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Analysis of Stressed Timoshenko Beams on Two Parameter Foundations

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Abstract

In the present work, the static and dynamic behaviors of a Timoshenko beam subjected to a static axial compression load and a dynamic lateral load resting on a two parameter foundation have been studied using Adomian Decomposition Method (ADM). After verifying the obtained expressions by comparing its results with those found in literature, it is used to calculate the critical loads in the static case, natural frequencies for free vibration and beam response in forced vibrations. Parametric study is conducted to investigate the influences of different beam and foundation parameters on the critical loads, the natural frequencies and the response of the beam.

Keywords: *forced vibration, natural frequency, stability parameter, Timoshenko beam, adomian decomposition method*

1. Introduction

Considerable attention has been given to the analysis of beams on elastic foundations which represent one of the most common civil and mechanical structural applications. A large number of studies have been devoted to the analysis of such models assuming different foundation models of various degrees of sophistication to capture the complex behavior of the foundation (Pasternak, 1954; Kerr, 1965; Vallabhan and Das, 1988).

On the other hand, numerous studies based on different theories (Bernoulli-Euler, Rayleigh, bending and shear, Timoshenko and modified Timoshenko) which use different mathematical treatments (Closed form solutions, analytical treatment and numerical solutions) have been carried out to investigate the static and dynamic behaviors of beams. Timoshenko beam theory is developed to take into account the shear deformations which affect the static and dynamic behavior when the beam depth/span ratio increases beyond a certain value (for $h/L \geq 0.1$). In addition, Timoshenko beam theory is based on the assumption that the plane normal to the neutral axis of the beam before deformation remains plane, but not normal to the neutral axis after deformation. Furthermore, the Timoshenko beam theory is applicable only for beams in which shear lag is insignificant.

Taha and Abohadima, (2008), Taha (2012) and Taha *et al.* (2013) used Bessel functions, elliptic integrals and differential quadrature method (DQM) to investigate the nonlinear static and dynamic behaviors of both prismatic and non-prismatic beams based on Bernoulli-Euler theory. Cheng *et al.* (1977, 1988) studied extensively Timoshenko beams using continuous models and stiffness matrix method. Geist and Mclaughlin (1997) discussed the phenomenon of double frequencies in Timoshenko beams at certain values of beam slenderness ratios. Kausel (2002) studied the invalidation of Bernoulli-Euler theory for the cases of free-free and pinned-free shear beams. Free vibration of Timoshenko beams on two-parameter foundation were examined by De Rosa (1995) who assumed that the second foundation parameter as a function of the beam properties. Aristizabal-Ochoa (2004) and Arboleda-Monsalve *et al.* (2008) presented a free vibration analysis of Timoshenko beam-column with generalized end conditions on an elastic foundation. Free and forced vibrations of Timoshenko beams, described by a single difference equation, have been studied by Majkut (2009). Kocaturk and Simsek (2005) had investigated the vibration of Timoshenko beams under various boundary conditions using Lagrange equations and Lagrange multipliers. The differential transform was applied to investigate the free vibration of beams resting on elastic foundation by Attarnejad *et al.* (2009) and Balkaya *et al.* (2009). Nguyen (2007, 2008) studied the vibration of stressed Timoshenko beams either fully or partially supported on an elastic foundation using the Finite Element Method (FEM). Chebychev polynomials were used by Ruta (2006) to study non-prismatic Timoshenko beams. Indeed, the governing equations which describe Timoshenko beam behavior are two coupled partial differential equations; one represents the lateral deflection and the other simulates the rotation of the cross section along the beam.

In recent years, much attention has been given to develop both mathematical and analytical techniques to solve the fourth-order boundary value problems (homotopy perturbation method, differential quadrature method DQM, differential transform

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method DTM and Adomian Decomposition Method ADM). It has been proved that the ADM yields accurate and stable solutions without discretization or linearization. In addition, the ADM is not affected by computation round off errors; and does not require a large computer memory or a long solution time (Adomian, 1994; Dehghan and Tatari, 2006; Yaman, 2006; Momani and Noor, 2007; Hsu *et al*., 2008; Shahba *et al*., 2012).

In this paper, the static and dynamic behavior of Timoshenko beam subjected to a static axial compression load and a dynamic lateral load resting on a two-parameter foundation will be investigated using the ADM.

The governing equations will be derived taking into account shear deformations and rotational inertia, then they will be solved using the ADM to obtain analytical expressions for the critical loads, natural frequencie and beam response. The boundary conditions at the beam ends in the case of Timoshenko beam will be treated in a different way than in the Bernoulli-Euler approach. The validity of the obtained expressions will be examined by comparing its results with those found in the literature, and then it will be used to investigate the significance of different beam-foundation characteristics on the natural frequencies, critical load and beam response.

2. Problem Formulation

2.1 Equations of Motion

The beam shown in Fig. 1(a) of length L, depth h and constant cross section, is assumed to be made of a homogenous linear elastic material with modulus of elasticity E and shear modulus G, axially-loaded with a constant load p and laterally-loaded with a dynamic load $q(x, t)$. The beam cross sectional area is A, the effective shear area $A_s = \kappa A$, the parameter κ is the shape parameter accounts for shear stress distribution, the second moment of area for the beam cross section about the axis of bending is I, the mass per unit of length $\mu = \rho A$, ρ is the volumetric density of the beam material and $r^2 = I/A$ is the radius of gyration of the beam cross section. The beam is resting on a two-parameter foundation with linear stiffness k_1 and shear stiffness k_2.

Applying the equations of translational and rotational motion for the differential element shown in Fig. 1(b), one obtains:

$$\frac{\partial V}{\partial x} + q(x,t) + k_1 y - k_2\frac{\partial^2 y}{\partial x^2} = -\mu\frac{\partial^2 y}{\partial t^2} \tag{1}$$

$$\frac{\partial M}{\partial x} + V - p\frac{\partial y}{\partial x} = -\mu r^2\frac{\partial^2 \theta}{\partial t^2} \tag{2}$$

where y, x, t, V, M, θ, γ and $\partial y/\partial x$ are beam lateral deflection, distance along the beam, time, shear force acting on the cross section at x, bending moment acting on the cross section at x, rotation of cross section of the beam at x, shear deformation of the beam cross section at x and slope of the neutral axis of the beam at x respectively.

The force-displacement relations for the beam element are:

$$V = A_s G\gamma \tag{3}$$

$$M = -EI\frac{\partial \theta}{\partial x} \tag{4}$$

where E is the modulus of elasticity and G is the shear modulus of the beam material.

The rotation of the cross section of the beam θ is the result of the rotation of the neutral axis of the beam $\partial y/\partial x$ and the shear deformation angle γ as indicted in Fig. 1(c), i.e.:

$$\theta = \frac{\partial y}{\partial x} + \gamma \tag{5}$$

Substitution of Eq. (3), Eq. (4) and Eq. (5) into Eq. (1) and Eq. (2) leads to:

$$A_s G\left(\frac{\partial \theta}{\partial x} - \frac{\partial^2 y}{\partial x^2}\right) = -\mu\frac{\partial^2 y}{\partial t^2} - q(x,t) - k_1 y + k_2\frac{\partial^2 y}{\partial x^2} \tag{6}$$

$$EI\frac{\partial^2 \theta}{\partial x^2} = A_s G\left(\theta - \frac{\partial y}{\partial x}\right) + \mu r^2\frac{\partial^2 \theta}{\partial t^2} - p\frac{\partial y}{\partial x} \tag{7}$$

Assuming harmonic excitation as:

$$q(x,t) = q(x)e^{i\Omega t} \tag{8}$$

For free vibration analysis, the value of $q(x)$ is set to be zero and the excitation frequency Ω is replaced by the natural frequency of the system ω.

Because the excitation is harmonic, then the steady state response is expected to be harmonic and the solutions for Eq. (6) and Eq. (7) can be assumed in the form:

$$y(x,t) = Y(x)e^{i\Omega t} \tag{9}$$

$$\theta(x,t) = \psi(x)e^{i\Omega t} \tag{10}$$

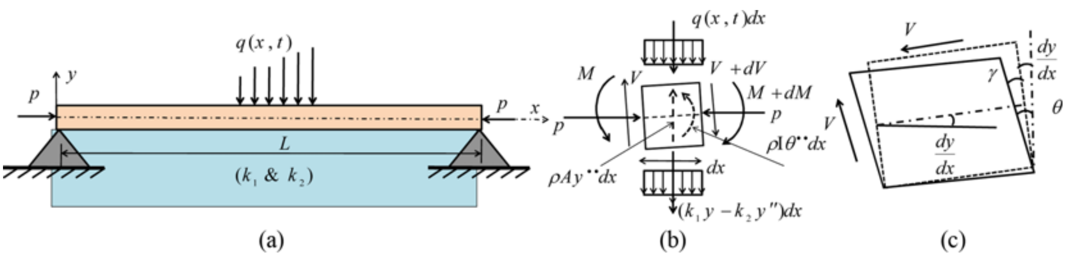

Fig. 1. Axially Compressed Timoshenko Beam Subjected to Lateral Dynamic Excitation and Resting on a Two-parameter Foundation: (a) Beam Configuration, (b) Element Forces, (c) Element Deformations

Thought, the present analysis can deal with any type of loading distribution along the beam (uniform, linear, nonlinear, etc.), but actually shear walls in high rise building which modeled as Timoshenko beam are built mainly to resist wind and earthquake loads whose distribution are best correlated by linear distributions.

Substituting Eqs. (8), (9) and (10) into Eqs. (6) and (7), assuming linear distribution of excitation along the beam axis $q_o(x)$ $=ax$, and eliminating all functions of time leads to:

$$A_sG\left(\frac{d\psi}{dx}-\frac{d^2Y}{dx^2}\right)-\mu\Omega^2Y+ax+k_1Y-k_2\frac{d^2Y}{dx^2}=0 \tag{11}$$

$$EI\left(\frac{d^2\psi}{dx^2}\right)-A_sG\left(\psi-\frac{dY}{dx}\right)+\mu r^2\Omega^2\psi+p\frac{dY}{dx}=0 \tag{12}$$

2.2 Dimensionless Analysis

Introducing dimensionless variables ξ and w as:

$$\xi=\frac{x}{L} \text{ and } w=\frac{Y}{L} \tag{13}$$

Substitution of Eq. (13) into Eq. (11) and Eq. (12) yields:

$$\frac{d\psi}{d\xi}-\left(\frac{k_2}{A_sG}+1\right)\frac{d^2w}{d\xi^2}+\left(\frac{k_1L}{A_sG}-\frac{\mu\Omega^2L^2}{A_sG}\right)w+\frac{aL^2}{A_sG}\xi=0 \tag{14}$$

$$\frac{EI}{A_sGL^2}\left(\frac{d^2\psi}{d\xi^2}\right)+\frac{dw}{d\xi}+\left(\frac{\mu r^2\Omega^2}{A_sG}-1\right)\psi+\frac{p}{A_sG}\frac{dw}{d\xi}=0 \tag{15}$$

In addition, introduce the following dimensionless parameters:

$$S^2=\frac{EI}{A_sGL^2},\ \lambda^2=\frac{\mu\Omega^2L^4}{EI},\ R^2=\frac{r^2}{L^2},\ g^2=\frac{aL^2}{A_sG},$$

$$f^2=\frac{pL^2}{EI},\ c_1^2=\frac{k_1L^2}{A_sG} \text{ and } c_2^2=\frac{k_2}{A_sG} \tag{16}$$

where S^2 is the bending to shear stiffness parameter, λ^2 is the frequency parameter, R^2 is the slenderness, g^2 is the lateral load parameter, f^2 is the axial-load parameter, c_1^2 is the foundation linear stiffness parameter and c_2^2 is the foundation shear stiffness parameter.

Substitution of the dimensionless parameters defined in Eq. (16) into Eqs. (14) and (15) yields:

$$\frac{d\psi}{d\xi}-(c_2^2+1)\frac{d^2w}{d\xi^2}+(c_1^2-\lambda^2S^2)w+g^2\xi=0 \tag{17}$$

$$S^2\frac{d^2\psi}{d\xi^2}+(1+f^2S^2)\frac{dw}{d\xi}+(\lambda^2S^2R^2-1)\psi=0 \tag{18}$$

Using Eq. (17) and Eq. (18), the rotation of the beam cross section can be expressed in terms of lateral displacement derivatives as:

$$\psi(\xi)=B_1w'''(\xi)+B_2w'(\xi)+B_3 \tag{19}$$

Then, the lateral displacement can be decoupled as:

$$w''''(\xi)+A_1w''(\xi)+A_2w(\xi)+A_3\xi=0 \tag{20}$$

where the constants A_i and B_i, $i=1, 2, 3$ are defined as:

$$A_1=\frac{\left(\lambda^2S^2+\lambda^2R^2+f^2-c_1^2+\lambda^2R^2c_2^2-\frac{c_2^2}{S^2}\right)}{(1+c_2^2)},$$

$$A_2=\frac{\left(\lambda^2R^2-\frac{1}{S^2}\right)(\lambda^2S^2-c_1^2)}{(1+c_2^2)},\text{ and } A_3=\frac{\left(\frac{g^2}{S^2}-g^2\lambda^2R^2\right)}{(1+c_2^2)}$$

$$B_1=\frac{S^2(c_2^2+1)}{1-S^2\lambda^2R^2},\ B_2=\frac{S^4\lambda^2-S^2c_1^2+f^2S^2+1}{1-S^2\lambda^2R^2} \text{ and}$$

$$B_3=\frac{S^2g^2}{S^2\lambda^2R^2-1} \tag{21}$$

The solution of Eq. (20) yields both the steady state amplitude distribution of a Timoshenko beam resting on a two-parameter foundation acted upon by an axial static load due to lateral excitation and yields the mode functions in case of free vibration ($g=0$). In addition, the obtained solution of Eq. (20) can be substituted into Eq. (19) to obtain the distribution of the rotation of the beam cross section along the beam. Actually, most practical applications require the determination of the lateral displacement along the beam. However, Eq. (19) which determines the shear deformation angle can be used to describe the amplitude of the rotation angle along the beam. This is a different approach than Bernoulli-Euler theory where the boundary conditions depend on the slope of the neutral axis of the beam at ends.

2.3 Boundary Conditions

The lateral displacement and the rotation of the beam cross section at the beam ends depend on the type of support at these ends.

Case (A): Pinned-pinned beams

The boundary conditions are:

$$w(0)=0,\ \psi'(0)=0,\ w(1)=0 \text{ and } \psi'(1)=0 \tag{22a}$$

Using Eq. (17), then:

$$w(0)=0,\ w''(0)=0,\ w(1)=0 \text{ and } w''(1)=g^2/(c_2^2+1) \tag{22b}$$

which different than the expression used in the case of the Bernoulli-Euler theory.

Case (B): Clamped-clamped beams

The displacement and cross section rotation are:

$$w(0)=0,\ \psi(0)=0,\ w(1)=0 \text{ and } \psi(1)=0 \tag{23a}$$

Using Eq. (19), the boundary conditions at $\xi=0$ and $\xi=1$ respectively can be expressed as:

$$B_1w'''(0)+B_2w'(0)+B_3=0 \tag{23c}$$

$$B_1w'''(1)+B_2w'(1)+B_3=0 \tag{23d}$$

It should be noted that for free vibration ($g=0$), the constant B_3 vanishes.

Case (C): Clamped-pinned beams

The displacement and the beam cross section rotation for C-P beams are defined as:

$$w(0)=0,\ \psi(0)=0,\ w(1)=0 \text{ and } \psi'(1)=0 \tag{24a}$$

Making use of Eq. (19) yields:

$$B_1 w'''(0)+B_2 w'(0)+B_3=0 \tag{24b}$$

$$w(1)=0 \text{ and } w''(1)=g^2/(c_2^2+1) \tag{24c}$$

3. Method of Solution

3.1 The Adomian Decomposition Method (ADM)

The ADM is used to obtain solution for the governing equation, Eq. (20), where the equation is rewritten as:

$$\ell w=-A_1\frac{d^2w}{d\xi^2}-A_2 w-A_3\xi,\ \text{where } \ell=\frac{d^4}{d\xi^4} \tag{25}$$

Applying the operator ℓ^{-1}, the inverse of the operator ℓ to both sides of Eq. (25), one obtains:

$$\ell^{-1}\ell w=-A_1\ell^{-1}\frac{d^2w}{d\xi^2}-A_2\ell^{-1}w-A_3\ell^{-1}\xi \tag{26}$$

where,

$$\ell^{-1}\ell w=\int_0^\xi\int_0^\xi\int_0^\xi\int_0^\xi\frac{d^4w}{d\xi^4}d\xi=w(\xi)-w(0)-\xi w'(0)$$
$$-\frac{\xi^2}{2!}w''(0)-\frac{\xi^3}{3!}w'''(0) \tag{27}$$

Substitution of Eq. (27) into Eq. (26) and rearranging the terms yields:

$$w(\xi)=w(0)+\xi w'(0)+\frac{\xi^2}{2!}w''(0)+\frac{\xi^3}{3!}w'''(0)$$
$$-A_1\ell^{-1}\frac{d^2w(\xi)}{d\xi^2}-A_2\ell^{-1}w(\xi)-A_3\ell^{-1}\xi \tag{28}$$

The solution for $w(\xi)$ can be approximated by a finite series according to the ADM as:

$$w(\xi)=\sum_{n=0}^{N}w_n(\xi) \tag{29}$$

where N is the number of terms chosen to achieve the required accuracy. Let the component $w_0(\xi)$ be defined as:

$$w_0(\xi)=w(0)+\xi w'(0)+\frac{\xi^2}{2!}w''(0)+\frac{\xi^3}{3!}w'''(0)-A_3\frac{\xi^5}{5!} \tag{30}$$

Substitution of Eq. (29) and Eq. (30) into Eq. (26), one obtains;

$$\sum_{n=0}^{N}w_n(\xi)=w_0(\xi)-A_1\ell^{-1}\sum_{n=0}^{N}w_n''(\xi)-A_2\ell^{-1}\sum_{n=0}^{N}w_n(\xi) \tag{31}$$

Then, the components $w_n(\xi)$ and $n=1,2,3,\ldots,N$ can be elegantly determined by using the recursive relation:

$$w_n(\xi)=-A_1\ell^{-1}w_{n-1}''(\xi)-A_2\ell^{-1}w_{n-1}(\xi) \tag{32}$$

The accuracy of the present solution is measured through the ratio of the incremental variation in the midpoint amplitude $w_N(0.5)$ to the total amplitude $\sum_{n=0}^{N-1}w_n(0.5)$ due to considering additional term in the series. Many attempts are employed to determine the number of terms N required to achieve the pre-assigned accuracy (0.5%), and it is found that $N=7$ is adequate (Omar, 2012). Then, the amplitude distribution of the Timoshenko beam is approximated by:

$$w(\xi)=\sum_{n=0}^{7}w_n(\xi) \tag{33}$$

Substitution of the boundary conditions at the beam ends into Eq. (33) yields a system of 4- algebraic equations in 4 unknowns $w(0), w'(0), w''(0)$ and $w'''(0)$. For the case of forced vibration, the system is solved and an analytic expression for the dimensionless lateral displacement is obtained. For the case of free vibration ($B_3=0$), the substitution of the boundary conditions into Eq. (33) yields a system of homogeneous equations. Then, replacing the excitation frequency Ω by the natural frequency of the system ω and recalling the eigen value analysis, an eigen value problem in two parameter (p and ω) is obtained. Furthermore, for the static case $\omega=0$, the solution of the eigen value problem yields the critical loads of the system p_{cr} while assigning any value for $p > p_{cr}$, the natural frequencies of the system can be calculated. Full expressions for $w_i(\xi)$, $i=1,2,\ldots,7$ are given for different cases of boundary conditions (Omar, 2012).

3.2 Verifications of Obtained Solutions

To verify the obtained expressions, values of the frequency

Table 1. Values of ($\sqrt{\lambda}$) for the Case of P-P Beam

$(\bar{k},\bar{k}_p)$	γ	FEM	ADM	$(\bar{k},\bar{k}_p)$	γ	FEM	ADM
(0, 0)	0.0	3.1347	3.1245	(100, 0.5)	0.0	3.9561	3.9492
	0.4	2.7589	2.7479		0.4	3.4818	3.4238
	0.8	2.0963	2.0837		0.8	2.6456	2.4897
(1, 0)	0.0	3.1428	3.1327	(100, 1)	0.0	4.1392	4.1306
	0.4	2.7660	2.7549		0.4	3.6430	3.5906
	0.8	2.1017	2.0889		0.8	2.6781	2.6010
(100, 0)	0.0	3.7433	3.7359	(100, 2.5)	0.0	4.5783	4.4490
	0.4	3.2945	3.2298		0.4	4.0294	3.9869
	0.8	2.5033	2.3731		0.8	3.0617	2.8943

parameter obtained using ADM are presented in Table 1 against results obtained from the Finite Element Method (FEM) based on Timoshenko beam theory (Nguyen, 2008). However, as the accuracy of the results obtained from ADM depend on the number of series terms taken into consideration, the accuracy of the FEM depend on the number of mesh points considered in the analysis. The loading ratio γ and foundation parameters $\bar{k}$ and $\bar{k}_p$ are defined as:

$$\gamma = \frac{p}{p_{cr}},\ \bar{k} = \frac{k_1 L^4}{EI} \text{ and } \bar{k}_p = \frac{k_2 L^2}{\pi^2 EI} \tag{34}$$

It is clear from Table 1 that the results obtained using ADM are compatible with those obtained from FEM.

4. Numerical Results and Analysis

4.1 Frequency Parameter λ

The frequency parameter λ calculated using the obtained ADM expressions for different values of the loading ratio γ and for different boundary conditions is illustrated in Fig. 2, Fig. 3 and Fig. 4. It is found that, as the applied axial load increases, the natural frequency of the system decreases. It is also found that the effect of the bending to shear stiffness S^2, which represents the effect of *h/L*, in decreasing the natural frequencies is significant for the values of $S^2 < 0.0015$. This effect is attributed to the additional shear deformations which increase the flexibility of the beam.

The influence of the foundation linear stiffness parameter $\bar{k}$ on the frequency parameter λ for both P-P and C-C beams is obtained and presented in Figs. 5 and 6. It is observed that as the foundation stiffness increases, the natural frequency of the system increases and that the effect of $\bar{k}$ is more noticeable for P-P case. The effect of the end conditions decreases as the foundation stiffness increases.

The effects of the foundation shear stiffness parameter $\bar{k}_p$ for beams of P-P and C-C boundary conditions are obtained and presented in Figs. 7 and 8. In addition, the effect of $\bar{k}_p$ is more significant for beams with P-P conditions.

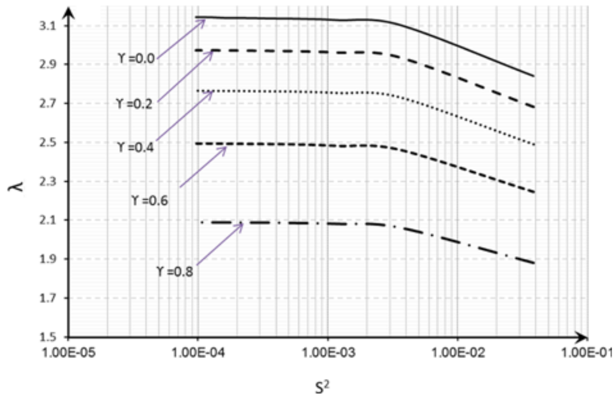

Fig. 2. Influence of the Loading Ratio (γ) and the Bending to Shear Stiffness (S^2) on the Natural Frequency for P-P Beams

4.2 Stability Parameter

Using the obtained expressions for static case ($\omega = 0$, $a = 0$), the values of the stability parameter f_b are calculated for different values of the system parameters. The stability parameter f_b is defined as:

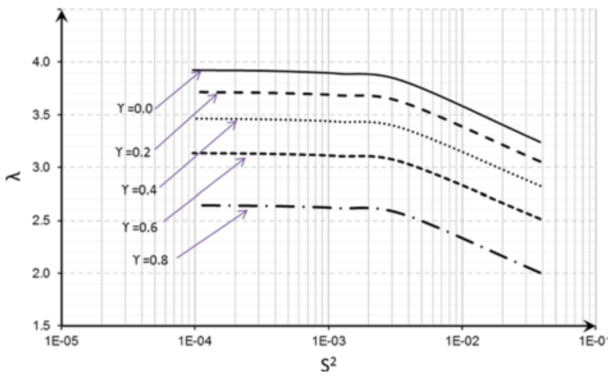

Fig. 3. Influence of the Loading Ratio (γ) and the Bending to Shear Stiffness (S^2) on the Natural Frequency for C-P Beams

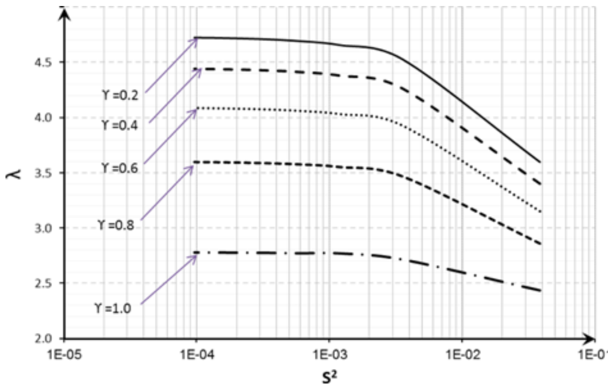

Fig. 4. Influence of the Loading Ratio (γ) and the Bending to Shear Stiffness (S^2) on the Natural Frequency for C-C Beams

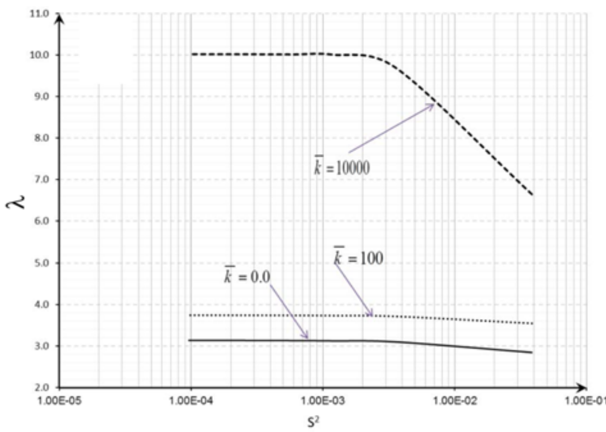

Fig. 5. Influence of the Foundation Linear Stiffness $\bar{k}$ and the Bending to Shear Stiffness (S^2) on the Natural Frequency for P-P Beams

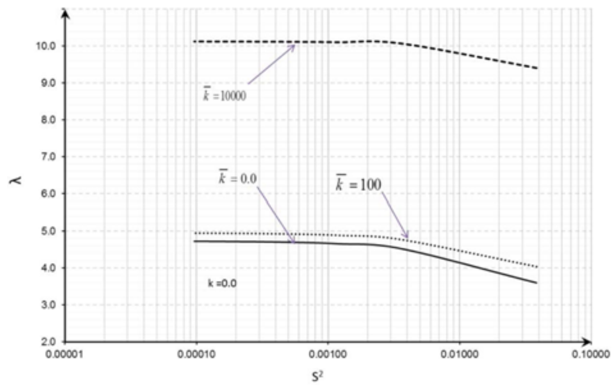

Fig. 6. Influence of the Foundation Linear Stiffness $\bar{k}$ and the Bending to Shear Stiffness (S^2) on the Natural Frequency for C-C Beam

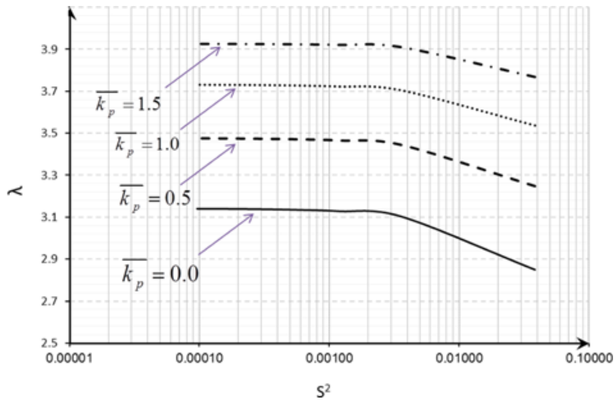

Fig. 7. Influence of the Foundation Shear Stiffness $\bar{k}_p$ and the Bending to Shear Stiffness (S^2) on the Natural Frequency for P-P Beams

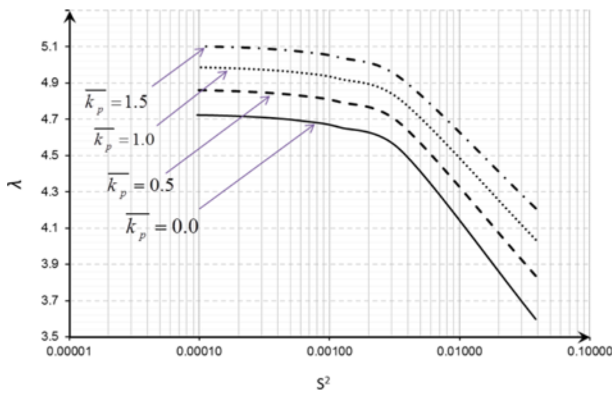

Fig. 8. Influence of the Foundation Shear Stiffness $\bar{k}_p$ and the Beam Bending to Shear Stiffness (S^2) on the Natural Frequency for C-C Beams

$$f_b^2 = \frac{p_{cr}L^2}{EI} \tag{35}$$

It is found that S^2 has a negligible effect on the value of the critical load of the system p_{cr}. This is because the considered deformation in the calculation of p_{cr} is of negligible value and the rotational inertia effect vanishes ($\omega = 0$).

The variations of f_b with the foundation stiffness parameters $\bar{k}$

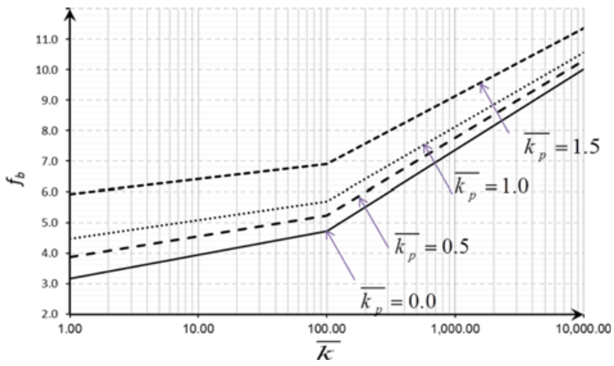

Fig. 9. Influence of Foundation Parameters ($\bar{k}$, $\bar{k}_p$) on the Stability Parameter (f_b) for P-P Beams

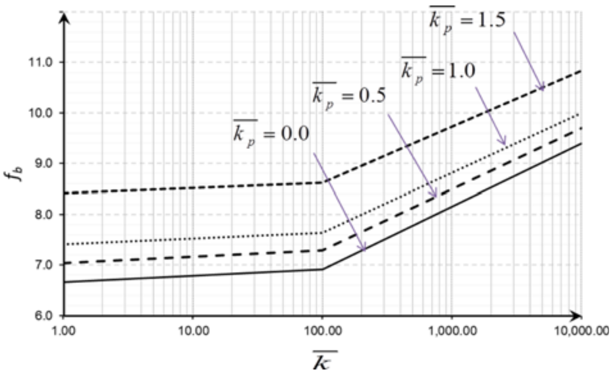

Fig. 10. Influence of Foundation Parameters ($\bar{k}$, $\bar{k}_p$) on the Stability Parameter (f_b) for C-C Beams

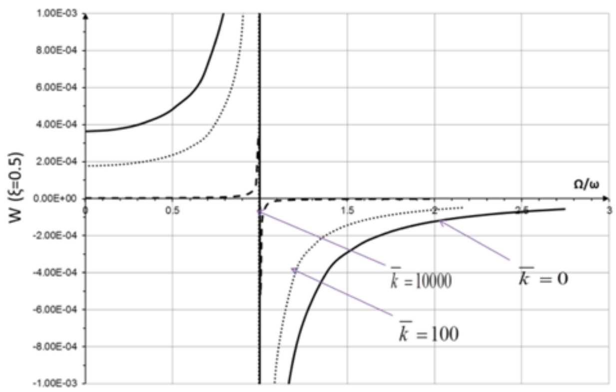

Fig. 11. Influence of the Frequency Ratio (Ω/ω) and the Foundation Linear Stiffness ($\bar{k}$) on the Mid Span Response of P-P Beams

and $\bar{k}_p$ are shown in Fig. 9 and Fig. 10. It is found that f_b increases as the foundation parameters increase, the effect of the foundation stiffness on f_b for P-P beams is larger than the effect for C-C beams and the influence of the foundation linear stiffness parameter is more noticeable for the value of $\bar{k}$ >100.

4.3 Forced Vibration

To investigate the significance of the different parameters on the beam response, the natural frequency for the proposed configuration ω is obtained then used to calculate the dimensionless excitation frequency Ω/ω. The amplitude of beam response at mid span is calculated (for the different values of frequency ratio Ω/ω, foundation parameters ($\bar{k}$, $\bar{k}_p$) and boundary conditions) then presented in Fig. 11. The resonance phenomenon is detected when the excitation frequency approaches the natural frequency of the system. Moreover, as the foundation stiffness increases, the response of the beam decreases.

5. Conclusions

The Adomian Decomposition Method (ADM) is used to investigate the static and dynamic behaviors of a Timoshenko beam resting on a two-parameter foundation and subjected to a static axial compression load and a dynamic lateral load with linear distribution. The governing differential equations of the beam vibration are decoupled, and then the boundary conditions for different types of end supports (P-P, C-C and C-P) are applied to obtain analytical expressions describing the lateral displacement of the beam. The obtained solutions are compared against those found in literature, and found in close agreement.

It is observed that as the bending to the shear stiffness parameter S^2 increases, the natural frequency λ for the beam decreases because considering the shear deformation increases the flexibility of the beam. It is also found that the frequency parameter λ increases as the foundation parameters increase and as the axial compression decreases. Furthermore, it is observed that the variation of bending to shear stiffness does not affect the stability parameter f_b of the beam. Also, the stability parameter f_b increases as the foundation stiffness parameters increase.

The forced vibration results indicate that the resonance conditions are approached as the system damping is neglected. Increasing the applied axial compression may be used to change the natural frequency and avoid resonance.

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