Simultaneous concealment of time delay signature in chaotic nanolaser with hybrid feedback

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A B S T R A C T
Simultaneous suppression of time delay (TD) signature in all observables of optical chaos emission, i.e. in both intensity and phase, has been achieved using two or more semiconductor laser (SL) sources or using two independent polarization components of single vertical cavity surface-emitting laser (VCSEL). In this paper, we examine for the first time the simultaneous TD feature suppression using a single nanolaser of single polarization state. The influence of hybrid optical/electrooptical feedback schemes employing either conventional, phase-conjugate, or grating mirror is evaluated. The concealment of TD signature is then investigated by means of the autocorrelation function. The results reveal the operational regions in each case with well eliminated TD signature in all observables. A secure communications system utilizing chaotic nanolaser is presented. Security analysis for a message transmitted in the form of a colored image is carried out which verifies the immunity of the system against possible statistical, brute force and differential attacks.

1. Introduction
Semiconductor laser (SL) sources have been considered the most common tool for generating optical chaos in the last two decades [1–6], compared with other laser sources such as gas lasers [7], solid-state lasers [8] and fiber lasers [9]. In fact, the fascinating features of optical chaos such as high sensitivity to initial states, random-like behavior, broad bandwidth, and possibility of achieving chaos synchronization between master/slave systems make it distinguished in many promising applications. These potential applications include physical encryption, in both frequency and time domains, for information signals [2–4] and [10–13], physical ultra-fast random number generators [14–17], chaotic radar [18], chaotic data [19], optical time-domain reflectometry [20,21], and optoelectronic logic gates [22].

Generally, there are two approaches which can be counted for generation of optical chaos [23]. In the first type, the free running laser source exhibits chaotic output. The more recent example is quantum-dot vertical cavity surface-emitting laser (VCSEL) where polarization chaos is induced through nonlinear coupling between two elliptically polarized modes [24]. However, the low dimensional and narrow bandwidth chaos obtained from this type [25] represents a main issue. The second mechanism for generation of optical chaos relies on adding perturbations to laser system’s parameters or state variables in the form of modulation [26], feedback or optical injection [27]. To enhance the security of chaos-based communication system, it is required to increase the complexity of optical attractor via employing high dimensional chaos generators. By incorporating the effects time delay (TD) on the laser system, high-dimensional optical chaos can be generated. This goal can be achieved via utilizing different types of all-optical [28–30] and electrooptical [31,32] feedbacks.

The main problem associated with the chaos-based encryption is that the high dimensionality of the chaos is not the unique factor that ensures the security of optical chaos cryptography [33], but also the prohibition of any crucial information related to the communication system from being extracted from the chaotic signal is essential. In other words, the SL internal parameters such as threshold current and wavelength along with external parameters e.g. the value of TD employed in optical chaos system are considered the primary secret keys. So, any eavesdropper who can spy out the TD signature is, at least theoretically, competent to reconstruct the chaotic system [34–37]. Another implication of existence of TD signature is diminishing of statistical performance of physical random bit generation [14] in addition to reduction of signal-to-noise ratio in chaotic radars [18].

Different designs of optical feedbacks are established to efficiently conceal TD signature in chaotic intensity series. Amongst them are optical feedbacks employing short-external-cavity regime [38], distributed feedbacks from a fiber Bragg grating [39], double optical feedbacks [40,41], double-cavity polarization-rotated optical feedback [6], and...
three cascaded VCSELs [25]. However, Nugimdo et al. [42] revealed that TD information in the intensity time series which are confirmed to be entirely hidden can be easily obtainable via the phase time series. This breakthrough leaves realization of a system with fully suppressed TD signature in all observables more demanding.

There have been some experimental and theoretical investigations of simultaneous concealment of TD feature in both the intensity and phase of transmitted output. For example, in [43] Nugimdo et al. employed a configuration composed of cross-feedback semiconductor ring laser in order to achieve this goal. Further, it was verified that the optical chaos that induces from two SLs with dual-path injection [44] or from a cascaded scheme of three SLs [45] exhibits TD information suppression in both the intensity and phase of chaotic emission. More recently, it is shown that nonlinearly-modulated feedback in a ring of chaotic SL systems induces smaller TD signature in intensity and phase than that of the chaos induced by the conventional optical feedback [46].

However, multiple SLs are not the only approach for TD signature suppression in all observables. A single VCSEL can also lead to simultaneous TD signature suppression using electro-optic and optical feedbacks of incommensurable delays [47]. In this hybrid feedback scheme, the interference between two independent polarization components of VCSEL emission, subject to chaotic differential phase modulation, leads to an enhanced feedback signal suppressing simultaneously the TD signature in intensity and phase.

Nowadays, the integrated nanophotonic devices provide a plethora of subtle applications in optical communications and information processing. Nanolasers have received much interest due to its potential applications in system-on-a-chip technologies [48]. In fact, the nanolaser system shows an enhanced dynamical performance due to a combination of physical factors including the Purcell spontaneous emission enhancement factor and enhanced spontaneous emission coupling. Optical chaos can be generated in nanolasers either by optical injection, conventional or phase conjugate optical feedbacks [48–50]. However, the limitations in size, complexity, and power consumption in a nanophotonic chaos chip renders the use of single nanolaser highly desirable. We therefore come to the not-yet-explored question: “can a single nanolaser emitting a beam of a well-defined polarization produce optical chaos with simultaneous TD signature suppression?”

In this paper, we attempt to answer that question. We first investigate the occurrence of TD feature in chaotic nanolasers exposed to different types of optical feedbacks. In particular, the cases where conventional, phase conjugate, and grating mirror feedbacks are considered. Second, we present a hybrid all-optical/electro-optic feedback scheme and assess its role in TD suppression in all observables. As an application, secure communications system based on a single nanolaser is proposed. The immunity of the presented encryption scheme is assessed against possible statistical, brute force and differential attacks.

The rest of the paper is organized as follows: The effects of different types of external optical feedback is examined in Section 2. The proposed schemes for TD suppression and secure communications are included in Section 3 and Section 4, respectively. Section 5 contains the conclusion.

2. Effects of external optical feedback

It is known that the difficulty of extracting TD information directly from time series results from the complexity of chaotic attractor, so that the existence of TD feature in each case is investigated via statistical means. Among the several statistical techniques which are suitable to quantitatively identify the TD signature in chaotic time series is the autocorrelation function (ACF). This technique has the advantage of being computationally efficient, robust, and immune to white noise [43,51]. The ACF can precisely measure how much a given time series waveform matches its time-shifted version such that the locations of peaks in ACF curve illustrate the presence of TD signature in chaotic output.

Here, the ACF for chaotic intensity and phase time series is defined as follows

\[ C(\Delta t) = \frac{\langle [\rho(t + \Delta t) - \langle \rho(t + \Delta t) \rangle] [\rho(t) - \langle \rho(t) \rangle] \rangle}{\langle [\rho(t + \Delta t) - \langle \rho(t + \Delta t) \rangle]^2 \rangle \langle [\rho(t) - \langle \rho(t) \rangle]^2 \rangle}, \]

where \( \rho(t) \) is either the intensity \( I(t) \) or the phase \( \varphi(t) \), \( \langle \rangle \) denotes the average of time series, and \( \Delta t \) is the time lag which is taken at 25 ps steps in numerical simulations. Now, let’s define \( m \) as being a mismatch coefficient and assume that \( \tau \) is the round trip time to be extracted. Therefore, if the modulus of ACF value in neighborhood of \( \tau \) has obviously larger value compared with contemporary values in other intervals, it reveals the existence of TD signature. To quantify the weakness or strength of TD signature mathematically, a good measure is then the peak signal to mean ratio (PSMR) which is determined as Max \( |C(\Delta t)| / \bar{C}(\Delta t) \), where \( \Delta t \in [\tau(1-m), \tau(1+m)] \). Because the PSMR values of intensity and phase time series can equally be used to unveil the time delay information, both PSMRs are combined in one variable, denoted as combined PSMR. It is reported that TD features are well suppressed when the value of PSMR is less than 4 [47].

Fig. 1 depicts the schematic diagram of the proposed setup. Following is the mathematical model for nanolaser subject solely to external optical feedback (i.e., electrooptic gain \( G = 0 \)) employing conventional mirror (CM), phase conjugate mirror (PCM), or a grating mirror (GM).

2.1. CM optical feedback

Let \( R(t) \), \( \varphi(t) \) and \( N(t) \) denote the photon density, the phase and the carrier density, respectively, all at the time \( t \). The rate equations describe a single semiconductor metal clad nanolaser in this case are expressed as [50]

\[
\frac{dI(t)}{dt} = \gamma \left( \frac{F \beta N(t)}{\tau_p} - \frac{g_{na}(N(t) - N_{na})}{1 + e^{I(t)}} \right) - \frac{1}{\tau_p} I(t) + 2k \sqrt{I(t)} (I(t) - \bar{\varphi}(t)) \cos(\alpha_0 t + \varphi(t) - \varphi(t - \tau)),
\]

\[
\frac{d\varphi(t)}{dt} = \frac{1}{2} \sqrt{I(t)} \left( \frac{N(t)}{I(t)} \right)^{1/2} \left( \frac{I(t) - \bar{\varphi}(t)}{I(t)} \right) \sin(\alpha_0 t + \varphi(t) - \varphi(t - \tau)),
\]

\[
\frac{dN(t)}{dt} = \frac{I(t)}{\tau_n} - \frac{N(t)}{\tau_n} \left( \beta F + 1 - \beta \right) - \frac{g_{na}(N(t) - N_{na})}{1 + e^{I(t)}},
\]

where the rate at which a proportion of laser field is reflected back from the mirror into the nanolaser active region is defined as the feedback.
rate $k$, which is given by

$$k = f(1 - R_e) \sqrt{\frac{R_{ex}}{R_s} \frac{c}{2nL}}. \tag{5}$$

Here, $R_e$ and $R_{ex}$ are the power reflectivities of the front laser facet and external mirror, respectively, $c$ is the speed of light in free space, $n$ is the refractive index, $L$ is the cavity length of nanolaser, and $f$ is feedback coupling fraction i.e. the fraction of the reflected optical field which couples back into the lasing mode. Other parameters of the system are described in Table 1.

### 2.2. PCM feedback

In this type of feedback, the phase conjugate mirror (PCM) reverses the phase of the reflected light, relative to the incident light, such that the net change in external cavity round trip phase is zero. Therefore, any phase shift induced due to the external cavity round trip time is exactly cancelled by the phase shift gained during the return path [48,52]. Several theoretical and experimental studies illustrated that employing PCM results in strong noise reduction compared to CM feedback [52]. Therefore, the transition to chaotic behavior in PCM initiated at larger values of feedback rates than values required for CM case. The mathematical model of a single nanolaser subject to an ideal PCM feedback with zero response time and no frequency change in reflected laser is given by [48]

$$\frac{dI(t)}{dt} = \gamma \left( \frac{F\beta N(t)}{\tau_n} + \frac{g_n(N(t) - N_0)}{1 + cI(t)}I(t) \right) - \frac{1}{\tau_p}I(t) + 2k \sqrt{I(t)I(t - \tau) \cos(\varphi(t) + \varphi(t - \tau))}, \tag{6}$$

$$\frac{d\varphi(t)}{dt} = \frac{a}{2} \sqrt{g_n(N(t) - N_{th})} - k \sqrt{\frac{I(t - \tau)}{I(t)}} \sin(\varphi(t) + \varphi(t - \tau) - \Delta\omega t). \tag{7}$$

$$\frac{dN(t)}{dt} = \frac{J_{dc}}{eV_a} - \frac{N(t)}{\tau_n}(F\beta + 1 - \beta) - \frac{g_n(N(t) - N_0)}{1 + cI(t)}I(t), \tag{8}$$

In this model, $R_{ex}$ is replaced by the PCM reflectivity $R_{pcm}$ in simulation. Also, the arbitrary phase change at PCM is set at zero change.

### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>Mode confinement factor</td>
</tr>
<tr>
<td>$Q$</td>
<td>Mode-quality factor</td>
</tr>
<tr>
<td>$F$</td>
<td>Cavity purcell factor</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Spontaneous emission coupling factor</td>
</tr>
<tr>
<td>$L$</td>
<td>External cavity length</td>
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<tr>
<td>$V_a$</td>
<td>Volume of active region</td>
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<td>$\varepsilon$</td>
<td>Electron charge</td>
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<td>$c$</td>
<td>Gain saturation factor</td>
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<td>Refractive index</td>
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<tr>
<td>$a$</td>
<td>Linewidth enhancement factor</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
</tr>
<tr>
<td>$J_{ex}$</td>
<td>DC bias current</td>
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<tr>
<td>$\lambda_b$</td>
<td>Threshold current</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Photon lifetime ($=\varphi_0/\omega_0$)</td>
</tr>
<tr>
<td>$g_n$</td>
<td>Differential gain</td>
</tr>
<tr>
<td>$N_0$</td>
<td>Transparency carrier density</td>
</tr>
<tr>
<td>$t_{c}$</td>
<td>Carrier lifetime</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>The angular frequency of the laser</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Wavelength</td>
</tr>
<tr>
<td>$D$</td>
<td>Distance from laser facet to external mirror</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>External cavity round trip time ($=2D/c$)</td>
</tr>
<tr>
<td>$\tau_d$</td>
<td>Optical delay time between laser and BS</td>
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<tr>
<td>$\tau_c$</td>
<td>Optical delay time between BS and external mirror</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>Total optical roundtrip time</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>Electronic time delay of electro-optic feedback</td>
</tr>
</tbody>
</table>

### Fig. 2

The intensity time series, in first column, and ACF of single nanolaser chaotic output when CM optical feedback is employed. The second and third columns show ACF of intensity and phase, respectively. The values of feedback coupling fraction are $f = 0.013$ (k = 65.43 GHz) in (a–c), $f = 0.02$ (k = 100 GHz) in (d–f) and $f = 0.031$ (k = 156 GHz) in (g–i).

### 2.3. GM feedback

Grating mirror is originally employed in optical feedback in order to select the oscillation frequencies in SLs or stabilize the laser oscillation. However, it can induce instabilities in SL at certain ranges of the feedback strengths with either small or large values [52]. Assume that $\alpha_0$ and $\Delta\omega = \alpha_0 - \alpha_0$ are the selected angular frequency of the grating and frequency detuning, respectively. For a small detuning and feedback strength, the rate equations of the model in this case are given by [52]:

$$\frac{dI(t)}{dt} = \gamma \left( \frac{F\beta N(t)}{\tau_n} + \frac{g_n(N(t) - N_0)}{1 + cI(t)}I(t) \right) - \frac{1}{\tau_p}I(t) + 2k \sqrt{I(t)I(t - \tau) \cos(\varphi(t) + \varphi(t - \tau) - \Delta\omega t)\cos(\alpha_0 \tau + \varphi(t) - \varphi(t - \tau) + \Delta\omega t)\cos(\alpha_0 \tau + \varphi(t) - \varphi(t - \tau) + \Delta\omega t)}, \tag{9}$$

$$\frac{d\varphi(t)}{dt} = \frac{a}{2} \sqrt{g_n(N(t) - N_{th})} - k \sqrt{\frac{I(t - \tau)}{I(t)}} \sin(\varphi(t) + \varphi(t - \tau) - \Delta\omega t). \tag{10}$$

$$\frac{dN(t)}{dt} = \frac{J_{dc}}{eV_a} - \frac{N(t)}{\tau_n}(F\beta + 1 - \beta) - \frac{g_n(N(t) - N_0)}{1 + cI(t)}I(t). \tag{11}$$

### 2.5. Numerical simulations

The previously described models of single nanolaser system are solved by the fourth order Runge-Kutta method [27] where the infinite dimensional nanolaser system with time delayed feedback is approximated by finite dimensional system. The following values of parameters [48,50]: $\gamma = 0.645$, $Q = 428$, $F = 14$, $\beta = 0.05$, $g_n = 1.64 \times 10^{-6}$ cm$^3$/s, $V_a = 3.96 \times 10^{-13}$ cm$^3$, $\tau_n = 1$ ns, $\tau_p = 0.36$ ps, $n = 3.4$, $\alpha = 5$, $J_{dc} = 21I_{th}$, $I_{th} = 1.1\text{mA}$, $N_0 = 1.1 \times 10^{18}$ cm$^{-3}$, $\epsilon = 2.3 \times 10^{-17}$ cm$^3$, $\lambda_0 = 1.591\mu\text{m}, f \in (0.09), R_s = 0.85, R_{ex} = 0.95. R_{pcm} = R_{ex}, D = 1.5 \text{ cm and } L = 1.39 \mu\text{m.}$

First, the case of CM optical feedback is examined. Fig. 2 illustrates the intensity time series and ACF for intensity and phase of nanolasers system chaotic emission. Three different values of feedback rate are considered. A clear peak in ACF plot are observed at time lag of 0.1 ns, for both observables i.e. the intensity and phase of chaotic output, which is the round trip time given by 2D/c. It is observed that some other peaks in ACF plot exist at multiples of round trip time, in particular, in ACF intensity graph. However, it is obvious that increasing the value of $k$, weaken the TD signature at the multiple values of feedback time.

Second, the case of generating chaos using PCM optical feedback is considered. Fig. 3 shows the results of numerical simulation of the...
Fig. 3. The same as in Fig. 2 for PCM optical feedback. The parameters values are \( f = 0.07 \) (k = 352.3 GHz) in (a–c), \( f = 0.11 \) (k = 553.6 GHz) in (d–f), and \( f = 0.15 \) (k = 755 GHz) in (g–i).

Fig. 4. The same as in Fig. 2 for GM optical feedback. The values of parameters used are \( f = 0.09 \) (k = 453 GHz) in (a–c), \( f = 0.13 \) (k = 654.3 GHz) in (d–f) and \( f = 0.15 \) (k = 755 GHz) in (g–i).

chaotic nanolaser system for different values of feedback gain. The first observation that can be extracted from simulation is the fast dynamics of the laser compared to the previous CM feedback. Also, the same TD signature exists at values of time lag equal feedback delay time and its doubled values. The strength of TD signature is boosted with the increase in feedback gain. The more interesting point is the appearance of more obvious ACF peaks in phase column in contrast to those arise for CM feedback.

Fig. 4 deems the case of GM optical feedback. It is clear that the maxima of ACF graph for phase time series is significantly relaxed for this type of optical feedback. However, the intensity ACF plots still have an obvious TD signature, especially at the value of 0.1 ns time lag.

From previous results, it is demonstrated that the presence of TD signature, at least in one of the two observables, is unavoidable for these different types of optical feedback. The dynamics of the laser light in PCM and GM feedbacks is faster than the dynamics in CM feedback output. The best statistical characteristics feature the GM feedback with its weak signature in phase and moderate TD signature in output intensity.

3. Proposed scheme for TD suppression

The most common techniques of optical chaos generation in previous research works mainly consider either all-optical feedback or electrooptic feedback in their mathematical model of the setup. However, in [53] Hizanidis et al. proposed a cascaded configuration of two types of feedback applied to laser light in order to improve the performance of the system.

The proposed scheme in this work simultaneously integrates both optical and electro-optic feedbacks into the rate equations of the model (see Fig. 1). Then, we study the performance of suggested hybrid-feedback technique for the three cases presented previously for single nanolaser, namely, when CM, PCM and GM are employed in optical feedback. Fig. 1 depicts the schematic diagram of the proposed setup. The rate equations of the optical system equipped with CM feedback are

\[
\frac{dI(t)}{dt} = \gamma \left( \frac{F \beta N(t)}{\tau_a} + \frac{g_a(N(t) - N_0)}{1 + e^{-I(t)}} \right) - \frac{1}{\tau_p} I(t) + 2k \sqrt{I(t)} (t - \tau_a) \cos(\omega_0 \tau + \phi(t) - \phi(t - \tau_a)) - G I(t - 2\tau_a - \tau_e - \tau_o),
\]

where parameters \( \tau_e, \tau_o, \tau_a, \) and \( \tau_o \) are described in Table 1.

Moreover, the rate equations for the case of PCM feedback are given as

\[
\frac{dI(t)}{dt} = \gamma \left( \frac{F \beta N(t)}{\tau_a} + \frac{g_a(N(t) - N_0)}{1 + e^{-I(t)}} \right) - \frac{1}{\tau_p} I(t) + 2k \sqrt{I(t)} (t - \tau_a) \cos(\omega_0 \tau + \phi(t) - \phi(t - \tau_a) - G I(t - 2\tau_a - \tau_e - \tau_o)),
\]

and finally for GM feedback, are

\[
\frac{dI(t)}{dt} = \gamma \left( \frac{F \beta N(t)}{\tau_a} + \frac{g_a(N(t) - N_0)}{1 + e^{-I(t)}} \right) - \frac{1}{\tau_p} I(t) + 2k \sqrt{I(t)} (t - \tau_a) \cos(\omega_0 \tau + \phi(t) - \phi(t - \tau_a) + \Delta \omega t - G I(t - 2\tau_a - \tau_e - \tau_o)),
\]

Numerical simulations are carried out to study the effects of the proposed hybrid feedback techniques on the strength of TD signature. In Figs. 5–7, the combined PSMR (intensity PSMR + phase PSMR) is calculated over a range of controlled system parameters for the three feedback mirrors. In particular, the values of feedback coupling fraction \( f \) along with optical round-trip time are varied. The electrooptic gain \( G \) is chosen such that the maximum induced phase in phase modulator is either (a) \( \pi/4 \), (b) \( \pi/2 \), (c) \( 3\pi/4 \) or (d) \( \pi \) as illustrated in each figure. In computational analysis we take \( \tau_e = (1/4)\tau_o, \tau_a = (3/4)\tau_o \) and \( \tau_e = 1 \) ns.

It is obvious that the TD signature in chaotic output of single nanolaser, subject to all types of optical feedback, can be effectively weakened simultaneously in both intensity and phase time series in some operational regions. This defines the appropriate range of parameters for system operation where the TD is concurrently concealed in all observables.
For example, results indicate that for $G$ corresponding to a maximum induced phase $= \pi/2$, the ranges $R1 = \left( \frac{75}{1000} \leq f \leq \frac{140}{1000}, 2.2 \leq D \leq 2.35 \right)$, $R2 = \left( \frac{140}{1000} \leq f \leq \frac{152}{1000}, 2.1 \leq D \leq 2.2 \right)$ and $R3 = \left( \frac{152}{1000} \leq f \leq \frac{170}{1000}, 2.4 \leq D \leq 2.55 \right)$ are appropriate for TD concealment when hybrid CM-Electrooptic feedbacks, PCM-Electrooptic feedbacks and GM-Electrooptic feedbacks are involved, respectively. Fig. 8 shows the intensity time series and ACFs of intensity and phase computed in appropriate TD-concealed regions for each configuration of optical feedback.

It can be noticed that the influence of $G$ on the PSMR values differ from one configuration scheme to another. For the hybrid CM-Electrooptic feedback, the values of $G$ that yield a maximum value of $\pi/2$ or $\pi$ in phase modulator is more appropriate whereas a maximum value of $\pi/4$ or $\pi$ of chaotic phase modulation is suitable for PCM-Electrooptic feedback. The better TD signature suppression achieved for GM-Electrooptic feedback is carried out at maximum phase modulation of $\pi/4$ and $\pi$ values.

Finally, the complexity of chaotic attractor can be quantified using Lyapunov exponents which measures the divergence of initially very close solution trajectories in phase space of the model, and therefore the degree of sensitive to initial conditions can be evaluated. More specifically, chaotic attractors should have at least one positive Lyapunov exponent as a proof for presence of deterministic chaos.

In addition, the degree of randomness and unpredictability of chaotic output can also be determined from the values of maximum Lyapunov exponents (MLE) where large positive values imply more randomness and unpredictability. In Fig. 9, the MLEs are computed for the optical system using the method presented in [54]. They are then tracked versus changes in feedback coupling fraction for all cases of optical feedback and also for the proposed hybrid-feedback setup. For the proposed scheme, the MLE value is obtained from the output intensity and phase time series when the maximum phase actuated by the phase modulator is $\pi/2$. Here, the MLEs of the conventional all-optical schemes are so close to that of the proposed hybrid scheme which verifies that the complexity of chaotic emission is not significantly affected in the proposed setup. Results show that all proposed schemes induce chaotic output for the range of feedback coupling used. However, the hybrid electrooptic-PCM optical feedback exhibits larger values of MLE than the other two types of optical feedback and then it shows more randomness and unpredictability in its output.
tance of this step can be understood by recalling that any eavesdropper, with an access to input and output of the encrypting system, can apply a sequence of inputs consist of the same plain image but differ only in one pixel each time. Then, the eavesdropper who employs the differential attacks can, at least in principle, break the encryption process, provided that parameters in the system do not change against the sequence of inputs. Therefore, the first step renders the system more immune against different types of differential attacks and ensures that any tiny perturbation in the plain image will cause a significant difference in the cipher image.

Assume that the feedback coupling fraction \( f \) is chosen as controlled parameter in transmitter system. Then it will be modified according to the following updating rule

\[
f = \tilde{f} + \frac{c}{M \times N} \left( \sum_{i=1}^{3} \sum_{j=1}^{N} P_{ij} \right)
\]

where plain image has dimension of \( M \times N \) pixels with three values assigned to each pixel, denoted as \( P_{ij} \in \{0, 1, 2, 3\} \). \( P \) for \( 1, 2, 3 \) correspond to the basic three colors; red, green and blue, respectively. Also, \( q_i \) work as a weight of the color \( i \), \( c \) is a constant which is carefully chosen such that the modified value of \( f \) lies within appropriate interval of \( f \) values with center at \( \tilde{f} \), see Figs. 5–7.

The center value \( f \) will be kept secret at both sides whereas the perturbing value is computed at the transmitter side and sent to receiver before encryption process.

2) A message \( m(t) \) is encoded into the chaotic emission of the transmitter nanolaser via additive chaos modulation. After beam splitting, a portion of the chaos-perturbed message is directed to the electrooptic feedback of the transmitter and the other is sent over the public channel to the receiver where a similar electrooptic feedback is fed by an identical portion. Symmetrical signal is therefore fed back to the nanolasers of the transmitter and receiver. The identical nanolasers used at both terminals verify their outputs are synchronized to each other and operate as symmetrical reference signals.

Mathematically, the rate equations of receiver system equipped with CM feedback are

\[
dI(t) = \gamma \left( \frac{F \beta N(t)}{\tau_p} + \frac{g_\beta (N(t) - N_0)}{1 + cI(t)} \right) - \frac{1}{\tau_p} I(t) + \frac{k \sqrt{I(t)I_f(t - \tau_p)}}{I(t)} \\
\times \cos(\omega_0 t + \phi_f(t) - \phi_f(t - \tau_p) - G I_f(t - 2 \tau_{\alpha1} - \tau_{\alpha2} - \tau_p)).
\]

\[
d\phi(t) = \frac{g}{2} \left( \frac{g_\beta (N(t) - N_0)}{1 + cI(t)} \right) - \frac{k \sqrt{I(t)I_f(t - \tau_p)}}{I(t)} \\
\times \sin(\omega_0 t + \phi_f(t) - \phi_f(t - \tau_p) - G I_f(t - 2 \tau_{\alpha1} - \tau_{\alpha2} - \tau_p)),
\]

\[
dN(t) = \frac{I_{dc}}{\tau_m} - \frac{N(t)}{\tau_m} (F \beta + 1 - \beta) - \frac{g_\beta (N(t) - N_0)}{1 + cI(t)} I(t),
\]

whereas the rate equations for PCM feedback are given as

\[
dI(t) = \gamma \left( \frac{F \beta N(t)}{\tau_p} + \frac{g_\beta (N(t) - N_0)}{1 + cI(t)} \right) - \frac{1}{\tau_p} I(t) + \frac{k \sqrt{I(t)I_f(t - \tau_p)}}{I(t)} \\
\times \cos(\phi_f(t) + \phi_f(t - \tau_p) - G I(t - 2 \tau_{\alpha1} - \tau_{\alpha2} - \tau_p)),
\]

\[
d\phi(t) = \frac{g}{2} \left( \frac{g_\beta (N(t) - N_0)}{1 + cI(t)} \right) - \frac{k \sqrt{I(t)I_f(t - \tau_p)}}{I(t)} \\
\times \sin(\phi_f(t) + \phi_f(t - \tau_p) - G I(t - 2 \tau_{\alpha1} - \tau_{\alpha2} - \tau_p)),
\]

\[
dN(t) = \frac{I_{dc}}{\tau_m} - \frac{N(t)}{\tau_m} (F \beta + 1 - \beta) - \frac{g_\beta (N(t) - N_0)}{1 + cI(t)} I(t),
\]

4. Application

The chaos-based image encryption techniques have witnessed a great interest, in terms of implementation and crypto-analysis during the last two decades (see 55–59 and references therein). In this section, we exploit the main advantages of optical chaos over the traditional chaos generated by electronic circuits. In particular, an optical chaos encryption scheme based on a single nanolaser is investigated for secure transmission of colored images. The schematic diagram of the complete secure communication system is shown in Fig. 10. The key steps in the encryption procedure are summarized as follows:

1) The proposed system is prepared to conduct the encryption process. The aim of this step is to perturb the values which are initially allocated to one or more of parameters that can be controlled in the system. In other words, the values of some selected parameters in secure communications system are not kept fixed and will be reassigned for each plain image undergoes encryption process.

The feedback coupling fraction and optical feedback time delay are examples of parameters that can be controlled and updated. The values of perturbations applied to controlled parameters are set up depending on the features extracted from pixel values in plain image. The import
and finally, for GM feedback they are

\[
\frac{d I(t)}{d t} = \gamma \left( F \beta N(t) + \frac{\epsilon N(t) - \epsilon N_0}{1 + \epsilon I(t)} I(t) \right) - \frac{1}{\tau_p} I(t) + 2k \sqrt{I(t)I(t - \tau_a)} \\
\times \cos(\alpha \tau + \varphi(t) - \varphi(t - \tau) + \Delta \omega t - G I(t - 2\tau_{a1} - \tau_{a2} - \tau_a)),
\]

(28)

\[
\frac{d \varphi(t)}{d t} = \frac{\alpha}{2} \gamma \epsilon N(t - \tau_{ah}) - k \frac{I(t - \tau_a)}{I(t)} \\
\times \sin(\alpha \tau + \varphi(t) - \varphi(t - \tau) + \Delta \omega t - G I(t - 2\tau_{a1} - \tau_{a2} - \tau_a)),
\]

(29)

\[
\frac{d N(t)}{d t} = \frac{J_{dc}}{\epsilon \Psi_a} - \frac{\epsilon N(t - \tau_{ah}) - \epsilon N_0}{1 + \epsilon I(t)} I(t),
\]

(30)

where the subscript \( t \) refers to transmitter system.

3) The given colored plain image of dimension \( M \times N \) is converted to a sequence of data bits. More specifically, for each pixel there are 24 bits which are allocated to its three color codes in sequential order. The pixels are taken in row by row order such that they give rise to a stream of \( 24 \times M \times N \) bits length. Then the information message is transformed onto an optical form.

4) The optical chaos generated from transmitter is employed as a carrier and a mask for hiding the message information. Notice that the amplitude of message bits is set smaller than the amplitude of chaotic carrier for effective concealment of information bits [27].

5) At the receiver side, the original information signal can be recovered by subtraction the chaotic output of the nanolaser from the received signal. Therefore, the plain image can be restored from decoded message after rearranging the pixels at their original order.

Fig. 11 shows a simulation example where a \( 512 \times 512 \) color image is taken as plain image. The original, encrypted and decrypted images are illustrated in Fig. 11, for the case of GM feedback at for \( f = 120 \), \( q_i = 0.5 \), \( i = 1, 2, 3 \), \( \epsilon = 1.54 \times 10^{-5} \) and \( D = 2.27 \) cm. Furthermore, the performance aspects of encryption scheme against possible attacks are checked as follows.

First, the performance of the system against statistical attacks is studied using image histogram analysis. The histogram is a tool to visualize the distribution of pixel intensity values within a given image. A good encryption scheme must render the encrypted image possess a uniform histogram in order to better oppose any statistical attacks. Fig. 12 illustrates the histograms, corresponding to each color scale in image’s pixels, for original, encrypted and decrypted images. It can be demonstrated that no features can be deduced through statistical attack on the encrypted image.

Second, regarding to the resistance against brute force attack, a good encryption technique should have a very large number of secret keys that can be utilized in encryption process. The controlled parameters \( f, c, \tau_{a1}, \tau_{a2}, \tau_a \) and \( \tau_{ah} \) extend over continuous ranges of suitable intervals that can be chosen during system operation. Therefore, they provide an enormous number of choices that constitute a large key space which is limited only by precision of equipment controlling the values of parameters. The secret keys sensitivity is highlighted in Fig. 13 where the value of \( c \) is slightly perturbed by adding \( 10^{-8} \) to its original value while other parameters of the system are fixed.

Third, it is known that high correlations, in horizontal, vertical and diagonal directions, occur for adjacent pixels in plain images. So, the successful secure communications system should obviously reduce the correlation among adjacent pixels in all directions. In our example, a randomly chosen \( 10^4 \) pairs of adjacent pixels are chosen in original and encrypted images. Then, the correlation coefficients for each group of adjacent pixels are evaluated for each color component of the pixels from the following relation

\[
R_p = \frac{\sum_{p=1}^{1000} (x_p^i - \overline{x_p})(y_p^i - \overline{y_p})}{\sqrt{\left(\sum_{p=1}^{1000} (x_p^i - \overline{x_p})^2\right)\left(\sum_{p=1}^{1000} (y_p^i - \overline{y_p})^2\right)}},
\]

(31)

\( p = 1, 2, 3 \)

where \( x_p^i \) and \( y_p^i \) refer to values of color \( p \) component of a pair \( i \) of pixels, \( \overline{x_p} \) and \( \overline{y_p} \) are the average intensity of color \( p \) through the chosen \( 10^4 \) pairs, and \( S \) refers to the direction which is considered for adjacent pixels. In particular, \( S = H, V, D \) denote horizontal, vertical and diagonal directions, respectively. The correlation coefficients in different directions are presented in Table 1 for plain image and in Table 2 for cipher image. It is obvious that correlation coefficients have been significantly reduced in encrypted image.

Finally, the immunity of the system against differential attacks is explored. More specifically, a slight modification of only one color component of a randomly chosen single pixel is applied on plain image. Then, a comparison between the new cipher image and the old cipher image
of upfertilized original image is made. In order to quantify the degree of convergence or divergence between two cipher images, the two measurements namely the unified average changing intensity (UACI) and the number of pixels change rate (NPCR) are used. The UACI for given two images measures the mean intensity of differences between particular color components in two images whereas NPCR gives the percentage of different pixel intensities between two images. They are defined by

$$UACIP = \frac{1}{M \times N} \sum_{i,j} \left( P_{ij}^p - Q_{ij}^p \right)^2$$

$$NPCRP = \frac{1}{M \times N} \sum_{i,j} B_{ij}^p \times 100,$$

where

$$B_{ij}^p = \begin{cases} 1 & P_{ij}^p = Q_{ij}^p \\ 0 & P_{ij}^p \neq Q_{ij}^p \\ i, j = 1, 2, 3 \end{cases}$$

The values of UACI and NPCR are given in Table 3. It is demonstrated that the present secure communication system results in good values for UACI and NPCR, see [55–59] and references therein.

5. Conclusion

We have addressed the problem of TD signature in intensity and phase of chaotic single nanolaser system subject to various feedback types. The proposed hybrid optical/electrooptic feedback scheme can suppress the TD signature concurrently in both intensity and phase of the chaotic emission at particular operation regions. Further, the hybrid feedback is found to preserve the complexity of dynamical behavior of chaotic nanolaser and to maintain the unpredictability and randomness in the optical chaos emission. A nanolaser-based secure communication system is proposed. The presented encryption scheme is verified to ensure security against possible statistical, brute force and differential attacks.

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