International Journal of Computer Techniques -– Volume 11 Issue 1, 2024

RESEARCH ARTICLE OPEN ACCESS

Control of a Rocket Pitch Angle using PD-PI controller, Feedback First-order Compensator and I-PD compensator

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Abstract:

The paper presents a PD-PI controller, a modified first-order and I-PD control compensators from the second generation for use to control a rocket pitch angle. The controller and compensator parameters are tuned for optimal characteristics including maximum percentage overshoot, settling time and steady-state error. A suitable performance index is selected for each controller/compensator. The proposed controller and compensators are compared with a tuned P-PD compensator used to control the same rocket variable in a previous research work. The best controller/compensator is assigned for both reference input and disturbance input.

Keywords — **Rocket pitch angle control, PD-PI controller, modified first-order compensator, I-PD compensator, controller/compensator tuning, control system performance.**

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I. INTRODUCTION

The PID controller is still in use till now even though it has some dynamic problems specially the kick associated with the step time response of the control system incorporating it. To solve the PID controller problems, the author introduced a second generation of controllers and compensators starting from 2014 till now. The author presented a large number of applications having difficult dynamics as a field for testing the applicability of the proposed controllers/compensators. Here I this work the author presents an unstable process from the rockets industry.

Roh, Cho, Ahn and Choi (2004) defined the KSR-III rocket and presented the structure of a designed attitude controller and gain scheduling. Their study covered also a stability analysis for the KSR-III rocket [1]. Jackson (2010) explored aspects of the missile flight control such as: role, sub-systems, types, design objectives and design challenges. He discussed also some of the APL's contributions to the field of missile flight control [2].Roy, Goswami, Sanyal and Sanyal (2013) investigated the pitch attitude control of a booster rocker. They developed the rocket mathematical mode and used MATLAB tools to facilitate their analysis. They considered the controlled rocket as a second-order dynamic system with an integrator and used a P-PD compensator to control it [3].

Zhang, Lv and Lei (2015) derived a longitudinal loop model and designed the missile longitudinal loop control system using a PID controller. They used a ½ transfer function for the pitch angle with an integrator. Their control system exhibited a maximum overshoot of about 15 % [4]. Losstomo, Seliadi and Djalal (2017) introduced a PID controller to control the pitch angle of a rocket. They tuned the controller parameters using an improved differential evolution algorithm (IDEA). They claimed that using the PID controller with IDEA tuning resulted in an enhanced performance of the pitch angle control system. The maximum overshoot of the pitch angle step time response reached about 15 % with the used of the tuned PID controller [5]. Lee, Ahn and Roh (2018) proposed an integrated design optimization framework for the gain schedule and bending filter for the longitudinal control of a rocket during its ascent field. They considered the dynamic model of the pitch/yaw motions of the rocket as six-order models. They adopted a PD controller with scheduled gain and bending filter parameter [6].

Kisabo, Adebimpe and Samuel (2019) presented the design, simulation and analysis of LQG and LQG/LTR control algorithms for pitch angle control of a sounding rocket. They presented eight different controllers with design, simulation and analysis. All synthesized controllers were analyzed using time response characteristics and compared with LQR and LQG control [7]. Fan et al. (2021) presented the design procedure of the control system for a boost glide rocket. They used an improved PID controller based on the small perturbation theory. They proposed the design and verification of a boost glide rocket attitude control system and used a third-order transfer function for the pitch angle. They presented the step time response of the control system for the rocket pitch angle having 8.36 % maximum overshoot and about 4 s settling time [8].

Sopegno, Livriri, Stefanovic and Volavanis (2023) studied the applicability, tuning and performance of some controllers used in a finless rocket during its boost phase. They studied the linear quadratic regulator (LQR), linear quadratic Gaussian (LQG) and proportional integral derivative (PID) controllers. They evaluated the controller performance in terms of overshoot, rise time, settling time and steady-state error. They concluded that the disturbances affecting the system were better handled and reduced using the PID controller [9]. , a novel Notch compensator to control a highly oscillating second-order process [9]. Zhang, Wen and Zhou (2023) established the dynamic model of a micro-sounding rocket and presented a control system based on using a PID controller aiming at controlling the position of the rocket. They used a filter associated with the derivative term of the PID controller [10].

II. THE CONTROLLED ROCKER AS A PROCESS

The rocket is considered as a 6-DOF rigid body with MDOF model. Out of which too many simplifications are applied to simplify its model and reduce it into SDOF models with coupled or separate loops. In such a case a lot of controllers are in applications with great variety in efficiency and success to suppress also the lot of disturbances facing any rocket or missile. It is well known in control engineering that process model is the first step towards process control. Researchers face the existence of a large variety of dynamics models for the rocket variables ranging from first-order to six-order models. The rocket under control is a booster one having a second-order + integrator transfer function with steady-state gain of 6 representing a rocket-actuator model, $G_p(s)$ given by [3]:

$$G_p(s) = 6/\{s(s^2+16s+100)\} \qquad (1)$$

Even the model looks simple; it resembles bad dynamics since it represents an unstable process. Its unit step time response is shown in Fig.1 as generated by the step command of MATLAB [11].

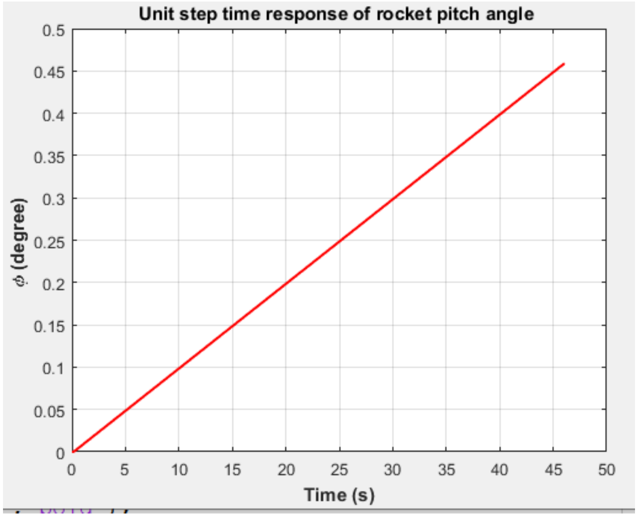


Fig.1 Step response of the rocket pitch angle.

Any proposed controller or compensator has to deal with the challenge of:

- Overcoming the instability of the rocket model and providing a stable control system.
- Providing good performance for the closed-loop control system of the pitch angle.
- Suppressing the disturbance effect on the pitch angle dynamics.

III. CONTROLLING THE ROCKET PITCH ANGLE USING A PD-PI CONTROLLER

The PD-PI controller was introduced by the author in 2014 as one of the good controllers of the second generation of the PID controllers. The author tested the performance of the PD-PI controller through its use in controlling first-order delayed processes [12], highly oscillating second-order process [13], integrating plus time-delay process [14], delayed double integrating process [15], third-order process [16] and boost-glide rocket engine [17].

The block diagram of the control system incorporating the rocket pitch angle loop and the controller for reference and disturbance inputs is shown in Fig.2.

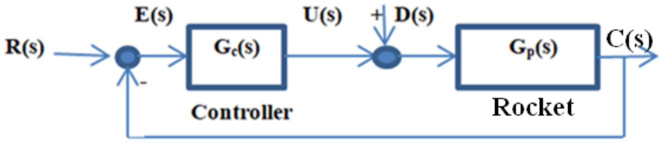


Fig.2 Rocket pitch angle control system using PD-PI controller.

The PD-PI controller is composed of a PD-control mode cascaded by a PI-control mode and has a transfer function, $G_{PDPI}(s)$ given by [17]:

$$G_{PDPI}(s) = [K_dK_{pc2}s^2+(K_dK_i+K_{pc1}K_{pc2})s+K_{pc1}K_i]/s \quad (2)$$

Where:

K_{pc1} = proportional gain of the PD-control mode.

K_d = derivative gain of the PD-control mode

K_{pc2} = proportional gain of the PI-control mode.

K_i = derivative gain of the PI-control mode

The PD-PI controller has four gain parameters to be tuned to satisfy the objectives of using the controller as stated when talking about the controlled rocket. It is tuned as follows:

- The transfer function of the closed-loop control system incorporating the PD-PI controller is derived using the block diagram in Fig.2, the rocket transfer function in Eq.1 and the controller transfer function in Eq.2.
- The step command 'step' of MATLAB is used to evaluate the step time response of the control system for reference input tracking [11].
- The MATLAB optimization toolbox is used to minimize an ITAE performance index [18].
- The tuned parameters of the PD-PI controller are as follows:

$$K_{pc1} = 0.58188 \quad , \quad K_d = 7.05802$$
$$K_{pc2} = 1.01281 \quad , \quad K_i = 10.39589 \qquad (3)$$

- Using the closed-loop transfer function of the closed-loop control system and the PD-PI controller gains in Eq.3, the reference input tracking unit step response is shown in Fig.3.

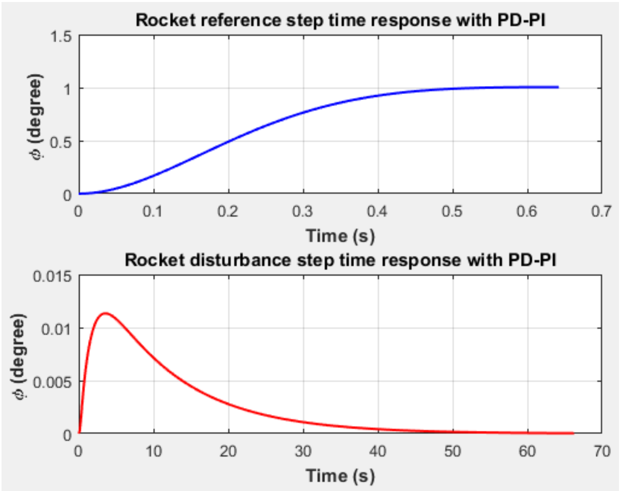


Fig.3 Pitch angle reference and disturbance step time response using PD-PI controller.

- Comments:
  - ➢ For reference input tracking:
    - ✚ The maximum overshoot is 0.0082 %.
    - ✚ The settling time is 0.5 s.
    - ✚ The steady-state error is -0.004 degree.
  - ➢ For disturbance input tracking:
    - ✚ The maximum time response is 0.011 degree.
    - ✚ The time of maximum time response is 3 s.
    - ✚ The settling time to zero is 50 s.

IV. CONTROLLING THE ROCKET PITCH ANGLE USING A FEEDBACK FIRST-ORDER COMPENSATOR

The feedback first-order compensator was introduced by the author in 2015 as one of the second generation control compensators. The author examined the validity of the proposed compensator through controlling a fractional time delay double integrating process [19], delayed double integrating process [20], second order processes [21], highly oscillating second order process [22] and aircraft pitch angle [23]. The author used the feedback first-order compensator either without any control mode in the forward path [20] or with a proportional control mode in the forward path [23]. The use of a control mode in the forward path depends on the dynamics of the controlled process and the interest of the control engineer in the suppression of the disturbance effect on the control system dynamics. Because of the special nature of rockets and missiles, disturbance may have serious effects on the operation of them and their effect has to be suppresses. Because of theis an integral mode is added to the control system in the forward path as illustrated in the block diagram of the control system of the rocket pitch angle shown in Fig.4.

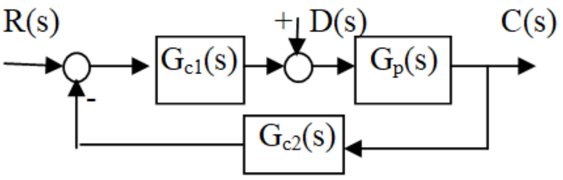


Fig.4 Rocket pitch angle control system using first-order compensator.

The first-order compensator is composed of a forward I-control mode of transfer function $G_{c1}(s)$ and a first-order compensator control mode of transfer function $G_{c2}(s)$ given by:

$$G_{c1}(s) = K_i/s \qquad (4)$$

and $$G_{c2}(s) = K_c(T_z s+1)/(T_p s+1) \qquad (5)$$

where: K_i = integral gain of the I-control mode.

K_c = gain of the first-order compensator.

T_z = zero time constant of the first-order compensator.

T_p = pole time constant of the first-order compensator.

The compensator has four gain constants to be tuned to provide the required performance of the closed-loop system of the pitch angle control of the rocket. This is performed as follows:

- Investigating the control system for the control of the rocket pitch angle, the author found that it is possible to suppress the disturbance effect and reach zero change in the pitch angle after disturbance application if the compensator gain K_c is set to a unit value.
- This reduces the tuning operation to adjusting only three parameters: K_i, T_z and T_p.
- The optimization toolbox of MATLAB is used to minimize an ITAE performance index and tune the compensator parameters. The tuning results are as follows:

  $$K_i = 1.65547 \ , \ T_z = 5.59464 \ \text{s}$$
  $$T_p = 1.45826 \ \text{s} \qquad (6)$$
- The block diagram of Fig.4 is used with the transfer functions in Eqs.1, 4 and 5 and the compensator parameters in Eq.6 to derive the closed-loop transfer functions of the control system for the reference and disturbance inputs.
- The closed-loop transfer functions are used to plot the unit step input step time response of the control system using the 'step' command of MATLAB as shown in Fig.5.

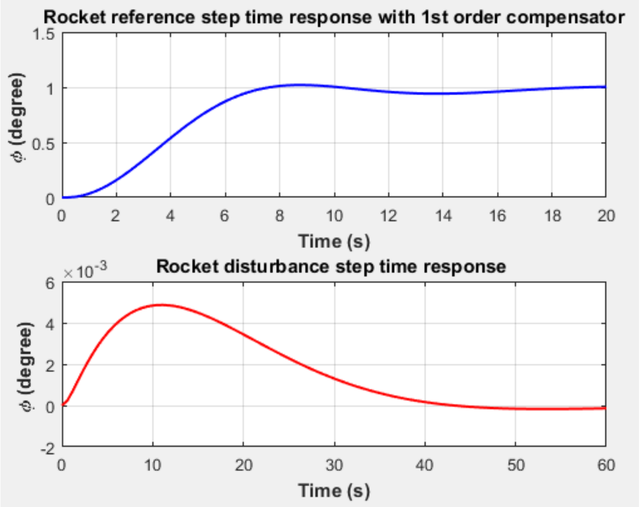


Fig.5 Pitch angle reference and disturbance step time response using first-order compensator.

- Comments:

➢ For reference input tracking:
- The maximum overshoot is 1.985 %.
- The settling time is 17.4 s.

  - The steady-state error is zero.
- For disturbance input tracking:
  - The maximum time response is 0.0048 degree.
  - The time of maximum time response is 11 s.
  - The settling time to zero is 50 s.

V. CONTROLLING THE ROCKET PITCH ANGLE USING AN I-PD COMPENSATOR

The I-PD compensator is one of the compensators proposed by the author from 2014 under the name '*second generation of control compensators*'. The author tested the proposed I-PD compensator through its application to control second-order processes [24] and greenhouse internal temperature [25]. The structure of the I-PD compensator proposed to control the pitch angle of the rocket is shown in Fig.6. An integral element of transfer function K_i/s is set in the feedforward path just after the error detector of the control system incorporating the compensator and the process. A PD element is set in the feedback path going to the error detector having a transfer function $K_{pc}+K_ds$.

The compensator parameters are:

K_{pc}: Proportional gain.
K_i: Integral gain.
K_d: Derivative gain.

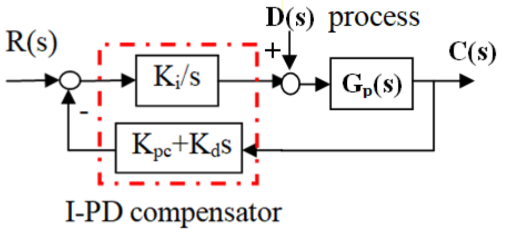


Fig.6 Rocket pitch angle control system using I-PD compensator.

Using the block diagram in Fig.6 and the process transfer function in Eq.1, the closed-loop transfer function of the closed-loop control system incorporating the I-PD compensator and the process (rocket pitch angle motion), the closed-loop transfer function of the control system with the reference input, $M_R(s)$ is derived as:

$$M_R(s)=6K_i /\{s^4+16s^2+6K_iK_ds+6K_{pc}K_i\} \qquad (7)$$

The transfer function of the control system considering the disturbance input D(s) with zero reference input, $M_D(s)$ is derived using the block diagram in Fig.6 and given by:

$$M_D(s) = s /\{s^4+16s^2+6K_iK_ds+6K_{pc}K_i\} \qquad (8)$$

Before going to the I-PD controller tuning we make a quick investigation of the transfer function of the closed-loop control system incorporating the I-PD compensator and the rocket pitch angle model. We find that the control system has a non-zero steady state error function of the proportional gain of the compensator. Furthermore, we find that we can eliminate completely this error if we set the proportional gain Kpc to a unit value. That is:

$$K_{pc} = 1 \qquad (9)$$

Now, the tuning process will be required to adjust only two parameters for good performance of the closed-loop control system with reference input. Eq.7 is used to tune the I-PD compensator using the MATLAB optimization toolbox and a ISTSE performance index. The result of this tuning process is outlined below:

$$K_i = 60.98388 \ , \quad K_d = 0.9738 \qquad (10)$$

The unit step time response of the control system for pitch angle control of the rocket is drawn using the step command of the MATLAB and using Eqs.7 through 10 as shown in Fig.7.

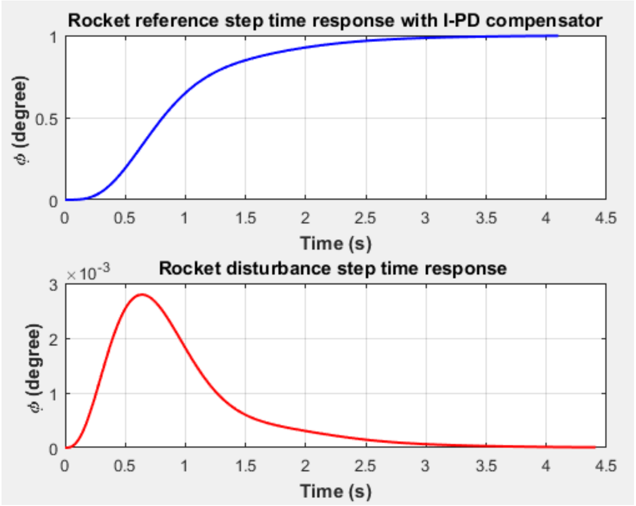


Fig.7 Pitch angle reference and disturbance step time response using an I-PD compensator.

- Comments:
- For reference input tracking:
  - The maximum overshoot is zero
  - The settling time is 2.825 s.
  - The steady-state error is zero.

- For disturbance input tracking:
  - The maximum time response is 0.0028 degree.
  - The time of maximum time response is 0.65 s.
  - The settling time to zero is 3.5 s.

VI. COMPARISON ANALYSIS

- To evaluate the effectiveness of using the proposed controller/compensators, the step time response for reference input is compared with that using a P-PD compensator [3] and shown in Fig.8.

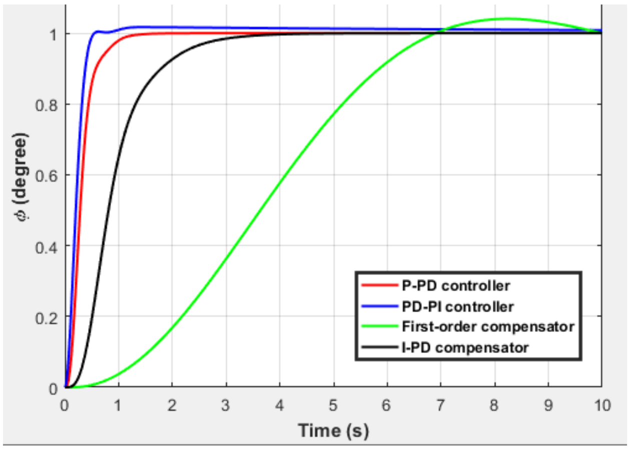


Fig.8 Comparison of reference input tracking step time response.

- A quantitative comparison for the time-based characteristics of the control systems handled in the present work to control the rocket pitch angle is given in Table 1 for a reference step input.

Table 1: Time-based characteristics of the pitch angle control system.

Controller/ compensator	Maximum overshoot (%)	Settling time (s)	Steady-state error (degree)
PD-PI controller	0.008	0.500	0
First-order compensator	1.985	17.400	0
I-PD compensator	0	2.825	0
P-PD compensator [3]	0	1.665	0.004

- One of the objectives of the proposed controllers/compensators is to suppress the disturbance time response. Have the proposed controllers/compensators succeeded in this aspect?. Fig.9 presents a graphical comparison for the disturbance step time response of the rocket pitch angle when controlled by the proposed three controller/compensators from the second generation and a P-PD compensator handled in reference [3].

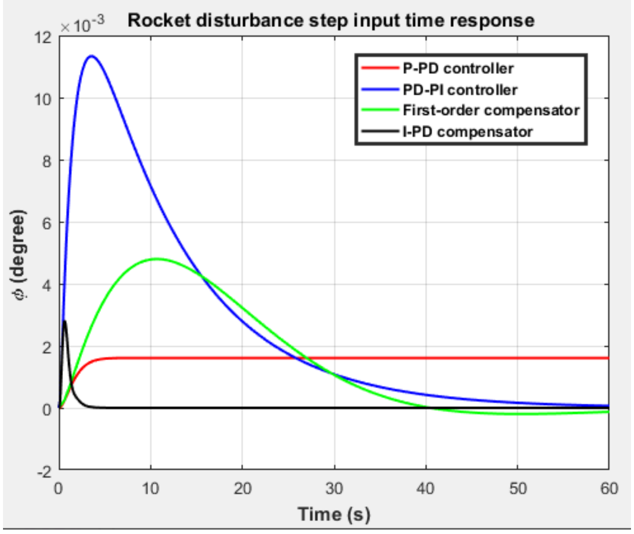


Fig.9 Comparison of disturbance input tracking step time response.

- A quantitative comparison for the time-based characteristics of the control systems handled in the present work to control the rocket pitch angle is given in Table 2 for a disturbance step input.

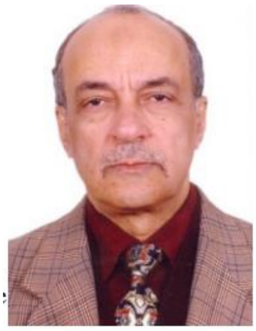
Table 2: Time based characteristics of the disturbance step time response of the rocket pitch angle.

Controller/ Compensator	Maximum time response (degree)	Time of maximum time response (s)	Settling time (s)	Steady-state time response (degree)
PD-PI controller	0.0110	3	50	0
First-order compensator	0.0048	11	50	0
I-PD compensator	0.0028	0.65	3.5	0
P-PD compensator [3]	0.0019	5	5	0.0019

VII. CONCLUSIONS

- This research work investigated the use of a PD-PI controller, first-order compensator and I-PD compensator all from the second generation of controllers and compensators to control the pitch angle of a rocket.
- The process under control (rocket pitch angle) is an example of unstable processes.
- The paper proposed one controller and two compensators to control the rocket unstable process: PD-PI controller, feedback first-order compensator and I-PD compensator.
- The performance of the proposed controller/compensators was compared with that of a P-PD compensator from previous research work.
- The PD-PI controller was superior in reducing the maximum overshoot to only 0.008 % and the settling time to only 0.5 s with zero steady-state error for the reference input tracking step time response.
- The I-PD compensator eliminate completely the maximum overshoot and the steady-state error for reference input tracking.
- The feedback first-order compensator could eliminate the steady-state error and keep the maximum overshoot at less than 2 % but with settling time of about 17.4 seconds for reference input tracking.
- The P-PD compensator could eliminate the maximum overshoot and keep the settling time less than 1.7 seconds, but it failed to eliminate the steady-state error of the control system.
- The PD-PI, feedback first-order compensator and I-PD compensator could suppress the disturbance input step time response to zero with maximum value of 0.011, 0.0048 and 0.0028 degrees respectively at a time of 3, 11 and 0.65 seconds respectively with settling time of 50, 50 and 3.5 seconds respectively.
- The P-PD compensator failed to suppress the disturbance step time response where it settled at 0.0019 degree value in 5 seconds.
- Regarding the reference input tracking step time response, the PD-PI controller was the best among the four controller/compensators studied in this work.
- Regarding the disturbance input tracking step time response, the I-PD compensator was the best among the four controller/compensators studied in this work.

BIOGRAPHY



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