Synthesis of Planar Mechanisms, Part V: Six Bar-Three Sliders Mechanism

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Abstract— This paper investigates the synthesis of 6 bar-3 sliders mechanism for motion generation having maximum time ratio and an assigned stroke. The mechanism output is a translation motion. The optimal dimensionless dimensions of the mechanism links are assigned using MATLAB optimization tool. The objective function to be maximized is the mechanisms time ratio subjected to four functional constraints controlling the performance of the mechanism. A desired dimensionless stroke between 3 and 8 is covers. In all the cases, the optimal dimensions are assigned and the transmission angel of the mechanism is kept between 45 and 135 degrees.

Index Terms— 6 bar-3 sliders mechanism, mechanism synthesis, optimal mechanism dimensions, motion generation, maximum time ratio.

I. INTRODUCTION

Six bar mechanisms are in use for various applications and find great attention from researchers regarding their analysis and synthesis. This series of research papers aims at synthesizing planar mechanisms for specific requirements using optimization tools.

Hongying, Zhixing, Dewei and Junsheng (2003) studied a numerical comparison method of planar 6-bar dwell mechanism synthesis. They illustrated their method by computer simulation on mechanism kinematics [1]. Collard, Fisette and Duysinx (2005) presented a simple approach to optimize the dimensions and positions of 2D mechanisms for path or function-generator synthesis. They discussed the question of finding the global optimum and presented three applications including the 6-bar steering linkage [2]. Soh, Gracia and McCarthy (2006) obtained designs for each of the known Watt and Stephenson 6-bar topologies except Watt II. They demonstrated the synthesis process with an example [3].

Dong and Wang (2007) presented an approach for optimum synthesis of 6-bar dwell mechanisms. Based on the adaptive curve fitting approach, a unified mathematical model was established for the synthesis of circular arc and straight line dwell mechanisms. They have given examples showing the simplicity, efficiency and accuracy of their technique [4]. Mehdigholi and Akbarnejad (2008) considered optimal synthesis of a special type of 4-bar linkages. Using the cognates of the 4-bar mechanism the obtained a Watt’s 6-bar mechanism generating straight and parallel motion. They used the genetic algorithm optimization method to find the optimal lengths of the mechanism [5]. Pennock and Israr (2009) investigated the kinematics of an adjustable 6-bar linkage. They showed how to determine the angle of oscillation of the output link for a specified position of the fixed pivot and investigated the extreme positions of the output link corresponding to the extreme positions of point on the coupler [6].

Eleashy, Elgayyar and Shabana (2012) presented a methodology to convert planar 3DOF open 4-bar chain into planar 1DOF 6-bar linkages. They could produce 7 different forms of 6-bar linkages [7]. Plecnik and McCarthy (2013) specified five positions of a planar RPR chain and solved the synthesis equations for two RR constraints to obtain a 6-bar linkage. They applied their synthesis procedure to the design of a linkage that generates a square pattern [8]. Agrawal, Upadhyay, Sharma and Sehgal (2013) investigated the kinematic analysis and synthesis of a Stephenson-III 6-bar linkage. They proposed the mechanism to be a dwell one and a double reciprocation one. They presented the effect of changing the dyad links orientation on the kinematic performance of the mechanism [9].

Hassaan (2014) synthesized a 6-bar planar linkage for a single dwell across 60 degrees of crank rotation. His optimal synthesis problem incorporated seven parameters to be optimally evaluated, objective function and three functional constrains controlling the performance of the mechanism [10]. Agarwal, Badduri and Bandyopadhyay (2015) introduced an approach for the synthesis of planar 6-bar mechanisms using multi-objective numerical optimization. They demonstrated the formulation and results in the context of a Stephenson III mechanism [11].

II. MECHANISM

The planar 6-bar mechanism under study is shown in Fig.1. It is a simplified mechanism of the quick return motion mechanism invented by the Arabic mechanical engineer Ibn Ismail Alzaray in the beginning of the 13th century AC [12]. A crank 2 rotates fully and joined to a slider 3 with a R-joint. Slider 3 drives an oscillating lever 4. Two other
sliders 5 and 6 are joined with each other through a R-joint. Slider 5 slider inside or over the oscillating lever, while the output slider 6 slides over the frame. There are two fixed R-joints in the mechanism, one at O and the second at Q. The mechanism has a unit degree of freedom. The vertical distance between the line of action of the output slider and joint Q is L. The dimensions of the mechanisms having constant length are the crank length $r_2$, frame length $r_1$ and frame dimension L.

III. PERFORMANCE PARAMETERS

The performance parameters of the mechanism are: mechanism stroke, time ratio, minimum transmission angle and maximum transmission angle. To derive those functional parameters, the mechanism is drawn in its two limiting positions as shown in Fig.2.

Corresponding to it slider 3 positions $A_1$ and $A_2$ respectively. The mechanism stroke is S (B1B2). The crank angle corresponding to the return stroke is $\theta$. The minimum transmission angle is $T_{A_{\text{min}}}$ at the limiting position of the oscillating lever. The maximum transmission angle is $T_{A_{\text{max}}}$ when the oscillating lever is in the vertical position (making 90 degrees with slider 6 centerline).

The mathematical models of the functional parameters of the mechanism in Fig.1 are as follows:

- **Time ratio, $T_{R}$:**
  
  The time ratio of a mechanism is defined as the ratio between the time of the forward stroke to the time of the return stroke. In terms of the crank angle, it is defined for a constant speed crank by (see Fig.2):

  \[ T_R = \frac{360 - \theta}{\theta} \]  

  Where:

  \[ \theta = 2\alpha \]  

  And

  \[ \alpha = \cos^{-1}\left(\frac{r_2}{r_1}\right) \]  

  Using normalized dimensions by referring all the dimensions to the crank length $r_2$. The normalized length of the frame, $r_{1n}$ becomes:

  \[ r_{1n} = \frac{r_1}{r_2} \]

  Combining Eqs.3 and 4 gives the angle $\alpha$ as:

  \[ \alpha = \cos^{-1}\left(\frac{1}{r_{1n}}\right) \]

- **Stroke, S:**
  
  The mechanism stroke $S$ using the trigonometric relations of the triangles in Fig.2 is given by:

  \[ S = 2L \tan \beta \]

  Where:

  \[ \beta = 90 - \alpha \]

  Dividing Eq.6 by $r_2$ provides the normal stroke $S_n$ in terms of the normalized length $L_n$ ($L/r_2$). That is:

  \[ S_n = 2L_n \tan \beta \]

- **Minimum and maximum transmission angle, $T_{A_{\text{min}}}$ and $T_{A_{\text{max}}}$:**
  
  Using the geometry of Fig.2, the minimum transmission angle is given by:

  \[ T_{A_{\text{min}}} = \beta \]

  And the maximum transmission angle is given by:

  \[ T_{A_{\text{max}}} = 90 \text{ degrees} \]

IV. OPTIMAL SYNTHESIS OF THE MECHANISM

1. Objective function: The mechanism is to be synthesized for maximum time ratio. That is the time ratio given by Eq.1 is to be maximized.

2. Functional constraints: Functional constraints are set in the optimization technique to control the performance of the mechanism during operation. Four functional constraints are used:
(i) Normalized stroke constraint:

\[ S_n \leq \lambda \]  \hspace{1cm} (11)

Where:

\( \lambda \) is a desired value for the normalized stroke.

(ii) Minimum transmission angle constraint:

\[ TA_{\text{min}} \geq 45 \text{ degrees} \]  \hspace{1cm} (12)

(iii) Output slider location constraint:

\[ L > r_1 + r_2 \]

In a normalized form:

\[ L_n > r_{fn} + 1 \]  \hspace{1cm} (13)

(iv) Fixed joints constraint:

\[ r_1 > r_2 \]

In a normalized form:

\[ r_{fn} > 1 \]  \hspace{1cm} (14)

3. Synthesis dimensions: The mechanism normalized dimensions used in the synthesis process are \( r_{fn} \) and \( L_n \).

4. Synthesis dimensions constraints: The constraints imposed on the synthesis normalized dimensions are as follows:

For \( r_{fn} \):

\[ 1 \leq r_{fn} \leq 10 \]  \hspace{1cm} (15)

For \( L_n \):

\[ 2 \leq L_n \leq 50 \]  \hspace{1cm} (16)

5. Optimization procedure: The objective function in Eq.1 is maximized subjected to the functional constraints of Eqs.11 through 14 and the dimensions constraints of Eqs.15 and 16. The MATLAB toolbox is used for this purpose through its command ‘fmincon’ [13].

6. Optimal mechanism synthesis results: The application of the detailed procedure above resulted in a very successful design for the 6 bar – 3 sliders mechanism. A sample of the results is given in Table 1 against the desired stroke of the mechanism.

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( r_{fn} )</th>
<th>( L_n )</th>
<th>TR</th>
<th>( S_n )</th>
<th>( TA_{\text{min}} ) (degrees)</th>
</tr>
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<tr>
<td>3</td>
<td>2.7621</td>
<td>3.8621</td>
<td>1.6173</td>
<td>3</td>
<td>68.7744</td>
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<td>1.9467</td>
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<td>61.0856</td>
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<td>4</td>
<td>1.7345</td>
<td>2.8346</td>
<td>2.2850</td>
<td>4</td>
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<tr>
<td>4.5</td>
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<td>2.6425</td>
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<td>49.5868</td>
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<tr>
<td>5</td>
<td>1.4198</td>
<td>2.5798</td>
<td>2.9799</td>
<td>5</td>
<td>45.2264</td>
</tr>
<tr>
<td>5.5</td>
<td>1.4142</td>
<td>2.7455</td>
<td>3</td>
<td>5.4910</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
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<td>3</td>
<td>5.9884</td>
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<tr>
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<td>3.2500</td>
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<td>3</td>
<td>8</td>
<td>45</td>
</tr>
</tbody>
</table>

7. The optimal dimensions of the 6 bar – 3 sliders planar mechanism is shown graphically in Fig.3 against its desired normalized stroke from 3 to 8.

8. The optimal time ratio (objective function) and normalized stroke (one of the functional constraints) against the desired normalized stroke of the mechanism is shown in Fig.4.

9. The optimal minimum and maximum transmission angles of the mechanism against the desired normalized stroke are shown in Fig.5. They are within the recommended range of 45 to 135 degrees [14].
V. OUTPUT MOTION OF THE OPTIMAL MECHANISM

The application of the optimization technique to a 6 bar 3 sliders mechanism has led to an optimal mechanism having a desired normalized stroke and specific transmission angle range for a maximum time ratio of the mechanism.

To investigate the normalized output displacement (slider 6 displacement) of the mechanism for one revolution of its crank, Fig.6 is generated using MATLAB for a desired normalized stroke of 4 (see Table 1). The maximum time ratio of the mechanism is 2.285, the normalized stroke is exactly 4 and the minimum transmission angle is 54.79 degrees (> 45 degrees).

VI. CONCLUSION

- The synthesis of a 6 bar – 3 sliders planar mechanism was investigated.
- The maximum time ratio of the mechanism was used as an objective function.
- The performance of the mechanism was controlled through four functional constraints.
- MATLAB optimization toolbox was used to assign the optimal dimensions of the mechanism.
- Normalized dimensions were used in the mechanism synthesis.
- Desired normalized stroke between 3 and 8 was assigned.
- The optimal mechanism dimensions were defined against the desired normalized stroke.
- The proposed procedure was completely successful, since the desired stroke was obtained and the transmission angle of the mechanism was within the recommended range for successful mechanism synthesis.

REFERENCES

BIOGRAPHY

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