CV-VIKOR: A New Approach for Allocating Weights in Multi-Criteria Decision Making Problems

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Abstract: Multi-Criteria analysis, often called Multi-Criteria Decision-Making (MCDM) or Multi-Criteria Decision Aid methods (MCDA), is a branch of a general class of Operations Research (OR) models which deal with the process of making decisions in the presence of multiple objectives. These methods, which can handle both quantitative and qualitative criteria, share the common characteristics of conflict among criteria, incommensurable units, and difficulties in design/selection of alternatives. The technique used in this paper named *Vlse Kriterijumska Optimizacija I KOmpromisno Resenje* in Serbian (VIKOR) is combined to the Coefficient of Variation (CV) to constitute a new approach called CV-VIKOR. The Coefficient of Variation (CV) is employed to allocate weights when no preference existed among the criteria considered. Also, a given numerical example is solved to illustrate the proposed method.

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1. Introduction

Multi-criteria decision making (MCDM) methods can select the best solution from several alternatives. Each candidate solution has multiple attributes with

different effects; each attribute is relevant to some criterion. When conflicting criteria exist, even the best solution cannot satisfy all criteria. The merit of MCDM techniques is that they consider both qualitative parameters as well as the quantitative ones, MCDM includes many solution techniques such as Simple Additive Weighting (SAW), Weighting Product(WP) [4], and Analytic Hierarchy Process (AHP) [6].

The compromise solution is a feasible solution that is the closest to the ideal solution, and a compromise means an agreement established by mutual concession. The compromise solution method, also known as (VIKOR) the *Vlse Kriterijumska Optimizacija I KOmpromisno Resenje* in Serbian was introduced as one applicable technique to implement within MADM [1]. In probability theory and statistics, the coefficient of variation (CV) is a normalized measure of dispersion of a probability distribution. In this paper, we try to tackle the problem of the preference absence among criteria, by using the Coefficient of Variation (CV) statistical measure. First, the weights are assigned to criteria by using CV method. Then, after the alternatives are ranked by the VIKOR method. The rest of the paper is structured as following; in section 2 the VIKOR method is illustrated, section 3 is made for the Coefficient of Variation method, a numerical example is presented in section 4, and finally section 5 is for conclusion.

2. VIKOR

A MCDM problem can be concisely expressed in a matrix format, in which columns indicate criteria (attributes) considered in a given problem; and in which rows list the competing alternatives. Specifically, a MCDM problem with m alternatives ($A_1, A_2, \ldots, A_m$) that are evaluated by n criteria ($C_1, C_2, \ldots, C_n$) can be viewed as a geometric system with m points in n-dimensional space. An element x_{ij} of the matrix indicates the performance rating of the i^{th} alternative A_i, with respect to the j^{th} criterion C_j, as shown in Eq. (1):

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{bmatrix} \end{array} \quad (1)$$

The VIKOR method was introduced as an applicable technique to implement within MCDM [5]. It focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. The compromise solution, whose foundation was established by Yu [9] and Zeleny [10] is a feasible solution, which is the closest to the ideal, and here "compromise" means an agreement established by mutual concessions.

The VIKOR method determines the compromise ranking list and the compromise solution by introducing the multi-criteria ranking index based on the particular measure of "closeness" to the "ideal" solution. The multi-criteria measure for compromise ranking is developed from the *Lp-metric* used as an aggregating function in a compromise programming method. The levels of regret in VIKOR can be defined as:

$$L_{p,i} = \{\sum_{j=1}^{n}[w_j(x_j^* - x_{ij})/(x_j^* - x_j^-)]^p\}^{1/p},\ 1 \le p \le \infty, \qquad (2)$$

where $i = 1,2,\ldots,m$. $L_{1,i}$ is defined as the maximum group utility, and $L_{\infty,i}$ is defined as the minimum individual regret of the opponent.

The procedure of VIKOR for ranking alternatives can be described as the following steps [3]:

Step 1: Determine that best x_j^* and the worst x_j^- values of all criterion functions, where $j = 1, 2,\ldots, n$. If the *jth* criterion represents a benefit then $x_j^* = \max_i f_{ij}$, $f_j^- = \min_i f_{ij}$.

Step 2: Compute the S_i (the maximum group utility) and R_i (the minimum individual regret of the opponent) values, $i = 1, 2,\ldots, m$ by the relations:

$$S_i = L_{1,i} = \sum_{j=1}^{n} w_j(x_j^* - x_{ij})/(x_j^* - x_j^-), \qquad (3)$$

$$R_i = L_{\infty,i} = \max_j [\sum_{j=1}^{n} w_j(x_j^* - x_{ij})/(x_j^* - x_j^-)], \qquad (4)$$

where w_i is the weight of the j^{th} criterion which expresses the relative importance of criteria.

Step 3: Compute the value Q_i, $i = 1,2,\ldots,m$, by the relation

$$Q_i = v(S_i - S^*)/(S^- - S^*) + (1-v)(R_i - R^*)/(R^- - R^*), \qquad (5)$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$, and v is introduced weight of the strategy of S_i and R_i.

Step 4: Rank the alternatives, sorting by the *S, R,* and *Q* values in decreasing order. The results are three ranking lists.

Step 5: Propose as a compromise solution the alternative (A') which is ranked the best by the minimum Q if the following two conditions are satisfied:

C1. "Acceptable advantage":

$Q(A'') - Q(A') \ge DQ$, where A'' is the alternative with second position in the ranking list by Q, $DQ = 1/(m - 1)$ and m is the number of alternatives.

C2. "Acceptable stability in decision making":

Alternative A' must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be: "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus" ($v \approx 0.5$), or "with vote" ($v < 0.5$). Here, v is the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility"). $v = 0.5$ is used in this paper. If one of the conditions is not satisfied, then a set of compromise solutions is proposed [3].

Recently, VIKOR has been widely applied for dealing with MCDM problems of various fields, such as environmental policy [7], data envelopment analysis [8], and personnel training selection [2].

3. Coefficient of Variation

The weight of the criterion reflects its importance in MCDM. In this paper, a new method is proposed to allocate weights in MCDM problems with no preference. The new method rely on the well known Coefficient of Variation (CV) to allocate the weights of different criteria. Range standardization was done to transform different scales and units among various criteria into common measurable units in order to compare their weights.

$$x'_{ij} = \frac{x_{ij} - \min_{1\le j\le n} x_{ij}}{\max_{1\le j\le n} x_{ij} - \min_{1\le j\le n} x_{ij}} \qquad (6)$$

$D'=(x')_{m\mathrm{x}n}$ is the matrix after range standardization; max x_{ij}, min x_{ij} are the maximum and the minimum values of the criterion (j) respectively, all values in D' are ($0 \le x'_{ij} \le 1$). So, according to the normalized matrix $D'= (x')_{m\mathrm{x}n}$, the Standard Deviation (σ_j) is calculated for every criterion independently as shown in Eq. (7):

$$\sigma_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(x'_{ij} - \bar{x}'_j)^2} \qquad (7)$$

where $\bar{x}'_j$ is the mean of the values of the j^{th} criterion after normalization and $j = 1,2,\ldots,n$.

After calculating (σ_j) for all criteria, the (CV) of the criterion (j) will be as shown in Eq.(8)

$$CV_j = \frac{\sigma_j}{\bar{x}'_j} \qquad (8)$$

The weight (W_j) of the criterion (j) can be defined as

$$W_j = \frac{CV_j}{\sum_{j=1}^{n} CV_j} \qquad (9)$$

where $j = 1,2,\ldots,n$.

4. Illustrative Example

In this section, an example of six alternatives to be ranked through comparing four criteria is presented in order to explain the method proposed. As shown in Table 1, the six alternatives and their performance ratings with respect to all criteria are presented. All criteria are from the maximization utility type (the maximum is better).

Table 1. Decision matrix

	C_1	C_2	C_3	C_4
Alternative 1	50	5	45	78
Alternative 2	32	8	60	56
Alternative 3	69	2	30	24
Alternative 4	54	7	86	56
Alternative 5	70	6	75	85
Alternative 6	92	1	62	22

In the above example, there is no preference among the criteria, no weights specified for them subjective by the decision maker, so the Coefficient of Variation (CV) method will be applied in this example. Table 2 illustrates the range standardization done to decision matrix as in Eq.(6).

Table 2. Range standardized decision matrix

	C_1	C_2	C_3	C_4
Alternative 1	0.3	0.5714	0.2679	0.8889
Alternative 2	0	1	0.5357	0.5397
Alternative 3	0.6167	0.1429	0	0.0317
Alternative 4	0.3667	0.8571	1	0.5397
Alternative 5	0.6333	0.7143	0.8036	1
Alternative 6	1	0	0.5714	0

Table 3 shows the values of Mean ($\bar{x}'_j$), Standard Deviaton (σ_j), the Coefficient of Variation (CV_j), and the weight assigned to each criterion (W_j) as shown in Eqs. (7-9).

Table 3. Weights assigned to criteria

	$\bar{x}'_j$	σ_j	CV_j	Weights
C_1	0.4861	0.3429	0.7055	0.239
C_2	0.5476	0.3981	0.7270	0.247
C_3	0.5298	0.3598	0.6791	0.230
C_4	0.5	0.4180	0.8360	0.284

By applying the procedure of VIKOR, we can calculate the S, R and Q values as shown in Table 4 in order to rank the alternatives. The fifth alternative should be selected because it has the minimum S, R, and Q values; also, the two conditions mentioned earlier in section 2 are satisfied.

Table 4. Ranking lists and scores

	S	*R*	*Q*	Rank
Alternative 1	0.47311	0.16839	0.42857	3
Alternative 2	0.47652	0.239	0.61117	4
Alternative 3	0.80832	0.27498	0.97704	6
Alternative 4	0.31738	0.15137	0.25651	2
Alternative 5	0.20338	0.08763	0	**1**
Alternative 6	0.62957	0.2840	0.85226	5

5. Conclusion

The Coefficient of Variation describes the dispersion of the values of criteria, giving the more dispersed values criteria much importance and more weight values. Also, being a normalized measure of dispersion, it suits the problem of allocating the weights in MCDM problems with no preference. In this paper, the CV-VIKOR proposed method is presented and illustrated. The new method employed the Coefficient of Variation to allocate the weights in MCDM problems. The proposed approach is illustrated by solving a numerical example, showing it is efficient and effective.

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