INTERNATIONAL JOURNAL OF COMPUTING SCIENCE, VOL. 1, NO. 3, April 2012
ISSN (Online): 2164-1374, ISSN (Print): 2164-1366, Published online April, 2012 (http://www.researchpub.org/journal/ijcs/ijcs.html)

A Multi-Objective Chaotic Harmony Search Technique for Structural Optimization

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***Abstract*—In this paper, a new Multi-Objective Evolutionary technique is introduced. The new method incorporates Harmony Search optimization to Chaos search. The well known Fitness Sharing method is employed to adopt the size of the external archive used by the technique during search. The proposed method is applied to Structural optimization which is one of the most challenging areas in Multi-Objective Optimization. The proposed technique is applied to two-bar truss problem, and the solution resulted shows superiority of the proposed method over the є-constraint method in terms of closeness and spread.**

***Index Terms*—Chaos, Harmony search algorithm, Multi-objective optimization, Structural optimization, Two-bar truss.**

I. Introduction

All areas of human interaction with its environment involve decision situations, daily we face hundreds of situations to take decisions about, and decision situations often involve multiple criteria or objectives. In many cases, objectives are incommensurable, meaning they are not comparable with respect to magnitude and value, and conflicting, meaning that the different objectives cannot be arbitrarily improved without decreasing the value of another.

Multi-Objective programming is a part of mathematical programming dealing with decision problems characterized by multiple and conflicting objective functions that are to be optimized over a feasible set of decisions. Such problems, referred to as Multi-Objective Programs (MOPs), are commonly encountered in many areas of human activity including engineering, management, and others [7].

The Harmony Search (HS) algorithm originally came from the analogy between music improvisation and optimization process [11]. This algorithm has been successfully applied to various discrete optimization problems such as traveling salesperson problem [11], tour routing [10], music composition [8], and water network design [9].

The scientific meaning of the term Chaotic System or Chaos for short is a phenomenon that has deterministic rules behind irregular appearances [2]. Chaos is a kind of common nonlinear phenomenon, which has diverse, complex and sophisticated native under apparent disorder. Chaotic motion is characterized by ergodicity, randomness, and 'regularity' which can traverse all status according to its own 'rule' without repetition [4].

A structure in mechanics is defined by J.E. Gordon [12] as "any assemblage of materials which is intended to sustain loads." Optimization means making things the best. Thus, structural optimization is the subject of making an assemblage of materials sustain loads in the best way. The two-bar truss problem is a typical structural optimization problem. In this paper, a new method is developed to solve the two-bar truss problem. The new method combined Harmony Search (HS) algorithm to chaos search in order to enhance exploration during search. The proposed method called Multi-Objective Chaotic Harmony Search (MOCHS) uses an external archive for keeping the *nondominated* solutions gained during search. A fitness sharing technique is used to fix the size of the external archive. The rest of this paper is organized as follows: section II is made for the Multi-Objective Optimization, section III is devoted to the Harmony Search algorithm, Chaos is presented in section IV, in section V, the new proposed method is illustrated, the two-bar truss problem is discussed in section VI, and finally section VII is made for conclusion.

II. Multi-Objective Optimization

A general multi-objective optimization problem consists of a number of objectives to be optimized simultaneously. In general, a k-objective minimization problem can be written as

$$min \ \{(f_1(x),..., f_k(x)): x \in X\} \qquad (1)$$

we usually assume that the set X is given implicitly in the form of constraints resulted in the feasible region in the decision space [3], i.e., $X := \{x \in R^n : g_j(x) \le 0, j = 1,...,s;\ h_j(x) = 0, j = 1,...,m\}$.

In Multi-Objective Optimization (MOO) the objective functions constitute a multi-dimensional space, in addition to the usual variable space. For each solution x in the variable space, there exists a point in the objective space, denoted by $f(x) = z = (z_1, z_2, \ldots, z_k)^T$. A mapping exists between an n-dimensional solution vector and a k-dimensional objective vector through the objective function, constraints, and variable bounds [3].

Definition 1 (Pareto Dominance): Without loss of generality in a minimization problem, a decision vector $x_1 \in X$ is said to *dominate* a decision vector $x_2 \in X$ *iff* the following two conditions are satisfied:

1. The decision vector x_1 is not worse than x_2 in all objectives, or $\forall\ i \in \{1,2,\ldots,k\} : f_i(x_1) \le f_i(x_2)$.
2. The decision vector x_1 is strictly better than x_2 in at least one objective, or $\exists\ i \in \{1,2,\ldots,k\} : f_i(x_1) < f_i(x_2)$.

Manuscript received March 9th, 2012.
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ISSN (Online): 2164-1374, ISSN (Print): 2164-1366, Published online April, 2012 (http://www.researchpub.org/journal/ijcs/ijcs.html)

If any of the above conditions is violated, then x_1 does not dominate x_2. A decision vector $x_1 \in X$ is called *Pareto-optimal* if there is no another $x_2 \in X$ that *dominates* it and in this case x_1 is called *nondominated* with respect to X ; also an objective vector is called *Pareto-optimal* if the corresponding decision vector is *Pareto-optimal*.

Definition 2 (Pareto Optimal Set): The Pareto Optimal Set P^* is defined by [16]:

$$P^* = \{x \in X \mid x \text{ is } \textit{pareto-optimal}\} \quad (2)$$

Many researchers had developed a lot of mathematical programming techniques to solve Multi-Objective Optimization problems, some representatives of this class of techniques are the weighting method, the ϵ-constraint method, and the goal programming; also some authors had adopted the Evolutionary Algorithms (EAs), Simulated Annealing (SA), Genetic Algorithms (GA), as well as Swarm Intelligence (SI) techniques to deal with Multi-Objective Problems [6].

III. HARMONY SEARCH ALGORITHM

The HS algorithm was originally developed by Geem et al. in 2001, and is based on natural musical performance processes that occur when a musician searches for a better state of harmony, such as during jazz improvisation. Jazz improvisation seeks to find musically pleasing harmony (a perfect state) as determined by an aesthetic standard, just as the optimization process seeks to find a global solution (a perfect state) as determined by an objective function. The pitch of each musical instrument determines the aesthetic quality, just as the objective function value is determined by the set of values assigned to each design variable [14]. The HS algorithm optimization procedure which is shown below consists of the following five steps

Step 1: Parameter Initialization

The HS algorithm parameters are also specified in this step. These are the harmony memory size (*HMS*), or the number of solution vectors in the harmony memory; harmony memory considering rate (*HMCR*); pitch adjusting rate (*PAR*); bandwidth distance (*bw*); and the number of improvisations (*NI*), or stopping criterion. The harmony memory (*HM*) is a memory location where all the solution vectors (sets of decision variables) are stored.

Step 2: Harmony Memory Initialization and Evaluation

The *HM* matrix is randomly generated as follows

$$x_{i,j}^{0} = x_j^{\min} + r_j \text{ x } (x_j^{\max} - x_j^{\min}) \quad (3)$$

where $i = 1,2,...,HMS$; $j = 1,2,...,N$; and $r_j \in [0,1]$ is a uniformly distributed random number generated new for each value of *j*. solution vectors in *HM* are analyzed, and their objective function values are calculated.

Step 3: New Harmony Improvisation

In this step, a new harmony vector is generated based on three rules. They are memory consideration, pitch adjustment, and random selection. The value of a design variable can be selected from the values stored in *HM* with a probability of harmony memory considering rate (*HMCR*). It can be further adjusted by moving to a neighbor value of a selected value from the *HM* with a probability of pitch adjusting rate (*PAR*). Or, it can be selected randomly from the set of all candidate values without considering the stored values in *HM*, with the probability of (1 - *HMCR*).

Step 4: Harmony Memory Update

The new better harmony vector is included in the *HM* and the worst harmony is excluded.

Step 5: Termination Criterion Check

The HS algorithm is terminated when the termination criterion (e.g. maximum number of improvisations) has been met. Otherwise, steps 3 and 4 are repeated.

IV. CHAOS

Chaos is a deterministic, random-like process found in nonlinear, dynamical system, which is non-period, non-converging and bounded. Moreover, it has a very sensitive dependence upon its initial condition and parameter. The nature of chaos is apparently random and unpredictable and it also possesses an element of regularity. Mathematically, chaos is randomness of a simple deterministic dynamical system and chaotic system may be considered as sources of randomness [1]. A chaotic map is a discrete-time dynamical system

$$z_{k+1} = f(z_k), \quad 0 < z_k < 1, \; k = 0,1,2,\ldots \quad (4)$$

running in the chaotic state. The chaotic sequence $\{z_k : k = 0,1,2,\ldots\}$ can be used as spread-spectrum sequence and as a random number sequence.

One-dimensional noninvertible maps are the simplest systems with capability of generating chaotic motion [17]. Here, two well-known one-dimensional maps are introduced. Later on, these maps are used in the chaotic searches.

The Logistic map: In 1976, Robert May pointed out that the logistic map led to chaotic dynamics. A logistic map is a polynomial map. It is often cited as an example of how complex behavior can arise from a very simple nonlinear dynamical equation [17]. This map is defined by

$$z_{k+1} = \mu \, z_k (1 - z_k) \quad (5)$$

Obviously, $z_k \in [0,1]$ under the conditions that the initial $z_0 \in [0,1]$, where k is the iteration number and $\mu = 4$.

The Circle map: The circle map [19] is represented by

$$z_{n+1} = z_n + d - (c/2\pi)\sin(2\pi z_n) \bmod(1) \quad (6)$$

where $c = 0.5$, $d = 0.2$, and $z_0 \in [0,1]$ generates chaotic sequence in [0,1].

Chaos has been extended to various optimization areas like in [5],[15], and [20] because it can more easily escape from local minima than other stochastic optimization algorithms. Recently, chaotic sequences have been adopted instead of random sequences and very interesting and somewhat good results have been shown in many applications.

V. MULTI-OBJECTIVE CHAOTIC HARMONY SEARCH TECHNIQUE

Some modifications should be done in order to extend the original HS algorithm to optimize the Multi-Objective version. The step of updating HM includes a method which is different from what is done in single search. The external archive being as well known technique used to handle the problem of increasing *nondominated* solutions over the steps is applied in this proposal. Diversification of solutions kept in the archive is maintained through updating the archive by using the fitness sharing method. The main idea of fitness sharing is to distribute a population of individuals along a set of resources [13]. When an individual *i* is sharing resources with other individuals, its fitness f_i is degraded in proportion to the number and closeness to individuals that surround it, and in this way promoting and maintaining diversity. In general Fitness sharing for an individual *i* is defined as:

ISSN (Online): 2164-1374, ISSN (Print): 2164-1366, Published online April, 2012 (http://www.researchpub.org/journal/ijcs/ijcs.html)

$$fshare_i = \frac{f_i}{\sum_{j=0}^{n} sharing_i^j} \quad (7)$$

where n is the number of individuals in the population.

The detailed steps of the Multi-Objective Chaotic Harmony Search technique (MOCHS) are as follows:

Step 1: Setting the HS algorithm parameters {*HMS*, *HMCR*, *NI*}.

Step 2: Initializing *HM* by iterating the selected chaotic maps until if reaches the *HMS*. This process produces *HMS* candidate solutions (vectors) randomly in the search space. **Step 3:** Generating new harmony improvisations based on the three updating rules as illustrated in section 3. In this algorithm *PAR* and *bw* values have not been fixed in *HS* and they have been modified by the selected chaotic maps as follows

$$PAR(t+1) = f(PAR(t)), \; 0 < PAR(t) < 1, \quad t = 1,2,\ldots \quad (8)$$

$$bw(t+1) = f(bw(t)), \; 0 < bw(t) < 1, \quad t = 1,2,\ldots \quad (9)$$

Step 4: Updating *HM*. In this step the main distinctions between single and multi-objective versions are illustrated due to the existence of the external archive in multi-objective case. In step 3, the new harmony memory generated by improvisation process, is combined with the existing harmony memory to form (2 x *HMS*) solution vectors. Then *non-dominated* sorting and ranking procedure is performed on the combined harmony memory. The *nondominated* solutions resulted are transferred to update the external archive. Fixing the size of the external archive is done using the fitness sharing scheme to maintain diversity among the solutions found in the archive. At each step, the top-k *nondominated* solutions ranked according to the fitness sharing scheme are combined to *HM*.

Step 5: Checking to ensure the termination criterion has been met. If so, print the solutions found in the external archive. Otherwise, steps 3 and 4 are repeated.

VI. TWO-BAR TRUSS PROBLEM

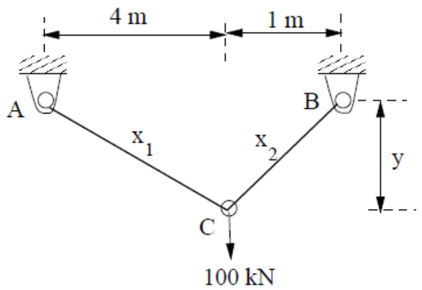

Fig. 1: The Two-Bar Truss Problem

This problem was originally studied using the ϵ-constraint method [18]. As shown in Fig. 1, the truss has to carry a certain load without elastic failure. Thus, in addition to the objective of designing the truss for minimum volume (which is equivalent to designing for minimum cost of fabrication), there are additional objectives of minimizing stresses in each of the two members AC and BC.

The two-objective optimization problem for three variables y (vertical distance between B and C in m), x_1(length of AC in m) and x_2(length of BC in m) is constructed as follows:

$$\begin{aligned} &\text{minimize } f_1(x) = x_1\sqrt{16+y^2} + x_2\sqrt{1+y^2} \\ &\text{minimize } f_2(x) = \max\;(\sigma_{AC}, \sigma_{BC}) \\ &S.T \quad \max\;(\sigma_{AC}, \sigma_{BC}) \le 1(10^5) \\ &\sigma_{AC} = \frac{20\sqrt{16+y^2}}{yx_1}, \quad \sigma_{BC} = \frac{80\sqrt{1+y^2}}{yx_2} \\ &1 \le y \le 3 \text{ and } x \ge 0 \end{aligned} \quad (10)$$

Only five solutions are resulted in the original study with the following spread: (0.004445 m^3, 89983 kPa) and (0.004833 m^3, 83268 kPa). In order to restrict solutions with stress in the above range, an additional constraint of maximum stress being smaller than $1(10^5)$ is added to the original problem.

Fig. 2 shows the pareto front produced using the proposed method. The solutions are spread in the following range: (0.00375 m^3, 99847 kPa) and (0.0537 m^3, 7685 kPa), which indicates the power of the proposed algorithm compared to the ϵ-constraint method. The ϵ-constraint method could not find wide variety of solutions in terms of the second objective.

In the second objective, MOCHS finds a solution with stress as low as 7685 kPa, whereas the ϵ-constraint method has found a solution with minimum stress of 83268 kPa, almost eleven times the minimum stress obtained by the proposed method. MOCHS solutions are better than ϵ-constraint solutions, both in terms of closeness to the optimum front and in their spread.

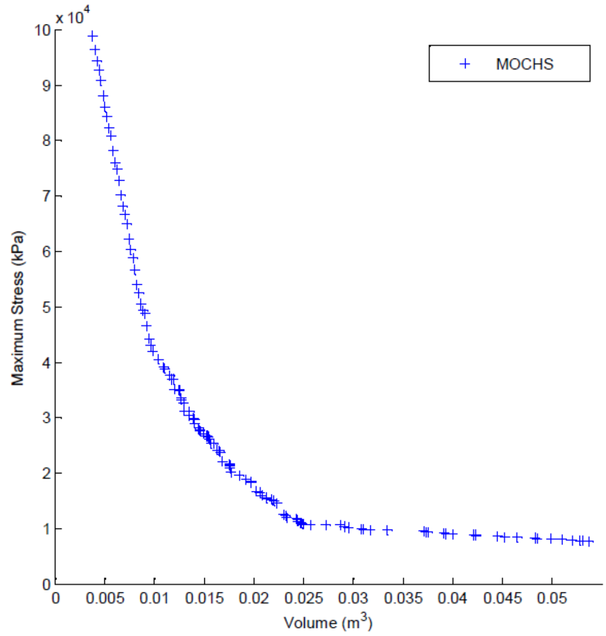

Fig. 2: Pareto front obtained by using MOCHS

VII. CONCLUSION

A new Multi-Objective version of the Harmony Search algorithm is developed using chaotic search combined to the original algorithm to enhance exploration during search. Also an external archive are incorporated to the algorithm in order to keep solutions found and maintain diversity during search. The proposed technique gives a much wider spread of solutions than the classical the ϵ-constraint method. The results are encouraging and suggests immediate application of the proposed method to other more complex engineering design problems.

ISSN (Online): 2164-1374, ISSN (Print): 2164-1366, Published online April, 2012 (http://www.researchpub.org/journal/ijcs/ijcs.html)

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