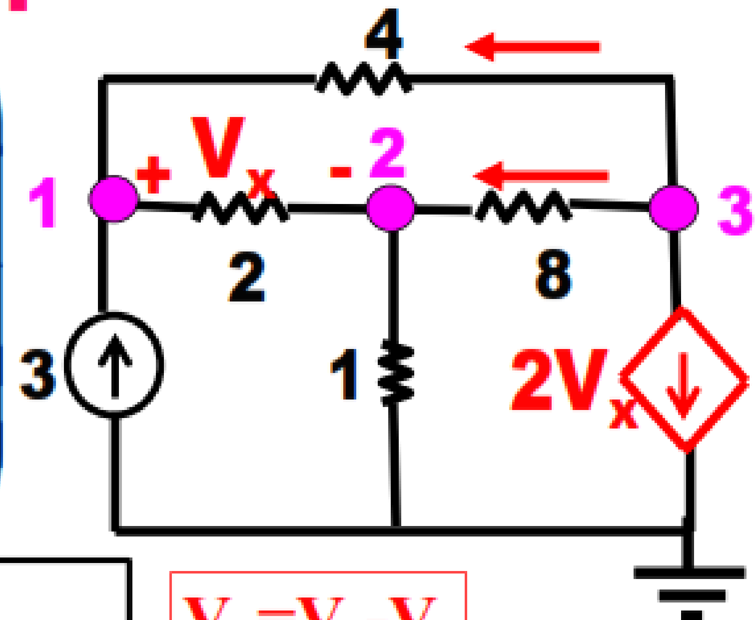
Node Analysis With Dependent Sources

$$\begin{pmatrix} 3/4 & -1/2 & -1/4 \\ -1/2 & 7/8 & -1/8 \\ & & \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$



$V_x = V_1 - V_2$

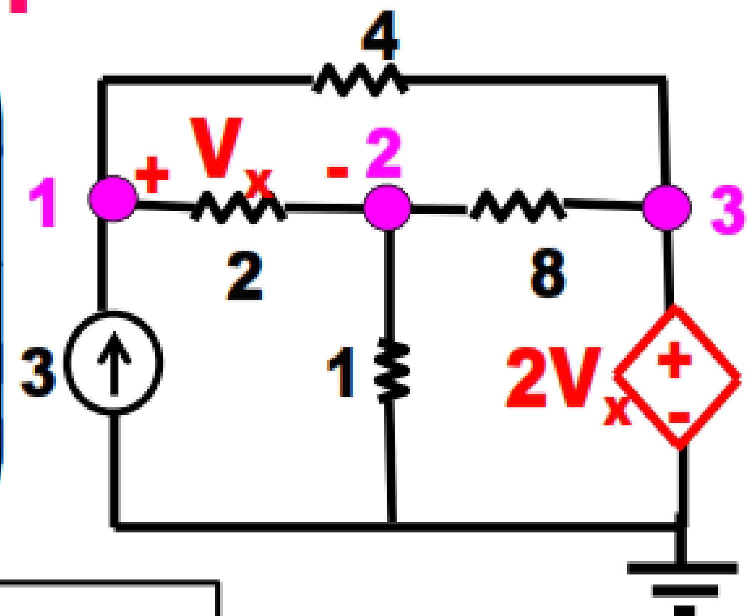
Node 3:

$\sum I_{out} = 0$

*****

Node Analysis With Dependent Sources

$$\begin{pmatrix} 3/4 & -1/2 & -1/4 \\ -1/2 & 7/8 & -1/8 \\ & & \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$



Node 3:

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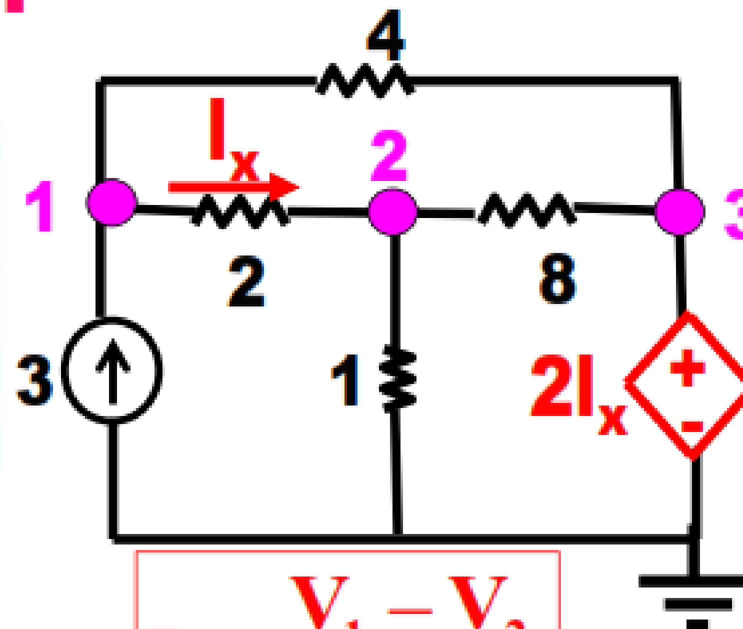
Node Analysis With Dependent Sources

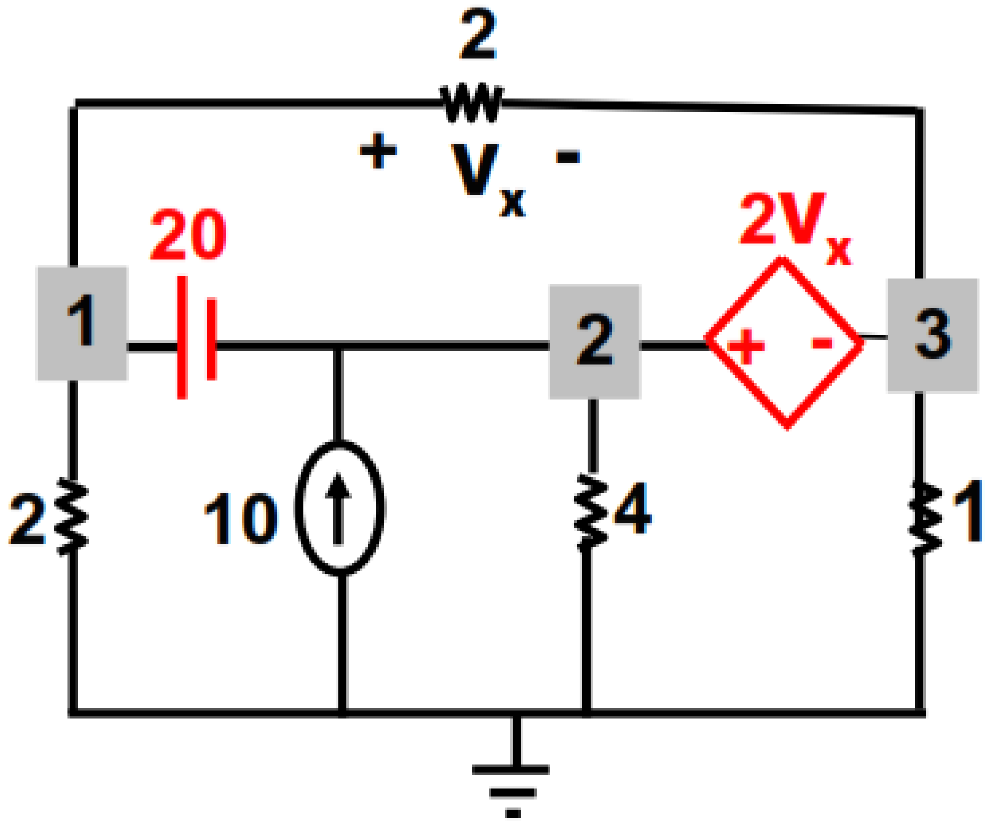
$$\begin{pmatrix} 3/4 & -1/2 & -1/4 \\ -1/2 & 7/8 & -1/8 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$



$$I_x = \frac{V_1 - V_2}{2}$$

Node 3:

Use **node analysis** to find **the node voltages** for the circuit shown.



Solution