Performance Evaluation of Hybrid DPSK-MPPM Techniques in Long-Haul Optical Transmission

**ABDULAZIZ E. EL-FIQI**\(^1,2,6,*\), **AHMED E. MORRA**\(^2,4\), **SALEM F. HEGAZY**\(^3,4\), **HOSSAM M. H. SHALABY**\(^1,5\), **SALAH S. A. OBAYYA**\(^4\), AND **KAZUTOSHI KATO**\(^6\)

\(^1\)Electronics and Communications Engineering Department, Egypt-Japan University of Science and Technology (E-JUST), Alexandria 21934, Egypt
\(^2\)Electronics and Electrical Communications Engineering Department, Faculty of Electronic Engineering, Menoufia University, Menouf 32952, Egypt
\(^3\)National Institute of Laser Enhanced Sciences, Cairo University, Giza 12613, Egypt
\(^4\)Center for Photonics and Smart Materials, Zewail City of Science and Technology, Giza 12588, Egypt
\(^5\)Electrical Engineering Department, Faculty of Engineering, Alexandria University, Alexandria 21544, Egypt
\(^6\)Graduate School of Information Science and Electrical Engineering, Kyushu University, Fukuoka 819-0395, Japan

*Corresponding author: abdulaziz.elfiqi@ejust.edu.eg

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In this paper, we evaluate the performance of hybrid differential phase shift keying-multipulse pulse position modulation (DPSK-MPPM) techniques in long-haul nonlinear-dispersive optical fiber channel. An expression for the nonlinear interaction variance is formulated analytically using the Gaussian noise (GN)-model. We drive upper-bound expressions, that include the fiber nonlinearity impact on the DPSK-MPPM system performance, for both bit- and symbol-error rates. The tightness of the bit-error rate upper bound is verified using a Monte Carlo simulation. To run the numerical analysis, we propose experimental and simulation setups for the hybrid DPSK-MPPM long-haul transmission system. Our results reveal that the hybrid DPSK-MPPM technique outperforms both traditional DPSK and MPPM techniques under amplified spontaneous-emission (ASE) noise (nonlinearity free) limit, yet it is less robust when fiber nonlinearity is considered. However, its performance surpasses them in both linear and forward-error correction (FEC)-limit nonlinear regions. Furthermore, we discuss the effect of some wavelength-division multiplexing (WDM) parameters on the optimal system performance. The nonlinear interference penalties on the maximum reachable distances by both hybrid and traditional modulation systems are also investigated at FEC limits. In particular, at an average launch power of -8 dBm, hybrid DQPSK-MPPM system with \(M = 8\) and \(n = 2\) outreaches traditional DQPSK system by 950 km.

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1. INTRODUCTION

Hybrid modulation techniques improve the poor power efficiency of high order modulation techniques, which represent a window to avoid the optical capacity crunch in single mode fibers (SMFs) [1]. In fact, these hybrid techniques can be used to increase the receiver sensitivity at a given bit-error rate (BER) [2]. Recently, several hybrid modulation techniques have been investigated for long-haul optical transmission [2–4]. For instance, Liu et al. demonstrated the transmission of a 6.23 Gbit/s PQ-4PPM (polarized multiplexed-quadrature phase shift keying in combination with 4-ary pulse position modulation) signal over a 370 km unrepeated ultra-large-area-fiber span with a total loss budget of 71.7 dB [2, 3]. Furthermore, Sjödin et al. carried out the first experimental realization of hybrid polarized multiplexed-2PPM-quadrature phase shift keying (PM-2PPM-QPSK) modulation over a long-haul transmission distance up to 13,000 km with a data rate of 42.8 Gbit/s [4].

To the best of our knowledge, the analytical evaluation of the performance of hybrid modulation techniques in long-haul nonlinear-dispersive optical channels has not been addressed yet. In [5], we introduced a hybrid differential phase shift keying-multipulse pulse position modulation (DPSK-MPPM) technique that is based on spectrally-efficient direct-detection differential binary/quadrature phase shift keying (DD-DBPSK/DD-DQPSK) techniques along with an energy-efficient multipulse pulse position modulation (MPPM) technique. The performances of these hybrid modulation schemes were evaluated under an optical amplifier noise limit, specifically amplified spontaneous-emission (ASE) noise [5]. However, in long-haul optical transmission, the nonlinearity impact becomes a significant limit of the overall system performance [6]. There is a plenty of approaches...
that have been proposed for modeling the effect of fiber nonlinearity [7–12]. One of the most reasonable models for addressing these nonlinear impacts in SMFs is the Gaussian noise model (GN-model). Such approach is four-wave-mixing (FWM)-based, which models the nonlinear interference as an additive Gaussian noise that is statistically independent from both the amplifier noise and the transmitted signal [13]. The validation of this approach has been assured over a wide range of system scenarios [9].

In this paper, we adopt the GN-model to address the effect of fiber nonlinearity on the performance of hybrid DPSK-MPPM systems. An analytical expression for the total noise variance is formulated for the DPSK-MPPM technique in nonlinear-dispersive optical channel. Both BER and SER expressions are developed to include the effect of fiber nonlinearity on the performance of DPSK-MPPM systems. Numerical evaluations are then carried out based on the proposed experimental and simulation scenarios for the DPSK-MPPM schemes. The results are compared to that of traditional DBPSK, DQPSK, and MPPM schemes. Furthermore, we discuss the effects of some WDM parameters on the optimal system performance. Finally, the nonlinear interference penalties on the maximum reachable distances by both hybrid and traditional modulation systems are investigated at FEC limits.

The rest of the paper is organized as follows. In Section 2, the GN-model specifications are highlighted. We also formulate an expression for the total nonlinear variance of the long-haul DPSK-MPPM optical transmission system in the same section. In addition, we develop expressions, which consider fiber nonlinearity impacts, of the SERs and BERs for DPSK-MPPM systems. In Section 3, experimental and simulation setups are introduced in order to numerical study the performance of the hybrid DPSK-MPPM long-haul transmission system. Section 4 is devoted for the performance evaluation and comparison of both hybrid and traditional systems in long haul transmission. Finally, the conclusion is given in Section 5.

2. NONLINEAR MODEL OF DPSK-MPPM SYSTEM

The effect of fiber nonlinearity in long-haul optical transmission is addressed by the GN-model [13]. Based on the model assumptions, the nonlinear frequency interaction is statistically independent from both the transmitted signal and other system noise [14]. In this model, the total system noise variance can be expressed as:

\[ \sigma^2 = \sigma_n^2 + \sigma_{nl}^2 \]  

(1)

where \( \sigma_n^2 \) is the complex optical amplifier noise variance and \( \sigma_{nl}^2 \) is the nonlinear interference variance of the complex transmitted signal. In the case of erbium-doped fiber amplifiers (EDFAs), we have \( \sigma_n^2 = (G - 1)\chi_{hv}B_n \), where \( B_n \) is the noise bandwidth, \( F \) is the amplifier noise factor, \( h \) is Planck’s constant, \( v \) is the center channel frequency, and \( G \) is the amplifier gain. Based on the GN-model, the complex transmitted signal should be spectrally sliced as a wavelength-division multiplexed (WDM) signal with \( N_{ch} \) channels. In other words, \( N_{ch} \) of DPSK-MPPM transmitted signals are wavelength-division multiplexed at the transmitter. Then, a long-haul nonlinear-dispersive SMF is used as the transmission channel. The channel is composed of \( N_s \) spans, each span is followed by an EDFA. At the receiver side, the \( N_{ch} \) modulated signals are wavelength-division demultiplexed and processed by separate DPSK-MPPM demodulators as shown in Fig. 1.

A. Nonlinear Interference Variance

The nonlinear interference variance \( \sigma_{nl}^2 \) is formulated from the nonlinear power spectral density (PSD) of single span system at a specific frequency \( f \) and distance \( z \) [7]:

\[ S_{nl}(f,z) = 2 \left( \frac{8}{27} \right) \gamma^2 \int_{-\infty}^{\infty} \left( H(f_1 + f_2 - f) \right)^2 \left( \frac{1 - e^{-2(2a-j4\pi|B_n|f_2)/2\gamma}}{2\gamma} \right)^2 \right] df_1 df_2, \]

(2)

where \( \delta_n \) is Kronecker delta function with \( p = 1 \) or \( p = 2 \) for single polarized- or dual polarized-multiplexed transmission, respectively. \( \beta_2 \) is the group-velocity dispersion (GVD), \( \alpha \) is the attenuation coefficient for SMF, and \( \gamma = 2\pi f_2/\lambda A_{eff} \) is the fiber nonlinearity coefficient, where \( A_{eff} \) is the core effective area, \( \lambda \) is the propagated wavelength, and \( \beta_2 \) is the nonlinear-index coefficient. \( f_1, f_2, \) and \( f_1 + f_2 - f \) are the pump frequencies of the FWM process that create the nonlinear interference signal at a frequency \( f \). \( H(f) \) is the transmitted filtered signal shape which is assumed to be flat as \( H(f) = \sqrt{\frac{MP_{tx}}{\pi B_n}} \text{rect}(f), \) where \( P_{tx} \) is the average launch power, \( B_{ch} \) is the channel bandwidth, which is equal to the sampling rate \( R_s \) at the Nyquist limit, \( M \) is the total number of slots per frame, and \( n \) is the number of active signal slots per frame [5].

We obtain a closed-form expression for the nonlinear interference variance \( \sigma_{nl}^2 \) through approximating the spectral bands into a square integration area with a side length of \( \sqrt{3B_n}/2, \) where \( B_n = B_g N_{ch} \) is the total WDM bandwidth with \( N_{ch} \) channels. This approximation is verified to be close to the exact integral evaluation [15]. Using the identities in [16], an analytical expression for the nonlinear interference variance is obtained through the integration of its power spectral density (PSD) over the optical noise bandwidth \( B_{n} \):

\[ e_{nl}^2 = \left( \frac{8}{27} \right) \delta_2 \gamma^2 \int_{-\infty}^{\infty} \left( \frac{MP_{tx}}{\pi B_{ch} N_{ch}} \right)^3 B_{n} N_{ch} \times \text{arcsinh} \left( \frac{3}{8 \gamma^2 L_{eff,a} |B_n| |B_n|} \right) \]

(3)

where \( L_{eff} = (1 - e^{-2\alpha L_s})/2\alpha \) and \( L_{eff,a} = 1/2\alpha \) are the effective and asymmetric-effective fiber lengths, respectively, for a fiber with a physical length \( L_s \) and an attenuation coefficient \( \alpha \).
that $r$ out of $n$ signal slots have this minimum power value, while the rest $n - r$ signal slots have power values higher than $P_{\text{min}}$. In this case, the symbol error arises when $s$ non-signal slots have a power value that reaches $P_{\text{min}}$ or above, while the rest $M - n - s$ non-signal slots have power values below $P_{\text{min}}$. Here, $s$ denotes the number of non-signal slots with power values greater than or equal to $P_{\text{min}}$. Therefore, the symbol error can be upper bounded as:

$$
\text{SER}_{\text{MPPM}} = \sum_{s=1}^{M-n} \sum_{r=0}^{n-s} \int_{0}^{\infty} \left( \frac{n}{r} \right) \text{BER}_{\text{DPSK}}(\xi) \left[ 1 - \text{BER}_{\text{DPSK}}(\xi) \right]^{n-r} 
$$

$$
\left\{ \left[ 1 - P_{0}\left(\frac{\sigma^{2}}{\sigma_{n}^{2}}\xi\right) \right]^{s} + \left[ P_{0}\left(\frac{\sigma^{2}}{\sigma_{n}^{2}}\xi\right) \right]^{s} \left[ 1 - \left(\frac{1}{r+1}\right)^{r} \right] \right\}^{M-n-s}$$

$$
\left[ P_{0}\left(\frac{\sigma^{2}}{\sigma_{n}^{2}}\xi\right) \right]^{s} \left[ p_{1}(\xi) \right]^{r} d\xi, 
$$

where $\xi = 2B_{n}P_{\text{min}}/B_{\text{ch}}\sigma^{2}$ and $p_{1}(\cdot)$ refers to the probability-density function (pdf) of the power in a signal slot, which follows a noncentral chi-squared $\chi^{2}$ distribution with noncentrality parameter equals to the peak power. Similarly, $p_{0}(\cdot)$ refers to the pdf of the power in a non-signal slot, which follows a $\chi^{2}$ distribution. In addition, $P_{0}(\cdot)$ and $p_{1}(\cdot)$ are the cumulative distributions corresponding to $p_{0}$ and $p_{1}$, respectively.

C. BER of DPSK-MPPM Systems under Nonlinear Effect

An upper bound on the BER of DPSK-MPPM techniques is expressed at any $q$-DPSK modulation level as [5]:

$$
\text{BER} \leq \frac{1}{N + n q} \left\{ \text{SER}_{\text{MPPM}} \left[ \frac{N2^{N}}{2(2^{N} - 1)} + \frac{q n}{2} \right] + q \left(1 - \text{SER}_{\text{MPPM}} \left[ \frac{1}{2} - \text{BER}_{\text{DPSK}} \right] \right) \times \text{SER}_{\text{MPPM}} + n q \text{BER}_{\text{DPSK}} \right\},
$$

where $\text{SER}_{\text{MPPM}}$ is the symbol-error rate of MPPM data bits, $\text{BER}_{\text{DPSK}}$ is the bit-error rate of DBPSK or DQPSK data bits on top of current MPPM frame. Also, $(N + n q)$ denotes the total number of transmitted bits per frame, where $N = \left\lfloor \log_{2}\left(\frac{M}{n}\right) \right\rfloor$ is the number of bits encoded using MPPM scheme, while $q$ is the DPSK modulation level, i.e., $q = 1$ for DBPSK and $q = 2$ in case of DQPSK.

BER expressions for both DBPSK and DQPSK can be extended in order to take the nonlinearity impacts as follows:

$$
\text{BER}_{\text{DBPSK}} = \frac{1}{2} \frac{MB_{n}P_{t}x}{8nB_{p}c^{2}\sigma^{2}x^{2}} \exp\left( -\frac{MB_{n}P_{t}x}{nB_{p}c^{2}\sigma^{2}x^{2}} \right),
$$

$$
\text{BER}_{\text{DQPSK}} = Q(e_{+} + e_{-}) + \frac{1}{2} \exp\left( -\frac{e_{+}^{2} + e_{-}^{2}}{2} \right) \left[ \frac{(e_{+}^{2} - e_{-}^{2})I_{1}(e_{+} + e_{-})}{4e_{+}e_{-}} - I_{0}(e_{+} + e_{-}) \right],
$$

where $e_{\pm} = \sqrt{\frac{MB_{n}P_{t}(1 \pm \sqrt{1/2})}{nB_{p}c^{2}}}$. $I_{\nu}(\cdot)$ is the $\nu$th order modified Bessel function of the first kind, and $Q(\cdot, \cdot)$ is the Marcum Q function.

3. EXPERIMENTAL AND SIMULATION SETUPS

In this section, we describe in details the proposed experimental and simulation setups that we adopt to run the numerical analysis of the hybrid DPSK-MPPM long-haul transmission system.

A. Transmitter Side

The experimental schematic for the transmitter is depicted in Fig. 2. A coherent laser source emits single-mode light pulses at a rate $1/T$, each pulse has a period $n T$, where $T$ and $r$ are frame and slot time durations, respectively. The pulsed laser emission has a coherence time longer than $2T$, thereby allowing phase information to be reserved among two sequential pulses. A straightforward approach to implement this coherent pulsed laser source is to act on the emission of a coherent continuous-wave diode laser by a fast optical switch. A subsequent traditional DPSK modulator encodes $q n$ phase bits per pulse.

The $q n$ bits modulated light pulse is fed to the MPPM modulator, which consists of a $K$-stages ultra-fast discrete delay line capable of applying up to $2^{K}$ discrete delay steps [5], where $L$ is the shortest length of polarization-maintaining single-mode (P FSM) fiber per delay stages. This discrete delay line modulates the position of each phase-modulated time-slot pulse into one of $M$ locations, where the number of available slot positions per frame $M \leq 2^{K}$. The synchronized electro-optic polarization switching within the delay line precisely chops the phase-modulated pulse into $n$ pulses. Both DPSK and MPPM modulators are controlled via the transmitter signal-processing unit (T-SPU), which is synchronized to the pulsed laser source, thereby carrying out the precise timing required by the hybrid DPSK-MPPM modulation. The input data of T-SPU is thus $\left[ \log_{2}\left(\frac{M}{n}\right) \right]$ bits. While DPSK data is forwarded by the T-SPU directly to the DPSK modulator, the MPPM data is manipulated first by the T-SPU to produce the $K$ delay controls. In addition, a subsequent control is used to unify the polarization of the delay output regardless of the introduced delay.

On the other hand, the simulation scenario is described in Fig. 3. The first $N = \left[ \log_{2}\left(\frac{M}{n}\right) \right]$ bits are encoded using MPPM scheme. Each MPPM optical pulse is then DPSK modulated using additional $q$ bits. The optical fiber is simulated as a nonlinear-dispersive channel. That is, complex noise is added to the transmitted signal as follows: both nonlinearity and ASE noise sources are added to the symbol signal slots (i.e., $\sigma^{2} = \sigma_{n}^{2} + \sigma_{c}^{2}$), whereas only ASE noise $\sigma_{c}^{2}$ is added to the symbol non-signal slots.

B. Receiver Side

At the receiver side, the received signal is split into two distinct arms, MPPM and DPSK receivers as shown in Fig. 2. In order to decode the phase information, the positions of the active $n$ slots should be defined first. A photodetector on the MPPM (upper) arm listens to the optical intensity along the frame period and feeds a subsequent $Z$-bits analog-to-digital converter (ADC). The ADC and its memory storage are triggered by the edge of time slot clock generated by the receiver signal-processing unit (R-SPU). This clock is precisely synchronized to the transmitter’s pulses by means of a reference non-encoded frame that is sent every while. The memory storage records the signal intensity within each time slot whether occupied or not. The R-SPU runs some comparison routine on the $M$ stored values resulting in a soft decision regarding the position of the most occupied $n$ time slots. On the DPSK arm, a two-frame delay holds the processing of the phase information until the best decision regarding the active $n$ time slots positions is made by the R-SPU. The R-SPU, knowing about the positions of the time slots, can adapt the receiver delay line to act by the complementary delay made by the transmitter for each signal slot, thereby realocating them in contiguous time slots.
The phase encoded signal is then split among two Mach-Zehnder interferometers (MZIs) whose unbalanced arms differ precisely by the time-slot period \( \tau \), while one of them involves \( \pi/2 \) phase shift between its two arms. Although only one MZI is sufficient for DBPSK decoding, the two MZIs are needed to run the phase compensation process as will be discussed later. The R-SPU eventually encapsulates the DPSK bits along with the MPPM bits, recovering back the completely sent frame data.

It is worth noticing that a receiver equipped with \( K \)-stages discrete delay line matched to that at the transmitter will certainly suffer different delay-induced phase shape (ideally matched delay line, however, can perfectly compensate for the delay-induced phase). In order to compensate this phase perturbation, the transmitter and receiver run an initial reconciliation routine as follows. The transmitter sends a training sequence, which has supposedly no phase information. This frame has \( M \) contiguous signal slots with the first slot traversing the fastest path along the PMSM fiber of all stages, while each following signal slot trains one of the delay steps in order. This training sequence is thus chopped owing to the different delay of each signal slot while conveying solely the phase accumulated by the delay line of the transmitter for each delay possibility. The R-SPU acts using the receiver delay-line on each frame slot by a delay value complementary to that of the transmitter, thereby recombining the signal slots. Then, using the two MZIs, the receiver measures the relative phase-accumulated (due to the delay lines of the transmitter and receiver) within each slot compared with the preceding one. The R-SPU then stores the absolute phase value corresponding to each delay step to be able to compensate for the delay-induced phase.

It should be remarked that the received signal in simulation setup in Fig. 3 is processed in a similar way as the experimental setup.

4. NUMERICAL RESULTS

Here we numerically study the performance of DPSK-MPPM modulation techniques in long-haul transmission. We assume a standard SMF with \( n = 0.22 \) dB/km, \( D = 16.7 \) ps/km·nm, and \( \gamma = 1.3 \) W⁻¹·km⁻¹. An EDFA with gain \( G = e^{2\alpha L_n} \), that compensates the fiber span loss, and noise figure 6 dB. The WDM specifications are: \( B_{ch} = R_S = 32 \) GHz and \( B_n = 12.48 \) GHz (0.1 nm is the reference resolution for optical signal-to-noise ratio (OSNR) calculation). The total fiber length is 1000 km with 100 km span length [14]. It should be mentioned that the quantization error of the ADC is ignored in our numerical evaluation.

Figure 4 shows the BERs versus average launch optical power per channel for different single polarized single channel DPSK-MPPM systems using both analytical expression and Monte Carlo simulation based on the schematic scenario in Fig. 3. The bits-to-symbol mapping and demapping of MPPM \((M = 4\) and \(n = 2\) and \(M = 8\) and \(n = 3\)) are carried out as in [19]. According to the setup in Fig. 2, these arrangements require a discrete delay line with \( K \geq \log_2 M \). That is \( K \geq 2 \) or 3 for \( M = 4 \) or 8, respectively. It can be noticed that for different DPSK-MPPM systems, the theoretical results are very tight to the simulation results, which justifies that our upper bound BER is very close to the exact BER expressions. The hybrid system performance is enhanced by increasing the launch power in the nonlinearity-free region, under the effect of the ASE noise only. By increasing the launch optical power, the nonlinearity becomes significant and results in the degradation of the hybrid system performance. It should be noticed that the optimal performance is achieved at an average launch power [20]:

\[
P_{\text{opt}} = 1.5^{0.2e} \frac{n}{M} \left| \frac{\pi L_{\text{eff}}, \beta_2}{2 \gamma^{1/2} \text{arcsinh} \left( \frac{\pi^2 L_{\text{eff}}, \beta_2}{2 \gamma^{1/2}} \right)} \right|
\]

This optimal average launch power (achieving the minimum BER) is
Fig. 4. BER versus average launch optical power per channel for both hybrid DBPSK-MPPM and DQPSK-MPPM systems: Analytical (solid) and simulation (dashed). The system has $N_s = 20$ with 100 km span length.

directly proportional to the active $n$ time slots and inversely proportional to the number of time slots per frame $M$. Indeed, it can be seen from Fig. 4 that as the ratio of $M/n$ increases, the optimal performance occurs at lower average power values.

Fig. 5. BER versus average launch optical power per channel for both hybrid DBPSK-MPPM and traditional MPPM systems, including nonlinearity-free limited case (dashed).

Figures 5 and 6 depict the BERs versus average launch optical power per channel for single channel hybrid DBPSK-MPPM systems compared with corresponding MPPM and DBPSK systems, respectively, for dual polarized-multiplexed transmission. Both $M$ and $n$ are chosen so as to ensure that all systems under comparison have same transmission data rate. In linear operation (nonlinearity-free limit case), the hybrid system performance is improved by increasing $M$ as the energy efficiency of the system is improved. Specifically, at BER = $10^{-3}$ (FEC requirement), there is an improvement of about 2.5 dB for DBPSK-MPPM ($M = 16$ and $n = 3$) when compared to traditional MPPM ($M = 16$ and $n = 5$) system and an improvement of about 0.5 dB for the hybrid system ($M = 22$ and $n = 6$) when compared to traditional DBPSK. The last improvement increases to about 1 dB at BER = $10^{-4}$. The reason behind this improvement is that when transmitting same data rate and bandwidth at a specific average optical power, hybrid systems have higher peak power per slot as compared to corresponding traditional MPPM and DBPSK systems. This leads to a higher SNR and improved BER.

After a specific peak power corresponding to the optimal average launched power, the nonlinearity impact becomes dominant over the ASE noise effect. Because of the proportionality of the nonlinear noise variance to $p_\text{peak}^2 = (MP_{tx}/n)^3$, the overall signal-to-noise ratio of the hybrid systems is reduced when compared to the traditional systems as the hybrid systems have higher peak power per slot, under the transmission of same data rate and bandwidth. Therefore, the hybrid DBPSK-MPPM system is affected more rapidly by fiber nonlinearity than traditional DBPSK and MPPM systems. Interestingly, at FEC limit, the fiber effect is not that sound and the improvement when using the hybrid system is the same as mentioned above. Specifically, it is still of about 2.5 dB for DBPSK-MPPM ($M = 16$ and $n = 3$) when compared to traditional MPPM ($M = 16$ and $n = 5$) and of about 0.5 dB for the hybrid system ($M = 22$ and $n = 6$) when compared to traditional DBPSK. Again, the last improvement increases to about 1 dB at BER = $10^{-4}$.

It worth observing that at very small average launch power, traditional DBPSK achieves better performance than that of hybrid system, Fig. 6. This is because in the case of very low launch power, the ASE noise has a dominant effect over the high peak power per slot benefit of the hybrid system. This leads to an increase in $\text{SER}_{\text{MPPM}}$, as it depends on the relative power difference between received signal and noise. However, the BER$_{\text{DBPSK}}$ symbols do not depend on the received signal power, but rather on the phase difference between the constituent received bits, which results in much higher noise margin for DBPSK system than MPPM techniques.

The aforementioned conclusions for the hybrid DBPSK-MPPM system apply for the hybrid DQPSK-MPPM system, as well. Specifically, from Figs. 7 and 8, at FEC limit, there is an improvement of about 2.5 dB for DQPSK-MPPM ($M = 36$ and $n = 3$) when compared to traditional MPPM ($M = 36$ and $n = 5$) system and an improvement of about 2 dB for the hybrid system ($M = 20$ and $n = 4$) when compared to traditional DBPSK. In addition, at FEC limit, after considering the fiber nonlinearity, the last improvements do not change.
Fig. 7. BER versus average launch optical power per channel for both hybrid DQPSK-MPPM and traditional MPPM systems, including nonlinearity-free limited case (dashed).

Fig. 8. BER versus average launch optical power per channel for both hybrid DQPSK-MPPM and traditional DBPSK and DQPSK systems, including nonlinearity-free limited cases (dashed). The inset is related to traditional DBPSK and DQPSK systems.

Fig. 9. Optimal BER versus number of WDM channels for different hybrid DBPSK- and DQPSK-MPPM systems.

Next, we study the effect of WDM characteristics on the performance of the hybrid system. Specifically, in Figs. 9 and 10, we show the effect of both the number of the WDM channels and channel bandwidth, respectively, on the optimal performance of the hybrid systems.

It can be noticed that the optimal system performance is degraded by increasing the number of WDM channels or channel bandwidth. The reason for that is the inverse proportionality between the SNR and both the number of WDM channels and channel bandwidth. Moreover, under the same transmission data rate and channel bandwidth, we can notice that by increasing the ratio of $M/n$ for the hybrid DBPSK-MPPM, the performance is degraded. This is not due to the decrease in the value of the peak SNR as the value of the peak SNR does not depend on the $M/n$ ratio at the optimal performance. Rather, increasing the ratio of $M/n$ will increase the SER_{MPPM} and as a result the performance is degraded. However, for the hybrid DQPSK-MPPM, under same transmission data rate and channel bandwidth, the performance is almost independent of the ratio of $M/n$. The reason behind that can be explained as follows. At the optimal performance, the BER_{DQPSK} dominates over the SER_{MPPM} and cancels the effect of the increase in the SER_{MPPM} while increasing the ratio of $M/n$. In addition, the optimal performance depends on the differential phase modulation level. It can be noticed that the optimal performance of DBPSK-MPPM system is better than that of the DQPSK-MPPM one at same $M = 8$ and $n = 2$. This is because DQPSK have double the transmission data rate of DBPSK at a specific bandwidth, which results in a degraded performance for DQPSK-MPPM when compared with DBPSK-MPPM at the same value of $M$ and $n$.

Fig. 10. Optimal BER versus WDM channel bandwidth for different hybrid DBPSK- and DQPSK-MPPM systems. The WDM system has $N_{ch} = 9$.

Figure 11 shows the nonlinear penalty on the maximum distance that can be reached by the hybrid and the traditional modulation systems at a BER of $10^{-3}$ (FEC-limit). As mentioned earlier, the hybrid modulation systems are more sensitive to fiber nonlinearity than traditional DPSK and MPPM systems. At low launch power, under same transmission rate and channel bandwidth, the maximum reach of the hybrid systems is better than that of traditional ones. Specifically, at an average transmitted power of $-10$ dBm, the DQPSK-MPPM system with $(M = 36$ and $n = 3$) reaches a distance more than that of MPPM system with $(M = 36$ and $n = 5$) by about 570 km. Moreover, at an average transmitted power of $-8$ dBm, the maximum reach of DQPSK-MPPM system with $(M = 20$ and $n = 4$) is longer than that of traditional DQPSK and DBPSK by about 1000 km and 345 km, respectively, and the maximum reach of DBPSK-MPPM with $(M = 16$
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