Car Dynamics Using Quarter Model And Passive Suspension, Part Iv: Destructive Miniature Humps (Bumps)

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ABSTRACT
This work presents three types of miniature humps or bumps. This covers polynomial, circular and trapezoidal bumps. The dynamics of a quarter-car model are investigated when crossing those humps to assess the destructive effect of such bumps to reach the conditions of ride comfort of drivers and passengers. The study assumed passive car suspension elements of linear characteristics. It covers car crossing speed between 0.25 and 10 km/h, and bump dimensions of 305 mm length and 57 mm height. A ride comfort diagram is presented using MATLAB simulation using the quarter-car model allowing the design of the simple harmonic hump for any desired hump-crossing speed in the range 0.25 to 10 km/h. The polynomial bump was superior for crossing speeds up to 0.65 km/h. Destructive effects are expected if speeds exceed 1.85 to 2.40 km/h depending on the bump type.

General Terms
Automotive engineering, vehicle dynamics, speed calming.

Keywords
Car dynamics, quarter-car model, Passive suspension system, Destructive miniature humps, Polynomial bump, circular bump, trapezoidal bump, ride comfort.

1. INTRODUCTION
Speed humps are in use to calm traffics through speed reduction. But going down in hump length to less than one meter makes the issue quite different. Here, we will be dealing with not ride comfort but with destructive matters to both vehicles, drivers and passengers. In this aspect, the analysis is for what is called bumps not humps. I call them miniature humps. In this engineering study the analysis of a quarter-car model dynamics will be studied during crossing such bumps trying to point out its serious effects if the crossing speed increases than 1 km/h.

Pau and Angins (2001) proposed a study for the effectiveness of 23 speed bumps installed in Cagliari (Italy). They
showed that one third of the vehicles crossed the bumps with speeds above the posted speed limit [1]. Johnson and Aloha (2002) discussed the design of a speed control device to slow passenger traffic in Oregon without slowing responding fire apparatus. The followed an action research methodology to identify national designs or standards for speed bumps which did not slow responding fire apparatus [2]. Salau, Adeyefa and Oke (2004) carried a vibration analysis to determine the effect of road bumps on vehicular system to reach a safe speed for bump crossing. They considered the vehicle as a single DOF excited by the road bump. They used a conical shaped bump in their analysis [3]. Astan et. al. (2005) reported that five cases were injured in inner city buses after crossing speed bumps. They concluded that drivers should be strongly warned and educated on the potential hazards of crossing bumps too fast and speed bumps have to be build according to tested standards [4].

Davis (2006) used a series of road test bumps of 25 x 300 mm and 50 x 300 mm dimensions [5]. Azman, King and Rahnejat (2007) investigated the vehicle dynamic response over single speed bumps. They showed that events caused by speed bumps can have implications for the design of anti-roll bars from ride comfort viewpoint [6]. Preis, Karzmerek, Griefahn and Gjestland (2008) investigated the effect of speed bumps on perceived annoyance. They concluded that in aggressive driving conditions the bump resulted in a significant increase annoyance and speed bumps cannot be considered as a noise reduction method [7]. Mohammedzadeh and Haidar (2009) used a two DOF model to simulate a vehicle crossing a speed bump. They studied the effect of vehicle speed and bump geometry on the bounce and pitch motions [8].

Namee and Witchayangkoon (2011) used bump dimensions of 76 to 150 mm height and 0.3 to 0.9 m length with crossing speeds of 8 km/h or less [9]. Huang, Liu, Zhang, Wang and Li (2011) evaluated the effect of speed bumps on speed reduction in local streets. They found that the reductions in mean speed were 6.7 to 7 km/h in two treatment sites [10]. Pirisi, Grimaccia, Mussetta and Zich (2012) presented the optimization of a tubular permanent magnet-linear generator for energy harvesting from vehicles to a grid. They investigated analytically and experimentally what is called 'power bump' [11]. Khademi, Renani, Mofarrah, Jedd and Yusof (2013) showed that the car speed is the most significant factor affecting the distance-time in comparison with other factors when assigning the best location of speed bumps [12].

Akanmu, Alabi and Agboola (2014) declared that the design and construction of speed bumps should be a responsibility of the government. They recommended the redesign of the existing speed humps with heights in excess of 210 mm. They recorded that the speed bumps in the studied area had heights up to 250 mm and length from < 0.3 up to 0.9 m [13]. Pozuelo, Ganchia and Draz (2014) developed a simulation program with MATLAB TM to analyze the influence of road irregularities on the vehicle components and occupants. They considered the vehicle dynamics, bump geometry and vehicle speed. They provided possible information to establish a set of guidelines for the proper design and installation of speed bumps [14].

Hassaan (2015) used a quarter-car model with passive suspension to investigate the car dynamics when crossing a novel simple harmonic hump. He covered crossing speeds from 5 to 30 km/h, hump height from 60 to 120 mm and hump length from 3 to 9 m. He set a ride comfort diagram to assist simple harmonic hump design for any desired crossing speed between 5 and 30 km/h [15]. Hassaan and Mohammed (2015) developed a 10 DOF model for a full-car. They used the model to simulate the effect of changing the suspension parameters on the vehicle dynamics when crossing a sinusoidal hump. They examined the effect of the hump crossing speed on the vehicle driver and passengers bounce [16].

2. STUDIED BUMPS

The author studied three types of speed bumps. Two of them are well known commercially known bumps. The third is a newly introduced bump. They are as follows:

2.1 Circular Bump

- Fig.1 shows a commercial circular bump from 'Eveluxusa Tech' [17].
Fig.1: Circular bump from Evelususa Tech [17].

- Dimensions: 45 mm height and 600 mm length.
- Material: PVC compound

2.2 Trapezoidal Bump
- Fig.2 shows a commercial trapezoidal speed bump from 'Innoplast' [18].

Fig.2: Trapezoidal bump from Innoplast [18].

- Dimensions: 50.8 mm height and 254 mm length.
- Material: Recycled plastic.

2.3 Polynomial Bump
- A polynomial speed hump was suggested by the author as a speed calming device providing better kinematics for the vehicle during crossing the hump [19].
- Using the profile design explained in reference [19], a polynomial speed bump with 50.8 mm height and 254 mm length will have the profile shown in Fig.3 as generated by MATLAB.
2.4 Bump Profiles

- Unified dimensions are used in the present research for the three types of speed bumps used in the simulation of the car dynamics when crossing the speed bump.
- The dimensions are 57 mm height and 305 mm length.
- The profiles of the circular, trapezoidal and polynomial speed bumps as generated by MATLAB are shown in Fig.4.
3. ANALYSIS

3.1 Quarter Car Model

A quarter-car model consists of the wheel and its attachments, the tire (of parameters \( m_1 \) for mass, \( k_1 \) for stiffness and \( c_1 \) for damping coefficient), the suspension elements (of parameters \( k_2 \) for stiffness and \( c_2 \) for the damping coefficient) and quarter the chassis and its rigidly connected parts (of mass \( m_2 \)).

The dynamic system is a standard two degree of freedom one having the dynamic motions:

- \( y \): The exciting motion at the ground.
- \( x_1 \): The unsprung mass motion.
- \( x_2 \): The sprung mass motion.

The driver and passenger seats are assumed rigidly connected to the chassis not to increase the degree of freedom of the dynamic system.

The parameters of the quarter-car model according to Florin, Ioan-Cosmin and Liliana are considered in this analysis except for the suspension damping coefficient \( c_2 \). Their parameter are given in Table 1 except the damping coefficient of the suspension which is set by the author [20].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 ) (kN/m)</td>
<td>Tire stiffness</td>
<td>135</td>
</tr>
<tr>
<td>( c_1 ) (kN/m)</td>
<td>Tire damping coefficient</td>
<td>1.4</td>
</tr>
<tr>
<td>( m_1 ) (g)</td>
<td>Un-sprung mass</td>
<td>49.8</td>
</tr>
<tr>
<td>( k_2 ) (kN/m)</td>
<td>Suspension stiffness</td>
<td>5.7</td>
</tr>
<tr>
<td>( c_2 ) (kN/m)</td>
<td>Suspension damping coefficient</td>
<td>15</td>
</tr>
<tr>
<td>( m_2 ) (g)</td>
<td>Sprung mass</td>
<td>466.5</td>
</tr>
</tbody>
</table>
3.2 Model Input
The input to the quarter-car model is a speed miniature hump or bump as an obstacle to the vehicle uniform motion in order to force the drive to slow down his speed or go into serious effects to his car, himself or the passengers with him. The profile of the bump excites the dynamics of the vehicle components and passengers. The vertical motion $y$ of the bump depends on the bump type as follows:

3.2.1 Circular bump
- The equation of the profile vertical deflection $y$ against the horizontal distance $x$ is given by [15]:

$$y = \sqrt{R^2 - (0.5L - x)^2} - R \cos \beta$$

where:
- $R =$ bump radius,
- $L =$ bump length
- $\beta =$ bump sector angle between terminal radii

3.2.2 Trapezoidal bump
- A trapezoidal bump consists of three segments of different equations. A line diagram showing the three segments and their dimensions is shown in Fig.5.

![Trapezoidal bump segments.](image)

Fig.5: Trapezoidal bump segments.

- The equations of the profile vertical deflection $y$ against the horizontal distance $x$ is given by:
  - For $0 \leq x \leq 114$ mm:
    $$y = 0.5 x$$

  (2)
  - For $114 \leq x \leq 191$ mm:
    $$y = 57 \text{ mm}$$

  (3)
  - For $191 \leq x \leq 305$ mm:
    $$y = -0.5 x + 152.5 \text{ mm}$$

  (4)
  - Otherwise, $x = 0$. 
3.2.3 Polynomial bump
For a polynomial bump with special characteristics, the equation of the profile vertical deflection $y$ against the horizontal distance $x$ is given by [19]:

$$y = a_1x^6 + a_2x^5 + a_3x^4 + a_4x^3 + a_5x^2 + a_6x + a_7$$  \hspace{1cm} (5)

where:
- $x$ = horizontal displacement from the hump starting point
- $a = $ polynomial coefficient (7 values)

The author set a MATLAB code to assign the seven parameters of Eq.5 depending on the bump height, length and characteristics.

3.3 Mathematical Model of the Quarter-car
Writing the differential equation of the unsprung and sprung masses of the quarter-car model yields the following two equations [21]:

$$m_1x_1'' + (c_1+c_2)x_1' - c_2x_2' + (k_1+k_2)x_1 - k_2x_2 = k_1y + c_1y'$$  \hspace{1cm} (6)

$$m_2x_2'' - c_2x_1' + c_2x_2' - k_1x_1 + k_2x_2 = 0$$  \hspace{1cm} (7)

where $x_1$ and $x_2$ are the dynamic motion of the unsprung and sprung masses respectively.

4. QUARTER-CAR MODEL DYNAMICS
- The state model of this dynamic problem is linear since the suspension parameters are assumed constant (linear characteristics).
- MATLAB is used to solve this problem using its command "ODE45" [22,23].
- The sprung mass motion is excited by the polynomial hump displacement only, i.e. zero initial conditions are set in the solution comment.
- The car speed is changed in the range: 0.25 to 1 km/h when crossing the studied bumps.
- The height of the three bumps is kept fixed at 57 mm.
- The length of the three bumps is kept fixed at 305 mm.
- The purpose of this research was to emphasise the effect of the vehicle crossing speed on the sprung mass displacement and the ride comfort in terms of the maximum sprung-mass acceleration in m/s$^2$.

4.1 Sprung-mass Displacement
The displacement of the sprung-mass as generated by MATLAB for a car velocity of 1 km/h for the three speed bumps under study using the system parameters in Table I and the quoted bump dimensions is shown in Fig.6.
4.2 Sprung-mass Maximum Displacements

- As clear from all the sprung-mass response of the quarter model as shown in Fig.6, the displacement reaches a maximum value then drops to a zero value as the car crosses the bump.
- The maximum value of $x_2$ is calculated in the same code generating the kinematics of the sprung mass using the commands 'max' of the MATLAB.
- The dynamic response of the sprung mass for the three types of speed bumps studied in this research work does not show any minimum value less than zero.
- The maximum displacements of the sprung-mass depend on the bump type and bump crossing speed.
- Fig.7 illustrates graphically this relation as generated by MATLAB.
4.3 Sprung-mass Acceleration
- The sprung-mass acceleration is the second derivative of its displacement with respect to time.
- The MATLAB command "diff" to differentiate the x2-t response twice producing the acceleration.
- Doing this, it didn't give any useful information.
- The author tried to overcome this pug by fitting an 8th order polynomial to the sprung mass velocity (dx2/dt) time response, then differentiated this polynomial analytically yielding the sprung-mass acceleration.
- A sample result of this procedure is shown in Fig.8 for the three bumps under study for a crossing speed of 1 km/h.

As clear from Fig.8, the polynomial bump provides smooth acceleration change.
- The polynomial bump starts with very small acceleration while the other two bumps start with a maximum value of 1.15 and 1.25 m/s² for the circular and trapezoidal bumps respectively.

4.3 Sprung-mass Maximum Acceleration
- The maximum acceleration of the sprung-mass depends on the bump profile and the bump-crossing speed.
- There are maximum and minimum acceleration values of the sprung mass.
- The maximum sprung mass acceleration is taken as the maximum of the absolute value of the acceleration.
- The ride comfort maximum acceleration is considered as 0.8 m/s² according to ISO 2631 [24].
- The effect of car bump crossing speed for the bumps under study is illustrated in Fig.8.
- The horizontal lines represent the 0.8, 1.6 and 2 m/s² limits of the uncomfortable acceleration ranges [24].
Fig. 8 Sprung mass maximum absolute acceleration.

- A zoomed plot for the maximum sprung mass acceleration is shown in Fig. 9.

Fig. 9: Zoomed sprung mass maximum absolute acceleration.

- Before 0.65 km/h, the three types of bumps provide comfortable ride with the polynomial bump leading the other two types.
If we consider the extremely uncomfortable range as a destructive range, then the polynomial bump comes first, then the circular, then the trapezoidal (with a 2.5 km/h crossing speed).

4.5 Maximum Car Speed for Ride Comfort
According to ISO 2631, the ride comfort range ends with 0.8 m/s² [24]. Using Fig.9, the maximum crossing speed to attain a ride comfort using the three types of bumps studied in this research can be assigned as:

- For a polynomial bump: 0.83 km/h.
- For a circular bump: 0.60 km/h.
- For a trapezoidal bump: 1.00 km/h.

5. CONCLUSIONS
- A quarter-car model with passive elements was used in this study to investigate the car dynamics during passing a polynomial, circular and trapezoidal bumps.
- A fixed speed bump dimensions of 57 mm height and 305 mm length were considered.
- Car speed between 1 and 10 km/h was considered during crossing the polynomial hump.
- The time response of the sprung mass did not show any undershoot.
- The sprung mass acceleration response had great variation from a bump type to another.
- The polynomial bump showed smooth acceleration variation with almost zero initial value.
- Ride comfort was considered through investigating the maximum sprung mass absolute acceleration during crossing the hump.
- The simulation of the quarter-car model using MATLAB showed that to have a ride comfort using a polynomial bump the crossing speed has to be < 0.8 km/h.
- The simulation of the quarter-car model using MATLAB showed that to have a ride comfort using a circular bump the crossing speed has to be < 0.6 km/h.
- The simulation of the quarter-car model using MATLAB showed that to have a ride comfort using a trapezoidal bump the crossing speed has to be < 1 km/h.
- A destructive effect was expected either on the vehicle or on the passengers if the crossing speed exceeds 1.85 km/h for a polynomial bump, 2.12 for a circular bump and 2.40 for a trapezoidal bump.

6. REFERENCES
[8]. A. Mohammedzadeh and S. Haidar, On the analysis and design of vehicle suspension system going over speed
bump, 2009, American Society for Engineering Education.


DEDICATION

Prof. Ahmed Ezzat

I dedicate this work to the sole of late Prof. Ahmed Ezzat Professor of System Dynamics at the Department of Mechanical Design & Production, Faculty of Engineering, Cairo University in the 1960’s.

Prof. Ezzat taught me System dynamics courses between 1968 and 1970.

He was the reason to love this specialization and join his department in 1970 as teaching assistant and conduct research in this field up to now.

BIOGRAPHY

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Has got his B.Sc. and M.Sc. from Cairo University in 1970 and 1974.

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