Disturbance Rejection with Highly Oscillating Second-order-like Process, Part IV: I-PD Controller

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Abstract—This paper investigates the possibility of using an I-PD controller for the purpose of disturbance rejection associated with a highly oscillating second-order-like process. The MATLAB control and optimization toolboxes are used to tune the I-PD controller parameters to achieve optimal performance of the linear control system excited by process unit step disturbance. Three objective functions based on the time response error of the control system are used, and the best function is assigned. The simulation results show the effectiveness of the I-PD controller and its tuning approach in rejecting the disturbance associated with such high oscillating processes. The ITAE objective function is appropriate for use with the I-PD controller and the highly oscillating process. It is possible to go with the maximum time response to as low level as $1.5 \times 10^{-4}$ at a time of only 3.4 ms. The studied controller is capable of competing with other types used in this series of papers such as PD-PI and PI-PD controllers.

Index Terms—Disturbance rejection, highly oscillating second-order-like process, I-PD controller, controller tuning, control system performance.

I. INTRODUCTION

This is a series of research papers investigating the use of specific controllers to control the disturbance rejection associated with highly oscillating second-order-like processes. The I-PD controller is one of successful controllers capable of rejecting disturbance associated with a highly oscillating second-order process.

Ogawa and Katayama (2001) presented a PID formula to provide critical damping response to a set-point change for a first order lag process with dead time. They established a model-based robust PID tuning method when the uncertainty of process model was reduced for the I-PD controller [1]. Shi and Lee (2004) presented tuning formulas for IMC controllers applied to second order plus dead time processes. They showed that the tradeoff between set-point response and disturbance rejection is limited by normalized dead time of the process for simple pole ones [2]. El-Sousy (2004) designed a robust controller for induction motor drive and field orientation control. The proposed controller was a 2DOF I-PD one model following speed controllers. He showed that the proposed controller achieved accurate and robust performance in the presence of load disturbance and motor parameter variations [3].

Tavakoli and Griffin (2005) presented formulae for optimal PI tuning for first order plus dead time processes. They applied the tuning formulae to two simulation examples to show the effectiveness of their tuning technique [4]. Shamsuzzoha and Lee (2007) proposed an optimal internal model control filter structure for several process models for improved disturbance rejection response using PID controller. The results demonstrated the robustness of the proposed control design with parameter uncertainty [5]. Shamsuzzoha and Lee (2008) proposed a design method for a PID cascaded with a lead-lag compensator for enhanced disturbance rejection of second order stable and unstable processes with time delay. Their proposed method illustrated greater robustness against process parameter uncertainty [6].

Rajnikanth and Latha (2010) discussed the role of the damping ratio in closed loop identification with a proportional controller. They proposed a particle swarm optimization based PID controller tuning for the identified transfer function model. They reported the improvement of the performance for set-point tracking, disturbance rejection and error minimization [7]. Tabboub (2011) proposed using an I-PD controller configuration to stabilize the process and achieve desired transient response without increasing unwanted zeros. He presented results from tracking and disturbance rejection control for MIMO systems to prove that I-PD control guaranteed full controllability and asymptotic tracking and disturbance rejection for second order processes [8]. Prasad, Varghese and Balakrishnan (2012) designed and tuned an I-PD controller for a first order lag plus time delayed model. They used particle swarm optimization to tune the controller and compared with other tuning methods [9].

Rao, Subramaniam and Satyaprasad (2014) presented a PID controller with internal model control tuning method (IMC-PID) filter for effective disturbance rejection and robust operation of first order process with time delay. They suggested IMC filter produced good disturbance rejection for a process having time delay $< 0.5$ second and $> 2$ second [10]. Hassaan (2014) investigated using an I-PD controller to generate an improved performance to set-point tracking when controlling a second order process having $85.4\%$ maximum overshoot. He tuned the controller using MATLAB optimization toolbox and an ISE objective function. He compared his results with ITAE standard forms tuning technique [11]. Yazdanparast, Shahbazi, Aghajani and Abed (2015) proposed an optimal active disturbance rejection control based on using asexual
reproduction optimization to control the temperature of nonlinear CSTR. They tuned the controller parameters using particle swarm optimization and compared the performance of the control system with that using PID controller tuned by the same optimization technique. Their proposed technique revealed robustness and better performance in both set-point and disturbance rejection [12]. Hassaan (2015) investigated using an I-PD controller for disturbance rejection associated with delayed double integrating processes. He used MATLAB optimization toolbox and five objective functions to tune the controller for time delay up to 5 s. He compared his results with those obtained using PD-PI and PIDF controllers for the same process [13].

II. PROCESS

The process is an un-delayed highly oscillating second-order-like process having the transfer function, \( G_p(s) \):

\[
G_p(s) = \frac{\omega_n^2}{(s^2 + 2\zeta \omega_n s + \omega_n^2)}
\]

(1)

Where:

\( \omega_n \) = process natural frequency \( \text{ (rad/s)} \).

\( \zeta \) = process damping ratio.

The process has an equivalent damping ratio of 10 rad/s and an equivalent damping ratio of 0.05. Those parameters of the equivalent second-order process generate a step time response having 85.4 % maximum overshoot. This high process overshoot represent a bad process dynamics and puts more obstacles in front of the proposed controller which is required to produce a satisfactory performance for the closed-loop control system during disturbance rejection.

III. CONTROLLER

The controller is an Integral – Proportional Derivative one (I-PD). It has the structure shown in Fig.1 for a control system with both reference and disturbance inputs [9].

![Fig.1. I-PD controller structure [9].](image)

The controller has a multi-loop structure as shown in Fig.1. In its forward path there is an integral controller and a proportional controller. In the feedback path there is a derivative controller. The I-PD has three parameters:

- Proportional gain: \( K_{pc} \).
- Integral time constant: \( T_i \).
- Derivative time constant: \( T_d \).

The block diagram of the closed loop control system incorporating the I-PD controller as shown in Fig.1 is for systems with a reference input \( R(s) \). For systems with another disturbance input associated with the process, the process block in Fig.1 will be replace with the modification given in Fig.2.

![Fig.2. Process with disturbance input.](image)

Using the block diagram in Fig.1, the I-PD has a transfer function \( G_c(s) \) given by [13]:

\[
G_c(s) = K_{pc} \frac{T_s T_d s^2 + 1}{(T_i s)}
\]

(2)

IV. CONTROL SYSTEM TRANSFER FUNCTION

The purpose of this study is to investigate the I-PD controller effectiveness in disturbance rejection of the highly oscillating second-order-like process. In this context, the control system input will be the disturbance \( D(s) \) and the output is the process output \( C(s) \). The reference input in the block diagram of Fig.1 will be set to zero. In this case, the closed loop transfer function \( C(s)/D(s) \) of the closed-loop system will be:

\[
\frac{C(s)}{D(s)} = \frac{b_0 s}{(s^2 + a_0 s^2 + a_1 s + a_2)}
\]

(3)

Where:

\( b_0 = \omega_n^2 \)

\( a_0 = 2\zeta \omega_n + K_{pc} T_d \omega_n^2 \)

\( a_1 = \omega_n^2 (1 + K_{pc}) \)

\( a_2 = K_{pc} \omega_n^2 / T_i \)

V. I-PD CONTROLLER TUNING

The I-PD controller is tuned as follows:

1. An error function is defined as the difference between the time response of the control system \( c(t) \) and a desired value. The desired value for disturbance rejection purpose is zero. Therefore, the error function is:

\[
e(t) = c(t)
\]

(4)

2. An objective function is assigned to be minimized by a multi-dimensional optimization technique. Here, three objective functions are investigated which are the ITAE, IAE and ISTSE [14] to [16].

3. The MATLAB control toolbox to define the time response of the closed-loop control system to a unit step disturbance input using its command ‘step’ [17].
4. The MATLAB optimization toolbox is used to minimize the objective functions yielding the three tuned controller parameters $K_{pc}$, $T_i$ and $T_d$ using its command ‘fminunc’ [18].

5. The MATLAB control toolbox is used to plot the step time response of the control system to a unit disturbance input using the command ‘step’ and extract some of the time based characteristics of the control system using the ‘stepinfo’ command [17].

6. The effect of changing the guessed values of the proportional gain of the controller on the dynamics of the control system due to the too local minima of the optimization problem is investigated.

7. A MATLAB code is written by the author to apply the mentioned steps.

8. A sample of the code outputs for the three objective functions ITAE, IAE and ISTSE is given in Table 1.

<table>
<thead>
<tr>
<th>Objective Function</th>
<th>$K_{pc}$</th>
<th>$T_i$ (s)</th>
<th>$T_d$ (s)</th>
<th>$c_{max}$</th>
<th>$T_{cmax}$ (ms)</th>
<th>$T_s$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITAE</td>
<td>6582.2</td>
<td>0.0066</td>
<td>0.0064</td>
<td>1.927*10^{-4}</td>
<td>3.6</td>
<td>0</td>
</tr>
<tr>
<td>IAE</td>
<td>1.0868</td>
<td>0.3848</td>
<td>0.4191</td>
<td>0.3546</td>
<td>411.7</td>
<td>0</td>
</tr>
<tr>
<td>ISTSE</td>
<td>233.83</td>
<td>0.0305</td>
<td>0.0042</td>
<td>0.0050</td>
<td>18.93</td>
<td>0</td>
</tr>
</tbody>
</table>

9. The ITAE objective function results in obtaining the best performance of the control system for disturbance rejection. Therefore, it is considered for the rest of the work.

10. The settling time is evaluated as the time after which the time response due to a unit disturbance input stays within a value of ± 0.05.

11. The values of the maximum time response $c_{max}$, time of maximum response $T_{cmax}$ and settling time $T_s$, all indicate the effectiveness of using the I-PD controller for disturbance rejection.

12. The time response of the control system due to a unit disturbance input for using the tuning parameters in Table 1 is shown in Fig.3 for the three objective functions used in the tuning process.

13. The effect of the controller parameter $K_{pc}$ on the control system dynamics with tuned I-PD parameters using the ITAE objective function for a unit disturbance input is shown in Fig.4.

14. The effect of the I-PD controller parameter $K_{pc}$ on the maximum time response $c_{max}$ and its corresponding time $T_{cmax}$ is shown in Figs.5 and 6.

Fig.3. Effect of objective function on disturbance time response.

Fig.4. Effect of controller parameter $K_{pc}$ on system time response.

Fig.5. Effect of controller gain $K_{pc}$ on maximum time response.

Fig.6. Effect of controller gain $K_{pc}$ on time of maximum time response.
VI. COMPARISON WITH OTHER RESEARCH WORK

To judge the effectiveness of using the I-PD controller for disturbance rejection associated with the highly oscillating second-order-like process, it has been compared with some other controllers used with the same process.

Fig. 7 shows a comparison between the time response of the control system during disturbance rejection using three different controllers: PD-PI [19], PI-PD [20] and I-PD (present) controllers. The I-PD controller is superior compared with the other PD-PI and PI-PD controllers.

![Time response comparison using three controllers](image)

Fig. 7. Time response comparison using three controllers.

VII. CONCLUSION

- The possibility of using an I-PD controller for disturbance rejection associated with a highly oscillating second-order-like process was investigated.
- The controller was tuned to adjust its three parameters for optimal performance using three objective functions and the MATLAB optimization technique.
- The best objective function was assigned which was the ITAE one.
- The effect of controller proportional gain on the disturbance time response was investigated for a range from 1000 to 7000.
- The I-PD controller could generate disturbance response of maximum value as low as 1.5x10^4 and time of maximum disturbance time response as low as 3.4 ms.
- The I-PD controller is succeeded to provide disturbance time response of zero settling time and zero steady-state error.
- The performance of the closed loop control system for disturbance rejection using a I-PD controller was compared with that using PD-PI and PI-PD controllers for the same purpose.
- The PI-P controller was superior compared with the other two controllers is rejecting the disturbance associated with the highly oscillating second-order-like process.

REFERENCES

**BIOGRAPHY**

Prof. Galal Ali Hassaan:
- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
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**DEDICATION**

- I dedicate this research work to my friend Mr. Nabil Gharip.
- Mr. Gharip is a General Manager in the Transformer Electricity Company, Bani Sweif, Egypt.
- I have known Mr. Gharip only few months ago, but I feel that I know him from 10’s of years ago.
- I am happy to know him and consider him as an intimate friend.
- His deeds show how great and useful he is for his community and society.
- This is why I dedicate this work to him.
- All the best to you Mr. Gharip and to your family.