

## Bayesian Analysis of Record Statistics Based on Generalized Inverted Exponential Model

Amal S. Hassan<sup>#</sup>, Marwa Abd-Allah<sup>#</sup>, Heba F. Nagy<sup>#</sup>

<sup>#</sup> Department of Mathematical statistics, Cairo University, 5 Dr. Ahmed Zewail Street, Giza, 12613, Egypt  
E-mail: dr.amalemoslamy@gmail.com, merostat@yahoo.com, heba\_nagy\_84@yahoo.com

**Abstract**— In some situations, only observations that are more extreme than the current extreme value are recorded. This kind of data is called record values which have many applications in a lot of fields. In this paper, the Bayesian estimators using squared error and LINEX loss functions for the generalized inverted exponential distribution parameters are considered depending on upper record values and upper record ranked set sampling. The Bayes estimates and credible intervals are derived by considering the independent gamma priors for the parameters. The Markov Chain Monte Carlo (MCMC) method is developed due to the lack of explicit forms for the Bayes estimates. A Simulation study is implemented to compute and compare the performance of estimators in both sampling schemes with respect to relative absolute biases, estimated risks and the width of credible intervals.

**Keywords**— upper record ranked set sample; Bayesian estimator; squared error (SE) loss function; linear exponential (LINEX) loss function; Markov Chain Monte Carlo.

### I. INTRODUCTION

Record data are very important in many situations when the observations are difficult to obtain or are destroyed in experimental tests. Record data arise in a wide variety of practical situations including industrial stress testing, meteorology, sports, hydrology and economics. Record values can be viewed as order statistics from a sample that's its size is determined by the values and the order of occurrence of the observations. A record value of some phenomenon is the largest (smallest) observation anyone has ever made. The theoretical contributions and inference for record values have been studied extensively in the literature. The reader may refer to [1], [2], [3] and [4].

According to [3], record values can be classified into lower record values (LRV) and upper record values (URV).

Let  $\{X_j, j \geq 1\}$  be a sequence of independent and identically distributed (iid) random variables, an observation  $X_j$  is called URV (LRV) if its value exceeds (lower than) all of the previous observations, i.e.,  $X_j > X_i (X_j < X_i)$  for every  $i < j$ .

Reference [5] presented a new sampling scheme, called record ranked set sampling (RRSS), for generating record data. The new scheme helps the scientists in situations where

the only observations that are going to be used are the last record data as in athletic, weather and Olympic data.

The upper record ranked set sampling (URRSS) can be described as follows: Suppose that there exist  $n$  independent sequential sequences of continuous random variables, the  $i^{th}$  sequence sampling is ceased when the  $i^{th}$  record value is observed. The only observations that are used for analysis are the last record value in each sequence. The last record value of the  $i^{th}$  sequence in this plane is denoted by  $U_{i,i}$ , then the available observations are  $U = (U_{1,1}, U_{2,2}, \dots, U_{n,n})^T$ , i.e.

$$\begin{aligned} 1: & \quad U_{(1)1} & \rightarrow U_{1,1} = U_{(1)1} \\ 2: & \quad U_{(1)2} \quad U_{(2)2} & \rightarrow U_{2,2} = U_{(2)2} \\ & \quad \vdots \quad \vdots \quad \ddots \quad \vdots \quad \vdots \\ n: & \quad U_{(1)n} \quad U_{(2)n} \quad \dots \quad U_{(n)n} & \rightarrow U_{n,n} = U_{(n)n} \end{aligned}$$

where,  $U_{(i)j}$  is the  $i^{th}$  record in the  $j^{th}$  sequence. It is recognized that  $U_{i,i}$ 's are independent random variables but not ordered.

Bayesian estimation based on record values has been considered by several researchers. Among them, [6]-[19]. Reference [20] discussed the Bayesian estimation problem

for the shape parameter of the weighted exponential distribution based on URRSS.

The applicability of the one parameter exponential distribution is the simplest and the most widely discussed distribution for lifetime data but it is restricted to a constant hazard rate. Most generalizations of the exponential distributions possess the constant, non-increasing, non-decreasing and bathtub hazard rates. But in many practical situations, the data shows the inverted bathtub hazard rate (initially increase and then decrease, i.e., unimodal). For such data types, another extension of the exponential distribution known as the one parameter inverted exponential distribution is provided, which have inverted bathtub hazard rate [21]. Reference [22] introduced the two-parameter generalized inverted exponential distribution (GIED) by adding a shape parameter to the inverted exponential distribution.

The probability density function (pdf) of the GIED with the shape parameter  $\alpha$  and the scale parameter  $\lambda$  takes the following form

$$f(x; \alpha, \lambda) = \frac{\alpha\lambda}{x^2} \left(1 - e^{-\frac{\lambda}{x}}\right)^{\alpha-1} e^{-\frac{\lambda}{x}} \quad (1)$$

$; x > 0, \alpha, \lambda > 0.$

The cumulative distribution function (cdf) is as follows

$$F(x; \alpha, \lambda) = 1 - \left(1 - e^{-\frac{\lambda}{x}}\right)^{\alpha}. \quad (2)$$

Recently, there has been a growing interest in the study of inference problems associated with record values and record ranked set samples via the Bayesian approach. The Bayesian estimation for the GIED based on URRSS hasn't been studied in the literature yet. The goal of this paper is to obtain and to compare the Bayes estimates and the average width of the posterior 95% credible intervals for the unknown parameters of GIED on the basis of URV and URRSS. These Bayes estimates and credible intervals width are obtained using independent gamma priors under symmetric (squared error (SE)) and asymmetric (linear exponential (LINEX)) loss functions through MCMC method. The procedures are illustrated through analysing a simulated data. The rest of the paper is organized as follows. Section (II) gives the Bayes estimates based on URV, the Bayes estimates under URRSS, MCMC approach and a simulation. Discussion and results of the simulation study appear in Section (III). Concluding remarks appear in Section (IV).

## II. MATERIAL AND METHOD

### A. Bayesian Estimators Based on URV

In this section, Bayesian estimators of the unknown parameters of the GIED under the assumption of independent gamma priors on both the shape and scale parameters are considered. Based on URV, the Bayes estimators cannot be obtained in explicit forms. Hence the MCMC technique is carried out to generate samples from the posterior distributions and consequently computing the Bayes estimators and construct the corresponding credible intervals. Here, two types of loss functions are considered

for Bayesian computation; symmetric one (SE) and asymmetric one (LINEX).

Let  $\underline{r} = (r_1, \dots, r_m)$  be a set of URV from GIED( $\alpha, \lambda$ ), the likelihood function according to [3], is given by

$$L_1 = f(r_m) \prod_{i=1}^{m-1} \frac{f(r_i)}{1-F(r_i)} \quad 0 < r_1 < \dots < r_m < \infty, \quad (3)$$

where,  $f(\cdot, \theta)$  and  $F(\cdot, \theta)$  are respectively the pdf and the cdf of GIED( $\alpha, \lambda$ ). The likelihood function of the observed URV is obtained, as follows

$$L_1 = (1 - e^{-r_m})^{\alpha} \prod_{i=1}^m \frac{\alpha \lambda}{r_i^2} e^{-\frac{\lambda}{r_i}} (1 - e^{-\frac{\lambda}{r_i}})^{-1} \quad (4)$$

Further, assuming that the prior of parameters  $\alpha$  and  $\lambda$  has a gamma distribution with parameters  $(a_1, b_1)$  and  $(a_2, b_2)$  respectively. Hence, assuming independence of parameters, the joint prior distribution of parameters, denoted by  $\pi(\alpha, \lambda)$ , is as follows

$$\pi(\alpha, \lambda) = \frac{1}{\Gamma(a_1)\Gamma(a_2)} \alpha^{a_1-1} \lambda^{a_2-1} e^{-b_1\alpha-b_2\lambda} \quad (5)$$

$; a_1, a_2, b_1, b_2, \alpha, \lambda > 0$

The expression for the joint posterior can be written as

$$\pi_1^*(\alpha, \lambda | \underline{r}) \propto \alpha^{m+a_1-1} \lambda^{m+a_2-1} e^{-b_1\alpha-b_2\lambda} (1 - e^{-r_m})^{\alpha} \prod_{i=1}^m r_i^{-2} e^{-\frac{\lambda}{r_i}} (1 - e^{-\frac{\lambda}{r_i}})^{-1}. \quad (6)$$

Hence, the marginal posterior distributions of  $\alpha$  and  $\lambda$ , based on URV, take the following forms

$$\pi_1^*(\alpha | \underline{r}) = C \alpha^{m+a_1-1} e^{-b_1\alpha} \int_0^{\infty} \lambda^{m+a_2-1} e^{-b_2\lambda} (1 - e^{-r_m})^{\alpha} \prod_{i=1}^m r_i^{-2} e^{-\frac{\lambda}{r_i}} (1 - e^{-\frac{\lambda}{r_i}})^{-1} d\lambda,$$

$$\pi_1^*(\lambda | \underline{r}) = C \lambda^{m+a_2-1} e^{-b_2\lambda} \prod_{i=1}^m r_i^{-2} e^{-\frac{\lambda}{r_i}} (1 - e^{-\frac{\lambda}{r_i}})^{-1} \int_0^{\infty} \alpha^{m+a_1-1} e^{-b_1\alpha} (1 - e^{-r_m})^{\alpha} d\alpha,$$

where,

$$C^{-1} = \int_0^{\infty} \int_0^{\infty} \alpha^{m+a_1-1} \lambda^{m+a_2-1} e^{-b_1\alpha-b_2\lambda} (1 - e^{-r_m})^{\alpha} \prod_{i=1}^m r_i^{-2} e^{-\frac{\lambda}{r_i}} (1 - e^{-\frac{\lambda}{r_i}})^{-1} d\alpha d\lambda.$$

Therefore, based on URV the Bayes estimates of the unknown parameters under SE loss function, denoted by  $\hat{\alpha}_{SE}$  and  $\hat{\lambda}_{SE}$ , can be obtained as posterior mean as follows

$$\hat{\alpha}_{SE} = E(\alpha | \underline{r}) = \int_0^{\infty} \alpha \pi_1^*(\alpha | \underline{r}) d\alpha, \quad (7)$$

and

$$\hat{\lambda}_{SE} = E(\lambda|\underline{r}) = \int_0^{\infty} \lambda \pi_1^*(\lambda|\underline{r}) d\lambda.$$

Additionally, the Bayesian estimators of  $\alpha$  and  $\lambda$  under LINEX loss function, denoted by  $\tilde{\alpha}_{LINEX}$  and  $\tilde{\lambda}_{LINEX}$ , are given by

$$\begin{aligned} \tilde{\alpha}_{LINEX} &= \frac{-1}{v} \log E[e^{-v\alpha}] \\ &= \frac{-1}{v} \log \left[ \int_0^{\infty} e^{-v\alpha} \pi_1^*(\alpha|\underline{r}) d\alpha \right], \end{aligned}$$

and

$$\begin{aligned} \tilde{\lambda}_{LINEX} &= \frac{-1}{v} \log E[e^{-v\lambda}] \\ &= \frac{-1}{v} \log \left[ \int_0^{\infty} e^{-v\lambda} \pi_1^*(\lambda|\underline{r}) d\lambda \right]. \end{aligned} \tag{8}$$

Generally, as observed, the analytical solution of integrations given by (7) and (8) is very difficult to obtain due to the complicated mathematical form. Therefore, the MCMC technique is employed to approximate these integrations. Therefore Metropolis-Hastings (M-H) method will be implemented which is a powerful MCMC technique to compute the Bayes estimates and credible intervals width.

### B. Bayesian Estimators Based on URRSS

This section discusses the Bayes estimates of the unknown shape and scale parameters of the GIED under the assumption of independent gamma priors defined in (5) based on URRSS using SE and LINEX loss functions.

Let  $r_u = (r_{1,1}, \dots, r_{m,m})$  be a set of observed URRSS, then the joint density function denoted by  $L_2$ , according to [5], is given by

$$L_2 = \prod_{i=1}^m \frac{(-\log \bar{F}(r_{i,i}; \theta))^{i-1}}{(i-1)!} f(r_{i,i}; \theta) \quad ; \theta \in \Theta, \tag{9}$$

where,  $\bar{F}(\cdot; \theta) = 1 - F(\cdot; \theta)$ ,  $\theta$  is real valued parameter,  $\Theta$  is the parameter space. Inserting the pdf (1) and cdf (2) in (9), then the likelihood function is obtained as follows

$$L_2 = \alpha^{\sum_{i=1}^m i} \lambda^m \left[ \prod_{i=1}^m \frac{1}{(i-1)! r_{i,i}^2} e^{\frac{-\lambda}{r_{i,i}}} \right] [-\log(1 - e^{\frac{-\lambda}{r_{i,i}}})]^{i-1} (1 - e^{\frac{-\lambda}{r_{i,i}}})^{\alpha-1}.$$

Under the assumption that  $\alpha$  and  $\lambda$  are independent, the expression for the joint posterior using gamma priors (5) can be written in the following form

$$\begin{aligned} \pi_2^*(\alpha, \lambda | r_u) &\propto \alpha^{\left(\sum_{i=1}^m i\right) + a_1 - 1} \lambda^{m + a_2 - 1} e^{-b_1 \alpha - b_2 \lambda} \\ &\prod_{i=1}^m \frac{e^{\frac{-\lambda}{r_{i,i}}}}{(i-1)! r_{i,i}^2} \left[ -\log(1 - e^{\frac{-\lambda}{r_{i,i}}}) \right]^{i-1} (1 - e^{\frac{-\lambda}{r_{i,i}}})^{\alpha-1}. \end{aligned}$$

Hence, the marginal posterior distributions of  $\alpha$  and  $\lambda$ , based on URRSS, can be expressed as follows

$$\begin{aligned} \pi_2^*(\alpha | r_u) &= K \alpha^{\left(\sum_{i=1}^m i\right) + a_1 - 1} e^{-b_1 \alpha} \prod_{i=1}^m \frac{1}{(i-1)! r_{i,i}^2} \\ &\int_0^{\infty} \lambda^{m + a_2 - 1} e^{\frac{-\lambda(b_2 + \frac{1}{r_{i,i}})}{r_{i,i}}} [-\log(1 - e^{\frac{-\lambda}{r_{i,i}}})]^{i-1} (1 - e^{\frac{-\lambda}{r_{i,i}}})^{\alpha-1} d\lambda, \\ \pi_2^*(\lambda | r_u) &= K \lambda^{m + a_2 - 1} e^{-b_2 \lambda} \prod_{i=1}^m \frac{1}{(i-1)! r_{i,i}^2} e^{\frac{-\lambda}{r_{i,i}}} \\ &[-\log(1 - e^{\frac{-\lambda}{r_{i,i}}})]^{i-1} \int_0^{\infty} \alpha^{\left(\sum_{i=1}^m i\right) + a_1 - 1} e^{-b_1 \alpha} (1 - e^{\frac{-\lambda}{r_{i,i}}})^{\alpha-1} d\alpha, \end{aligned}$$

where,

$$K^{-1} = \prod_{i=1}^m \frac{1}{(i-1)! r_{i,i}^2} \int_0^{\infty} \int_0^{\infty} \alpha^{\left(\sum_{i=1}^m i\right) + a_1 - 1} \lambda^{m + a_2 - 1} e^{-b_1 \alpha - b_2 \lambda} e^{\frac{-\lambda}{r_{i,i}}} [-\log(1 - e^{\frac{-\lambda}{r_{i,i}}})]^{i-1} (1 - e^{\frac{-\lambda}{r_{i,i}}})^{\alpha-1} d\alpha d\lambda.$$

Therefore, based on URRSS, the Bayes estimates of the unknown parameters under SE loss function, denoted by  $\hat{\alpha}_{SE}$  and  $\hat{\lambda}_{SE}$ , can be obtained as the posterior mean as follows

$$\hat{\alpha}_{SE} = \int_0^{\infty} \alpha \pi_2^*(\alpha | r_u) d\alpha, \tag{10}$$

and

$$\hat{\lambda}_{SE} = \int_0^{\infty} \lambda \pi_2^*(\lambda | r_u) d\lambda.$$

Similarly, the Bayes estimators of  $\alpha$  and  $\lambda$  under LINEX loss function, denoted by  $\tilde{\alpha}_{LINEX}$  and  $\tilde{\lambda}_{LINEX}$ , are given by

$$\tilde{\alpha}_{LINEX} = \frac{-1}{v} \int_0^{\infty} e^{-v\alpha} \pi_2^*(\alpha | r_u) d\alpha, \tag{11}$$

and

$$\tilde{\lambda}_{LINEX} = \frac{-1}{v} \int_0^{\infty} e^{-v\lambda} \pi_2^*(\lambda | r_u) d\lambda,$$

where,  $v$  is a real number.

Again, the integrals (10) and (11) cannot be reduced to a closed form due to its difficult mathematical form. So, M-H algorithm is used to compute the Bayes estimator under the SE and LINEX loss functions.

### C. Simulation Study

In this section, a numerical study is performed in order to examine and compare the behaviour of the Bayes estimators for the two parameters of the GIED( $\alpha, \lambda$ ) based on URV and URRSS. The Bayes estimates are obtained using gamma priors under SE and LINEX loss functions. The major difficulty in the implementation of the Bayesian procedure is that of obtaining the posterior distribution. MCMC is simply an iterative procedure drawing values from the posterior distributions of the parameter in the concerned model. The M-H algorithm is one of the most famous subclasses of

MCMC method in Bayesian literature to simulate the deviates from the posterior density and produce the good approximate results. Here, M-H algorithm will be used via R program.

To compare the estimators, MCMC simulations are performed for different sample sizes under SE and LINEX loss functions. For each simulation, the number of records are selected as  $n = 3, 4, 5, 6, 7, 8$  and the parameter values are selected as  $(\alpha, \lambda) = (2, 2), (0.5, 0.5), (0.5, 1.5), (1.5, 0.5), (1, 2)$  and  $(2, 1)$ . The hyper-parameters for gamma priors are selected as  $a_1 = a_2 = 4$  and  $b_1 = b_2 = 1$ . Also, we take  $(v = -2, 2)$ . All the results are based on the number of replications  $(NR) = 5000$ . Evaluating the performance of the estimates is considered through some measurements of accuracy, so it is convenient to use the relative absolute biases (Rabs) and estimated risks (ERs) of the Bayes estimates which are computed as follows:

$$\text{Rabs}(\text{estimator}) = \frac{|\text{estimator} - \text{true value}|}{\text{true value}},$$

$$\text{ERs}(\text{estimator}) = \frac{\sum_{i=1}^{NR} (\text{average} - \text{population parameter})^2}{NR}.$$

Here is how M-H algorithm works; let  $g(\cdot)$  be the density of the subject distribution. The M-H algorithm proceeds as follows

```

Initialize a starting value  $x_0$  and the number of samples
N
for  $i = 2$  to N
    set  $x = x_{i-1}$ 
    generate u from  $U(0,1)$ 
    generate y from  $g(\cdot)$ 
    if  $u \leq \frac{\pi_\alpha(y)g(x)}{\pi_\alpha(x)g(y)}$  then
        set  $x_i = y$ 
    else
        set  $x_i = x$ 
    end if
end for

```

### III. RESULTS AND DISCUSSION

The simulation results are summarized in Tables (I-VI) and represented through Fig. (1-6). From these tables and figures, the following observations can be made

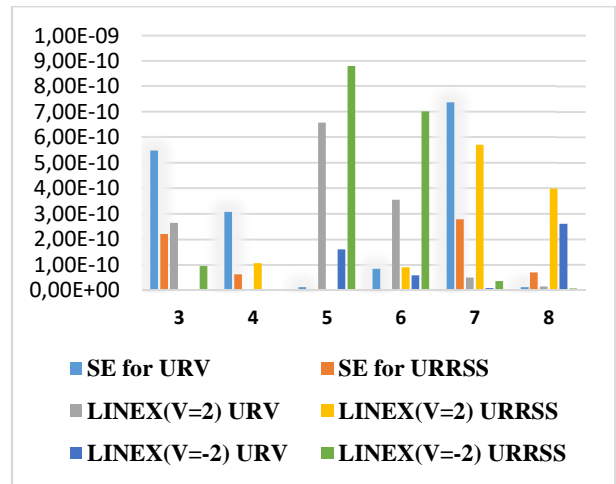


Fig. 1 ERs of  $\alpha$  under SE and LINEX loss functions based on URV and URRSS at  $(\alpha, \lambda) = (2, 2)$

Fig.(1) shows that the ERs of  $\hat{\alpha}_{SE}$  are less than the ERs of the corresponding  $\hat{\alpha}_{SE}$  for all the number of records except at  $n = 8$ . Also, the ERs of  $\tilde{\alpha}_{LINEX}$  when  $v = 2$  are less than the ERs of the corresponding;  $\tilde{\alpha}_{LINEX}$  for  $n = 3, 5, 6$  and when  $v = -2$  the ERs of  $\tilde{\alpha}_{LINEX}$  are less than the ERs of the corresponding  $\tilde{\alpha}_{LINEX}$  for all the number of records except at  $n = 8$ .

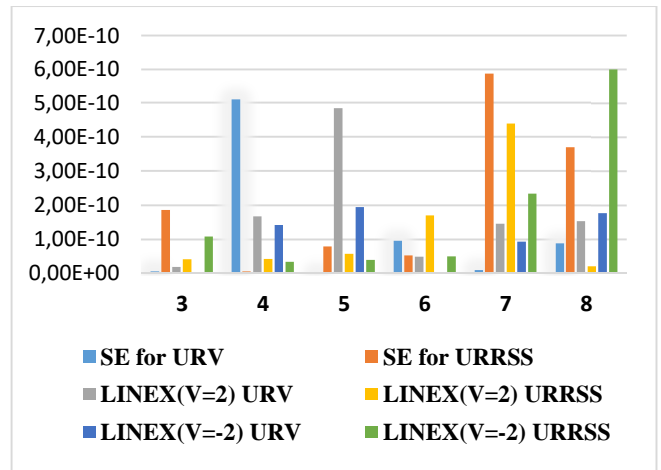


Fig. 2 ERs of  $\lambda$  under SE and LINEX loss functions based on URV and URRSS at  $(\alpha, \lambda) = (2, 2)$

Fig. (2), shows that the ERs of  $\hat{\lambda}_{SE}$  are less than the ERs of the corresponding  $\hat{\lambda}_{SE}$  for all the number of records except at  $n = 4, 6$ . Also, the ERs of  $\tilde{\lambda}_{LINEX}$  when  $v = 2$  are less than the ERs of the corresponding  $\tilde{\lambda}_{LINEX}$  for  $n = 4, 5, 8$ , but when  $v = -2$ , ERs of  $\tilde{\lambda}_{LINEX}$  are less than the ERs of the corresponding  $\tilde{\lambda}_{LINEX}$  for all the number of records except at  $n = 4, 5$ .

TABLE I  
 BAYES ESTIMATES, Rab and ER of  $\alpha$  and  $\lambda$  BASED ON URV and URRSS for  $(\alpha, \lambda) = (2, 2)$ .

Number of records/parameters		3		4		5		6		7		8		
		$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	
URV	SE	Estimates	1.9983	1.9998	1.9988	1.9984	1.9997	2.0001	2.0007	1.9993	1.9981	2.0002	2.0003	1.9993
		Rab	0.0008	0.0001	0.0006	0.0008	0.0001	0.0000	0.0003	0.0003	0.0010	0.0001	0.0001	0.0003
		ER	1.09E-03	2.57E-04	7.06E-04	8.55E-04	2.82E-04	2.68E-04	4.12E-04	3.79E-04	1.08E-03	2.16E-04	2.67E-04	4.16E-04
		Width	0.0039	0.0016	0.0025	0.0022	0.0018	0.0020	0.0016	0.0013	0.0030	0.0012	0.0018	0.0016
	LINEX $v=(2)$	Estimates	1.9988	2.0003	2.0000	2.0009	2.0018	1.9984	1.9987	1.9995	1.9995	1.9991	2.0003	1.9991
		Rab	0.0006	0.0002	0.0000	0.0005	0.0009	0.0008	0.0007	0.0002	0.0003	0.0004	0.0001	0.0004
		ER	2.65E-10	1.85E-11	1.10E-13	1.68E-10	6.59E-10	4.85E-10	3.55E-10	4.86E-11	5.07E-11	1.46E-10	1.49E-11	1.54E-10
		Width	0.0018	0.0015	0.0014	0.0021	0.0028	0.0024	0.0020	0.0017	0.0020	0.0018	0.0021	0.0020
	LINEX $v=(-2)$	Estimates	1.9999	2.0001	2.0000	1.9992	1.9991	1.9990	2.0005	1.9999	1.9998	1.9993	2.0011	1.9991
		Rab	0.0001	0.0000	0.0000	0.0004	0.0004	0.0005	0.0003	0.0000	0.0001	0.0003	0.0006	0.0005
		ER	3.00E-12	1.52E-12	7.50E-14	1.42E-10	1.60E-10	1.95E-10	5.91E-11	1.54E-12	8.25E-12	9.32E-11	2.62E-10	1.77E-10
		Width	0.0012	0.0010	0.0013	0.0017	0.0013	0.0018	0.0021	0.0016	0.0015	0.0019	0.0029	0.0018
URRSS	SE	Estimates	1.9990	1.9990	2.0006	2.0002	2.0002	1.9994	2.0000	2.0005	2.0012	1.9983	2.0006	1.9986
		Rab	0.0005	0.0005	0.0003	0.0001	0.0001	0.0003	0.0000	0.0003	0.0006	0.0009	0.0003	0.0007
		ER	2.20E-10	1.87E-10	6.32E-11	5.69E-12	5.31E-12	7.90E-11	1.66E-15	5.28E-11	2.79E-10	5.88E-10	6.99E-11	3.71E-10
		Width	0.0019	0.0013	0.0018	0.0020	0.0012	0.0012	0.0013	0.0010	0.0018	0.0030	0.0021	0.0031
	LINEX $v=(2)$	Estimates	2.0000	2.0005	2.0007	2.0005	1.9998	2.0005	1.9993	1.9991	2.0017	2.0015	2.0014	2.0003
		Rab	0.0000	0.0002	0.0004	0.0002	0.0001	0.0003	0.0003	0.0005	0.0008	0.0007	0.0007	0.0002
		ER	7.98E-14	4.19E-11	1.07E-10	4.22E-11	4.94E-12	5.76E-11	9.06E-11	1.71E-10	5.72E-10	4.41E-10	3.99E-10	2.11E-11
		Width	0.0021	0.0010	0.0013	0.0009	0.0012	0.0016	0.0008	0.0019	0.0027	0.0028	0.0024	0.0018
	LINEX $v=(-2)$	Estimates	2.0007	2.0007	2.0002	1.9996	2.0021	1.9996	1.9981	1.9995	2.0004	2.0011	1.9998	1.9983
		Rab	0.0003	0.0004	0.0001	0.0002	0.0010	0.0002	0.0009	0.0002	0.0002	0.0005	0.0001	0.0009
		ER	9.54E-11	1.08E-10	5.33E-12	3.43E-11	8.80E-10	3.96E-11	7.01E-10	4.98E-11	3.66E-11	2.34E-10	7.59E-12	6.00E-10
		Width	0.0012	0.0015	0.0015	0.0016	0.0039	0.0015	0.0022	0.0014	0.0013	0.0022	0.0016	0.0025

TABLE II  
 BAYES ESTIMATES, Rab and ER of  $\alpha$  and  $\lambda$  BASED on URV and URRSS for  $(\alpha, \lambda) = (0.5, 0.5)$ .

Number of records/parameters		3		4		5		6		7		8		
		$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	
URV	SE	Estimates	0.4997	0.5007	0.5003	0.5009	0.5006	0.5000	0.4991	0.5007	0.4996	0.4985	0.5005	0.5002
		Rab	0.0006	0.0015	0.0006	0.0018	0.0012	0.0001	0.0018	0.0014	0.0008	0.0030	0.0010	0.0004
		ER	2.07E-11	1.10E-10	1.58E-11	1.56E-10	7.49E-11	2.57E-13	1.57E-10	9.99E-11	3.40E-11	4.56E-10	4.59E-11	8.09E-12
		Width	0.0009	0.0017	0.0019	0.0011	0.0021	0.0015	0.0022	0.0013	0.0010	0.0033	0.0025	0.0019
	LINEX $v=(2)$	Estimates	0.5009	0.4988	0.5008	0.5001	0.4998	0.4992	0.5001	0.4992	0.4992	0.5004	0.5010	0.5006
		Rab	0.0018	0.0023	0.0015	0.0002	0.0005	0.0015	0.0003	0.0015	0.0016	0.0008	0.0020	0.0011
		ER	1.55E-10	2.72E-10	1.15E-10	2.05E-12	1.07E-11	1.15E-10	4.10E-12	1.15E-10	1.27E-10	3.01E-11	2.04E-10	6.41E-11
		Width	0.0015	0.0018	0.0019	0.0025	0.0018	0.0016	0.0019	0.0026	0.0021	0.0017	0.0018	0.0016
	LINEX $v=(-2)$	Estimates	0.5002	0.5008	0.4994	0.5012	0.4996	0.5012	0.5001	0.4999	0.5012	0.5002	0.5004	0.4994
		Rab	0.0004	0.0016	0.0012	0.0024	0.0008	0.0025	0.0001	0.0002	0.0024	0.0005	0.0008	0.0011
		ER	8.40E-12	1.24E-10	6.96E-11	2.83E-10	3.47E-11	3.09E-10	9.49E-13	1.33E-12	2.88E-10	1.23E-11	3.22E-11	6.43E-11
		Width	0.0011	0.0014	0.0026	0.0017	0.0018	0.0025	0.0009	0.0014	0.0026	0.0013	0.0015	0.0013
URRSS	SE	Estimates	0.4987	0.4991	0.4989	0.5000	0.4993	0.5008	0.5005	0.4987	0.5003	0.4985	0.5000	0.5003
		Rab	0.0026	0.0018	0.0021	0.0001	0.0015	0.0016	0.0009	0.0026	0.0007	0.0030	0.0001	0.0006
		ER	3.43E-10	1.55E-10	2.24E-10	3.21E-13	1.10E-10	1.22E-10	4.17E-11	3.30E-10	2.35E-11	4.51E-10	2.81E-13	1.86E-11
		Width	0.0022	0.0012	0.0023	0.0010	0.0015	0.0023	0.0027	0.0018	0.0011	0.0017	0.0013	0.0011
	LINEX $v=(2)$	Estimates	0.5015	0.4999	0.5005	0.5002	0.5006	0.4998	0.4985	0.5005	0.5003	0.4995	0.5011	0.4999
		Rab	0.0030	0.0002	0.0010	0.0004	0.0012	0.0005	0.0029	0.0009	0.0007	0.0009	0.0021	0.0003
		ER	4.60E-10	2.69E-12	5.49E-11	1.00E-11	7.76E-11	1.04E-11	4.34E-10	4.48E-11	2.20E-11	4.14E-11	2.22E-10	4.22E-12
		Width	0.0019	0.0013	0.0018	0.0017	0.0015	0.0012	0.0035	0.0016	0.0021	0.0012	0.0026	0.0014
	LINEX $v=(-2)$	Estimates	0.5008	0.5008	0.5007	0.5004	0.4995	0.5007	0.5002	0.5011	0.4997	0.5013	0.5008	0.5005
		Rab	0.0016	0.0015	0.0015	0.0009	0.0011	0.0014	0.0005	0.0021	0.0006	0.0027	0.0015	0.0011
		ER	1.25E-10	1.18E-10	1.05E-10	3.87E-11	5.83E-11	9.44E-11	1.17E-11	2.20E-10	1.95E-11	3.64E-10	1.19E-10	5.56E-11
		Width	0.0026	0.0013	0.0011	0.0012	0.0013	0.0013	0.0018	0.0016	0.0012	0.0026	0.0022	0.0014

TABLE III  
 BAYES ESTIMATES, Rab and ER of  $\alpha$  and  $\lambda$  BASED on URV and URRSS for  $(\alpha, \lambda) = (0.5, 1.5)$ .

Number of records/parameters		3		4		5		6		7		8		
		$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	
URV	SE	Estimates	0.4974	1.5006	0.5002	1.5006	0.4996	1.4990	0.4987	1.5000	0.5006	1.5006	0.5001	1.5014
		Rab	0.0051	0.0004	0.0003	0.0004	0.0008	0.0007	0.0026	0.0000	0.0012	0.0004	0.0002	0.0009
		ER	1.32E-09	6.64E-11	4.89E-12	6.39E-11	2.87E-11	2.12E-10	3.42E-10	1.59E-17	7.43E-11	8.07E-11	2.50E-12	3.85E-10
		Width	0.0037	0.0009	0.0014	0.0018	0.0010	0.0013	0.0021	0.0022	0.0018	0.0015	0.0014	0.0021
	LINEX $v=(2)$	Estimates	0.4989	1.5001	0.4988	1.4990	0.5001	1.4999	0.4993	1.4994	0.5009	1.4998	0.5005	1.5015
		Rab	0.0021	0.0001	0.0024	0.0006	0.0002	0.0001	0.0015	0.0004	0.0018	0.0001	0.0011	0.0010
		ER	2.23E-10	2.26E-12	2.76E-10	1.88E-10	1.45E-12	2.19E-12	1.11E-10	8.35E-11	1.68E-10	8.47E-12	5.77E-11	4.66E-10
		Width	0.0021	0.0015	0.0017	0.0010	0.0015	0.0018	0.0017	0.0014	0.0027	0.0022	0.0014	0.0027
	LINEX $v=(-2)$	Estimates	0.4992	1.4990	0.4994	1.4996	0.4992	1.5006	0.4985	1.5004	0.4984	1.4981	0.5001	1.5001
		Rab	0.0015	0.0007	0.0012	0.0003	0.0017	0.0004	0.0029	0.0003	0.0031	0.0013	0.0001	0.0001
		ER	1.19E-10	1.98E-10	6.76E-11	3.35E-11	1.42E-10	6.20E-11	4.34E-10	3.54E-11	4.92E-10	7.31E-10	8.82E-13	2.70E-12
		Width	0.0011	0.0027	0.0012	0.0015	0.0016	0.0019	0.0023	0.0022	0.0038	0.0034	0.0015	0.0023
URRSS	SE	Estimates	0.4989	1.5007	0.4990	1.4988	0.4999	1.4990	0.4993	1.5006	0.4979	1.5000	0.4999	1.4985
		Rab	0.0022	0.0005	0.0020	0.0008	0.0001	0.0007	0.0014	0.0004	0.0041	0.0000	0.0002	0.0010
		ER	2.46E-10	9.53E-11	1.99E-10	2.97E-10	1.06E-12	2.03E-10	9.30E-11	6.61E-11	8.43E-10	1.04E-13	2.51E-12	4.25E-10
		Width	0.0024	0.0019	0.0012	0.0023	0.0017	0.0015	0.0020	0.0020	0.0031	0.0022	0.0035	0.0022
	LINEX $v=(2)$	Estimates	0.5005	1.5002	0.4994	1.5010	0.4995	1.5014	0.5002	1.4994	0.5008	1.4999	0.4992	1.4989
		Rab	0.0010	0.0002	0.0012	0.0007	0.0009	0.0010	0.0003	0.0004	0.0015	0.0000	0.0016	0.0008
		ER	5.01E-11	1.10E-11	6.90E-11	1.99E-10	4.16E-11	4.11E-10	4.74E-12	6.84E-11	1.14E-10	1.10E-12	1.35E-10	2.57E-10
		Width	0.0013	0.0011	0.0021	0.0015	0.0012	0.0017	0.0026	0.0021	0.0014	0.0011	0.0017	0.0021
	LINEX $v=(-2)$	Estimates	0.5014	1.5005	0.5004	1.4998	0.5001	1.5002	0.4996	1.5009	0.5008	1.5006	0.4997	1.4992
		Rab	0.0028	0.0003	0.0008	0.0001	0.0002	0.0001	0.0008	0.0006	0.0017	0.0004	0.0006	0.0006
		ER	4.06E-10	4.89E-11	3.59E-11	5.33E-12	2.51E-12	7.06E-12	3.35E-11	1.49E-10	1.43E-10	7.51E-11	1.96E-11	1.38E-10
		Width	0.0022	0.0021	0.0014	0.0020	0.0013	0.0019	0.0023	0.0024	0.0017	0.0017	0.0023	0.0020

TABLE IV  
 BAYES ESTIMATES, Rab and ER of  $\alpha$  and  $\lambda$  BASED ON URV and URRSS for  $(\alpha, \lambda) = (1, 2)$ .

Number of records/parameters		3		4		5		6		7		8		
		$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	
URV	SE	Estimates	1.0006	2.0008	1.0006	1.9999	1.0002	2.0020	1.0003	2.0010	0.9986	1.9991	0.9990	2.0003
		Rab	0.0006	0.0004	0.0006	0.0000	0.0002	0.0010	0.0003	0.0005	0.0014	0.0004	0.0010	0.0002
		ER	6.27E-11	1.41E-10	6.74E-11	9.29E-13	7.23E-12	7.83E-10	2.37E-11	2.17E-10	4.07E-10	1.52E-10	1.94E-10	2.23E-11
		Width	0.0014	0.0014	0.0018	0.0016	0.0012	0.0041	0.0011	0.0012	0.0036	0.0018	0.0022	0.0021
	LINEX $v=(2)$	Estimates	1.0006	1.9984	1.0005	2.0004	1.0009	1.9998	1.0013	1.9981	0.9999	2.0015	0.9990	1.9995
		Rab	0.0006	0.0008	0.0005	0.0002	0.0009	0.0001	0.0013	0.0009	0.0001	0.0007	0.0010	0.0002
		ER	7.60E-11	4.85E-10	4.44E-11	3.30E-11	1.76E-10	8.57E-12	3.20E-10	7.02E-10	1.73E-12	4.26E-10	2.01E-10	4.85E-11
		Width	0.0012	0.0029	0.0019	0.0021	0.0018	0.0013	0.0024	0.0025	0.0018	0.0024	0.0024	0.0018
	LINEX $v=(-2)$	Estimates	1.0007	1.9988	1.0002	2.0011	0.9989	2.0005	0.9985	1.9999	0.9996	1.9998	1.0013	2.0004
		Rab	0.0007	0.0006	0.0002	0.0005	0.0011	0.0002	0.0015	0.0000	0.0004	0.0001	0.0013	0.0002
		ER	9.38E-11	2.92E-10	1.05E-11	2.41E-10	2.45E-10	4.57E-11	4.73E-10	1.39E-12	3.93E-11	1.19E-11	3.34E-10	3.68E-11
		Width	0.0025	0.0019	0.0012	0.0034	0.0019	0.0011	0.0024	0.0014	0.0016	0.0012	0.0028	0.0011
URRSS	SE	Estimates	1.0005	2.0004	1.0008	2.0003	1.0001	1.9984	1.0004	1.9997	1.0002	2.0001	1.0001	1.9998
		Rab	0.0005	0.0002	0.0008	0.0002	0.0001	0.0008	0.0004	0.0001	0.0002	0.0000	0.0001	0.0001
		ER	6.01E-11	3.46E-11	1.19E-10	2.23E-11	3.70E-12	5.38E-10	2.51E-11	1.35E-11	1.15E-11	7.43E-13	9.65E-13	9.31E-12
		Width	0.0016	0.0010	0.0020	0.0017	0.0019	0.0031	0.0014	0.0012	0.0017	0.0013	0.0013	0.0019
	LINEX $v=(2)$	Estimates	1.0000	1.9985	1.0010	2.0008	0.9999	2.0008	1.0010	1.9993	1.0003	2.0007	1.0004	1.9990
		Rab	0.0000	0.0008	0.0010	0.0004	0.0001	0.0004	0.0010	0.0003	0.0003	0.0003	0.0004	0.0005
		ER	4.18E-13	4.58E-10	2.01E-10	1.23E-10	3.86E-12	1.24E-10	1.92E-10	9.47E-11	1.57E-11	9.78E-11	2.77E-11	2.17E-10
		Width	0.0015	0.0025	0.0022	0.0034	0.0018	0.0012	0.0015	0.0021	0.0012	0.0016	0.0010	0.0020
	LINEX $v=(-2)$	Estimates	0.9996	1.9994	1.0010	1.9999	1.0004	2.0001	1.0020	1.9994	1.0010	1.9992	1.0003	1.9999
		Rab	0.0004	0.0003	0.0010	0.0001	0.0004	0.0001	0.0020	0.0003	0.0010	0.0004	0.0003	0.0000
		ER	3.94E-11	7.27E-11	1.82E-10	4.13E-12	2.64E-11	2.24E-12	7.93E-10	8.32E-11	2.10E-10	1.34E-10	1.76E-11	6.29E-13
		Width	0.0012	0.0026	0.0023	0.0019	0.0018	0.0009	0.0026	0.0019	0.0022	0.0012	0.0020	0.0012



TABLE V  
 BAYES ESTIMATES, Rab and ER of  $\alpha$  and  $\lambda$  BASED on URV and URRSS for  $(\alpha, \lambda) = (1.5, 0.5)$ .

Number of records/parameters		3		4		5		6		7		8		
		$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	
URV	SE	Estimates	1.4996	0.4993	1.5003	0.5007	1.4998	0.5009	1.4988	0.5003	1.4995	0.5004	1.5012	0.4996
		Rab	0.0003	0.0014	0.0002	0.0013	0.0001	0.0017	0.0008	0.0005	0.0003	0.0007	0.0008	0.0008
		ER	3.36E-11	1.03E-10	1.40E-11	8.64E-11	5.27E-12	1.53E-10	2.79E-10	1.40E-11	4.43E-11	2.72E-11	2.98E-10	3.34E-11
		Width	0.0015	0.0017	0.0015	0.0017	0.0014	0.0015	0.0029	0.0008	0.0014	0.0021	0.0021	0.0015
	LINEX $v=(2)$	Estimates	1.4990	0.5001	1.5011	0.4999	1.4999	0.4997	1.4981	0.5010	1.5004	0.5015	1.5001	0.5006
		Rab	0.0006	0.0002	0.0007	0.0001	0.0001	0.0005	0.0013	0.0021	0.0003	0.0031	0.0000	0.0011
		ER	1.82E-10	1.73E-12	2.22E-10	6.80E-13	4.45E-12	1.39E-11	7.45E-10	2.18E-10	3.68E-11	4.68E-10	5.87E-13	6.54E-11
		Width	0.0019	0.0020	0.0030	0.0025	0.0015	0.0022	0.0033	0.0032	0.0012	0.0026	0.0009	0.0011
	LINEX $v=(-2)$	Estimates	1.5006	0.5001	1.4991	0.4998	1.5001	0.5013	1.5000	0.4994	1.5001	0.5010	1.4988	0.4998
		Rab	0.0004	0.0002	0.0006	0.0003	0.0001	0.0027	0.0000	0.0011	0.0000	0.0020	0.0008	0.0004
		ER	8.37E-11	2.37E-12	1.62E-10	5.79E-12	1.77E-12	3.52E-10	2.06E-14	6.07E-11	1.09E-12	2.09E-10	3.01E-10	9.02E-12
		Width	0.0019	0.0014	0.0023	0.0017	0.0023	0.0021	0.0009	0.0014	0.0017	0.0020	0.0025	0.0022
URRSS	SE	Estimates	1.5001	0.4999	1.5006	0.5000	1.4989	0.5001	1.4996	0.4994	1.4993	0.4998	1.4988	0.5014
		Rab	0.0001	0.0001	0.0004	0.0001	0.0007	0.0002	0.0003	0.0012	0.0005	0.0004	0.0008	0.0027
		ER	1.72E-12	7.77E-13	6.48E-11	1.50E-13	2.41E-10	1.66E-12	3.03E-11	7.13E-11	1.03E-10	9.59E-12	3.01E-10	3.72E-10
		Width	0.0009	0.0011	0.0022	0.0018	0.0021	0.0021	0.0013	0.0026	0.0017	0.0017	0.0021	0.0019
	LINEX $v=(2)$	Estimates	1.4985	0.5009	1.5007	0.4992	1.4992	0.4993	1.5003	0.4994	1.5004	0.5001	1.4992	0.4988
		Rab	0.0010	0.0017	0.0005	0.0016	0.0005	0.0013	0.0002	0.0013	0.0003	0.0002	0.0005	0.0023
		ER	4.31E-10	1.50E-10	1.08E-10	1.35E-10	1.31E-10	8.90E-11	1.60E-11	8.34E-11	3.24E-11	1.96E-12	1.28E-10	2.73E-10
		Width	0.0022	0.0020	0.0021	0.0029	0.0013	0.0023	0.0013	0.0011	0.0011	0.0014	0.0022	0.0015
	LINEX $v=(-2)$	Estimates	1.5003	0.4998	1.4992	0.4985	1.5002	0.5017	1.4999	0.4996	1.5011	0.4992	1.4981	0.5003
		Rab	0.0002	0.0004	0.0006	0.0031	0.0001	0.0033	0.0001	0.0008	0.0007	0.0015	0.0012	0.0007
		ER	2.15E-11	7.58E-12	1.41E-10	4.72E-10	9.32E-12	5.54E-10	3.57E-12	2.94E-11	2.23E-10	1.17E-10	7.00E-10	2.37E-11
		Width	0.0015	0.0008	0.0015	0.0025	0.0015	0.0029	0.0015	0.0010	0.0019	0.0019	0.0027	0.0028

TABLE VI  
 BAYES ESTIMATES, Rab and ER of  $\alpha$  and  $\lambda$  BASED ON URV and URRSS for  $(\alpha, \lambda) = (2, 1)$ .

Number of records/parameters		3		4		5		6		7		8		
		$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	$\alpha$	$\lambda$	
URV	SE	Estimates	1.0006	2.0008	1.0006	1.9999	1.0002	2.0020	1.0003	2.0010	0.9986	1.9991	0.9990	2.0003
		Rab	0.0006	0.0004	0.0006	0.0000	0.0002	0.0010	0.0003	0.0005	0.0014	0.0004	0.0010	0.0002
		ER	3.18E-10	9.90E-11	3.19E-12	6.61E-11	3.07E-11	4.74E-12	4.15E-10	6.86E-11	6.54E-14	1.35E-11	2.17E-10	3.21E-10
		Width	0.0014	0.0014	0.0018	0.0016	0.0012	0.0041	0.0011	0.0012	0.0036	0.0018	0.0022	0.0021
	LINEX $v=(2)$	Estimates	1.0006	1.9984	1.0005	2.0004	1.0009	1.9998	1.0013	1.9981	0.9999	2.0015	0.9990	1.9995
		Rab	0.0006	0.0008	0.0005	0.0002	0.0009	0.0001	0.0013	0.0009	0.0001	0.0007	0.0010	0.0002
		ER	2.65E-11	5.90E-10	8.24E-11	9.90E-13	8.54E-12	1.16E-10	6.63E-12	2.93E-11	1.53E-10	9.15E-13	6.21E-12	5.89E-12
		Width	0.0012	0.0029	0.0019	0.0021	0.0018	0.0013	0.0024	0.0025	0.0018	0.0024	0.0024	0.0018
	LINEX $v=(-2)$	Estimates	1.0007	1.9988	1.0002	2.0011	0.9989	2.0005	0.9985	1.9999	0.9996	1.9998	1.0013	2.0004
		Rab	0.0007	0.0006	0.0002	0.0005	0.0011	0.0002	0.0015	0.0000	0.0004	0.0001	0.0013	0.0002
		ER	1.12E-10	2.06E-11	1.20E-10	1.25E-12	2.99E-11	1.45E-10	9.47E-11	1.79E-14	4.44E-10	3.01E-10	2.67E-10	1.66E-10
		Width	0.0025	0.0019	0.0012	0.0034	0.0019	0.0011	0.0024	0.0014	0.0016	0.0012	0.0028	0.0011
URRSS	SE	Estimates	1.0005	2.0004	1.0008	2.0003	1.0001	1.9984	1.0004	1.9997	1.0002	2.0001	1.0001	1.9998
		Rab	0.0005	0.0002	0.0008	0.0002	0.0001	0.0008	0.0004	0.0001	0.0002	0.0000	0.0001	0.0001
		ER	5.35E-10	1.46E-11	1.33E-12	2.53E-10	2.77E-10	5.24E-10	1.89E-10	1.56E-10	3.08E-10	4.88E-10	4.62E-10	2.42E-13
		Width	0.0016	0.0010	0.0020	0.0017	0.0019	0.0031	0.0014	0.0012	0.0017	0.0013	0.0013	0.0019
	LINEX $v=(2)$	Estimates	1.0000	1.9985	1.0010	2.0008	0.9999	2.0008	1.0010	1.9993	1.0003	2.0007	1.0004	1.9990
		Rab	0.0000	0.0008	0.0010	0.0004	0.0001	0.0004	0.0010	0.0003	0.0003	0.0003	0.0004	0.0005
		ER	7.36E-11	2.86E-11	1.05E-10	5.31E-12	2.41E-11	4.21E-13	2.84E-11	1.42E-15	6.81E-11	1.03E-10	1.16E-11	2.38E-11
		Width	0.0015	0.0025	0.0022	0.0034	0.0018	0.0012	0.0015	0.0021	0.0012	0.0016	0.0010	0.0020
	LINEX $v=(-2)$	Estimates	0.9996	1.9994	1.0010	1.9999	1.0004	2.0001	1.0020	1.9994	1.0010	1.9992	1.0003	1.9999
		Rab	0.0004	0.0003	0.0010	0.0001	0.0004	0.0001	0.0020	0.0003	0.0010	0.0004	0.0003	0.0000
		ER	4.74E-10	4.83E-11	2.55E-11	5.51E-11	6.88E-12	1.17E-10	4.31E-10	2.11E-12	5.11E-11	1.82E-11	2.14E-12	1.21E-11
		Width	0.0012	0.0026	0.0023	0.0019	0.0018	0.0009	0.0026	0.0019	0.0022	0.0012	0.0020	0.0012

From Table I one can observe that the width of the Bayes credible intervals for  $\hat{\alpha}_{SE}$  is shorter than the corresponding  $\hat{\alpha}_{SE}$  for all the number of records except at  $n = 8$ . Also the width for  $\tilde{\alpha}_{LINEX}$  when  $\nu = 2$  is shorter than that of the corresponding  $\tilde{\alpha}_{LINEX}$  for  $n = 4, 5, 6$  but when  $\nu = -2$ , the width of the Bayes credible intervals for  $\tilde{\alpha}_{LINEX}$  is shorter than that of the corresponding  $\tilde{\alpha}_{LINEX}$  in case of  $n = 4, 5, 6$ .

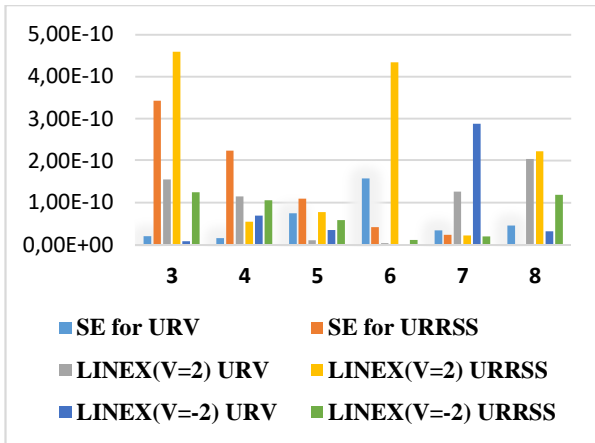


Fig. 3 ERs of  $\alpha$  under SE and LINEX loss functions based on URV and URRSS at  $(\alpha, \lambda) = (0.5, 0.5)$

From Fig. (3) one can observe that the ERs of  $\hat{\alpha}_{SE}$  are less than the ERs of corresponding  $\hat{\alpha}_{SE}$  for  $n = 6, 7, 8$ , also ERs of  $\tilde{\alpha}_{LINEX}$  when  $\nu = 2$  are less than the ERs of corresponding  $\tilde{\alpha}_{LINEX}$  for all the number of records except at  $n = 4, 7$ , but when  $\nu = -2$ , ERs of  $\tilde{\alpha}_{LINEX}$  is shorter than that ERs of corresponding  $\tilde{\alpha}_{LINEX}$  for all number of records except at  $n = 7$ .

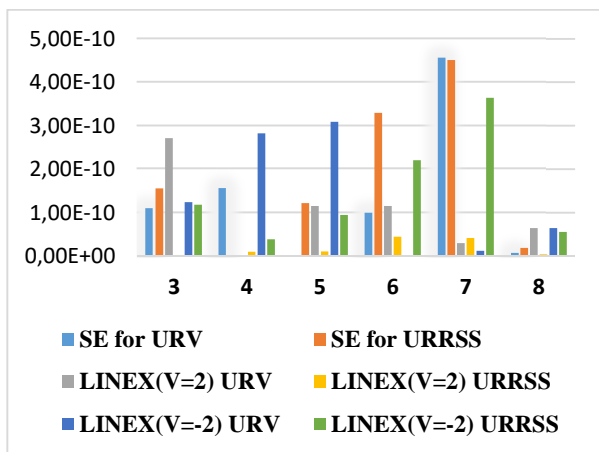


Fig. 4 ERs of  $\lambda$  under SE and LINEX loss functions based on URV and URRSS at  $(\alpha, \lambda) = (0.5, 0.5)$

Fig. (4) shows that the ERs of  $\hat{\lambda}_{SE}$  are less than the ERs of the corresponding  $\hat{\lambda}_{SE}$  for all number of records except at  $n = 4, 7$ . But the ERs of  $\tilde{\lambda}_{LINEX}$  when  $\nu = 2$  are less than the

ERs of the corresponding  $\tilde{\lambda}_{LINEX}$  for all number of records except at  $n = 4, 7$ . On the other hand, the ERs of  $\tilde{\lambda}_{LINEX}$  when  $\nu = -2$  are less than the ERs of the corresponding  $\tilde{\lambda}_{LINEX}$  for all number of records except at  $n = 6, 7$ .

From Table II one can observe that, the width of the Bayes credible intervals for  $\hat{\lambda}_{SE}$  is shorter than that of the corresponding  $\hat{\lambda}_{SE}$  for all number of records except at  $n = 5, 6$ . While the width of credible intervals for  $\hat{\alpha}_{SE}$  is shorter than that of the corresponding  $\hat{\alpha}_{SE}$  for all number of records except at  $n = 5, 8$ . Also the width of the Bayes credible intervals for  $\tilde{\lambda}_{LINEX}$  when  $\nu = 2$  is shorter than that corresponding of  $\tilde{\lambda}_{LINEX}$ .

For  $n \leq 5$  the Rabs of  $\hat{\alpha}_{SE}$  are less than that of the corresponding  $\hat{\alpha}_{SE}$ , but the Rabs of  $\hat{\lambda}_{SE}$  is less than that of the corresponding  $\hat{\lambda}_{SE}$  for all number of records except at  $n = 4$ . However the Rabs for  $\tilde{\alpha}_{LINEX}$  when  $\nu = -2$  are less than that of the corresponding  $\tilde{\alpha}_{LINEX}$  for all number of records except at  $n = 4$  according to Table II.

Table III shows that the ERs of  $\hat{\lambda}_{SE}$  are less than the ERs of the corresponding  $\hat{\lambda}_{SE}$  for all number of records except at  $n = 5, 7$ , the ERs of  $\tilde{\lambda}_{LINEX}$  when  $\nu = -2$  are less than the ERs of the corresponding  $\tilde{\lambda}_{LINEX}$  for all number of records except at  $n = 6, 8$ . The ERs of  $\tilde{\alpha}_{LINEX}$  when  $\nu = 2$  are less than the ERs of the corresponding  $\tilde{\alpha}_{LINEX}$  for all number of records except at  $n = 5, 8$  and the ERs of  $\tilde{\alpha}_{LINEX}$  when  $\nu = -2$  are less than the ERs of the corresponding  $\tilde{\alpha}_{LINEX}$  for all number of records except at  $n = 3, 8$ .

As shown in Table III, the width of the Bayes credible intervals for  $\hat{\lambda}_{SE}$  is shorter than that of the corresponding  $\hat{\lambda}_{SE}$  for all number of records except at  $n = 6$ . The width of the credible intervals for  $\tilde{\lambda}_{LINEX}$  when  $\nu = 2$  is shorter than that width of the credible intervals for the corresponding  $\tilde{\lambda}_{LINEX}$  for all number of records except at  $n = 4, 6$ . But when  $\nu = -2$  the width of the credible intervals for  $\tilde{\alpha}_{LINEX}$  is shorter than that of the corresponding  $\tilde{\alpha}_{LINEX}$  for all number of records except at  $n = 5, 7$ .

Table IV illustrates that the ERs of  $\hat{\lambda}_{SE}$  are less than the ERs of the corresponding  $\hat{\lambda}_{SE}$  for all number of records except at  $n = 4$ . The ERs of  $\hat{\alpha}_{SE}$  are less than the ERs of the corresponding  $\hat{\alpha}_{SE}$  for all number of records except at  $n = 4, 6$ . While, the ERs of  $\tilde{\lambda}_{LINEX}$  when  $\nu = -2$  are less than the ERs of the corresponding  $\tilde{\lambda}_{LINEX}$  for all number of records except at  $n = 6, 8$  and the ERs of  $\tilde{\alpha}_{LINEX}$  when  $\nu = 2$

are less than the ERs of the corresponding  $\tilde{\alpha}_{LINEX}$  for all number of records except at  $n = 4, 7$ .

The width of the Bayes credible intervals for  $\hat{\lambda}_{SE}$  is shorter than that of the corresponding  $\hat{\lambda}_{SE}$  for all number of records except at  $n = 4$ . While, the width of the credible intervals for  $\tilde{\lambda}_{LINEX}$  when  $\nu = 2$  is shorter than that of the corresponding  $\tilde{\lambda}_{LINEX}$  for all number of records except at  $n = 4, 8$  according to Table IV.

Table V shows that the ERs of  $\hat{\alpha}_{SE}$  are less than the ERs of the corresponding  $\hat{\alpha}_{SE}$  for all number of records except at  $n = 3, 6$ . While, the ERs of  $\hat{\lambda}_{SE}$  are less than the ERs of the corresponding  $\hat{\lambda}_{SE}$  for all number of records except at  $n = 6, 8$ . The ERs of  $\tilde{\lambda}_{LINEX}$  when  $\nu = 2$  are less than the ERs of the corresponding  $\tilde{\lambda}_{LINEX}$  for all number of records except at  $n = 6, 7$ . While, when  $\nu = -2$  the ERs of  $\tilde{\lambda}_{LINEX}$  are less than the ERs of the corresponding  $\tilde{\lambda}_{LINEX}$  for all number of records except at  $n = 6, 7$ .

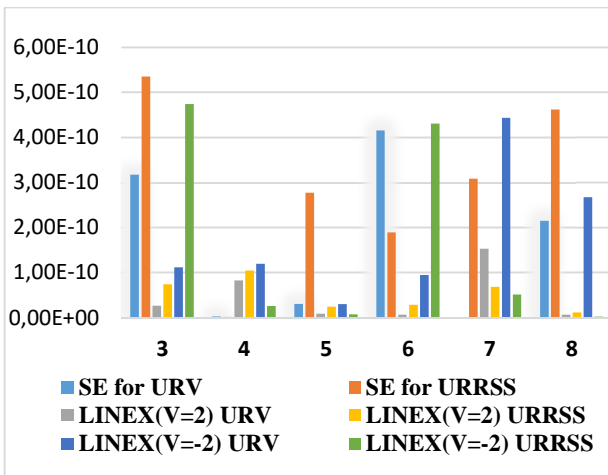


Fig. 5 ERs of  $\alpha$  under SE and LINEX loss functions based on URV and URRSS at  $(\alpha, \lambda) = (2, 1)$

From Fig. (5), the ERs of  $\hat{\alpha}_{SE}$  are less than the ERs of the corresponding  $\hat{\alpha}_{SE}$  for all the number of records except for  $n = 4, 6$ . Also, when  $\nu = 2$  the ERs of  $\tilde{\alpha}_{LINEX}$  are less than of the corresponding  $\tilde{\alpha}_{LINEX}$  for all number of records except at  $n = 7$ . But when  $\nu = -2$ , the ERs of the  $\tilde{\alpha}_{LINEX}$  are less than the ERs of the corresponding  $\tilde{\alpha}_{LINEX}$  for all the number of records except at  $n = 3, 6$ .

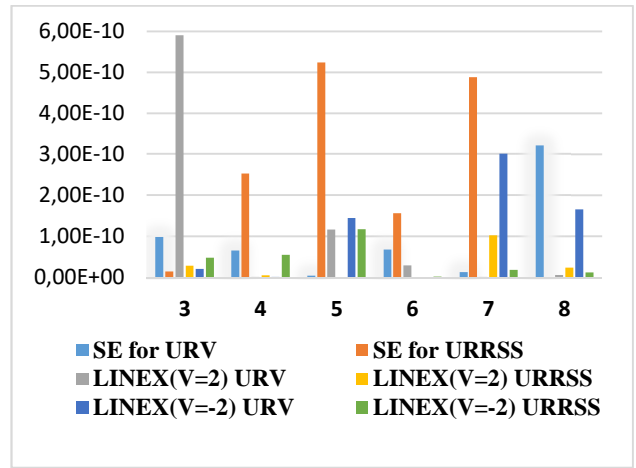


Fig. 6 ERs of  $\lambda$  under SE and LINEX loss functions based on URV and URRSS at  $(\alpha, \lambda) = (2, 1)$

Fig. (6) shows that the ERs of  $\hat{\lambda}_{SE}$  are less than of the corresponding  $\hat{\lambda}_{SE}$  for all number of records except at  $n = 3, 8$ . Also, the ERs of  $\tilde{\lambda}_{LINEX}$  when  $\nu = 2$  are less than the ERs of the corresponding  $\tilde{\lambda}_{LINEX}$  for  $n = 3, 5, 6$ . The ERs of  $\tilde{\lambda}_{LINEX}$  when  $\nu = -2$  are less than the ERs of the corresponding  $\tilde{\lambda}_{LINEX}$  for  $n = 5, 7, 8$ .

Table VI shows that, the width of the Bayes credible intervals for  $\hat{\alpha}_{SE}$  is shorter than that corresponding  $\hat{\alpha}_{SE}$  for all number of records except at  $n = 7, 8$ . also, The width of the Bayes credible intervals for  $\tilde{\alpha}_{LINEX}$  when  $\nu = 2$  is shorter than that corresponding  $\tilde{\alpha}_{LINEX}$  for all number of records except at  $n = 3, 8$

#### IV. CONCLUSION

In this paper, we presented how to develop Bayes estimates in the context of upper record values and upper record ranked set sampling from generalized inverted exponential distribution under symmetric and asymmetric loss functions.

Based on the URV and URRSS, it is observed that the Bayes estimators cannot be obtained in explicit forms. Therefore, the MCMC technique has been used to generate posterior samples.

We observe from the numerical study that the relative absolute biases, estimated risks and widths of confidence intervals are very small based on the two sampling schemes for both SE and LINEX loss functions.

Generally, the Bayes estimates under LINEX loss function when  $\nu = -2$  perform better than the Bayes estimates under LINEX loss function when  $\nu = 2$  in case of URV in approximately most of situations. While the Bayes estimates under LINEX loss function when  $\nu = 2$  perform better than estimates under LINEX loss function when  $\nu = -2$  in case of URRSS in approximately most of situations.

## REFERENCES

- [1] K. N. Chandler, "The distribution and frequency of record values," *Journal of the Royal Statistical Society*, vol. 14(2), pp. 220-228, 1952.
- [2] H. N. Nagaraja, "Record values and related statistics - a review," *Communications in Statistics - Theory and Methods*, vol. 17(7), pp. 2223-2238, 1988.
- [3] B. C. Arnold, N. Balakrishnan, and H. N. Nagaraja, *Records*, ser. Wiley series in probability and statistics, Canada: John Wiley & Sons, Inc., 1998.
- [4] M. Ahsanullah and V. B. Nevzorov, *Records via probability theory*, ser. Atlantis studies in probability and statistics, Tampa, USA: Atlantis Press, 2015, vol. 6.
- [5] M. Salehi and J. Ahmadi, "Record ranked set sampling scheme," *Metron*, vol. 72(3), pp. 351-365, 2014.
- [6] M. A. M. Ali Mousa, Z. F. Jaheen and A. A. Ahmad, "Bayesian estimation, prediction and characterization for the Gumbel model based on records," *Statistics: A Journal of Theoretical and Applied Statistics*, vol. 36(1), pp. 65-74, 2002.
- [7] Z. F. Jaheen, "A Bayesian analysis of record statistics from the Gompertz model," *Applied Mathematics and Computation*, vol. 145(2-3), pp. 307-320, 2003.
- [8] M. Doostparast and J. Ahmadi, "Statistical analysis for geometric distribution based on records," *Computers & Mathematics with Applications*, vol. 52(6-7), pp. 905-916, 2006.
- [9] A. A. Soliman and F. M. Al-Aboud, "Bayesian inference using record values from Rayleigh model with application," *European Journal of Operational Research*, vol. 185(2), pp. 659-672, 2008.
- [10] A. Baklizi, "Likelihood and Bayesian estimation of using lower record values from the generalized exponential distribution," *Computational Statistics & Data Analysis*, vol. 52(7), pp. 3468-3473, 2008.
- [11] M. D. Habib, A. M. Abd-Elfattah and M. A. Selim, "Bayesian and Non-Bayesian estimation for exponentiated-Weibull distribution based on record values," *The Egyptian Statistical Journal*, vol. 54(2), pp. 47-59, 2010.
- [12] M. Nadar and A. S. Papadopoulos, "Bayesian analysis for the Burr type XII distribution based on record values," *STATISTICA*, vol. 71(4), pp. 421-435, 2011.
- [13] M. Doostparast, M. G. Akbari and N. Balakrishna, "Bayesian analysis for the two-parameter Pareto distribution based on record values and times," *Journal of Statistical Computation and Simulation*, vol. 81(11), pp. 1393-1403, 2011.
- [14] S. Dey and T. Dey, "Bayesian estimation of the parameter and reliability of a Rayleigh distribution using records," *Model Assisted Statistics and Applications*, vol. 7(2012), pp. 81-90, 2012.
- [15] F. Kızılaslan and M. Nadar, "Classical and Bayesian analysis for the generalized exponential distribution based on record values and times," *Bilim ve Teknoloji Dergisi B-Teorik Bilim*, vol. 2, pp. 111-120, 2013.
- [16] S. Dey, T. Dey, M. Salehi and J. Ahmadi, "Bayesian inference of generalized exponential distribution based on lower record values," *American Journal of Mathematical and Management Sciences*, vol. 32(1), pp. 1-18, 2015.
- [17] R. M. El-Sagheer, "Bayesian estimation based on record values from exponentiated Weibull distribution: An Markov Chain Monte Carlo approach," *American Journal of Theoretical and Applied Statistics*, vol. 4(1), pp. 26-32, 2015.
- [18] J. I. Seo and Y. Kim, "Bayesian inference on extreme value distribution using upper record values," *Communications in Statistics - Theory and Methods*, vol. 46(15), pp. 7751-7768, 2016.
- [19] S. Dey, T. Dey, and D. J. Lockett, "Statistical inference for the generalized inverted exponential distribution based on upper record values," *Mathematics and Computers in Simulation*, vol. 120, pp. 64-78, 2016.
- [20] Z. Khoshkhou Amiri and S. M. T. K. MirMostafae. "Estimation of the shape parameter of the weighted exponential distribution under the record ranked set sampling plan," in *Proc. Second Seminar on Reliability Theory and its applications*, 2016, p. 115.
- [21] S. K. Singh, U. Singh and M. Kumar, "Estimation of parameters of generalized inverted exponential distribution for progressive type-II censored sample with binomial removals," *Journal of Probability and Statistics*, vol. 2013, pp. 1-12, 2013.
- [22] A. M. Abouammoh and A. M. Alshingiti, "Reliability estimation of generalized inverted exponential distribution," *Journal of Statistical Computation and Simulation*, vol. 79(11), pp. 1301-1315, 2009.