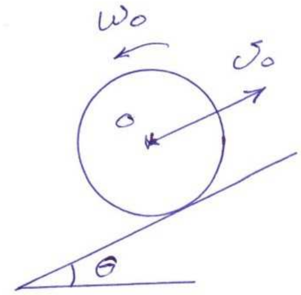
Momentum & Impulse



Problem (1):

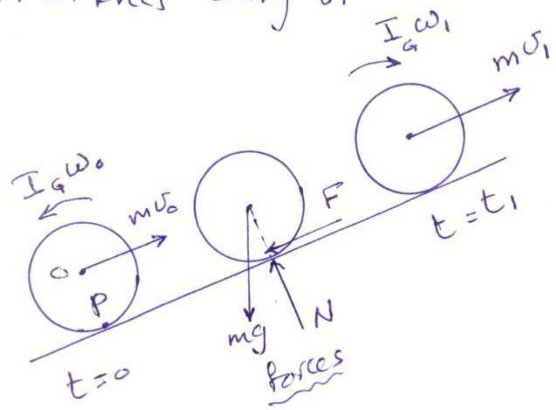
- Disc (m, r)
- initially: υ_o, ω_o
- $\mu = 0.1$, $\theta = 30^\circ$

Req: i) sliding time t_s

ii) T_1, T_2, x_o during t_1

iii) Work of external forces during t_1

Solution



$$\underline{\upsilon}_P = \upsilon_o \underline{i} + \omega_o r \underline{i}$$
$$= (\upsilon_o + \omega_o r)\underline{i}$$
$$\neq 0$$

$\therefore$ disc slides on inclined surface

$F = \mu N$ and opposite to $\underline{\upsilon}_P$ $(\swarrow)$

at $t = t_1$ sliding stops and rolling starts, then

$$\upsilon_1 = \omega_1 r \quad \text{----} \circledast$$

(kinematical condition)

Momentum - Impulse equations

$$\nearrow \quad \int_0^{t_1} (-F - mg\sin\theta)\,dt = m v_1 - m v_0 \quad \text{----} \; (1)$$

$$\nwarrow \quad \int_0^{t_1} (N - mg\cos\theta)\,dt = 0 \quad \text{----} \; (2)$$

$$(G \circlearrowright \quad \int_0^{t_1} (Fr)\,dt = I_G \omega_1 - (-I_G \omega_0) \quad \text{----} \; (3)$$

Eqs solution

from (2) : $N = mg\cos\theta \equiv \text{constant}$

from (1) : $(F + mg\sin\theta)\,t_1 = m(v_0 - \omega_1 r)$

$\therefore \; (\mu mg\cos\theta + mg\sin\theta)\,t_1 = m(v_0 - \omega_1 r) \quad \text{----} \; (4)$

from (3) : $F r t_1 = \frac{1}{2} m r^2 (\omega_1 + \omega_0)$

$\therefore \; \mu mg\cos\theta \; r t_1 = \frac{1}{2} m r^2 (\omega_1 + \omega_0) \quad \text{----} \; (5)$

Solving (4) & (5) for t_1 , ω_1 ; then :

$$t_1 = \frac{v_0 - \omega_1 r}{g(\sin\theta + \mu\cos\theta)} = \frac{(\omega_1 + \omega_0) r}{2\mu g\cos\theta}$$

$\omega_1 = 3.383$ r/sec and $t_1 = 1.474$ sec

$\boxed{A}$ $\boxed{A}$

$$T_1 = \frac{1}{2} m v_o^2 + \frac{1}{2} I_G \omega_o^2$$

$$= \frac{1}{2}(20)(10)^2 + \frac{1}{2}\left[\frac{1}{2}(20)(0.4)^2\right](3)^2$$

$$= 1007.2 \text{ joule} \rightsquigarrow \boxed{C}$$

$$T_2 = \frac{1}{2} m v_1^2 + \frac{1}{2} I_G \omega_1^2$$

$$= \frac{1}{2}(20)(0.4 * 3.383)^2 + \frac{1}{2}\left[\frac{1}{2}(20)(0.4)^2\right](3.383)^2$$

$$= 27.467 \text{ joule} \rightsquigarrow \boxed{A}$$

$$W_{1-2} = T_2 - T_1 = 27.467 - 1007.2$$

$$= -979.733 \text{ joule} \quad \boxed{D}$$

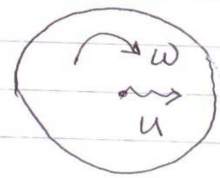
Distance covered by disc center:

As all forces acting on the disc are constant, then the accelerations are also constant. Then the disc center makes linear motion with constant acc.

$$v = v_o + at \rightsquigarrow a = \frac{10 - 0.4(3.383)}{1.47} = -5.8$$

$$X = v_o t + \frac{1}{2} a t^2 \rightsquigarrow X = 8.35 \text{ m.}$$

Problem (2):



A disc (m, r) travelling with center velocity u and rotates with angular velocity ω.
Suddenly a point A on the mid point of its radius set to be fixed. Calculate the Impulse of reaction at A and the loss in kinetic energy.

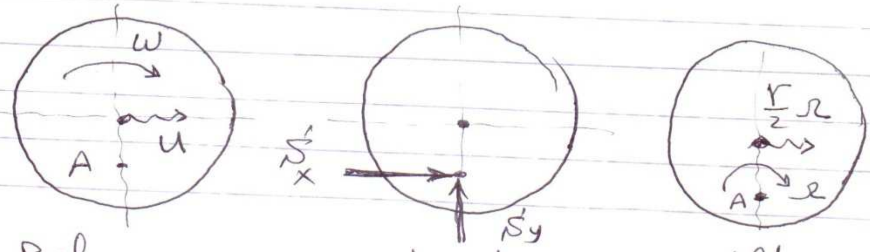
Solution

Kinetic energy before impulse

$$T_1 = \frac{1}{2} m u^2 + \frac{1}{2} I_G \omega^2$$

$$= \frac{1}{2} m u^2 + \frac{1}{2}\left(\frac{1}{2} m r^2\right)\omega^2 = \frac{1}{2} m u^2 + \frac{1}{4} m r^2 \omega^2$$

During Impulse



Before Impulse After

Impact Equations:

$\rightarrow \quad \acute{S}_x = \frac{m\Omega r}{2} - mu \quad \ldots\ldots (1)$

$\uparrow \quad \acute{S}_y = 0 - 0 \quad \ldots (2)$

$(A \curvearrowright \quad 0 = I_A \Omega - (mu\frac{r}{2} + I_G \omega) \quad \ldots (3)$

Note that: equation (3) may be written as:

$(A \curvearrowright \quad 0 = \{I_G \Omega + \frac{m\Omega r}{2} \cdot \frac{r}{2}\} - [\frac{mur}{2} + I_G \omega]$

$$0 = \Omega[I_G + m(\tfrac{r}{2})^2] - [\frac{mur}{2} + I_G \omega]$$

$$= \Omega \quad I_A - [mu\frac{r}{2} + I_G \omega]$$

and may be calculated about G as follow:

$(G \curvearrowright \quad -\acute{S}_x \cdot \frac{r}{2} = I_G \Omega - I_G \omega$

sub. from (1): $-(\frac{m\Omega r}{2} - mu)\frac{r}{2} = I_G \Omega - I_G \omega$

$$\therefore 0 = \Omega[I_G + m(\tfrac{r}{2})^2] - (\frac{mur}{2} + I_G \omega)$$

which is the same as equation (3)

$$I_A = \tfrac{1}{2} m r^2 + m(\tfrac{r}{2})^2 = \tfrac{3}{4} m r^2$$

$$I_G = \tfrac{1}{2} m r^2$$

Sub. in (3) :

$$\Omega = \frac{4}{3mr^2}\left[\frac{mru}{2} + \frac{1}{2}mr^2\omega\right]$$

$$= \frac{2u}{3r} + \frac{2}{3}\omega$$

$$\therefore \quad S'_x = \frac{mr}{2}\left(\frac{2u}{3r} + \frac{2}{3}\omega\right) - mu$$

$$= \frac{1}{3}mu + \frac{1}{3}mr\omega - mu$$

$$= \frac{1}{3}mr\omega - \frac{2}{3}mu$$

Kinetic Energy :

$$T_2 = \frac{1}{2} I_A \Omega^2 = \frac{3}{8}mr^2\left(\frac{2u}{3r} + \frac{2}{3}\omega\right)^2$$

loss in T

$$\Delta T = T_1 - T_2$$

$$= \frac{1}{2}mu^2 + \frac{1}{4}mr^2\omega^2 - \frac{3}{8}mr^2\left(\frac{2u}{3r} + \frac{2}{3}\omega\right)^2$$

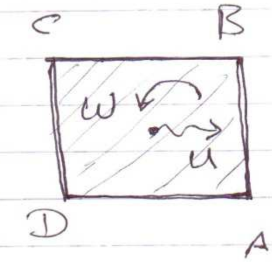
For $m = 20\text{ kg}$ $\quad r = 1.2\text{ m}$ $\quad u = 5\text{ m/sec}$ $\quad \omega = 4\text{ r/s}$

$$\Omega = \frac{2(5)}{3(1.2)} + \frac{2}{3}(4) = 5.444$$

$S'_x = -34.667$ N $\qquad \Delta T = 45.067$ joule

Problem (3):

A square plate (m, L) is travelling on a horizontal smooth surface with center velocity u m/sec ($\rightarrow$) and rotates with angular velocity ω_1 (C.C.W) rad/sec. In the shown instant, the corner A is setting to be fixed. Calculate: T_1, T_2, ω_2 v_C after impulse and impulse of reaction at A.



Solution:

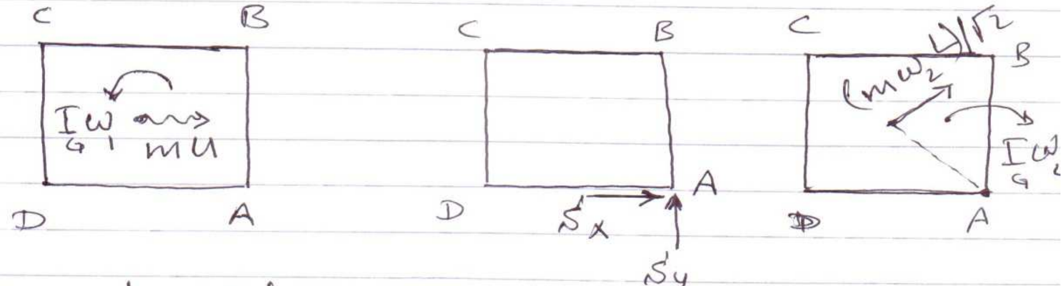
Before Impulse:

$$T_1 = \frac{1}{2} m u^2 + \frac{1}{2} I_G \omega_1^2$$

$$I_G = \frac{1}{6} m L^2$$

$$\therefore T_1 = \frac{1}{2} m u^2 + \frac{1}{12} m L^2 \omega_1^2$$

Impact Equations:



momentum Before Impulse   Impulse   Momentum After Impulse

$$\rightarrow \quad S'_x = \frac{1}{2} m \omega_2 L - m u \quad \text{----} \; (1)$$

$$\uparrow \quad S'_y = \frac{1}{2} m \omega_2 L - 0 \quad \text{---} \; (2)$$

$$(A) \quad 0 = I_G \omega_2 + \frac{m\omega_2 L^2}{2} - \left(\frac{muL}{2} - I_G \omega_1\right) \quad \text{----} \; (3)$$

from (3) : $\omega_2\left[\frac{1}{6} m L^2 + \frac{1}{2} m L^2\right] = \frac{1}{2} m u L - \frac{1}{6} m L^2 \omega_1$

$$\omega_2 = \frac{3}{2mL^2}\left(\frac{1}{2} m L u - \frac{1}{6} m L^2 \omega_1\right)$$

$$\therefore \omega_2 = \frac{3u}{4L} - \frac{1}{4}\omega_1$$

Sub. in ① & ②

$$\acute{S}_x = \frac{1}{2} mL \left(\frac{3u}{4L} - \frac{1}{4}\omega_1 \right) - mu$$

$$= \frac{3}{8} mu - \frac{1}{8} mL\omega_1 - mu$$

$$= -\frac{5}{8} mu - \frac{1}{8} mL\omega_1$$

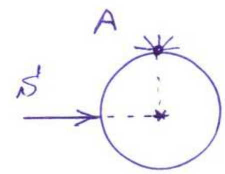
$$\acute{S}_y = \frac{1}{2} mL \left(\frac{3u}{4L} - \frac{1}{4}\omega_1 \right)$$

$$= \frac{3}{8} mu - \frac{1}{8} mL\omega_1$$

$$v_c = \omega_2 \frac{L}{\sqrt{2}} = \frac{L}{\sqrt{2}} \left(\frac{3u}{4L} - \frac{1}{4}\omega_1 \right)$$

$$= \frac{3}{4\sqrt{2}} u - \frac{1}{4\sqrt{2}} L\omega_1$$

Problem (4):



Data:

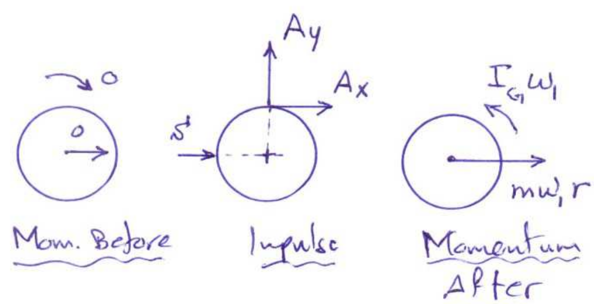
- $m = 20 kg$ $r = 0.4 m$
- $\mathcal{S} = 50$ N.sec

Req:

- Impulse of reaction at A
- ω and T just after impulse
- $\mathcal{S}_{min}$ to cause complete revolution

Solution

A) Impulse:



Impact Equations:

$\rightarrow$ $\mathcal{S} + A_x = m\omega_1 r - 0$ — ①

$\uparrow$ $A_y = 0 - 0$ — ②

$\circlearrowleft A$ $\mathcal{S} \cdot r = [I_G \omega_1 + m\omega_1 r^2] - 0$ --- ③

$\therefore \mathcal{S} = \frac{3 m r \omega_1}{2}$

Eqs solution

from ③ $\omega_1 = \dfrac{S \cdot r}{I_G + mr^2} = \dfrac{2S}{3\,mr} = 4.167$ --- ④

sub into ① $A_x = 33.33 - 50 = -16.67$

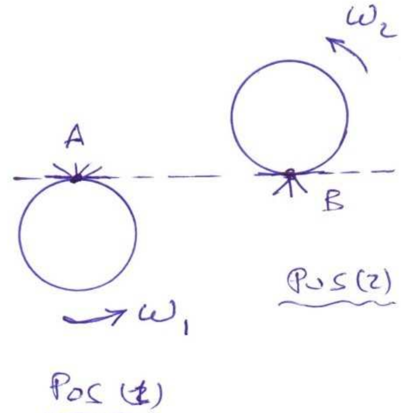
from ② $A_y = 0$

$$T\ (\text{after impulse}) = \tfrac{1}{2} I_A \omega_1^2 = \tfrac{1}{2}\left(\tfrac{3}{2} m r^2\right)(\omega_1)^2$$

$$= 41.667 \text{ joule}$$

Minimum impulse to complete a revolution

for $S_{min} \leadsto \omega_2 = 0$



Pos (1)

$T_1 = \tfrac{1}{2} I_A \omega_1^2$

$V_1 = -mgr$

Pos (2)

$T_2 = 0 \qquad V_2 = mgr \qquad W_{1-2}^{n.c.f} = 0$

$\therefore\ \tfrac{1}{2} I_A \omega_1^2 - mgr = mgr + 0 \leadsto \omega_1^2 = \dfrac{2mgr}{I_A} = \dfrac{4g}{3r}$

from ④ $S_{min} = \tfrac{3}{2} m r \omega_1 = 1.5\, mr \sqrt{\dfrac{4g}{3r}} = 69.28$ N.sec